

# Trinification from a complete $E_6$ GUT model

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Based on [2305.16398] and [2403.20278].

In collaboration with: K.S. Babu, B. Bajc

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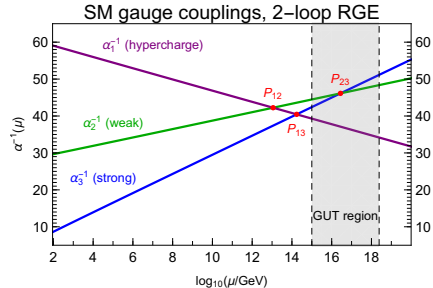


## Motivation — $E_6$ GUT

- Do gauge couplings in the Standard Model (SM) unify at high  $E$ ?

Window of opportunity :

- $> 10^{15}$  GeV (proton decay)
- $< 2.4 \cdot 10^{18}$  GeV (gravity — r. Planck scale)



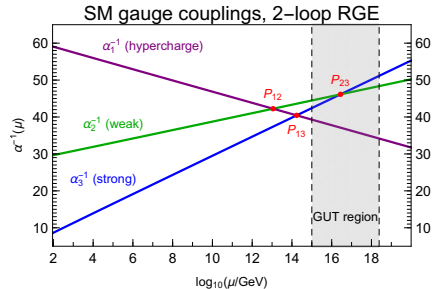
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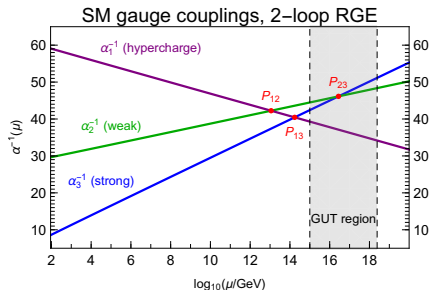
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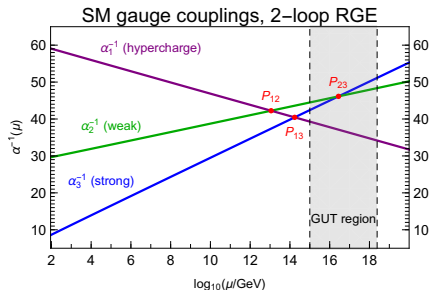
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Minimal choices for  $G$ :

$$SU(5) \subset SO(10) \subset E_6$$



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  - (a) must contain  $SU(3)^3$  singlet(s)
  - (b) only 650 leads to possibly perturbative theory, 2430 too big



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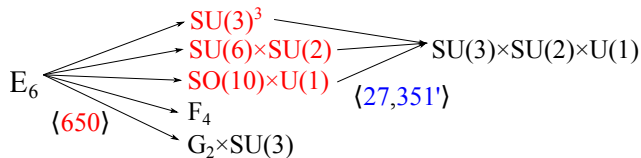
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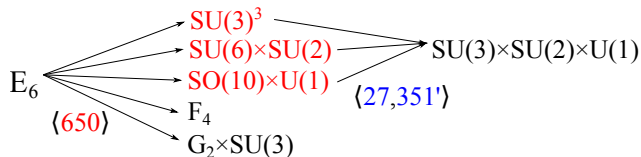
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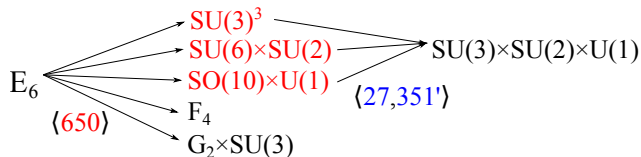
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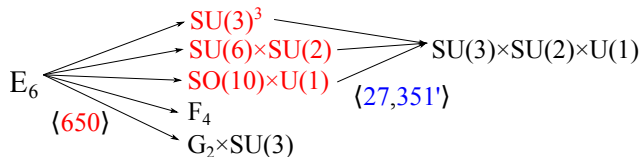
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- Realistic, analog of the SO(10) case:  $16_F 16_F 10 + 16_F 16_F \overline{126}$





