Formal Theory

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ICHEP 2024, Prague, July 23, 2024

”Formal Theory”

What is ”Formal Theory”?

~~Formal as in ...”Theory, that is not useful for any actual physics”?~~

Theory, which develops a fundamental understanding of Quantum Field Theory and Quantum Gravity, consolidating insights from various areas of theoretical physics and mathematics.

Disclaimer: Often this requires studying idealized systems, that are simpler than real world setups. But we will see many avenues to connect to the latter.

"Formal Theory"

A (biased[a]) selection of topics with exciting, recent progress:

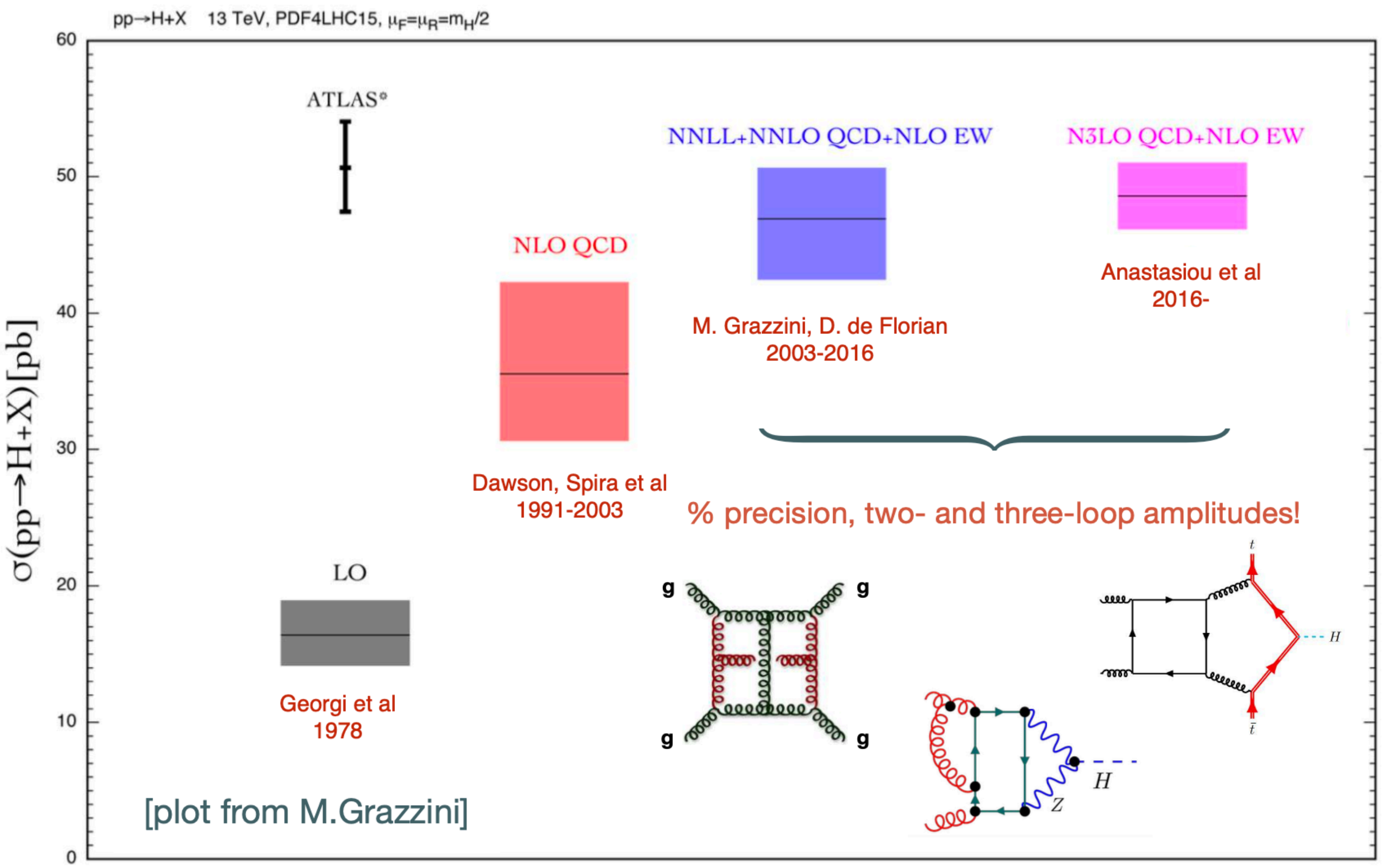
0. Scattering Amplitudes: QFT, Strings, Gravity
   ⇒ Formal Parallel Session at ICHEP organized by Jusinskas, Trnka, Volovich.
1. **Quantum Gravity constraints on IR physics**
2. **Holography**:
   (a) AdS/CFT precision holography
   (b) Quantum Information and Quantum Gravity
   (c) Flat-Space holography: Celestial and Carollian holography
3. **Generalized symmetries: Non-invertible symmetries**

[a] 25 mins to cover hep-th: 2023 totals: 3834 articles + 3452 cross-lists

0. Scattering Amplitudes

Scattering Amplitudes: From Formal to Precision Physics

Amplitudes are of direct relevance for current **Collider Physics**:
LHC requires % precision predictions: QCD+ EW, higher loops and legs.



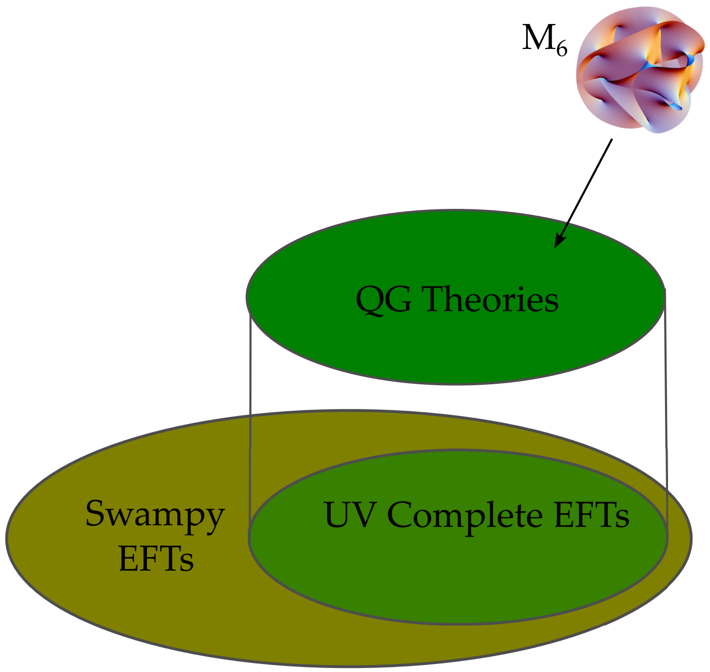
Current status of amplitudes in QCD: 2→2 @ 3L, 2 → 3 @ 2L

