Gravitational waves from composite dark sectors

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Strongly coupled dynamics: outlook

- **Important physical examples of gauge fields are realised in Nature (QCD and electroweak interactions)**
- **Non-perturbative QCD phenomena are far from being understood (e.g. quark confinement, mass gap, QCD phase transitions, hot/dense QCD phenomena etc)**
- **Non-abelian gauge (Yang-Mills) fields are present in most of UV completions of the Standard Model (e.g. GUTs, string/EDs compactifications etc)**
- **Confining dark Yang-Mills sectors are often considered as a possible source of Dark Matter in the Universe (e.g. dark glueballs)**
- **Pure gluons**

  $\Rightarrow$ confinement-deconfinement phase transition
- **Gluons + fermions**

$\Longrightarrow$
- Fermions in fundamental representation $\Rightarrow$ chiral phase transition
- Fermions in adjoint rep. $\Rightarrow$ confinement & chiral phase transition
- Fermions in 2-index symmetric rep. $\Rightarrow$ confinement & chiral phase transition

Polyakov Loop Model for pure gluons I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB 72 (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x}, \tau)\, \mathrm{d}\tau \right]$$

- The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for pure gluons II

- Convenient to define the trace of the Polyakov loop as an order parameter for the $Z(N)$ symmetry

$$\ell\,(\vec{x}) = \frac{1}{N}\mathrm{Tr}_c[\mathbf{L}]\,,$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \to e^{i\phi}\ell, \qquad \phi = \frac{2\pi j}{N}, \qquad j = 0, 1, \cdots, (N-1)$$

- The expectation value of ℓ i.e. $< \ell >$ has the important property:

$$\langle \ell \rangle = 0 \quad (T < T_c, \text{ Confined})\,; \qquad \langle \ell \rangle > 0 \quad (T > T_c, \text{ Deconfined})$$

- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle \ell \rangle = \exp\left(i\frac{2\pi j}{N}\right)\ell_0, \qquad j = 0, 1, \cdots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \to 1$ as $T \to \infty$

Effective PLM potential

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\rm PLM}^{(\rm poly)} = T^4 \left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \cdots - b_3\left(\ell^N + \ell^{*N}\right)\right)$$

$$\text{where} \quad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 + a_4\left(\frac{T_0}{T}\right)^4$$

"$\cdots$" represent any required lower dimension operator than ℓ^N i.e. $(\ell\ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

$$V_{\rm PLM}^{(\rm 3log)} = T^4 \left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right)\right)$$

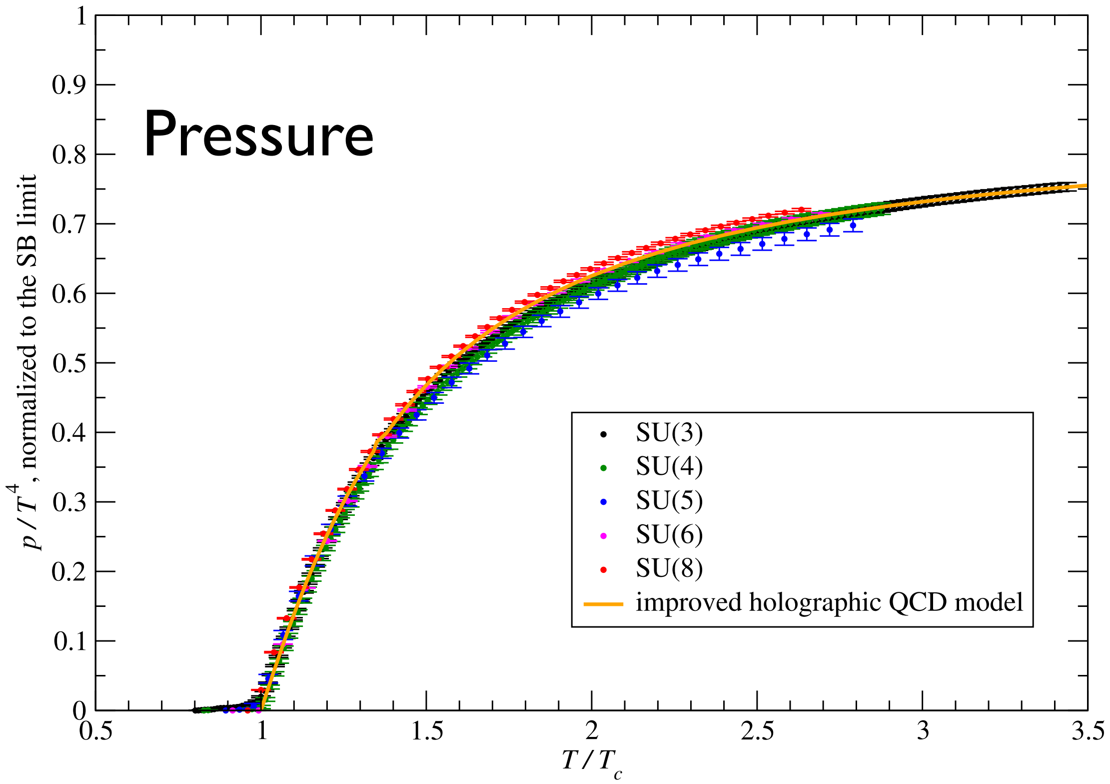
$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

