Collinearly Enhanced YFS MC Approach to Precision High Energy Collider Physics

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Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$
- Using FCC as an example, factors of improvement from ~ 5 to ~ 100 are needed from Theory
- Resummation is a key to such improvements in many cases: Today, we discuss amplitude-based resummation following the YFS methodology
- YFS $\rightarrow$ 'no limit to precision'
- See 1989 CERN Yellow Book article by Berends *et al.*

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Introduction

- YFS methods are exact in the infrared but treat the collinear logs perturbatively in the $\bar{\beta}_n$ residuals
- DGLAP-based collinear factorization treats the collinear logs to all orders but has a non-exact IR limit - see Stefano's talk and references therein ('all roads lead to Rome')
- Today, we investigate improving the collinear limit of YFS theory
- A Key Point: Exact Amplitude-Based Resummation Realized on Evt-by-Evt Basis – Enhanced Precision for a Given Level of Exactness: LO, NLO, NNLO, ...

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Review of Exact Amplitude-Based Resummation Theory

$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \quad (1)$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons.

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Review of Exact Amplitude-Based Resummation Theory

Here,

$$\begin{aligned} {\rm SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\rm QCED} &= \int \frac{d^3k}{k^0}\left(e^{-iky} - \theta(K_{max}-k^0)\right)\tilde{S}^{nls}_{QCED} \end{aligned} \quad (2)$$

where K_{max} is "dummy" and

$$\begin{aligned} B^{nls}_{QCED} &\equiv B^{nls}_{QCD} + \frac{\alpha}{\alpha_s}B^{nls}_{QED}, \\ \tilde{B}^{nls}_{QCED} &\equiv \tilde{B}^{nls}_{QCD} + \frac{\alpha}{\alpha_s}\tilde{B}^{nls}_{QED}, \\ \tilde{S}^{nls}_{QCED} &\equiv \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED}. \end{aligned} \quad (3)$$

"nls"$\equiv$ DGLAP-CS synthesization.

Shower/ME Matching: $\tilde{\bar{\beta}}_{n,m} \rightarrow \hat{\tilde{\bar{\beta}}}_{n,m}$ --*KKMCee, KKMChh, Herwiri,...*

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Improving the Collinear Limit in YFS Theory

- Basic Formula for CEEX/EEX realization of the YFS resummation of $e^+e^- \rightarrow f\bar{f} + n\gamma,\ f = \ell, q,\ \ell = e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau,\ q = u,d,s,c,b,t$:

$$\sigma = \frac{1}{\rm flux}\sum_{n=0}^{\infty}\int d{\rm LIPS}_{n+2}\,\rho_A^{(n)}(\{p\},\{k\}), \quad (4)$$

- $$\rho^{(n)}_{\rm CEEX}(\{p\},\{k\}) = \frac{1}{n!}e^{Y(\Omega;\{p\})}\bar{\Theta}(\Omega)\frac{1}{4}\sum_{{\rm helicities}\ \{\lambda\},\{\mu\}}\left|\mathcal{M}\begin{pmatrix}\{p\}\{k\}\\ \{\lambda\}\{\mu\}\end{pmatrix}\right|^2. \quad (5)$$

By definition, $\Theta(\Omega,k) = 1$ for $k \in \Omega$ and $\Theta(\Omega,k) = 0$ for $k \notin \Omega$, with $\bar{\Theta}(\Omega;k) = 1 - \Theta(\Omega,k)$ and

$$\bar{\Theta}(\Omega) = \prod_{i=1}^{n}\bar{\Theta}(\Omega,k_i).$$

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Improving the Collinear Limit in YFS Theory

- For Ω defined with the condition $k^0 < E_{\rm min}$, the YFS infrared exponent reads

$$\begin{aligned} Y(\Omega;p_a,\ldots,p_d) = &\, Q_e^2 Y_\Omega(p_a,p_b) + Q_f^2 Y_\Omega(p_c,p_d) \\ &+ Q_eQ_f Y_\Omega(p_a,p_c) + Q_eQ_f Y_\Omega(p_b,p_d) \\ &- Q_eQ_f Y_\Omega(p_a,p_d) - Q_eQ_f Y_\Omega(p_b,p_c). \end{aligned} \quad (6)$$

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Improving the Collinear Limit in YFS Theory

- Here

$$\begin{aligned} Y_\Omega(p,q) \equiv &\, 2\alpha\tilde{B}(\Omega,p,q) + 2\alpha\Re B(p,q) \\ \equiv &-2\alpha\frac{1}{8\pi^2}\int\frac{d^3k}{k^0}\Theta(\Omega;k)\left(\frac{p}{kp}-\frac{q}{kq}\right)^2 \\ &+2\alpha\Re\int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2. \end{aligned} \quad (7)$$

- Fundamental Idea of YFS: isolate and resum to all orders in α the infrared singularities so that these singularities are canceled to all such orders between real and virtual corrections.
  What collinear singularities are also resummed in the YFS resummation algebra?