Formal Session: Amplitudes Talks at ICHEP

Formal theory tools geared towards QCD-like theories (super-YM, etc) that have fed directly into phenomenologically relevant results:

- **BCFW Recursion Relations** and **Unitarity Methods**, Mathematical tools: Drummond, Parisi
- **Amplituhedron**: for $N = 4$ Super-Yang Mills, but very powerful in producing integrands.
- **String Amplitudes**: quantum gravity, applications to holography Monteiro, Schlotterer, Brown^2, Klisch, Lipstein
- **Double Copy:** Color-Kinematics Duality when relating gauge to gravity scattering Brandhuber, Carrasco, Chen,Travaglini.
- **Surface-ology**: Most recent development: promising **Loop integrals for (in principle) any colored theory** using curves on fat graphs.

1. Quantum Gravity (QG) Constraints on IR Physics



Traditionally: Top-down

10d string theory $\mathcal{T}_{10}$ compactified to 4d gives EFT, e.g. SM spectrum + X (where X is usually $N = 1$ susy, exotics)

$$\mathbb{R}^{1,9} \rightarrow \mathbb{R}^{1,3} \times M_6$$

Defines at low energies a UV completable EFT $(\mathcal{T}_{10}, M_6)$.

Modern approach: Bottom-up/Swampland Program

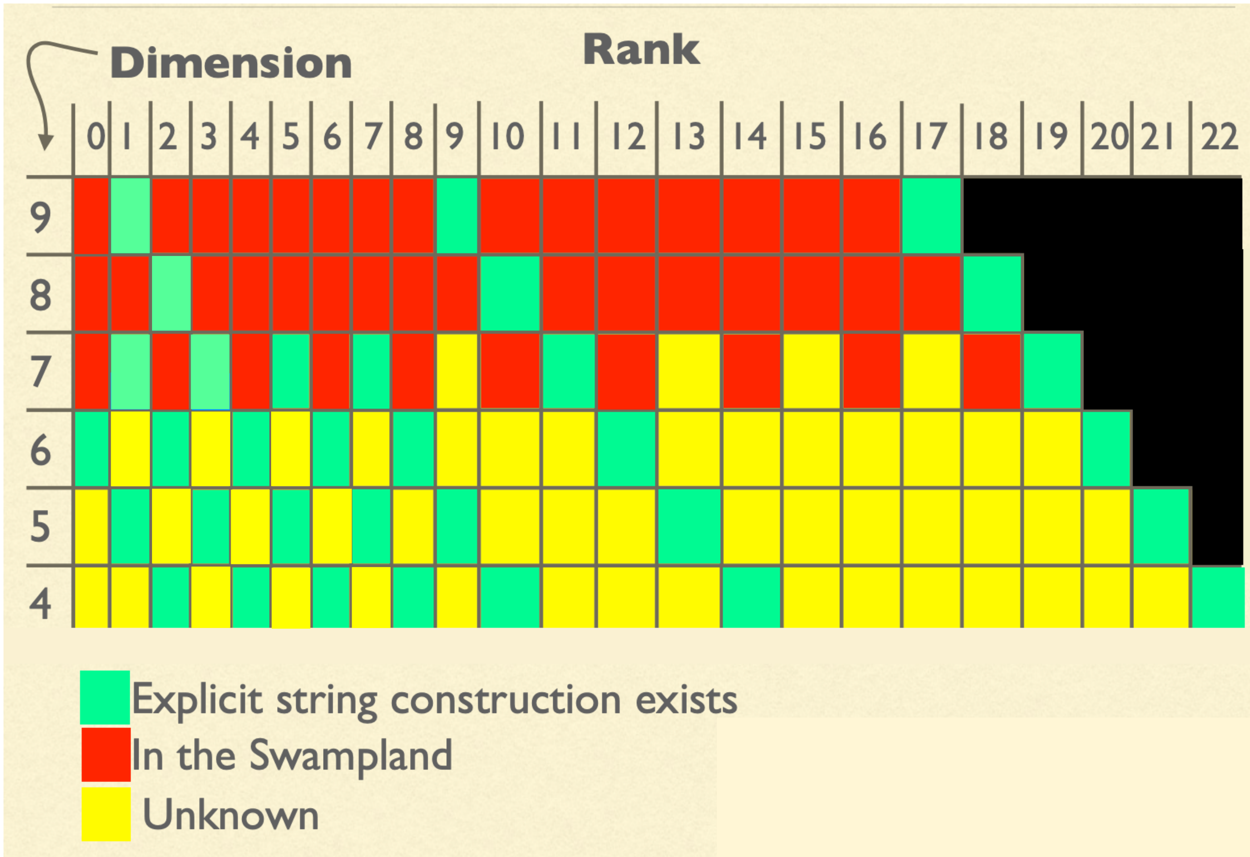
Question 1: Given an EFT, can it be embedded into a consistent theory of QG?
Question 2: String Universality? Do all QGs come from string theory?

- **Distance conjecture** ”infinite distance in parameter space results in infinite tower of states”, i.e. breakdown of EFT.
- **Weak Gravity** ”gravity is weakest force”
- **No global symmetries** ”all global symmetries gauged or broken”

Some recent progress on both questions – albeit in theories with so far little pheno relevance.

String universality: Abelian gauge theories with 16 susies

Example: 8d and 9d abelian gauge theories with 16 susies. Find agreement of top down and bottom up and confirms string universality.