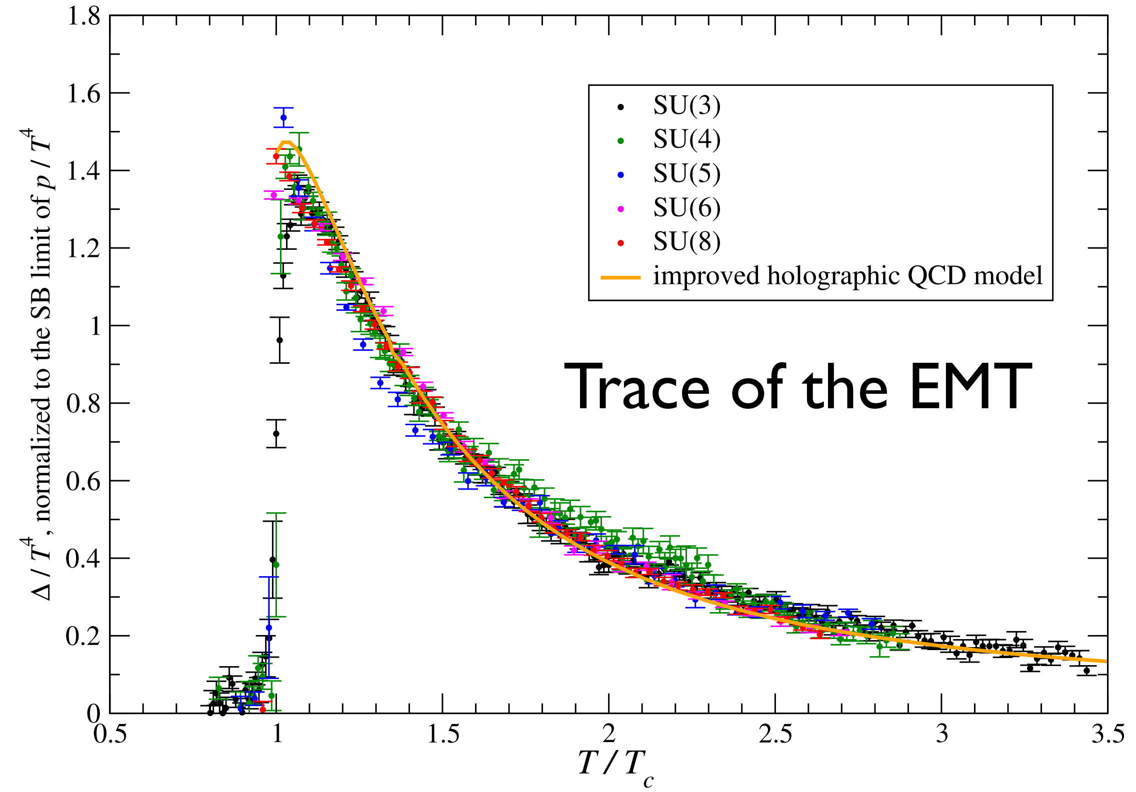
- The a_i, b_i coefficients in $V_{\rm PLM}^{(\rm poly)}$ and $V_{\rm PLM}^{(\rm 3log)}$ are determined by fitting the lattice results