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Improving the Collinear Limit in YFS Theory

- Focusing on the s-channel and s'-channel contributions, we have

$$\begin{aligned} Y_e(\Omega_I;p_1,p_2) &= \gamma_e \ln\frac{2E_{min}}{\sqrt{2p_1p_2}} + \frac{1}{4}\gamma_e + Q_e^2\frac{\alpha}{\pi}\left(-\frac{1}{2}+\frac{\pi^2}{3}\right), \\ Y_f(\Omega_F;q_1,q_2) &= \gamma_f \ln\frac{2E_{min}}{\sqrt{2q_1q_2}} + \frac{1}{4}\gamma_f + Q_f^2\frac{\alpha}{\pi}\left(-\frac{1}{2}+\frac{\pi^2}{3}\right), \end{aligned} \quad (8)$$

where

$$\gamma_e = 2Q_e^2\frac{\alpha}{\pi}\left(\ln\frac{2p_1p_2}{m_e^2}-1\right), \quad \gamma_f = 2Q_f^2\frac{\alpha}{\pi}\left(\ln\frac{2q_1q_2}{m_f^2}-1\right), \quad (9)$$

$\Rightarrow$ The YFS exponent resums the collinear big log term $\frac{1}{2}Q^2\frac{\alpha}{\pi}L$ to the infinite order in both the ISR and FSR contributions.

- Can this be improved to the result of Gribov and Lipatov to exponentiate $\frac{3}{2}\frac{\alpha}{\pi}L$ via the QED form-factor?

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Improving the Collinear Limit in YFS Theory

- The YFS form factor derivation illustrated in Fig. 1

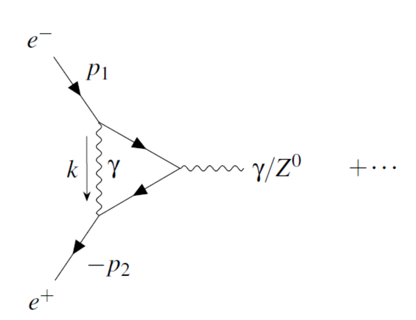

Fig. 1: Virtual corrections which generate the YFS infrared function B. Self-energy contributions are not shown.

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Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ the amplitude factor, for $A = \gamma$ or Z,

$$\mathcal{M}_\mu = \frac{\int d^4k}{(2\pi)^4}\frac{-i}{k^2+i\varepsilon}\bar{v}(p_2)(-iQ_ee)\gamma^\alpha\frac{i}{-\not{p}_2-\not{k}-m+i\varepsilon}(-ie)\gamma_\mu(v_A-a_A\gamma_5)\frac{i}{\not{p}_1-\not{k}-m+i\varepsilon}(-iQ_ee)\gamma_\alpha u(p_1) \quad (10)$$

- Scalarising the fermion propagator denominators $\Rightarrow$

$$\mathcal{M}_\mu = -ie\frac{\int d^4k(-iQ_e{}^2e^2)}{(2\pi)^4}\frac{1}{k^2+i\varepsilon}\bar{v}(p_2)\gamma^\alpha\frac{-\not{p}_2-\not{k}+m}{k^2+2kp_2+i\varepsilon}\gamma_\mu(v_A-a_A\gamma_5)\frac{\not{p}_1-\not{k}-m}{k^2-2kp_1+i\varepsilon}\gamma_\alpha u(p_1). \quad (11)$$

- Using the equations of motion

$$\begin{aligned} (\not{p}_1-\not{k}-m)\gamma_\alpha u(p_1) &= \{(2p_1-k)_\alpha - \tfrac{1}{2}[\not{k},\gamma_\alpha]\}u(p_1), && (a) \\ \bar{v}(p_2)\gamma^\alpha(-\not{p}_2-\not{k}+m) &= \bar{v}(p_2)\{-(2p_2+k)^\alpha + \tfrac{1}{2}[\not{k},\gamma^\alpha]\}, && (b). \end{aligned} \quad (12)$$

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Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ Contribution to $2Q_e^2\alpha B(p_1,p_2)$ corresponding to the cross-term in the virtual IR function on the RHS of eq.(7):

$$2Q_e^2\alpha B(p_1,p_2)|_{\text{cross-term}} = \int d^4k\frac{(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\frac{(2p_1-k)(2p_2+k)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}. \quad (13)$$

This term, together with the two squared terms in $2\alpha Q_e^2B(p_1,p_2)$, leads to the exponentiation of $\frac{1}{2}Q_e^2\frac{\alpha}{\pi}L$.

