

KIRILL MELNIKOV

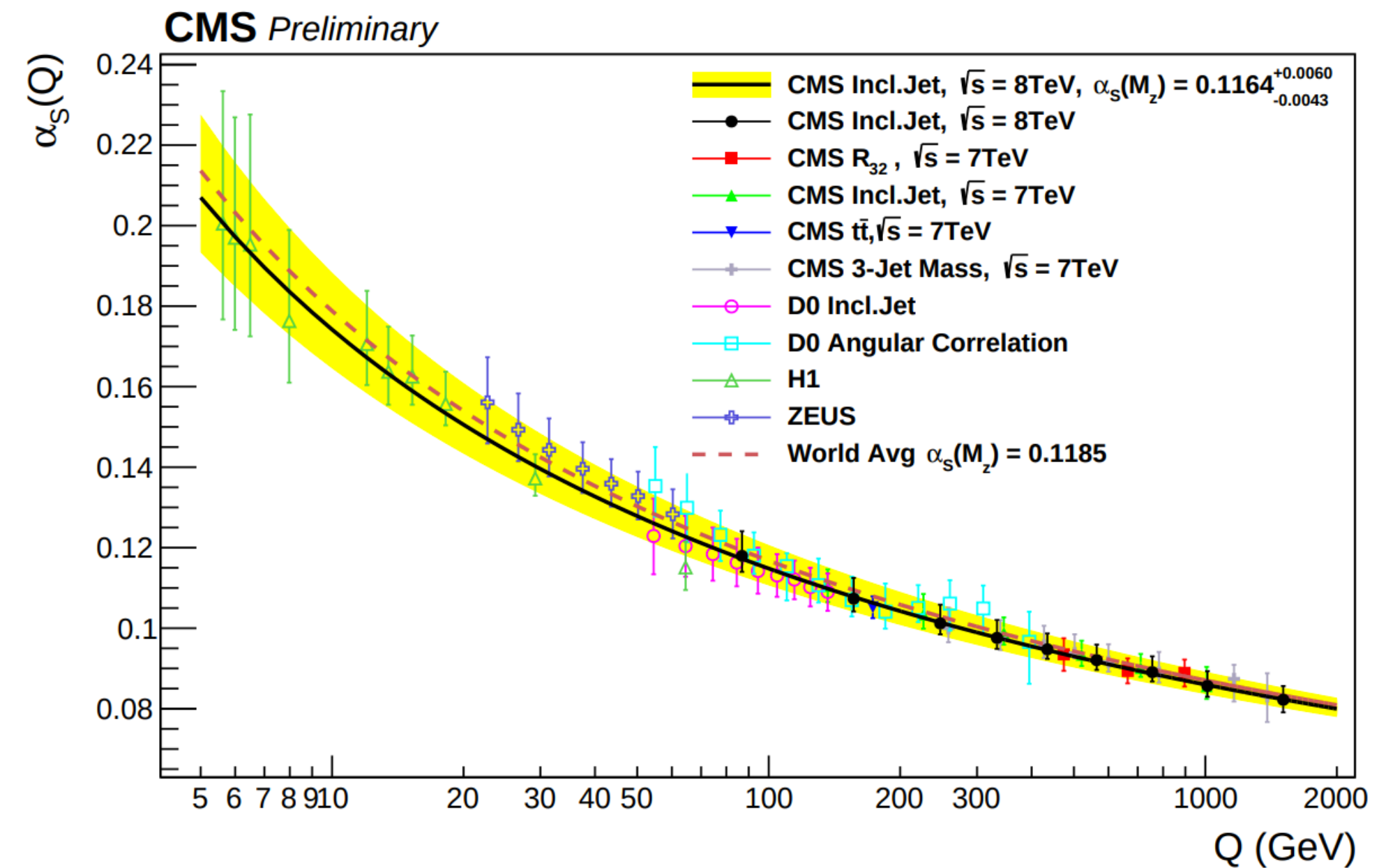
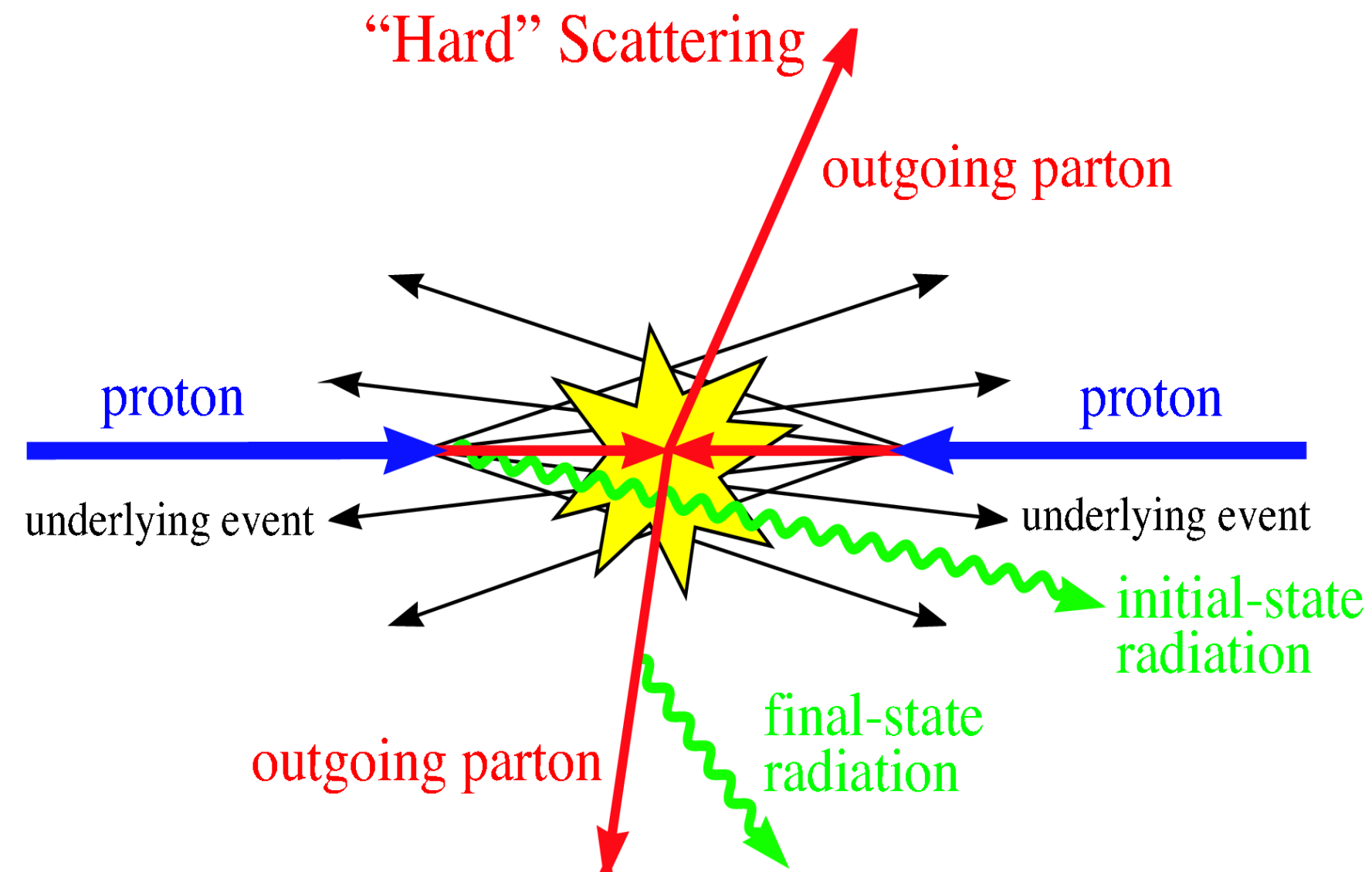
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# STATUS OF STANDARD MODEL QCD PREDICTIONS