Fitting the PLM potential to the lattice data

Lattice data

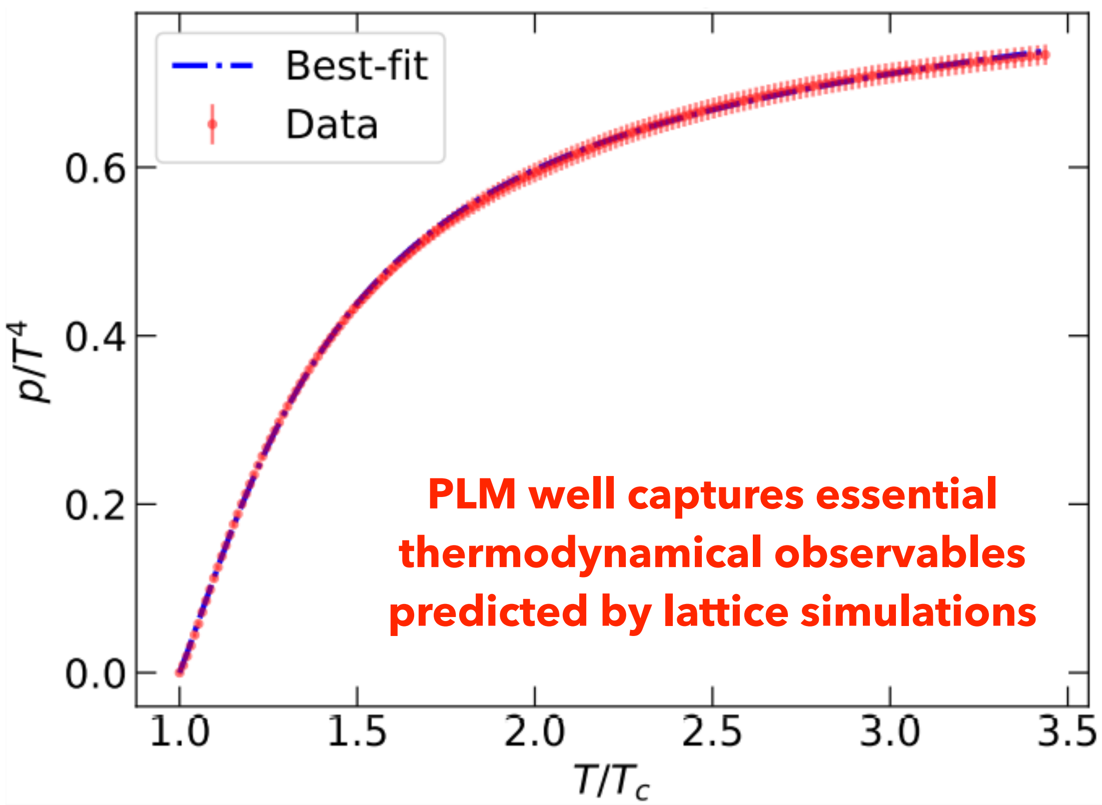
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001