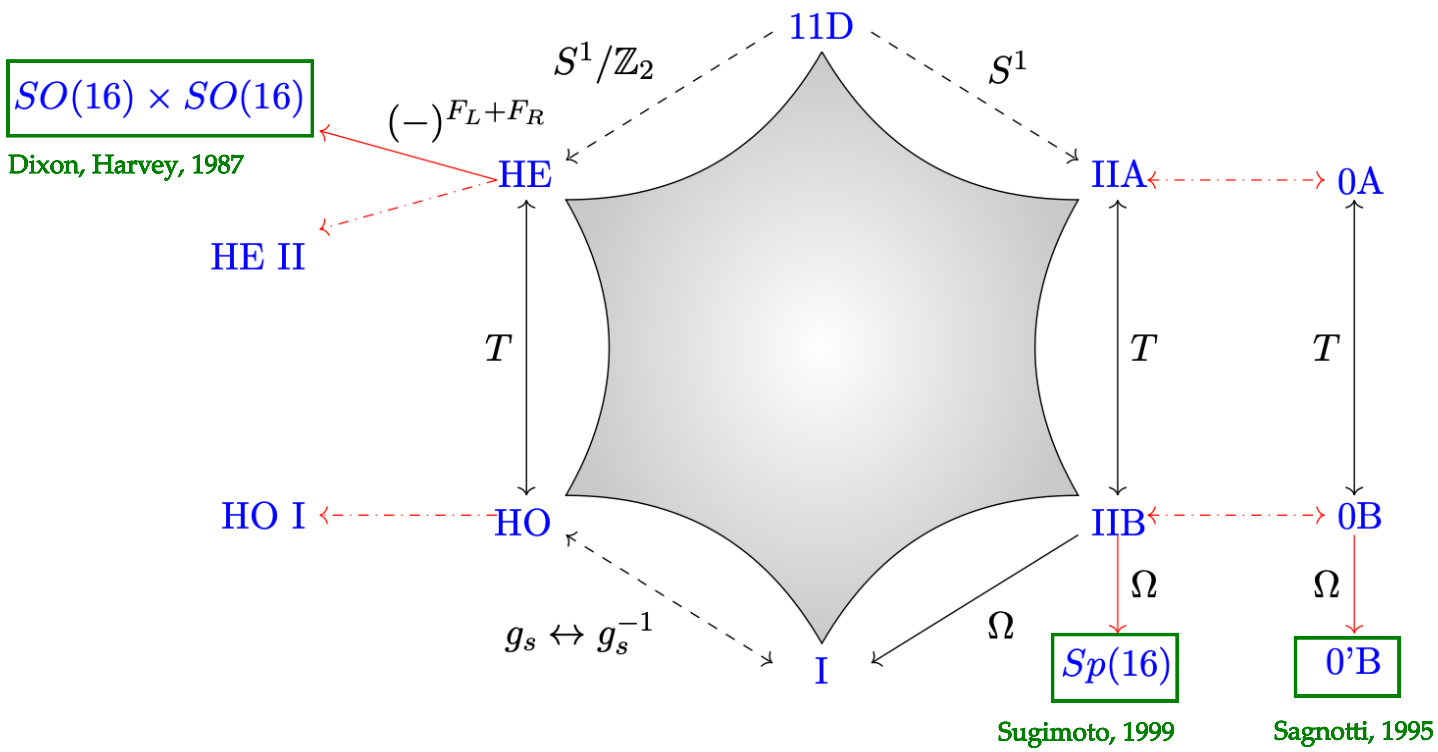
[modified from: Montero: Strings 2024 talk]

Similar arguments, with less stringent results thus far for 8 and 4 supersymmetries.

Status: Sharpening theoretical tools.

What about no susy?

What is the status of string models with no supersymmetry? Start in 10d.



[from 2310.06895]

There are 3 non-supersymmetric tachyon-free string theories in 10d.

10d Non-Supersymmetric Tachyon-Free Strings

$SO(16)^2$ Het (1987)	$Sp(16)$ Sugimoto (1999)	0'B Sagnotti (1995)
$E_8 \times E_8$ Het$/(-1)^F$	IIB + $O9^+$ + 32 $\overline{D9}$	IIB$/(-1)^F/\Omega$
Heterotic g, ϕ, B	Closed strings: $\mathcal{N} = 1$ susy	Closed: metric dilaton
No gravitino	Open strings: non-susy	non-susy
$SO(16)^2$ gauge group	$Sp(16)$ gauge group	$U(32)$ gauge group

Recent progress:

1. Completeness proven: these are **all** 10d non-susy heterotic theories [Boyle Smith, Lin, Tachikawa ,Zheng, 2023]
2. [Basile, Debray, Delgado, Montero, 2023] showed using cobordism theory, **all local and global anomalies cancel**, i.e. not connected to identity: Prediction of new extended objects (branes)
3. Analysis of moduli stabilization [Sagnoti et al, (2023, 24)]
4. New $d < 10$ non-supersymmetric compactifications [Baykara, Tarazi, Vafa, 2024]

Status: systematic progress exploring non-susy string theories.