ICHEP 2024, Prague, July 18-25 2024

# HARD COLLISIONS CAN BE DESCRIBED FROM FIRST PRINCIPLES

Studies of hadron collisions with large momentum transfer allow us to explore heaviest particles in the Standard Model and search for new particle and interactions. Such collisions are amenable to a rigorous theoretical descriptions based on first principles.



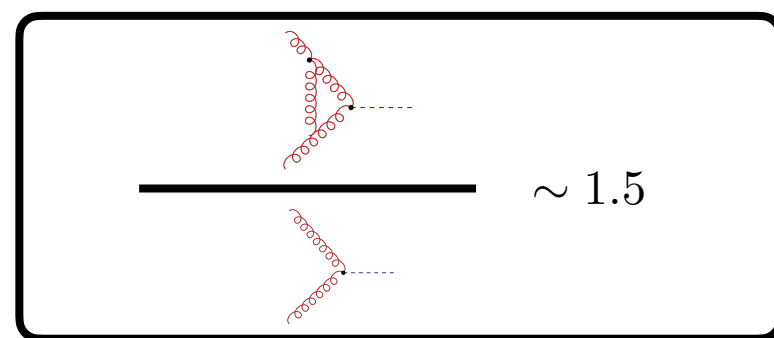
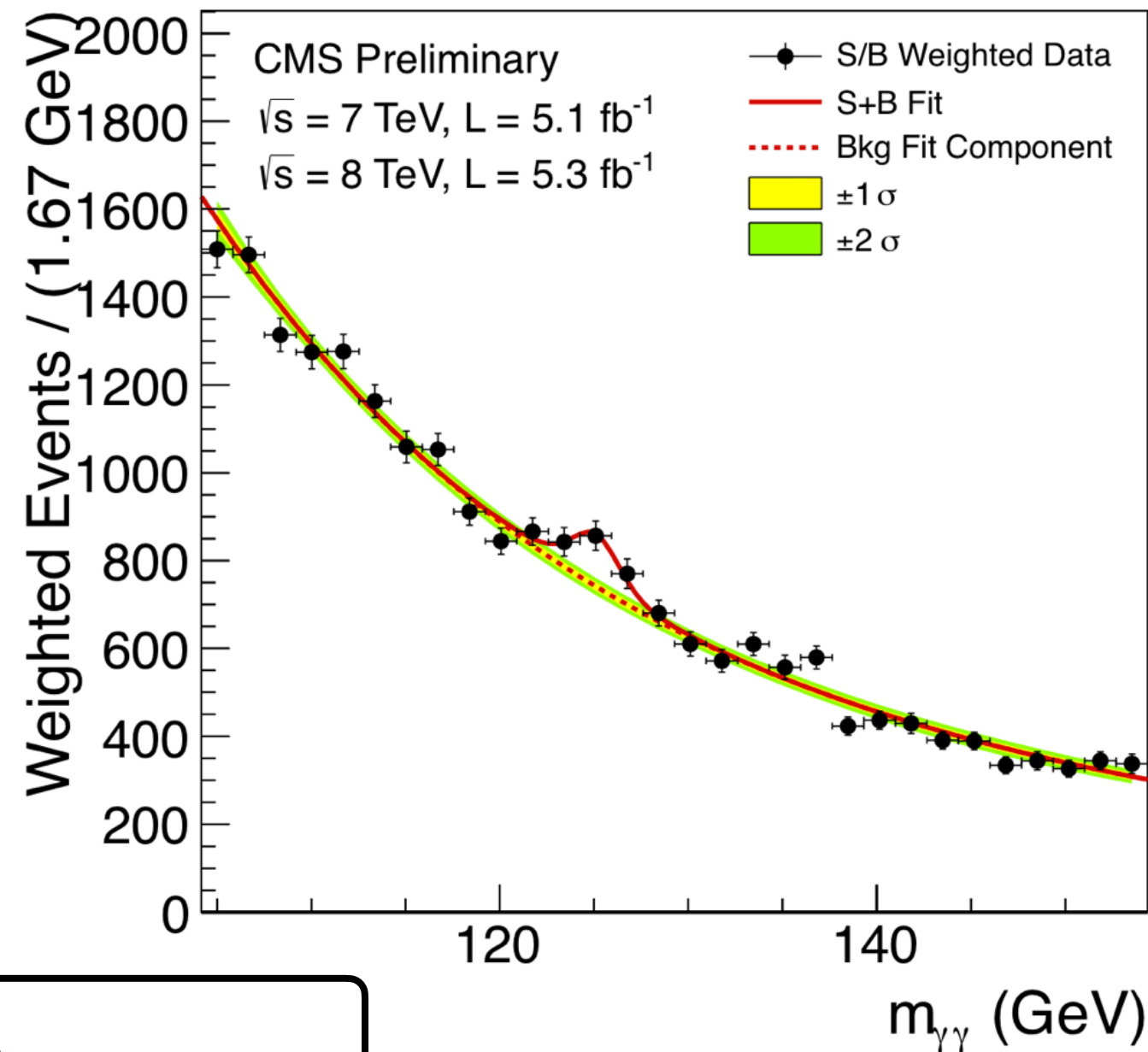
$$\mathcal{L}_{\text{QCD}} = \sum \bar{q}_j (i\hat{D} - m_j) q_j - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

$$d\sigma_{\text{hard}} = \sum_{ij \in \{q,g\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}).$$

$$d\sigma_{ij} = d\sigma_{ij,\text{LO}} (1 + \alpha_s \Delta_{ij,\text{NLO}} + \alpha_s^2 \Delta_{ij,\text{NNLO}} + \dots)$$

