

Beam dynamics corrections to measurements of the muon anomalous magnetic moment

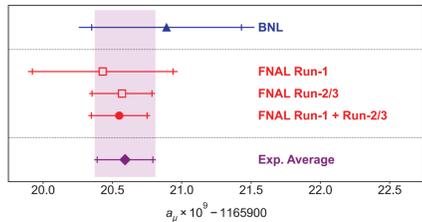
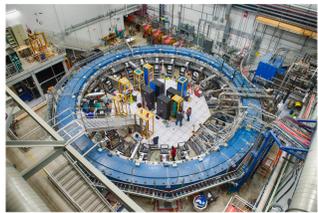


The Muon g-2 experiment

- Goal:** Aim to measure the muon magnetic anomaly a_μ with a precision of 140 parts per billion (ppb)
- Method:** Measure the frequency difference ω_a between the cyclotron motion ω_c and the spin precession ω_s of the muon in a highly uniform magnetic field B

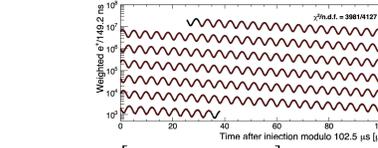
$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$$

- Significance:** Hope to shed light on the long-standing discrepancy between Standard Model calculations and previous experimental results.



Magnetic Storage Ring at Fermilab Experimental values of Muon g-2 [1,3]

Corrections to g-2 measurement



Anomalous precession frequency obtained from the modulation of the positron time spectrum

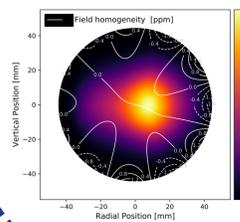
Beam Dynamics Corrections

E-field & vertical motion: Spin precesses slower than in basic equation

Phase changes over each muon fill: Phase acceptance, differential decay, and muon losses

$$a_\mu \sim \frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m} \frac{1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}}{1 + B_k + B_q}$$

Transient magnetic fields: Quad vibrations and kicker eddy current



The magnetic field distribution was obtained using NMR probes and then convolved with the tracker-measured muon distribution.

Summary of Run-2/3 values [4]

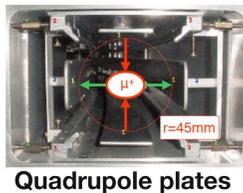
Quantity	Correction (ppb)	Uncertainty (ppb)
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{ml}	0	3
Total	580	40

Electric field and vertical motion[2,4]

- Generic spin-motion is described by the BMT equation:

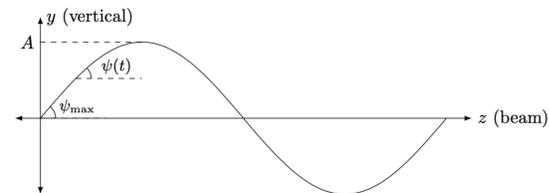
$$\frac{d(\hat{\beta} \cdot \vec{S})}{dt} = -\frac{q}{m} \vec{S}_T \cdot \left[a_\mu \hat{\beta} \times \vec{B} + \beta \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{E}}{c} \right]$$

- Muons travel in E-field from focusing quadrupoles: experience a motional magnetic field in their rest frame
- The second term vanishes at "magic" momentum ($p_\mu = 3.094$ GeV/c), but not all muons are with this momentum
- The Electric-field Correction, C_e** comes from momentum distribution measured using i) timing data from calorimeters and ii) beam radial spread data from trackers



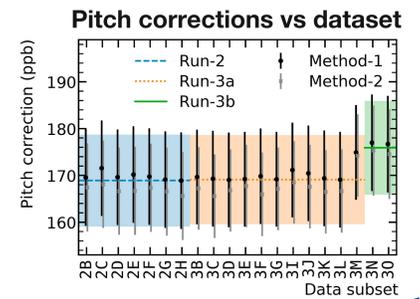
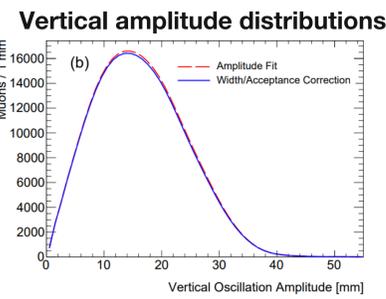
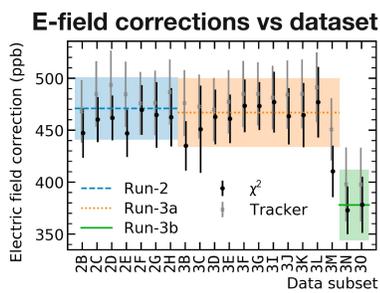
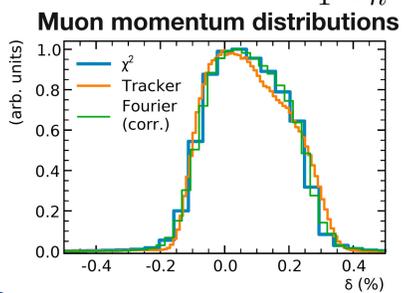
Quadrupole plates