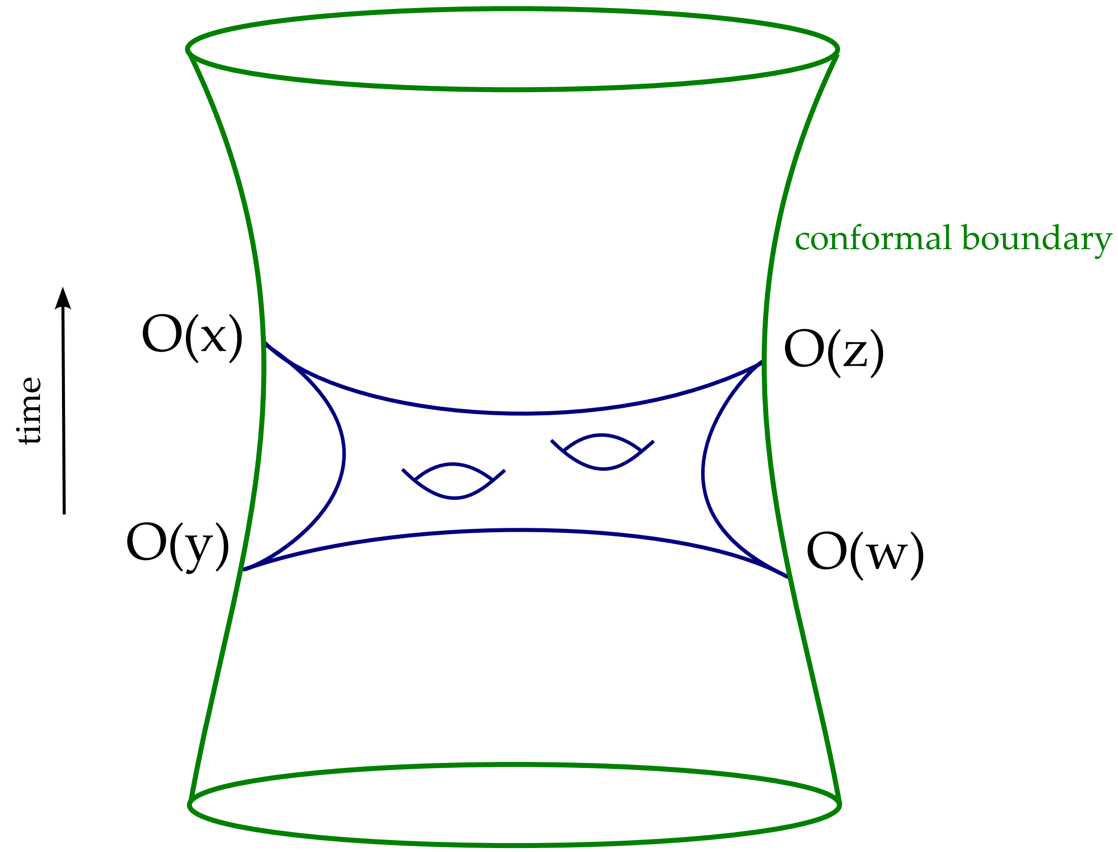
2. Holography

Why is Holography important? Gauge/gravity and Strong/weak dualities. Provides conceptual and computational window into quantum gravity and strongly-coupled QFTs alike.

2.1. AdS/CFT Precision Holography

AdS/CFT

String theory on $\text{AdS}_{d+1} \times X$ $\longleftrightarrow$ CFT_d on boundary of AdS_{d+1}

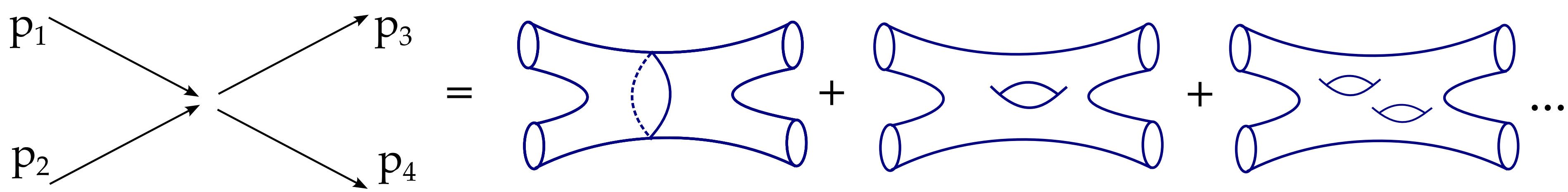


Precision (in coupling constants on both sides) lab for quantum gravity and strongly-coupled CFTs alike.

- Conformal group = isometries of AdS.
- Parameters: string coupling $g_s = \frac{1}{N_c}$ and length $\frac{R^2}{\alpha'} = \sqrt{\lambda} = \sqrt{g_{YM}^2 N_c}$

1. $\text{AdS}_3/\text{CFT}_2$: exact duality proven [Eberhardt, Gaberdiel, Gopakumar]
   $\text{AdS}_3 \times S^3 \times T^4$ (with 1 unit of NSNS flux) dual to $\text{Sym}^N(T^4)$ 2d CFT.

2. $\text{AdS}_5/\text{CFT}_4$: Example of deriving quantum gravity from QFT:
   Reconstructing string amplitudes from QFT:
   Construct string amplitudes in $\text{AdS}_5 \times S^5$ (HARD). E.g. 4-graviton scattering in $\text{AdS}_5 \times S^5$:



This is an expansion in string-loops, i.e. $1/N_c$. Even tree-level in curved spacetimes in very difficult.

Tree-level: Virasoro-Shapiro amplitude has an expansion in α' or 't Hooft coupling $1/\sqrt{\lambda}$

$$A(S,T) = A^{(0)}(S,T) + \frac{A^{(1)}(S,T)}{\sqrt{\lambda}} + \cdots$$

where the flat space amplitude is

$$A^{(0)}(S,T) = \frac{1}{U^2} \int d^2z |z|^{-2S-2} |1-z|^{-2T-2} = -\frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)}$$

Quantizing the string in $\mathrm{AdS}_5 \times S^5$ is notoriously difficult (RR-fluxes). Usually people leave it at supergravity level.

Using insights from

- conformal bootstrap and localization
- integrability
- number-theory

[Alday, Hansen, Silva] constucted (conjecturally) the subleading terms and wrote them in terms of a world-sheet-type integral with insertions:

$$A_4^{AdS}(S,T) \sim \int d^2z |z|^{-2S}|1-z|^{-2T} W_0(z,\bar{z}) \left(1 + \frac{S^2}{R^2} W_3(z,\bar{z}) + \frac{S^4}{R^4} W_6(z,\bar{z}) + \cdots\right)$$

W_n's are single-valued polylogarithms of weight n

New progress on String Field Theory in backgrounds like $\mathrm{AdS}_5 \times S^5$ should soon be able to test this.

Status: AdS/CFT is now a precision lab for strongly-coupled QFTs and quantum gravity. Not real world holographic duals, but many important lessons learned from these correspondences.

2.2. Quantum Information and Quantum Gravity

Lessons:

1. Ordinary quantum systems, e.g. spin chains, can have emergent gravitational properties.

   SYK/JT: N Majorana fermions in 1+1d with random couplings

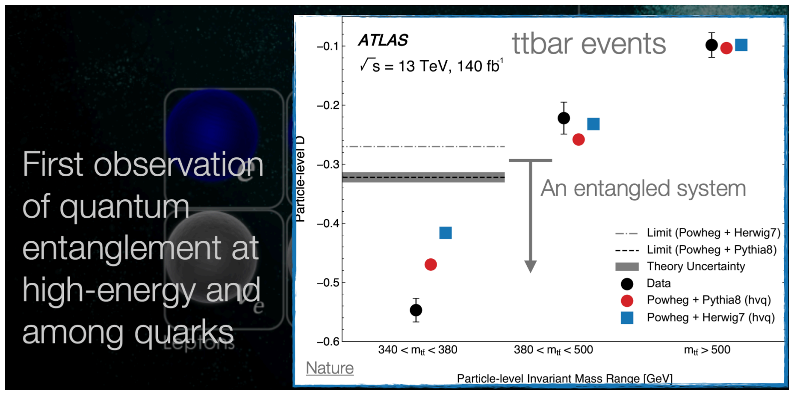
   $$H_{\text{SYK}} = \sum_{i_1, i_2, \ldots, i_q} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}$$

   At large N, emergent gravity:

   $\Rightarrow$ Emergent conformal symmetry with $c \approx \frac{N^2}{2}$

   $\Rightarrow$ Entropy is akin to black holes: $S \approx \frac{N^2}{4} \log T$.