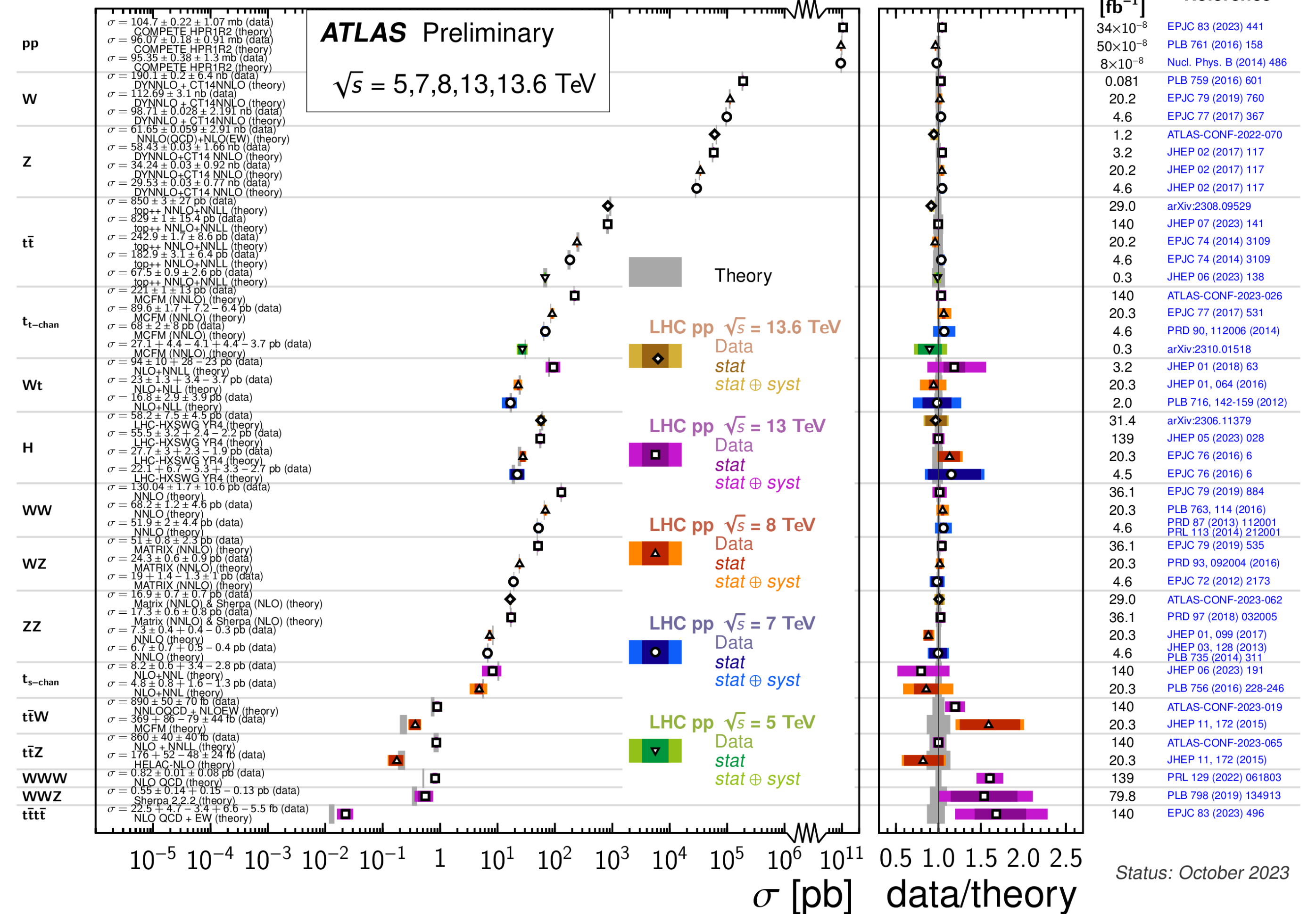
# PERTURBATIVE QCD FACILITATES INTERPRETATION OF LHC MEASUREMENTS

Perturbative QCD is very well understood by now. We use it for an unambiguous interpretation of all LHC measurements.



48.58 pb =	16.00 pb	(+32.9%)	(LO, rEFT)
	+ 20.84 pb	(+42.9%)	(NLO, rEFT)
	- 2.05 pb	(-4.2%)	((t, b, c), exact NLO)
	+ 9.56 pb	(+19.7%)	(NNLO, rEFT)
	+ 0.34 pb	(+0.7%)	(NNLO, 1/m <sub>t</sub> )
	+ 2.40 pb	(+4.9%)	(EW, QCD-EW)
	+ 1.49 pb	(+3.1%)	(N <sup>3</sup> LO, rEFT)

## Standard Model Total Production Cross Section Measurements



# PERTURBATIVE QCD AS QUANTUM FIELD THEORY

Demands of the LHC physics program have shaped the development of QCD as a perturbative Quantum Field Theory and kept QCD practitioners on their toes.

## Higher order cross sections: The first wishlist

### An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+ \leq 5j$	$WW+ \leq 5j$	$WWW+ \leq 3j$	$t\bar{t}+ \leq 3j$
$W + b\bar{b} \leq 3j$	$W + b\bar{b}+ \leq 3j$	$WWW + b\bar{b}+ \leq 3j$	$t\bar{t} + \gamma+ \leq 2j$
$W + c\bar{c} \leq 3j$	$W + c\bar{c}+ \leq 3j$	$WWW + \gamma\gamma+ \leq 3j$	$t\bar{t} + W+ \leq 2j$
$Z+ \leq 5j$	$ZZ+ \leq 5j$	$Z\gamma\gamma+ \leq 3j$	$t\bar{t} + Z+ \leq 2j$
$Z + b\bar{b}+ \leq 3j$	$Z + b\bar{b}+ \leq 3j$	$ZZZ+ \leq 3j$	$t\bar{t} + H+ \leq 2j$
$Z + c\bar{c}+ \leq 3j$	$ZZ + c\bar{c}+ \leq 3j$	$WZZ+ \leq 3j$	$t\bar{b} \leq 2j$
$\gamma+ \leq 5j$	$\gamma\gamma+ \leq 5j$	$ZZZ+ \leq 3j$	$b\bar{b}+ \leq 3j$
$\gamma + b\bar{b} \leq 3j$	$\gamma\gamma + b\bar{b} \leq 3j$		single top
$\gamma + c\bar{c} \leq 3j$	$\gamma\gamma + c\bar{c} \leq 3j$		
	$WZ+ \leq 5j$		
	$WZ + b\bar{b} \leq 3j$		
	$WZ + c\bar{c} \leq 3j$		
	$W\gamma+ \leq 3j$		
	$Z\gamma+ \leq 3j$		

circa 20 years ago, NLO calculations are requested

## Wishlist, the 2022 edition

process	known	desired
$pp \rightarrow t\bar{t}$	NNLO <sub>QCD</sub> + NLO <sub>EW</sub> (w/o decays)	N <sup>3</sup> LO <sub>QCD</sub>
	NLO <sub>QCD</sub> + NLO <sub>EW</sub> (off-shell effects) NNLO <sub>QCD</sub> (w/ decays)	
$pp \rightarrow t\bar{t} + j$	NLO <sub>QCD</sub> (off-shell effects)	NNLO <sub>QCD</sub> + NLO <sub>EW</sub> (w/o decays)
	NLO <sub>EW</sub> (w/o decays)	
$pp \rightarrow t\bar{t} + 2j$	NLO <sub>QCD</sub> (w/o decays)	NLO <sub>QCD</sub> + NLO <sub>EW</sub> (w/o decays)
$pp \rightarrow V$	N <sup>3</sup> LO <sub>QCD</sub>	N <sup>3</sup> LO <sub>QCD</sub> + N <sup>(1,1)</sup> LO <sub>QCD</sub> ⊗EW N <sup>2</sup> LO <sub>EW</sub>
	N <sup>(1,1)</sup> LO <sub>QCD</sub> ⊗EW	
	NLO <sub>EW</sub>	
$pp \rightarrow VV'$	NNLO <sub>QCD</sub> (w/o decays)	NLO <sub>QCD</sub> (gg channel, w/ massive loops) N <sup>(1,1)</sup> LO <sub>QCD</sub> ⊗EW
	+ NLO <sub>EW</sub> (gg channel)	
$pp \rightarrow V + j$	NNLO <sub>QCD</sub> + NLO <sub>EW</sub>	hadronic decays
$pp \rightarrow t\bar{t} + j$	NLO <sub>QCD</sub> + NLO <sub>EW</sub> (QCD component)	NNLO <sub>QCD</sub>
	NLO <sub>QCD</sub> + NLO <sub>EW</sub> (EW component)	
$pp \rightarrow 2 \text{ jets}$	NNLO <sub>QCD</sub>	N <sup>3</sup> LO <sub>QCD</sub> + NLO <sub>EW</sub>
	NLO <sub>QCD</sub> + NLO <sub>EW</sub>	
$pp \rightarrow 3 \text{ jets}$	NNLO <sub>QCD</sub> + NLO <sub>EW</sub>	