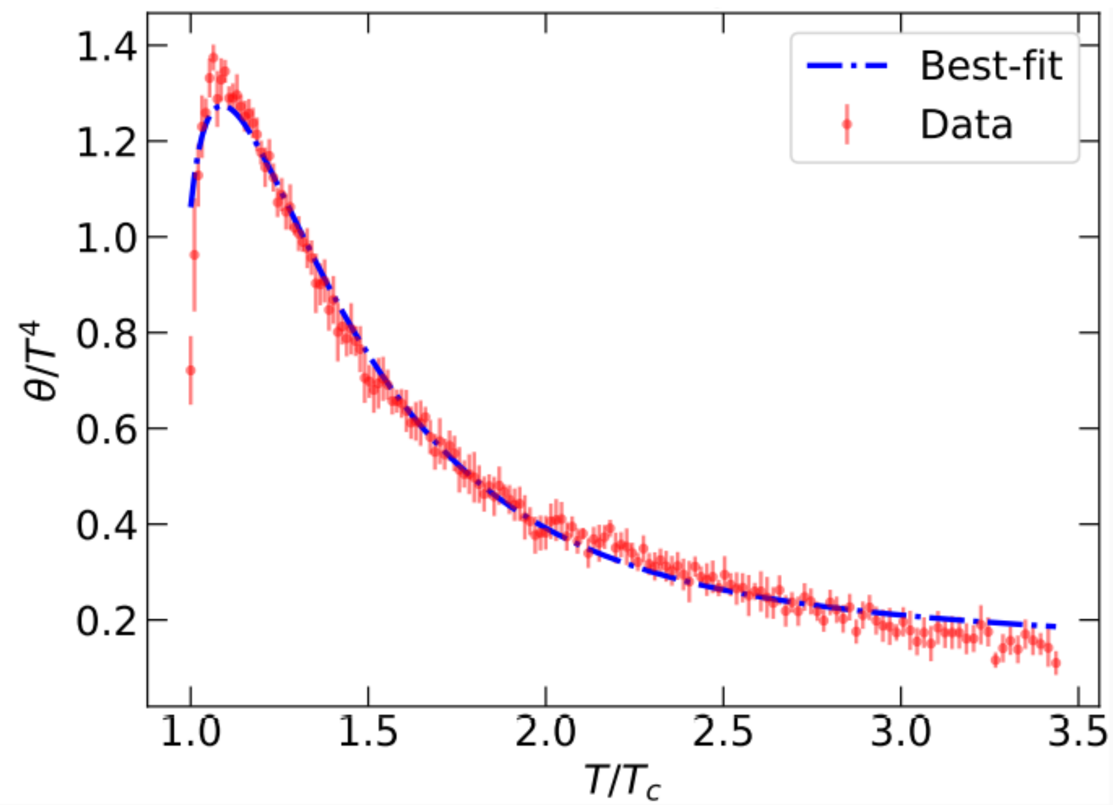




Best fit of the PLM potential

Huang, Reichert, Sannino and Wang, PRD 104 (2021) 035005





Including fermions: the PQM model

B. Schaefer, J. Pawlowski, J. Wambach PRD 76 (2007) 074023
B. Schaefer, M. Wagner, PPNP 62 (2009) 391
RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- The Polyakov quark meson model (PQM) is widely used as an effective theory to study the first order chiral phase transition
- The Lagrangian of the PLSM where mesons couple to a spatially constant temporal background gauge field reads

$$\mathcal{L} = \bar{q}\left(i\not{D} - g\left(\sigma + i\gamma_5 T^a \pi_a\right)\right) q + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \pi_a\right)^2$$
$$- V_{\mathrm{PLM}}^{(\mathrm{poly})} + V_{\mathsf{LSM}} + V_{\mathrm{medium}}\,, \mathrm{where}\ \not{D} = \gamma_\mu \partial_\mu - i\gamma_0 A_0$$

- V_{LSM} under symmetry $SU(N_f) \times SU(N_f)$ with N_f flavours reads

$$V_{\mathsf{LSM}} = \frac{1}{2}\left(\lambda_\sigma - \lambda_a\right) \mathrm{Tr}\left[\Phi^\dagger \Phi\right]^2 + \frac{N_f}{2} \lambda_a \mathrm{Tr}\left[\Phi^\dagger \Phi \Phi^\dagger \Phi\right] - m^2 \mathrm{Tr}\left[\Phi^\dagger \Phi\right]$$
$$- 2\left(2N_f\right)^{N_f/2-2} c\left(\det \Phi^\dagger + \det \Phi\right)$$

where the meson field Φ is a $N_f \times N_f$ matrix defined as

$$\Phi = \frac{1}{\sqrt{2N_f}}\left(\sigma + i\eta'\right) I + \left(a_a + i\pi_a\right) T^a\,, I \equiv \ \mathrm{identity\ matrix}$$