2. Quantum info concepts have become key tools in hep-th:
   (a) Spacetime emerges from quantum entanglement: ER= EPR, entangled particles are connected by wormholes.
   (b) Holography: Entanglement entropy = Area (minimal surface)$/(4G_N)$.
   (c) hep-ph/ex applications: measurement of top-quark entanglement



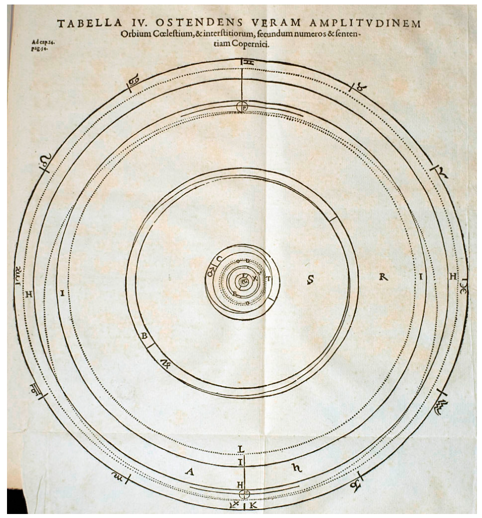
[ATLAS, see Dunford's ICHEP talk]

3. Full quantum gravitational path integral requires sum over all topologies of spacetime. This is well established now in AdS/CFT duality. For the future: relevant for cosmology (inflationary models e.g.).

2.3. Flat Space Holography

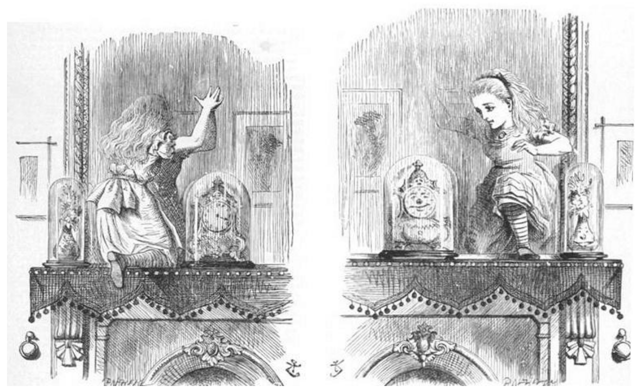
Is there a holographic dual to quantum gravity in flat space? (or asymptotically flat spacetimes (AFS)).

Two recently developed approaches for gravity in AFS spacetimes:



- **Celestial holography**:
  $(d+2)$-dim AFS dual to d-dim CFT on celestial sphere at future/past null-infinity $\mathscr{I}^{\pm}$
  "4d gravity is dual to 2d CFT on a celestial sphere S^2"

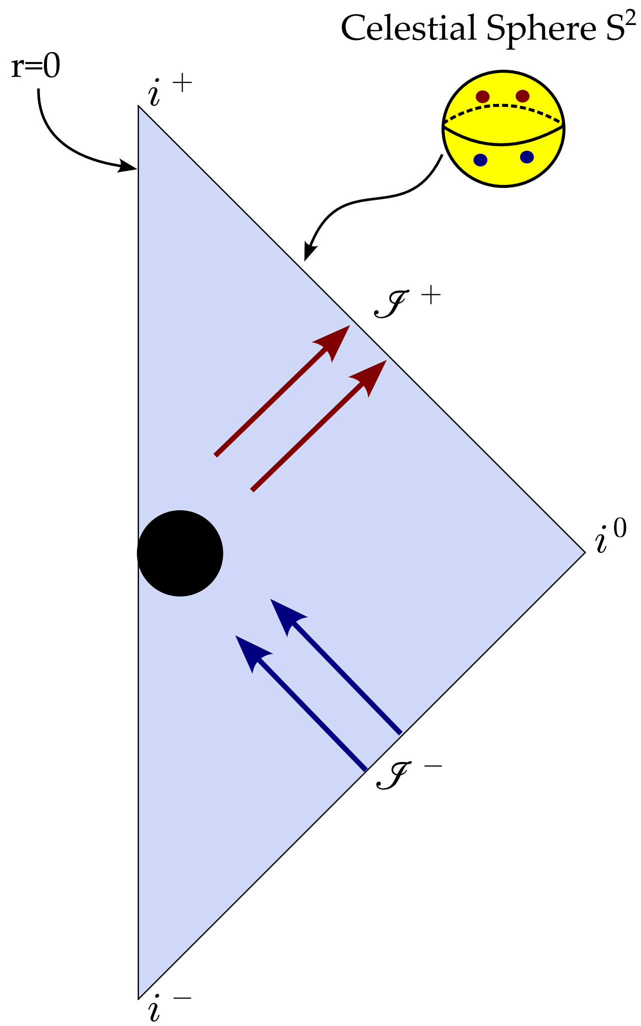
- **Carrollian holography**:
  $(d+1)$-dim AFS dual to d-dim Carrollian field theory on $\mathscr{I}^{\pm}$
  "4d gravity is dual to a 3d Carrollian[a] $(c \to 0)$ field theory at null infinity"



[a] Peculiar causal structure, like in Lewis Carroll's "Through the Looking Glass"

Celestial Holography

- 4d Lorentz group = conformal group on $S^2 \subset$ null infty $\mathscr{I} = \mathbb{R} \times S^2$.
- Symmetries = Bondi-Metzner-Sachs (BMS) symmetries extended to $w_{1+\infty}$
- Scattering amplitudes in 4d become correlators in 2d celestial CFT (CCFT)!



S-matrix between $\mathscr{I}^-$ and $\mathscr{I}^+$ becomes correlator in 2d CCFT:

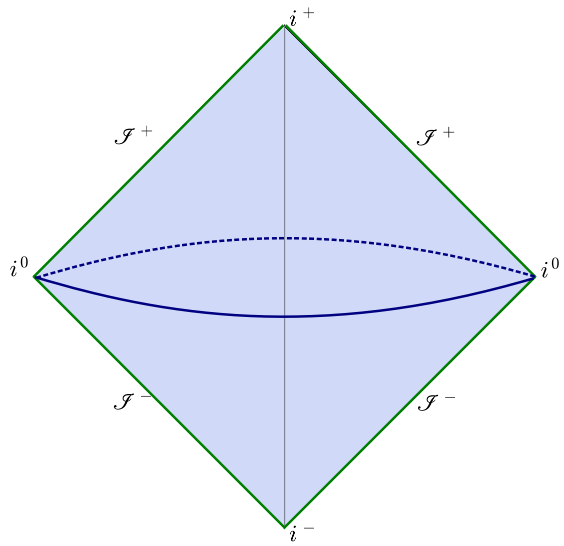
$$\langle p_n^{\text{out}} \ldots |\mathcal{S}| \ldots p_1^{\text{in}} \rangle \longleftrightarrow \quad \langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$$