and many many more...

Huss, Huston, Jones, Pellen

# CHALLENGES IN DESCRIBING HARD COLLISIONS AT THE LHC

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Two very different challenges need to be addressed for improving theoretical framework that we use to describe hard hadron collisions. We need to overcome:

1) **technical problems**: develop efficient methods to describe quark and gluon collisions to higher and higher orders in QCD perturbation theory

$$d\sigma_{ij} = d\sigma_{ij,\text{LO}} \left( 1 + \alpha_s \Delta_{ij,\text{NLO}} + \alpha_s^2 \Delta_{ij,\text{NNLO}} + \dots \right)$$

2) **conceptual problems**: find **systematically-improvable** description of proton-to-partons and partons-to-hadrons transitions, which are relevant for initial and final stages of the process. **This problem can only be addressed if a better understanding of non-perturbative power  $\mathcal{O}(\Lambda_{\text{QCD}})$  corrections in collider processes is achieved.**

$$d\sigma_{\text{hard}} = \sum_{ij \in \{q,g\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}) \left( 1 + \mathcal{O}(\Lambda_{\text{QCD}}^n / Q^n) \right)$$

$$N_c \left( \frac{\alpha_s}{\pi} \right)^2 \sim \frac{\Lambda_{\text{QCD}}}{Q}, \quad \Lambda_{\text{QCD}} \sim 0.3 \text{ GeV}, \quad Q \sim 30 \text{ GeV}$$

## **PERTURBATIVE CHALLENGES**

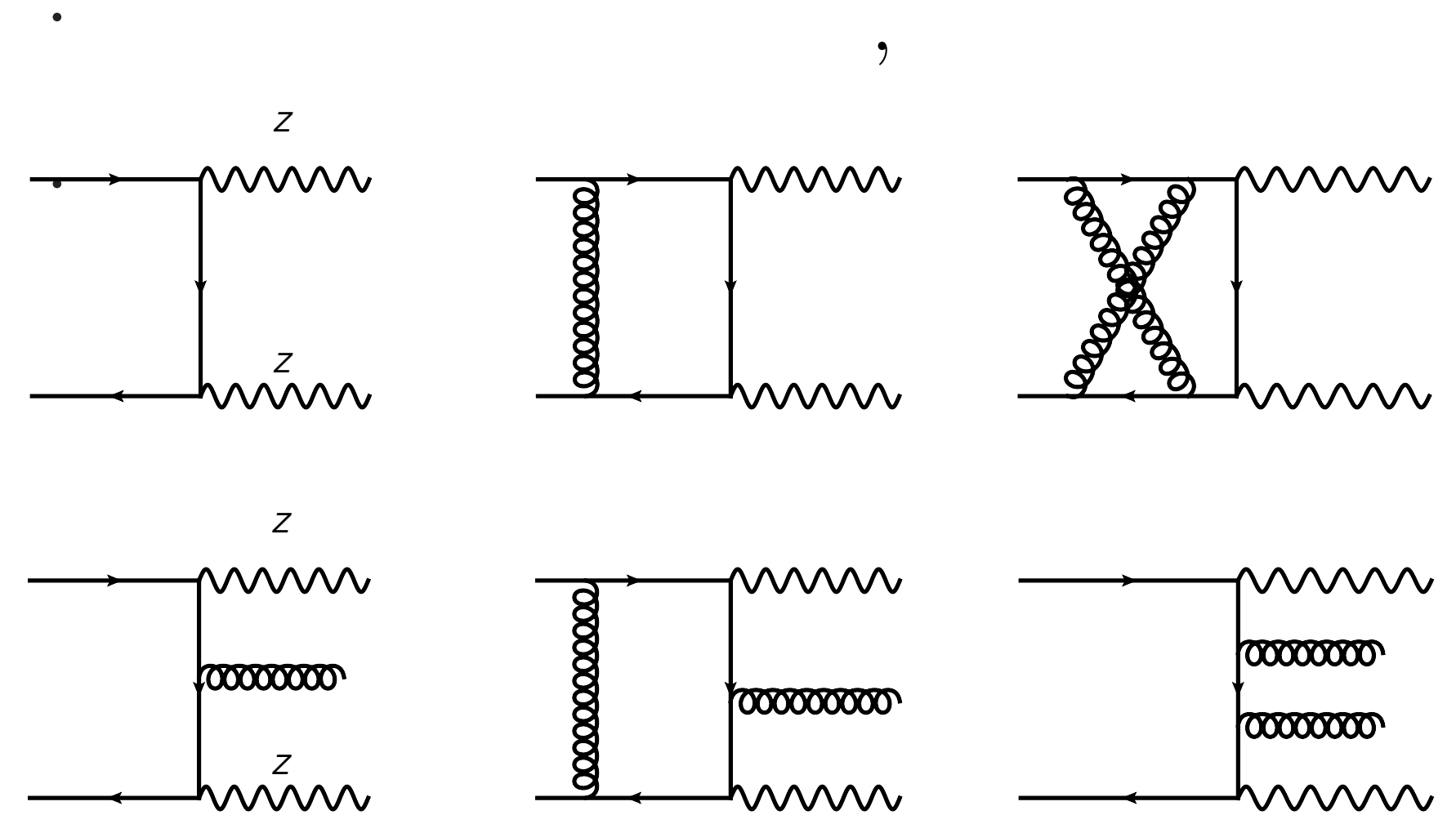
# FIXED ORDER CHALLENGES

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Any perturbative computation in higher orders of QCD requires calculation of loop amplitudes and real-emission contributions.

To perform phenomenologically-relevant computations, we need to:

- 1) figure out how to compute virtual loop amplitudes;
- 2) understand how to integrate infra-red divergent contributions over partonic phase spaces;
- 3) implement the emerging procedure into efficient numerical codes;



$$d\sigma_{ij} = d\sigma_{ij,\text{LO}} \left( 1 + \alpha_s \Delta_{ij,\text{NLO}} + \alpha_s^2 \Delta_{ij,\text{NNLO}} + \dots \right)$$

# LOOP AMPLITUDES

The problem of computing loop amplitudes is the problem of calculating divergent integrals of rational functions in Minkowski space.