Thermal corrections: the CJT Method

J. Cornwall, R. Jackiw, E. Tomboulis PRD 10 (1974) 2428
G. Amelino-Camelia, PRD 47 (1993) 2356
RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159

- Cornwall, Jackiw and Tomboulis (CJT) first proposed a generalized effective action $\Gamma\left(\phi, G\right)$ of composite operators, where the effective action not only depends on $\phi(x)$ but also on the propagator $G(x, y)$
- The effective action becomes the generating functional of the two-particle irreducible (2PI) vacuum graphs rather than the conventional 1PI diagrams
- The CJT method is equivalent to summing up the infinite class of “daisy" and “super daisy" graphs and is thus useful in studying such strongly coupled models beyond mean-field approximation
- The PQM with the CJT method compared to other model computations such as holography and the PNJL model, can bridge perturbative and non-perturbative regimes of the effective theory

First-order phase transitions and bubble's nucleation

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$$

- The generic three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)^2 + V_{\text{eff}}(\rho, T)\right] ,$$

where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$

- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use percolation temperature T_p: the temperature at which 34% of false vacuum is converted
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*) e^{\beta(t-t_*)}$$

Phase transition characteristics

Huang, Reichert, Sannino, Wang
PRD 104 (2021) 035005

- The inverse duration time then follows as

$$\beta = -\frac{\mathrm{d}}{\mathrm{d}t}\frac{S_3(T)}{T}\bigg|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T\frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T=T_*},$$

where we used that $\mathrm{d}T/\mathrm{d}t = -H(T)T$.

- We define the strength parameter α from the trace of the energy-momentum tensor θ weighted by the enthalpy

$$\alpha = \frac{1}{3}\frac{\Delta\theta}{w_+} = \frac{1}{3}\frac{\Delta e - 3\Delta p}{w_+}, \qquad \Delta X = X^{(+)} - X^{(-)}, \text{ for } X = (\theta, e, p)$$

$(+)$ denotes the meta-stable phase (outside of the bubble) while $(-)$ denotes the stable phase (inside of the bubble).

- The relations between enthalpy w, pressure p, and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}, \qquad e = \frac{\partial p}{\partial \ln T} - p, \qquad p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors

Gravitational wave spectrum: an outlook

C. Caprini et al., JCAP 03, 024, 1910.13125

- Contributions from bubble collision and turbulence are subleading
  The GW spectrum from sound waves is given by

$$h^2\Omega_{\text{GW}}(f) = h^2\Omega_{\text{GW}}^{\text{peak}} \left(\frac{f}{f_{\text{peak}}}\right)^3 \left[\frac{4}{7} + \frac{3}{7}\left(\frac{f}{f_{\text{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\text{peak}} \simeq 1.9 \cdot 10^{-5}\,\text{Hz} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \left(\frac{T}{100\,\text{GeV}}\right) \left(\frac{\tilde{\beta}}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\text{GW}}^{\text{peak}} \simeq 2.65 \cdot 10^{-6} \left(\frac{v_w}{\tilde{\beta}}\right) \left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \Omega_{\text{dark}}^2 \qquad \Omega_{\text{dark}} = \frac{\rho_{\text{rad,dark}}}{\rho_{\text{rad,tot}}}$$