## Breaking possibilities

- Considerations for intermediate vacua:
  - (a) There can be different **embeddings** of SM group into intermediate-symmetry group
  - (b) **Unification** cannot happen yet at intermediate scale (cf. bottom-up RGE)
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- Viable vacua:

name	intermediate symmetry	viable?
trinification	$SU(3)_C \times SU(3)_L \times SU(3)_R$	✓✓✓
standard	$SU(6)_{CL} \times SU(2)_R$	✓
flipped	$SU(6)_{CL} \times SU(2)_{R'}$	—
LR-flipped	$SU(6)_{CR} \times SU(2)_L$	✓
standard	$SO(10) \times U(1)$	—
flipped	$SO(10)' \times U(1)'$	✓



## Unification analysis — minimal cases

- effective theory for each of the viable vacua: [K. Babu, B. Bajc, VS, 2403.20278]
  - (a) assume **extended survival hypothesis** (ESH)
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    - SO(10) spinorial irreps (16, 144) have no VEV
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    - vector-like fermions in  $10 \subset 27_F$  do not mix with  $16 \subset 27_F$
- After careful considerations... the effective intermediate-scale models are:

vacuum	scalars with ESH + $\mathbb{Z}_2^\psi$	vacuum	scalars with ESH + $\mathbb{Z}_2^\psi$
$3_C 3_L 3_R \times LR$	$2 \times (1, \bar{3}, 3) + (1, \bar{6}, 6)$	$6_{CL} 2_R$	$(15, 1) + (\bar{2}\bar{1}, 3) + (\bar{6}, 2) + (84, 2)$
$3_C 3_L 3_R \times CL$	$2 \times (1, \bar{3}, 3) + (1, \bar{6}, 6)$ $+ 2 \times (\bar{3}, 1, \bar{3}) + (\bar{6}, 1, \bar{6})$	$6_{CR} 2_L$	$(15, 1) + (\overline{105}', 1) + (\bar{6}, 2) + (84, 2)$
$3_C 3_L 3_R \times CR$	$2 \times (1, \bar{3}, 3) + (1, \bar{6}, 6)$ $+ 2 \times (3, 3, 1) + (6, 6, 1)$	$10' 1'$	$(16, +1) + (126, +2) + (10, -2)$



## Unification analysis — results

- Unification: bottom-up via 2-loop RGE

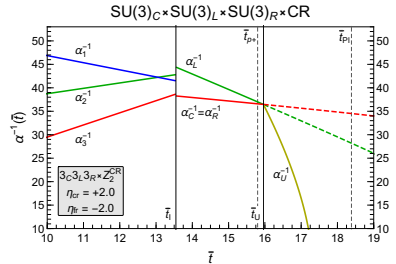
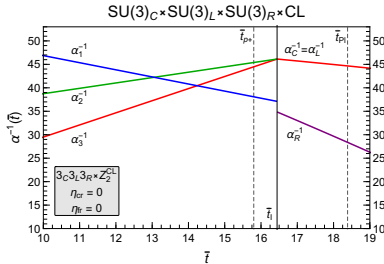
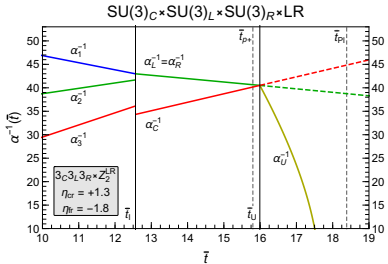