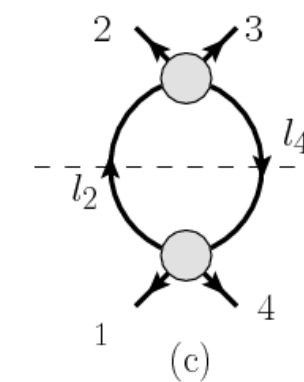
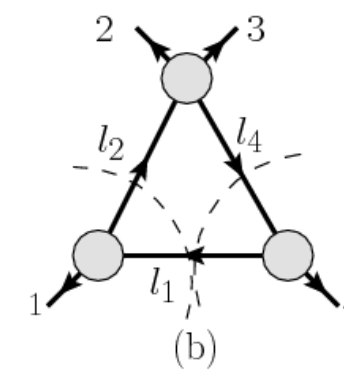
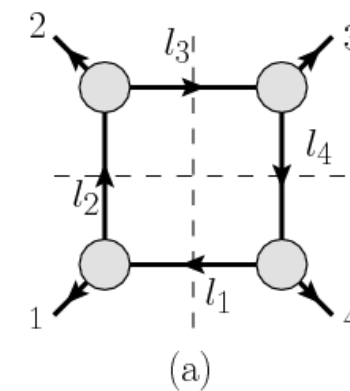
$$\int \prod \frac{d^d k_i}{(2\pi)^d} \frac{\text{Num}(\{p\}, \{k\})}{((p_1 + k_1)^2 - m_1^2)((p_2 + k_2)^2 - m_2^2) \dots}$$

Different techniques to address this problem were developed over time, from analytic to numerical.

www.edwardtufte.com

$$\sum c_n(s_{ij}, m_k^2) I_k = 0$$

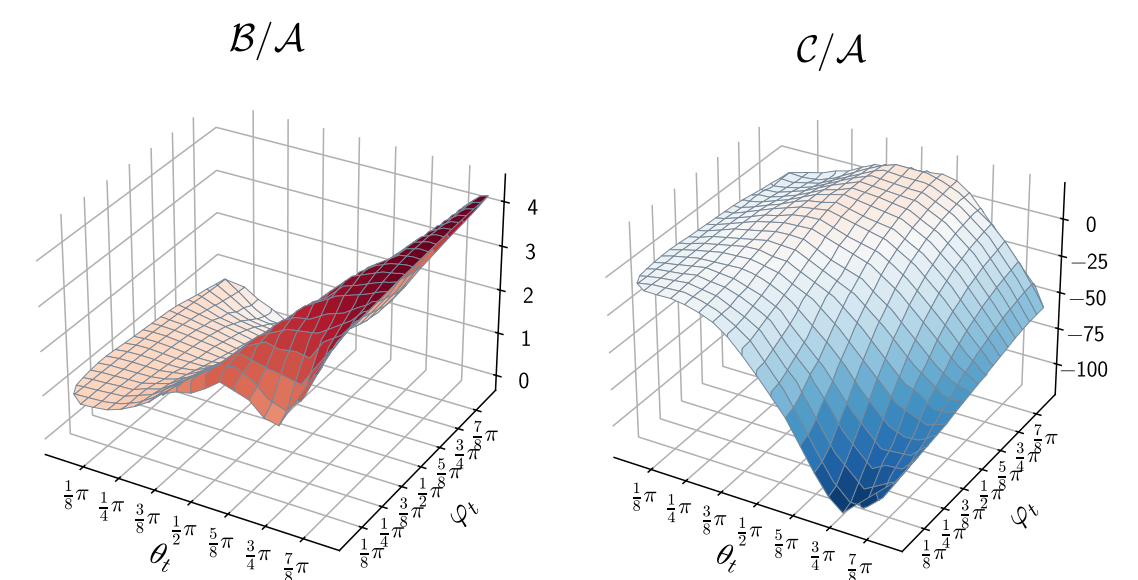
$$s_i \frac{\partial}{\partial s_i} \vec{I} = \epsilon \hat{A}(\{s\}) \vec{I}$$



Integration-by-parts

Differential equations

Generalized unitarity



$$G(\{a_n, \vec{a}_{n-1}\}, x) = \int_0^1 \frac{dt}{t - a_n} G(\{\vec{a}_{n-1}, t\})$$

$$K(x, a) = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-at^2)}}$$

Numerics: integration, solution of differential equations

Classes of functions, from Goncharov polylogarithms, to elliptic integrals.

Chetyrkin, Tkachov, Laporta, Smirnov, von Manteuffel, Lee, Maierhoefer, Usovitsch, Uwer, Abreu, Cordero, Ita, Page, Zeng; Badger, Hartano, Peraro, Sotnikov, Zola, Gehrman, Henn, Chicherin, Tancredi, Caola, Buncioni, Devoto, Chen, Czakon, Poncelet, Greiner, Heinrich, Kerner, Jones, Liu, Ma, C.Y.Wang, Moriello, Steinhauser, Schönwald, Anastasiou, Sterman, Hirschi



# INTEGRATING REAL EMISSION CONTRIBUTIONS

Real emission contributions are integrated over partonic phase spaces with the help of **subtraction and slicing methods**.

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int [|\mathcal{M}|^2 F_J - S] d\phi_4 + \int S d\phi_d$$

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int_0^\delta [|\mathcal{M}|^2 F_J d\phi_d]_{\text{simp}} + \int_\delta^1 |\mathcal{M}|^2 F_J d\phi_4 + \mathcal{O}(\delta)$$