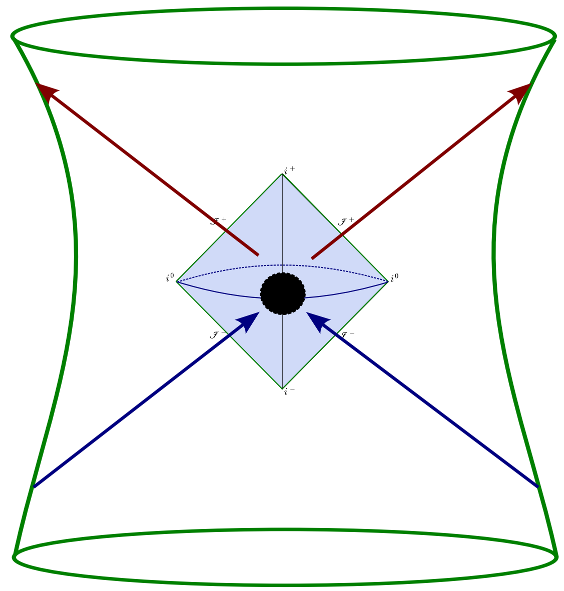
BMS symmetry of CCFT and implies via Ward identities soft graviton theorems

Gravitational memory effects (permanent displacement of test particles (detectors) due to the passage of gravitational waves), correspond to change of state in CCFT.

Directly related to soft theorems/BMS symmetries.

Carrollian Holography





$d+1$ dimensional AFS gravity dual to d-dimensional Carrollian field theory living on $\mathscr{I} \simeq \mathbb{R} \times S^{d-1}$.

1. Carrollian limit $c \to 0$ of Poincaré group: lightcones collapse along t-axis, so spacetimes events are causally disconnected.
2. Symmetries:
   BMS algebra is the conformal Carrollian algebra
3. **Limit from AdS/CFT:** $c \to 0$ of relativistic CFTs dual to flat-space local patch of AdS. Many studies of conformal to Carrollian (non-relativistic) limits. Using AdS results to get flat space results!

Status: Very active area. Many open questions (e.g. what precisely is the CCFT?), great potential to learn about flat space QG, and scattering.

3. Non-Invertible Symmetries

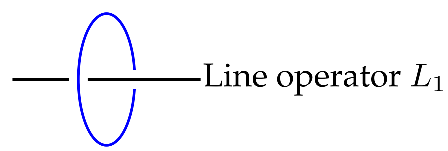
New Structures: Generalized Symmetries

Symmetries are vital – spectrum, interactions, SSB phases, anomalies, etc.

Usually: Symmetries = Groups, acting on point-like operators.

There are new symmetries in town: Relax acting on points and form group:

- Higher form symmetries: [Gaiotto, Kapustin, Seiberg, Willett]
  Example: Line operators charged under 1-form symmetry (”center symmetry $\mathbb{Z}_N$ of $SU(N)$ gauge group”).



  Physics: Confinement = 1-form symmetry preserving phase.

- Non-invertible symmetries: [Kaidi, Ohmori, Zheng][Choi, Cordova, Hsin, Lam, Shao][Bhardwaj, Bottini, SSN, Tiwari] $a, b \in \mathcal{S}$ a non-invertible symmetry, have no inverse and compose as follows:

$$a \cdot b = n_{c_1} c_1 + \cdots + n_{c_k} c_k \,, \qquad c_i \in \mathcal{S}, n_{c_i} \in \mathbb{N}$$

Higher-form and Non-invertible symmetries are ubiquitous in 4d QFT!

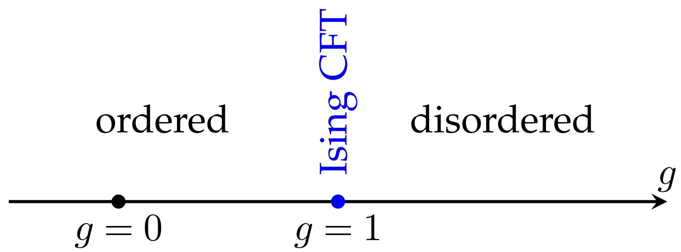
Non-Invertible Symmetries in the Ising CFT

Transverse field **Ising model**: $\mathcal{H} = (\mathbb{C}^2)^L$ with nearest neighbor Hamiltonian

$$H = -\sum_j \sigma_j^z \sigma_{j+1}^z - g \sum_j \sigma_j^x \,.$$

There is a $\mathbb{Z}_2$ spin flip symmetry $\eta = \prod_j \sigma_j^x$.

- $g = 0$: two ground states, $|\uparrow^L\rangle$ and $|\downarrow^L\rangle$: "ordered phase"
- $g \gg 1$: ground state preserves the $\mathbb{Z}_2$: "disordered phase"
- $g = 1$: critical Ising CFT at $c = 1/2$.



Kramers-Wannier duality N:
$\sigma_i^x \to \sigma_j^z \sigma_{j+1}^z$ and $\sigma_j^z \sigma_{j+1}^z \to \sigma_{j+1}^x$, corresponds to $g \to g^{-1}$.
At $g = 1$: symmetry of the critical Ising CFT, which is non-invertible

$$N^2 = 1 + \eta$$

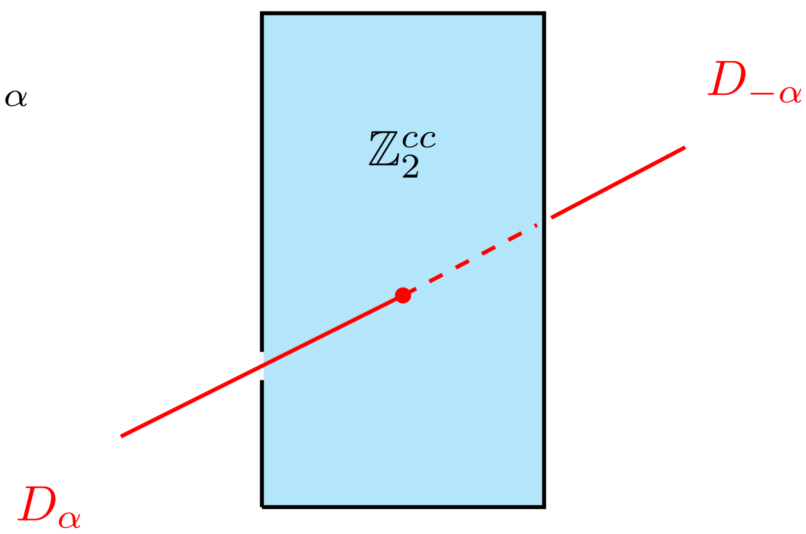
Non-Invertible Symmetries in $d = 4$

- 4d Kramers-Wannier duality symmetries:
  [Kaidi, Ohmori, Zheng][Choi, Cordova, Hsin, Lam, Shao]

$$\text{QFT} \cong \text{QFT}/D \qquad \Rightarrow \qquad \text{non-invertible 0-form symmetry}$$