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Improving the Collinear Limit in YFS Theory

- The two commutator terms on the RHS of eq.(12), usually dropped, can be analyzed further: possible IR finite collinearly enhanced improvement of the YFS virtual IR function B.
- Isolate the collinear parts of k via the change of variables

$$k = c_1p_1 + c_2p_2 + k_\perp \quad (14)$$

where $p_1k_\perp = 0 = p_2k_\perp$, $\Rightarrow$ we have the relations

$$\begin{aligned} c_1 &= \frac{p_1p_2}{(p_1p_2)^2-m^4}p_2k - \frac{m^2}{(p_1p_2)^2-m^4}p_1k \xrightarrow[CL]{} \frac{p_2k}{p_1p_2} \\ c_2 &= \frac{p_1p_2}{(p_1p_2)^2-m^4}p_1k - \frac{m^2}{(p_1p_2)^2-m^4}p_2k \xrightarrow[CL]{} \frac{p_1k}{p_1p_2}, \end{aligned} \quad (15)$$

BAYLOR *CL* denotes the collinear limit $\equiv \mathcal{O}(m^2/s)$ dropped.

Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ $(2p_1-k)^\alpha$ in eq.(12(a)) combines with the commutator term in eq.(12(b)) to produce

$$\begin{aligned} \bar{v}(p_2)\{(2p_1-k)_\alpha\tfrac{1}{2}[\not{k},\gamma^\alpha]\}\gamma_\mu(v_A-a_A\gamma_5)u(p_1) &= \bar{v}(p_2)[\not{k},\not{p}_1]\gamma_\mu(v_A-a_A\gamma_5)u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)[c_2\,\not{p}_2,\not{p}_1]\gamma_\mu(v_A-a_A\gamma_5)u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)(-2c_2p_1p_2)\gamma_\mu(v_A-a_A\gamma_5)u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)(-2p_1k)\gamma_\mu(v_A-a_A\gamma_5)u(p_1). \end{aligned} \quad (16)$$

- Similarly, $-(2p_2+k)^\alpha$ in eq.(12 (b)) combines with the commutator term in eq.(12(a)) to produce

$$\begin{aligned} \bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)\{-(2p_2+k)^\alpha(-\tfrac{1}{2}[\not{k},\gamma_\alpha])\}u(p_1) &= \bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)[\not{k},\not{p}_2]u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)[c_1\,\not{p}_1,\not{p}_2]u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)(2c_1p_1p_2)u(p_1) \\ &\xrightarrow[CL]{} \bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)(2p_2k)u(p_1). \end{aligned}$$

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Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ Shift on the RHS of eq.(13):
- What does the term quadratic in the commutator (C^2) contribute?
- Superficial UV divergence $\Rightarrow$ Cannot naively drop $k_\perp$
- Proceed directly: we need

$$2Q_e^2\alpha B(p_1,p_2)|_{C^2}\mathcal{M}_{B\mu} \equiv \frac{\int d^4k(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\left.\frac{\frac{1}{4}\bar{v}(p_2)[\not{k},\gamma^\alpha]\gamma_\mu[\not{k},\gamma_\alpha](-ie)(v_A-a_A\gamma_5)u(p_1)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}\right|_{CL}, \quad (19)$$

where we define

$$\mathcal{M}_{B\mu} = -ie\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)u(p_1). \quad (20)$$

- *CL* now further restricted to contributions singular as $m^2/s \rightarrow 0$.

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Improving the Collinear Limit in YFS Theory

- Using standard methods, we need

$$I_\mu = 2\int_0^1 d\alpha_1\int_0^{1-\alpha_1}d\alpha_2\frac{\int d^nk'(iQ_e{}^2e^2)}{8\pi^4}\left.\frac{\frac{1}{4}\bar{v}(p_2)N_\mu^{\lambda\lambda'}[\frac{k'^2}{n}g_{\lambda\lambda'}+\Delta_\lambda\Delta_{\lambda'}](-ie)(v_A-a_A\gamma_5)u(p_1)}{[k'^2-\Delta^2+i\varepsilon]^3}\right|_{CL}, \quad (21)$$

where $\Delta = \alpha_1p_1 - \alpha_2p_2$.