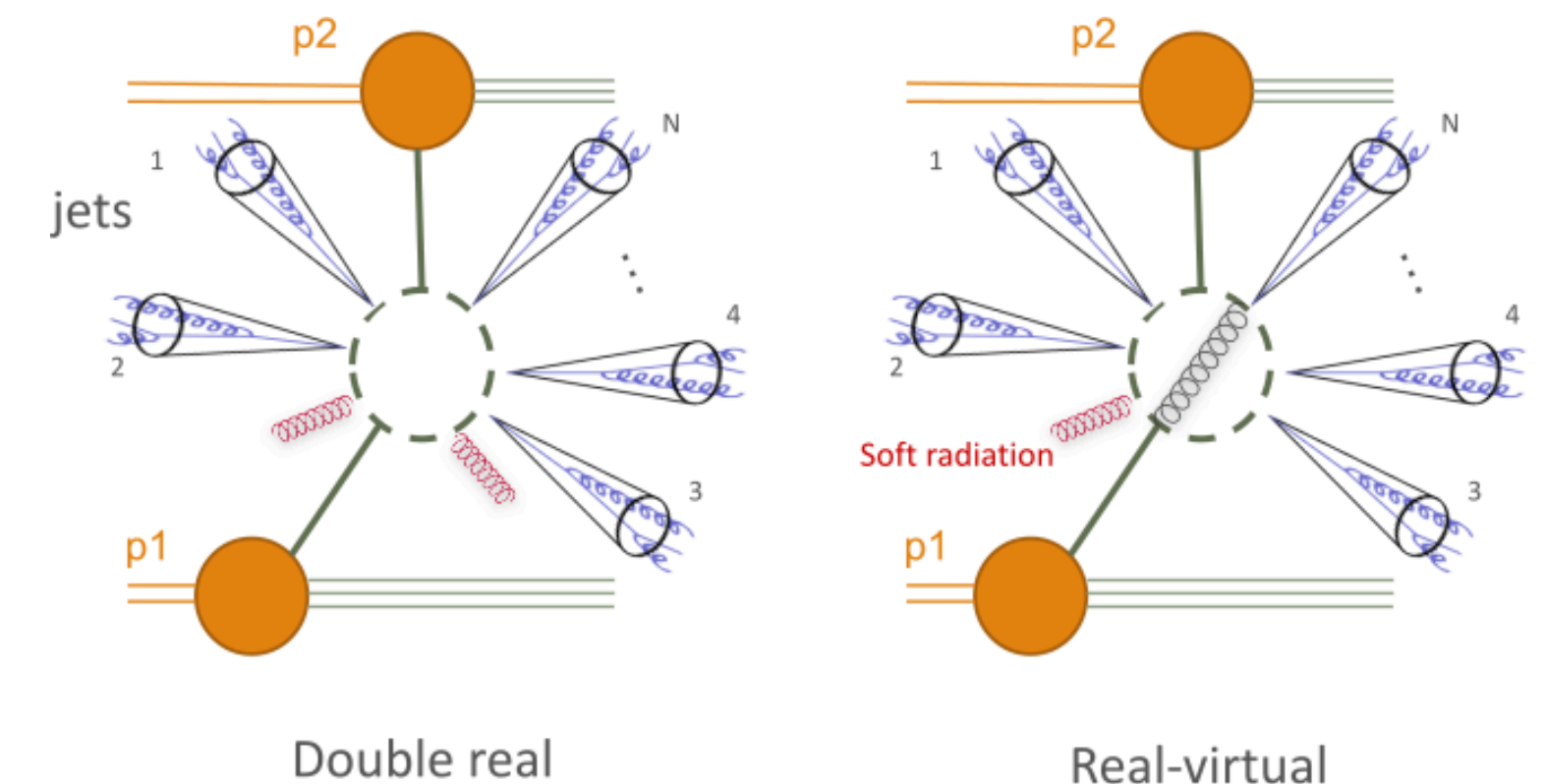
In both cases, one needs to know singular limits of amplitudes squared, and one should be able to integrate subtraction/slicing terms over singular parts of phase spaces. Integrals of subtraction and slicing terms contain infra-red divergencies that should cancel with similar divergencies in loop contributions.

Gehrmann, Glover, Czakon, Caola, Roentsch, K.M., Troscanyi, Somogyi, Del Duca, Duhr, Kardos, Magnea, Bertolotti, Pelliccioli, Uccirati, Torrielli, Signorile-Signorile, Catani, Grazzini, Boughezal, Petriello, Tackmann, Gaunt, Stahlhofner

In recent years, extensions of existing NNLO slicing and subtraction methods appeared, where such cancellations are demonstrated analytically for arbitrary collider processes.

Magnea, Bertolotti, Pelliccioli, Uccirati, Torrielli, Signorile-Signorile, Tagliabue, Devoto, Roetsch, Melnikov; Bell, Dehnadi, Mohrmann, Rahn, Pedron, Agarwal

These developments bring us one step closer to the formulation of an ultimate subtraction scheme at NNLO which will be amenable to a straightforward automation and will enable the construction of general-purpose codes, capable of computing real-emission contributions to arbitrary cross sections through NNLO.



# A HIGHLY-DEVELOPED THEORY OF PARTONIC COLLISIONS

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A highly-developed theory of partonic collisions, that can be used to describe complicated collider processes, is available.

Leading order computations are automated; it is a solved problem. [Madgraph etc.](#)

Modern NLO computations are possible for processes with up to 6 final-state particles. They incorporate electroweak corrections and are often matched to parton showers allowing one to simulate realistic events. They include realistic final states (for unstable intermediate particles) and all interferences between (resonance) signal and (non-resonance) background.

[Worek, Pozzorini, Denner etc.; OpenLoops etc.](#)

NNLO QCD computations have become available for many interesting processes. The limiting factors currently are availability of virtual loop amplitudes and the efficiency of implementation of subtraction schemes in numerical codes.

[Gehrmann, Glover, Huss, Czakon, Mitov, Poncelet, Williams, Roentsch, Caola, Catani, Grazzini](#)

The current focus is on computing two-loop loop amplitudes for processes with three (some massive) final-state particles.

$$pp \rightarrow V + jj, pp \rightarrow VV + j, pp \rightarrow t\bar{t}j, pp \rightarrow t\bar{t}H$$

First N3LO QCD computations appeared (Higgs cross section and rapidity distribution in gluon fusion, Drell-Yan cross section and rapidity distributions). Amplitudes for 2->2 partonic processes are known; current frontier are 2->2 amplitudes with one massless and one massive final-state particle.

[Anastasioiu, Duhr, Mistlberger, Gehrmann, Glover, Caola, Tancredi, Devoto, Buncioni](#)

$$pp \rightarrow jj, pp \rightarrow \gamma\gamma \quad pp \rightarrow Vj, pp \rightarrow Hj$$