- Gauging charge conjugation [Bhardwaj, Bottini, SSN, Tiwari]
  Example: $O(2)$ gauge theory as $U(1)/\mathbb{Z}_2^{\text{cc}}$.
  # There is a 1-form symmetry: $D_\alpha := e^{i\alpha \int *F}$.
  # $\mathbb{Z}_2^{\text{cc}}$: $D_\alpha \to D_{-\alpha}$

$$D_\alpha^{\text{inv}} = D_\alpha \oplus D_{-\alpha}$$



Example: ABJ Anomaly as a Non-Invertible Symmetry

In 4d QED with massless charge 1 Dirac fermion

$$\mathcal{L}_{\text{QED}+\Psi} = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\left(\partial_\mu - iA_\mu\right)\gamma^\mu \Psi$$

the axial current $j_\mu = \frac{1}{2}\bar{\Psi}\gamma_5\gamma_\mu\Psi$ is not conserved due to the ABJ anomaly

$$d \star j = \frac{1}{8\pi^2} F \wedge F$$

Define an operator dressed by 3d Topological QFT that has opposite anomaly

$$\mathcal{N}_{\frac{1}{N}}(M_3) = \int [Da] \exp\left(\int_{M_3} \frac{2\pi i}{N} \star j + \frac{iN}{4\pi} a da + \frac{i}{2\pi} a dA\right) .$$

It is topological, but satisfies non-invertible fusion

$$\mathcal{N}_{\frac{1}{N}} \times \mathcal{N}^\dagger_{\frac{1}{N}} = \mathcal{C} = \text{condensation defect for 1-form symmetry}$$

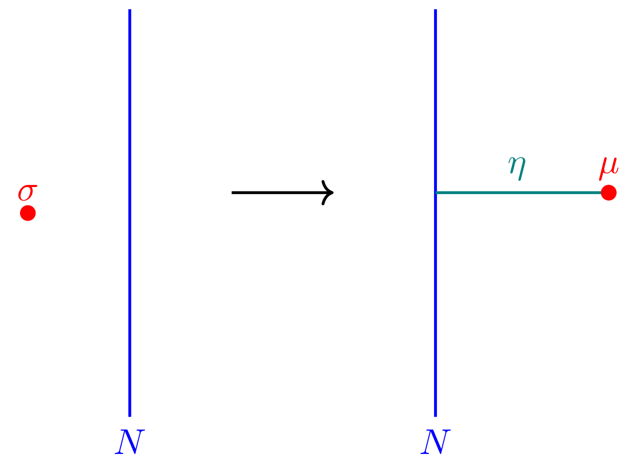
- [Choi, Lam, Shao][Cordova, Ohmori] application to pion decay
- [Cordova, Hong, Koren, Ohmori] Z′ with non-invertible chiral symmetry, gives breaking by exponentially small amount, application to neutrino masses

Status: implications so far mostly known, but promising direction.

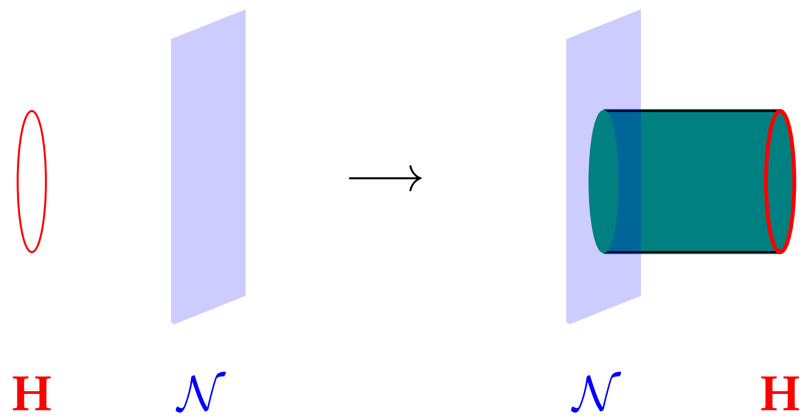
Physical Implications of Non-Invertible Symmetries 1.

They map genuine operators to non-genuine, i.e. endpoints of extended (topological) defects, or: order to disorder operators.

Example: Ising CFT $N^2 = 1 + \eta$



Example: Witten effect. 4d $SO(3)$ SYM has non-invertible symmetry, maps 't Hooft loop to flux attached 't Hooft loop

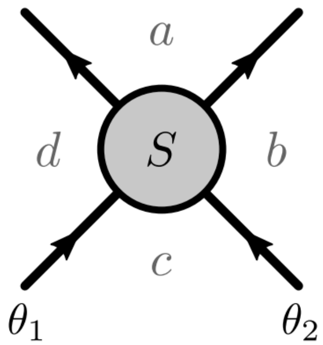


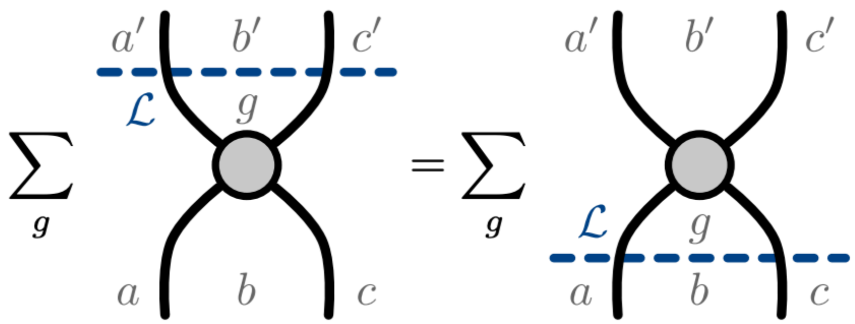
Physical Implications 2.: Modified Crossing Relations

Non-invertible symmetries lead to modified crossing relations for S-matrices!