- Muons oscillate vertically so the $\hat{\beta} \times \vec{B}$ term is reduced



- This vertical motion (pitch) causes the vertical spin precession
- The Pitch Correction, C_p** is calculated from the amplitude distribution of the vertical motion measured with straw trackers

$$C_p = \frac{n \langle y^2 \rangle}{2 R_0^2} = \frac{n \langle A^2 \rangle}{4 R_0^2}$$



Phase changes over each muon fill[2,4]

- If there are effects that change the g-2 phase of the detected e^+ over time

$$\cos(\omega_a t + \phi(t)) = \cos(\omega_a t + \phi_0 + \phi' t + \dots) = \cos((\omega_a + \phi')t + \phi_0 + \dots)$$

- These make us mismeasure ω_a with no indications that we're getting it wrong!

Muon Losses Correction, C_{ml}

- Muons lost during a storage period can also lead to a change in the muon momentum distribution
- Phase is also correlated with muon momentum owing to the momentum-dependent phase advance in upstream beamline components

$$C_{ml} = -\frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a} \frac{d\phi_0}{dt} = \frac{1}{\omega_a} \frac{d\phi_0}{dp} \left(\frac{dp}{dt} \right)_{ml}$$

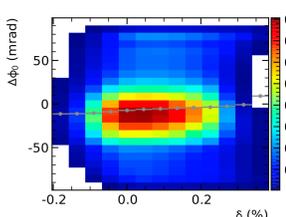
Differential Decay Correction, C_{dd}

- Phase changes due to the coupled effects from ϕ - p correlation & p - t correlation

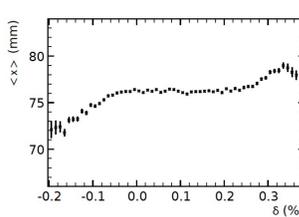
$$C_{dd} = -\frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a} \frac{d\phi_0}{dt} = \frac{1}{\omega_a} \frac{d\phi_0}{dp} \left(\frac{dp}{dt} \right)_{dd}$$

- Can be decomposed into 3 categories:
 - beamline (4 revolutions in the Delivery Ring)
 - p - x (beam injection through an inflector)
 - p - t_0 (longitudinal phase variation and kickers t_0 -dependent momentum acceptance)

Phase-Momentum Correlation



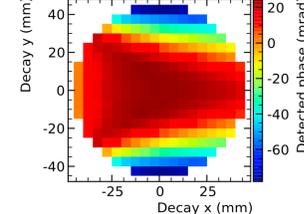
Position-Momentum Correlation



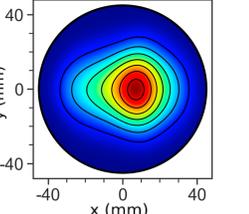
Phase-Acceptance Correction, C_{pa}

- Average detected g-2 phase changes with decay position
- The muon position distribution changes over time
- Correction is calculated using simulated data including PA phase vs time

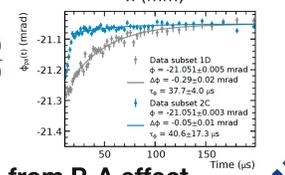
Position-dependent Phase



Beam 2D Distribution



$$\phi_{pa}^c(t) = \arctan \left[\frac{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j)}{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j)} \cdot \frac{A^c(x_i, y_j) \cdot \sin[\phi_{pa}^c(x_i, y_j)]}{A^c(x_i, y_j) \cdot \cos[\phi_{pa}^c(x_i, y_j)]} \right]$$



Phase vs time from P-A effect

References

- [1] Muon g-2 collaboration, *Phys. Rev. Lett.* **126** (2021) 14, 141801
- [2] Muon g-2 collaboration, *Phys. Rev. Accel. Beams* **24** (2021) 4, 044002
- [3] Muon g-2 collaboration, *Phys. Rev. Lett.* **131** (2023) 16, 161802
- [4] Muon g-2 collaboration, arXiv:2402.15410 (2024)



Acknowledgments

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