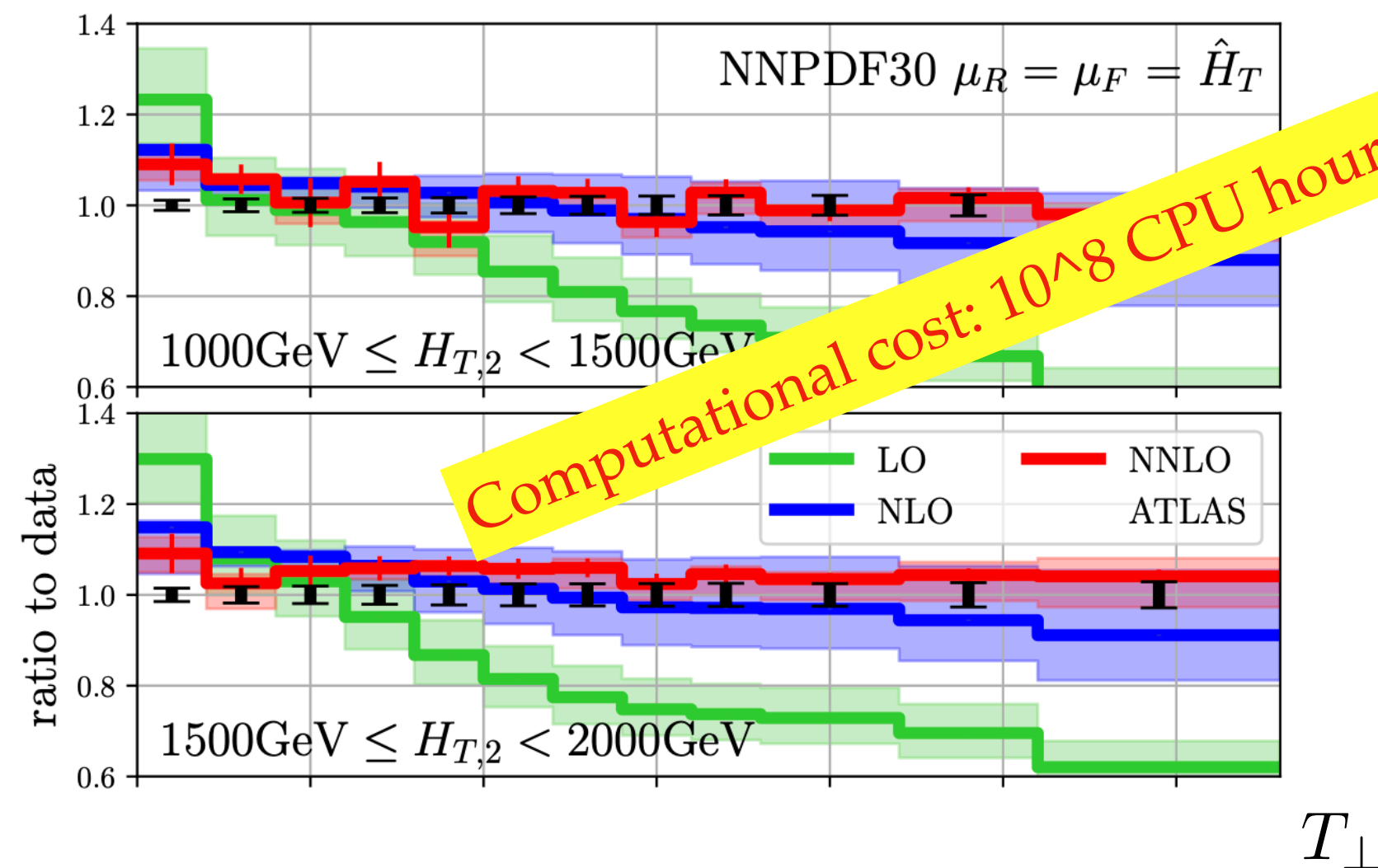
**TECHNICAL PROGRESS LEADS TO BETTER PHENOMENOLOGY**

LHC experiments can measure the running of the strong coupling constant at very high energies. A useful observable is the transverse energy-energy correlator for 3j events. NLO results for this observable were known since quite some time. Pushing them to the next level — NNLO — was an enormous adventure.

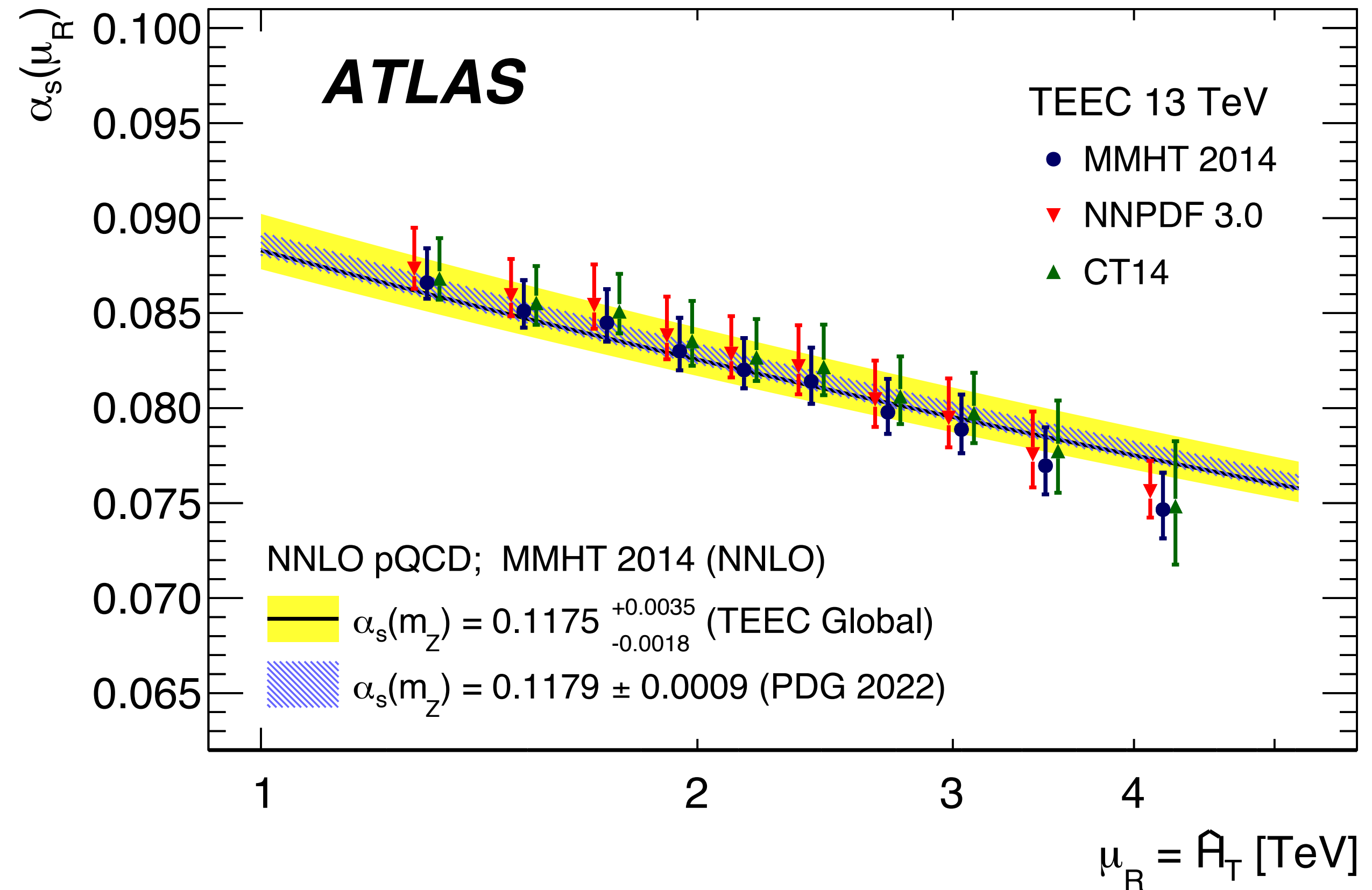


Two-loop amplitudes: Chicherin, Sotnikov, Gehrmann, Zhang, Henn, Wasser, Zola, Abreu et al.