- Equations of motion $\Rightarrow$ term involving Δ is not collinearly enhanced.
- The term contracted with $g_{\lambda\lambda'}$ gives us

$$I_\mu = \left\{\frac{-3Q_e^2\alpha}{4\pi}\mathcal{M}_{B\mu}\right\}\Big|_{CL} \equiv 0 \quad (22)$$

- $\Rightarrow$ No collinearly enhanced contribution from I_μ.
- Eq.(18) gives the complete collinear enhancement of B.

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Improving the Collinear Limit in YFS Theory

- We have

$$\begin{aligned} 2\alpha Q_e^2\Delta B(p_1,p_2) &= \frac{\int d^4k(iQ_e^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\frac{2p_1k-2p_2k}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)} \\ &= 2\int_{x_i\geq 0, i=1,2,3}d^3x\,\delta(1-x_1-x_2-x_3)\frac{\int d^4k'(iQ_e^2e^2)}{8\pi^4}\frac{2(p_1-p_2)p_x}{(k'^2-d+i\varepsilon)^3} \end{aligned} \quad (23)$$

where $d = p_x^2$ with $p_x = x_1p_1 - x_2p_2$.

- $\Rightarrow$ We get

$$2Q_e^2\alpha\Re\Delta B(p_1,p_2) = Q_e^2\frac{\alpha}{\pi}L. \quad (24)$$

- We see that indeed the entire term $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ is now exponentiated by our collinearly improved YFS virtual IR function B_{CL}

$$\begin{aligned} B_{CL} &= B + \Delta B \\ &= \int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left[\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2-\frac{4pk-4qk}{(2pk-k^2)(2qk+k^2)}\right] \end{aligned} \quad (25)$$

See S. Jadach, Durham talk, 2002, for integrated form of B_{CL}.

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Improving the Collinear Limit in YFS Theory

- What about the real YFS IR algebra? Collinear enhancement desired in some applications
- We again isolate collinearly enhanced contributions by using the representation in eq.(14) for k, respecting the condition $k^2 = 0$. $\Rightarrow$ Maintain $0 = (c_1^2+c_2^2)m^2 + 2c_1c_2p_1p_2 - |k_\perp|^2$.
- $\Rightarrow$ Collinear enhancement of $\tilde{B}$:

$$2\alpha Q_e^2\tilde{B}_{CL} = \frac{-\alpha Q_e^2}{4\pi^2}\int\frac{d^3k}{k_0}\left\{\left(\frac{p_1}{kp_1}-\frac{p_2}{kp_2}\right)^2 + \frac{1}{kp_1}\left(2-\frac{kp_2}{p_1p_2}\right) + \frac{1}{kp_2}\left(2-\frac{kp_1}{p_1p_2}\right)\right\}. \quad (26)$$

- Agreement with Berends *et al.*

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Improving the Collinear Limit in YFS Theory

- What about CEEX?
- In the use of amplitude-level isolation of real IR divergences, K-S photon polarization vectors $\Rightarrow$

$$\mathcal{M}_\mu = \mathcal{M}_{B\mu}\mathfrak{s}_{CL,\sigma}(k), \quad (27)$$

with

$$\mathfrak{s}_{CL,\sigma}(k) = \sqrt{2}Q_ee\left[-\sqrt{\frac{p_1\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_1-\sigma >}{2p_1k} + \delta_{\lambda-\sigma}\sqrt{\frac{k\zeta}{p_1\zeta}}\frac{< k\sigma|\hat{p}_1\lambda >}{2p_1k} + \sqrt{\frac{p_2\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_2-\sigma >}{2p_2k} + \delta_{\lambda\sigma}\sqrt{\frac{k\zeta}{p_2\zeta}}\frac{< \hat{p}_2\lambda|k-\sigma >}{2p_2k}\right]. \quad (28)$$

Here, $\zeta \equiv (1,1,0,0)$ and $\hat{p} = p - \zeta m^2/(2\zeta p)$.

- Upon taking the modulus squared of $\mathfrak{s}_{CL,\sigma}(k)$ we see that the extra non-IR divergent contributions reproduce the known collinear big log contribution which is missed by the usual YFS algebra.

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SUMMARY: Enhanced the toolbox available to extend the CEEX YFS MC method to the other important processes at present and future colliders.