Example: (1+1)d CFTs have non-invertible symmetries $\mathcal{L}$ e.g. Ising model.

Relevant, integrable deformations can preserve some of $\mathcal{L}$.
$\Rightarrow$ IR are gapped vacua. $\mathcal{L}$ constrains S-matrix of kinks through Ward ids:

$S^{ab}_{dc}(\theta) =$ 



[Copetti, Lucia Cordova, Komatsu] showed: crossing incompatible with symmetry/integrability/unitarity. Consistency implies modified crossing

$$S^{ab}_{dc}(\theta) = \sqrt{\frac{d_a d_c}{d_b d_d}}\, S^{bc}_{ad}(i\pi - \theta)\,, \qquad d_a = \langle \mathcal{L}_a \rangle \tag{1}$$

Modified crossing expected in (3+1)d (e.g. massive fermion-dyon scattering).

Status: Modified crossing direct implication of non-invertible symmetries.
Compelling if in particular extendable to higher dims.

Physical Implications 3.: Constraining Phases of Matter

Non-invertible Symmetries lead to new IR phases, and new second order Phase Transitions!

Landau paradigm:
A continuous (2nd order) phase transition is a symmetry breaking transition.

Gapped Phases: G spontaneously broken (SSB) to subgroup H. Phase has $|G/H|$ vacua, which are acted upon by the broken symmetry.

Beyond Landau:
$\mathcal{S}$ be a non-invertible symmetry, then there are new gapped and gapless phases [Bhardwaj, Bottini, Pajer, SSN][Bhardwaj, Pajer, SSN, Warman]

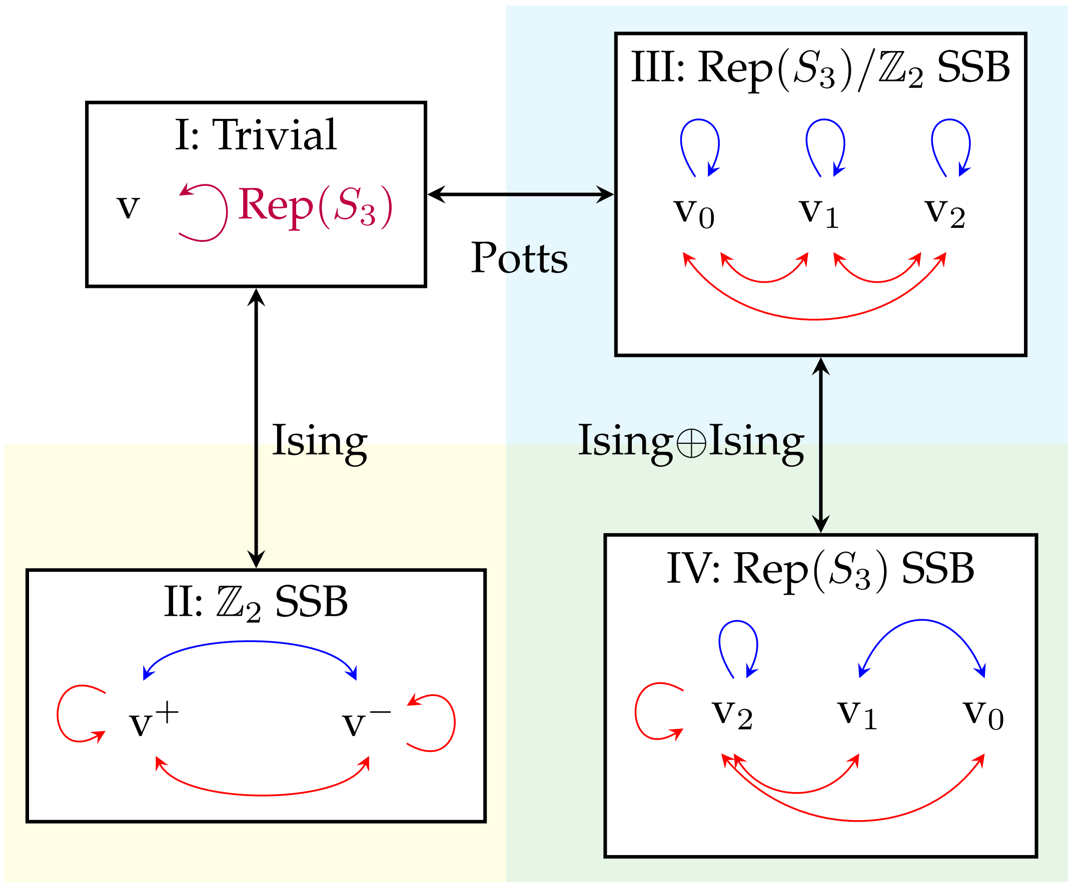
- Determined gapped (topological) phases, order parameters
- Gapless phase transitions between $\mathcal{S}$-symmetric gapped phases:

$$\boxed{\mathcal{S} \text{ Gapped Phase}} \quad \longleftarrow \quad \boxed{\text{CFT}} \quad \longrightarrow \quad \boxed{\mathcal{S} \text{ Gapped Phase'}}$$

$\Rightarrow$ **Categorical Landau Paradigm** [Bhardwaj, Bottini, Pajer, SSN]

Example: New Phases from Non-Invertible Symmetries

$\text{Rep}(S_3)$= $\{1, \sigma, E\}$ with $\sigma^2 = 1$, $E^2 = 1 \oplus \sigma \oplus E$ [Bhardwaj, Pajer, SSN, Warman]



Confirmed by lattice models and numerics [Bhardwaj, Bottini, SSN, Tiwari][Chatterjee, Aksoy, Wen]

Status: huge number of predictions, currently discussing how to use "quantum experiments" (using cold atoms) to test these.

Formal Theory – Status Report

Formal Theory is very much alive and well. Many unexpected, new directions and developments.

- ICHEP Formal Theory session: impressive progress on **Scattering Amplitudes** – string theory (Monteiro, Schlotterer), QFT/holography (Brown^2, Carrasco, Drummond, Klisch, Lipstein, Parisi), gravitational waves (Brandhuber,Chen,Travaglini), in particular the latter, having direct phenomenological implications.
- **Holography** continues to be a huge source of new approaches to QFT and QG alike: precision results in AdS/CFT, new flat space holography proposals.
- **Generalized (non-invertible) symmetries** in QFTs and spin systems: revolutionarizes classification of phases. Direct (as in collaborative and computational) relevance for cond-mat.
- **Quantum Gravity constraints on EFT**: the swampland/landscape program is making tremendous progress exploring which EFTs are UV completed within QG/string theory.