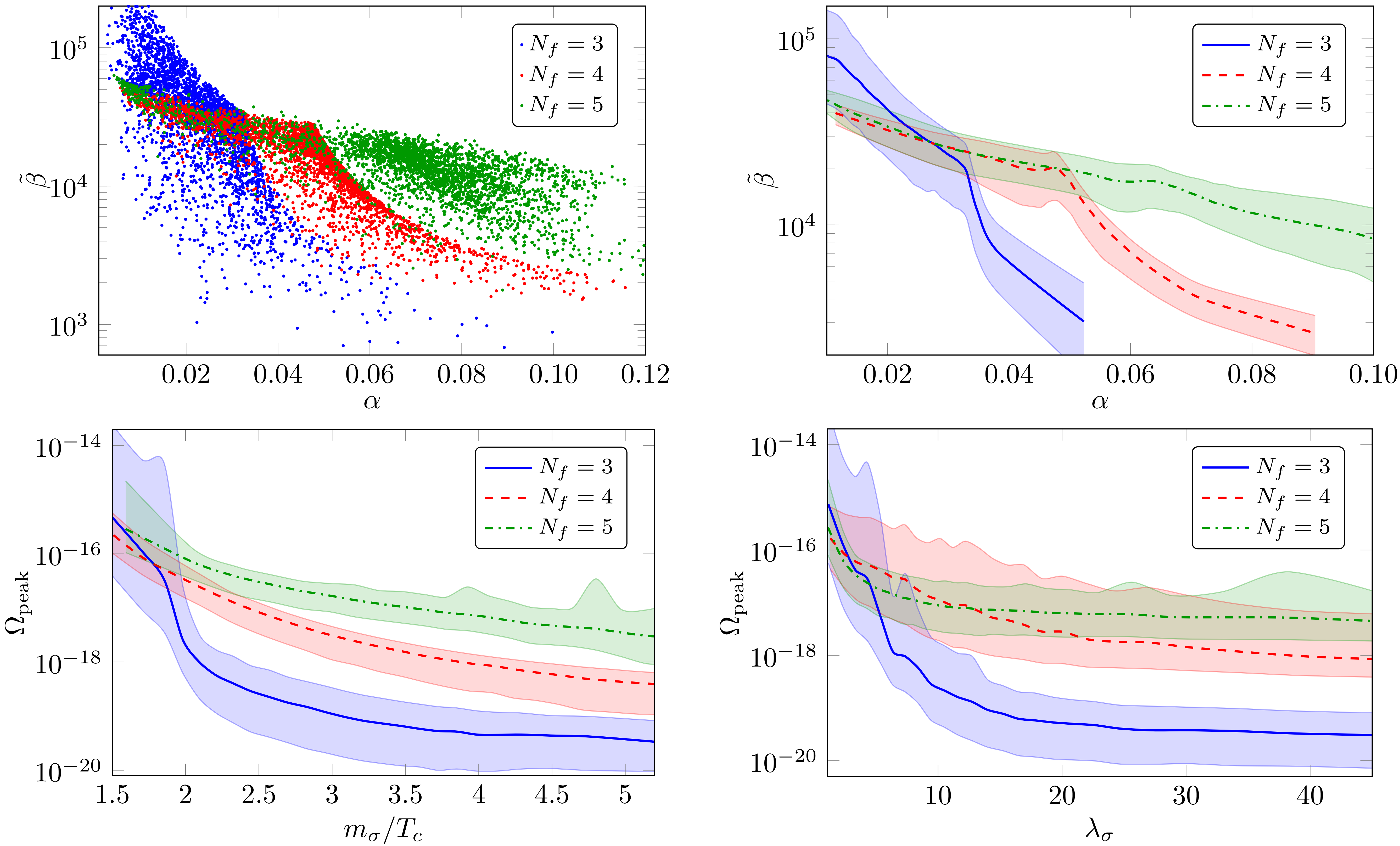
- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.
- The efficiency factor for the sound waves κ_{sw} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\text{sw}} = \sqrt{\tau_{\text{sw}}}\,\kappa_v \qquad \tau_{\text{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \;\text{for}\; \beta >> 1 \qquad \kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98+\alpha}}$$

where $\bar{U}_f$ is the root-mean-square fluid velocity $\quad \bar{U}_f^2 \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$

Phase diagram and gravitational waves in the PQM model

RP, Reichert, Sannino and Wang, JHEP 02 (2024) 159



The strongest signal we found can almost reach the LISA sensitivity

Summary:

- We developed a new approach based upon the well-established thermal EFT and the existing lattice results to explore phase structure and PTs in confining gauge theories incorporating confinement effects and non-perturbative self-interactions

- We analysed the phase transitions in the Polyakov-loop extended LSM utilising the CJT method and computed the resulting primordial gravitational wave spectra showcasing an enhancement for weak sigma self-interactions and light sigma meson

- Inclusion of the Polyakov loop enhances the strength of the chiral phase transition compared to LSM

- The PLSM represents an important framework for analysis of various cosmological implications of strongly coupled dynamics in consistency with lattice simulations