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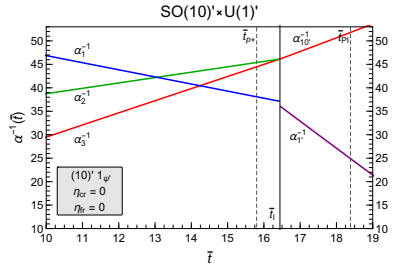
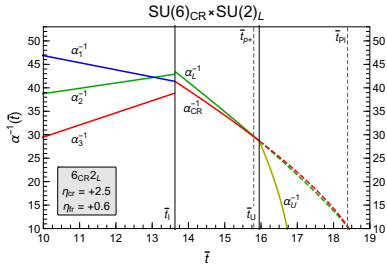
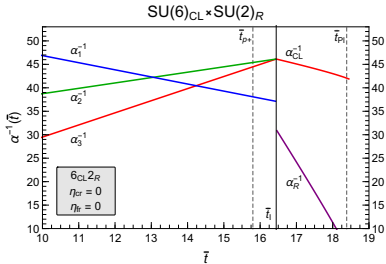
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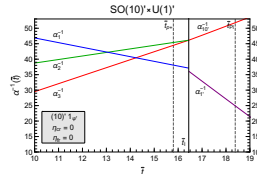
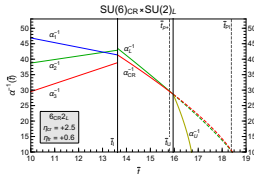
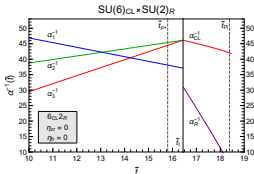
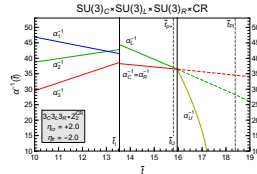
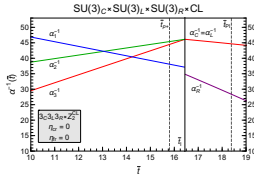
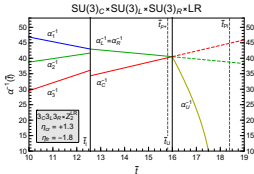
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- Viable cases:  $3C3L3R \times LR$ ,  $3C3L3R \times CR$ ,  $6CR2L$



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- Gauge mediators of proton decay in  $E_6$ :  $X$ ,  $X'$ ,  $X''$

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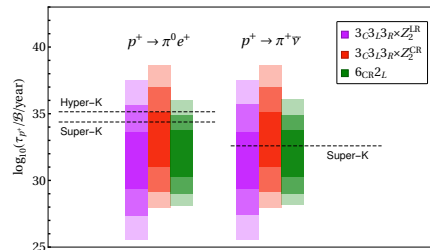
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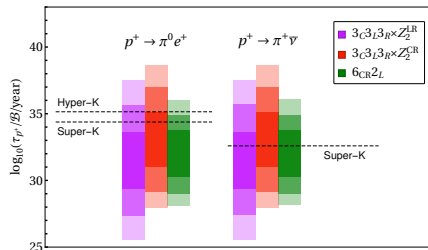
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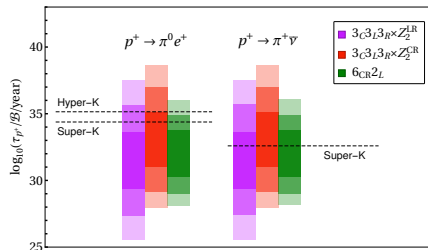
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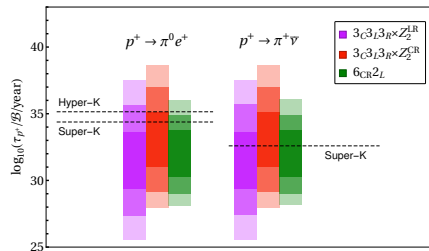
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- Large uncertainties in proton lifetime:  $\tau_{p^+} \propto M_{GUT}^4$  ( $M_{GUT}$  varies with spectrum)







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**Thank you for your attention!**