Subtraction scheme: Czakon

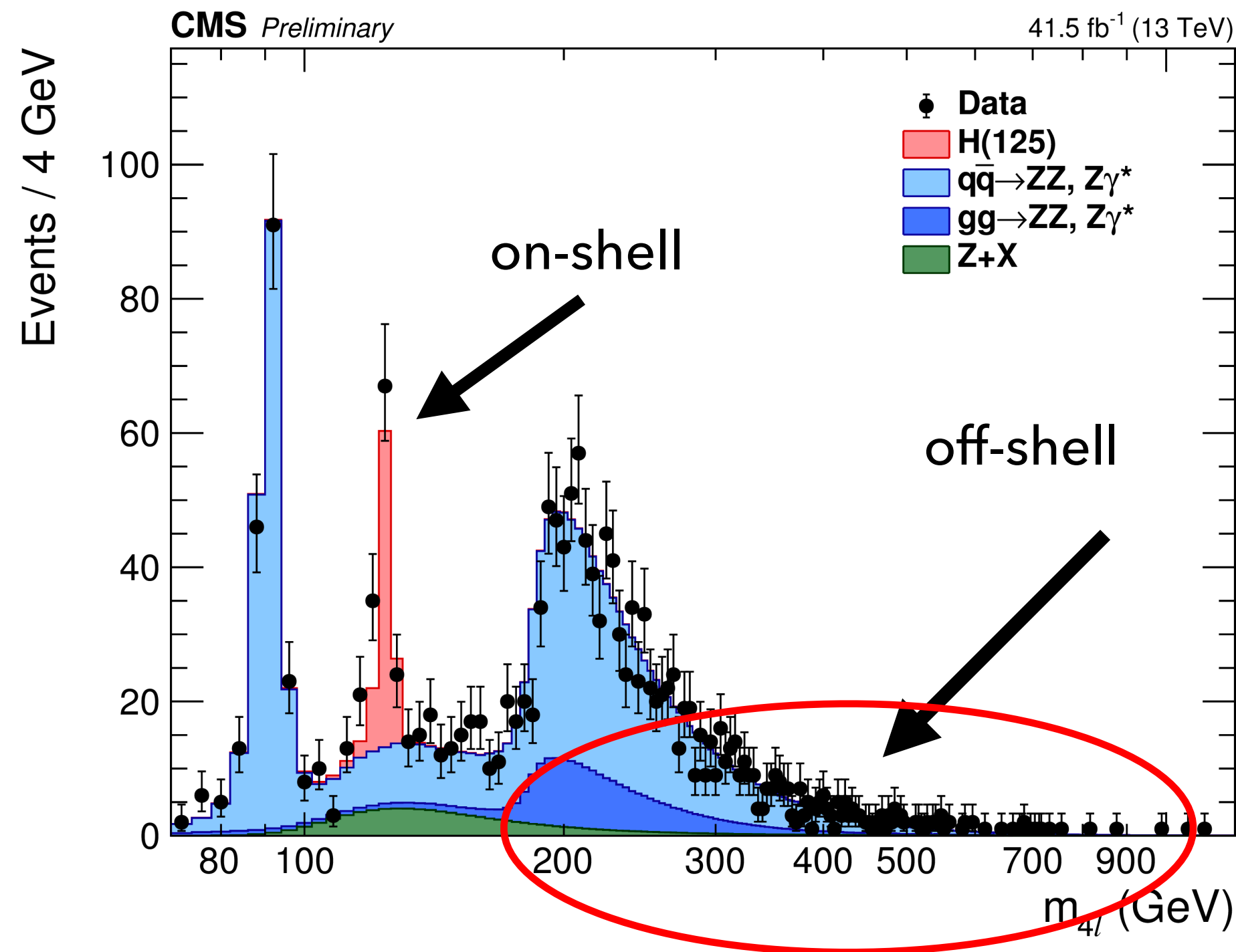


Alvarez, Cantero, Czakon, Lorente, Mitov, Poncelet

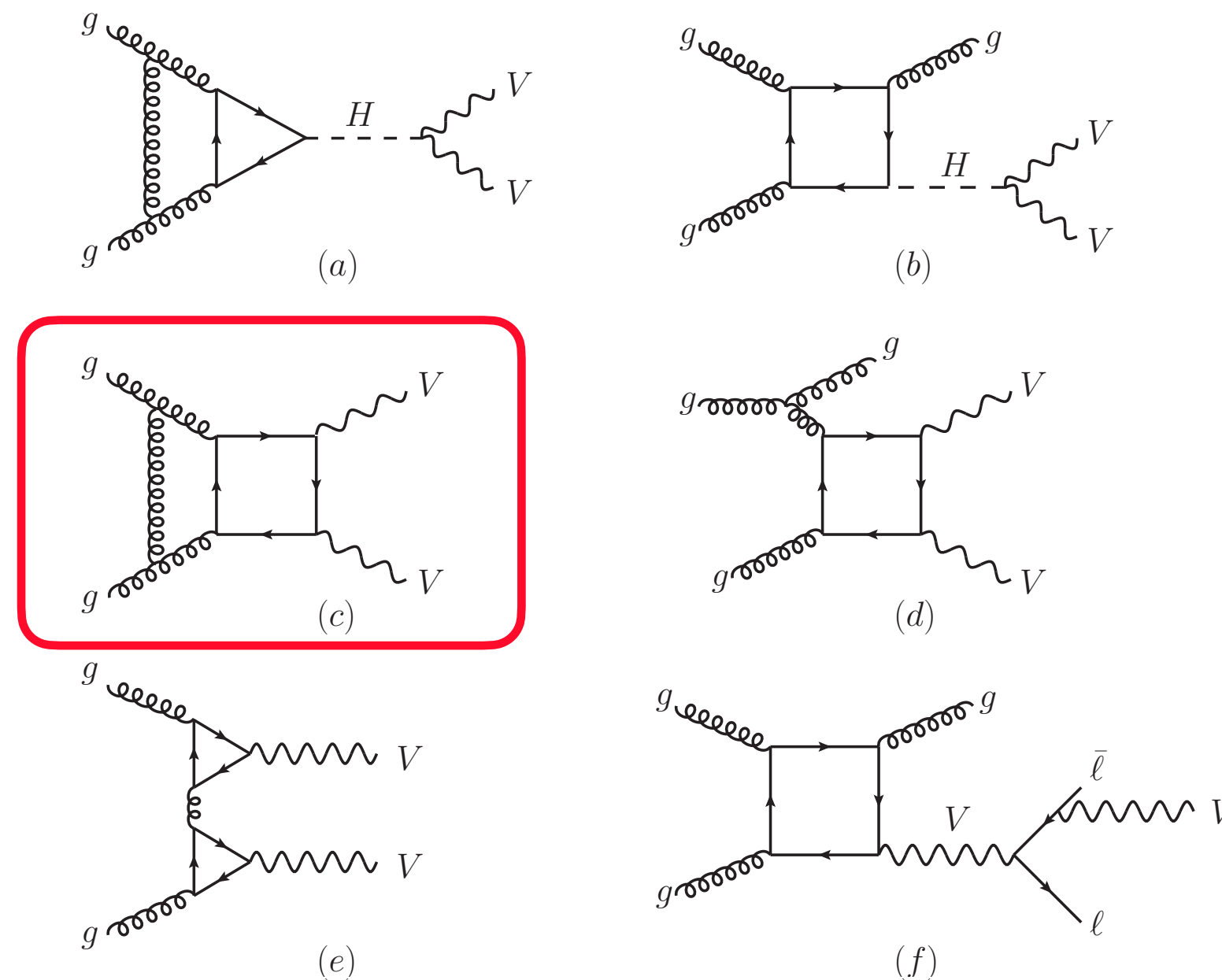


# THE HIGGS WIDTH: FULL NNLO RESULTS FOR IRREDUCIBLE BACKGROUND

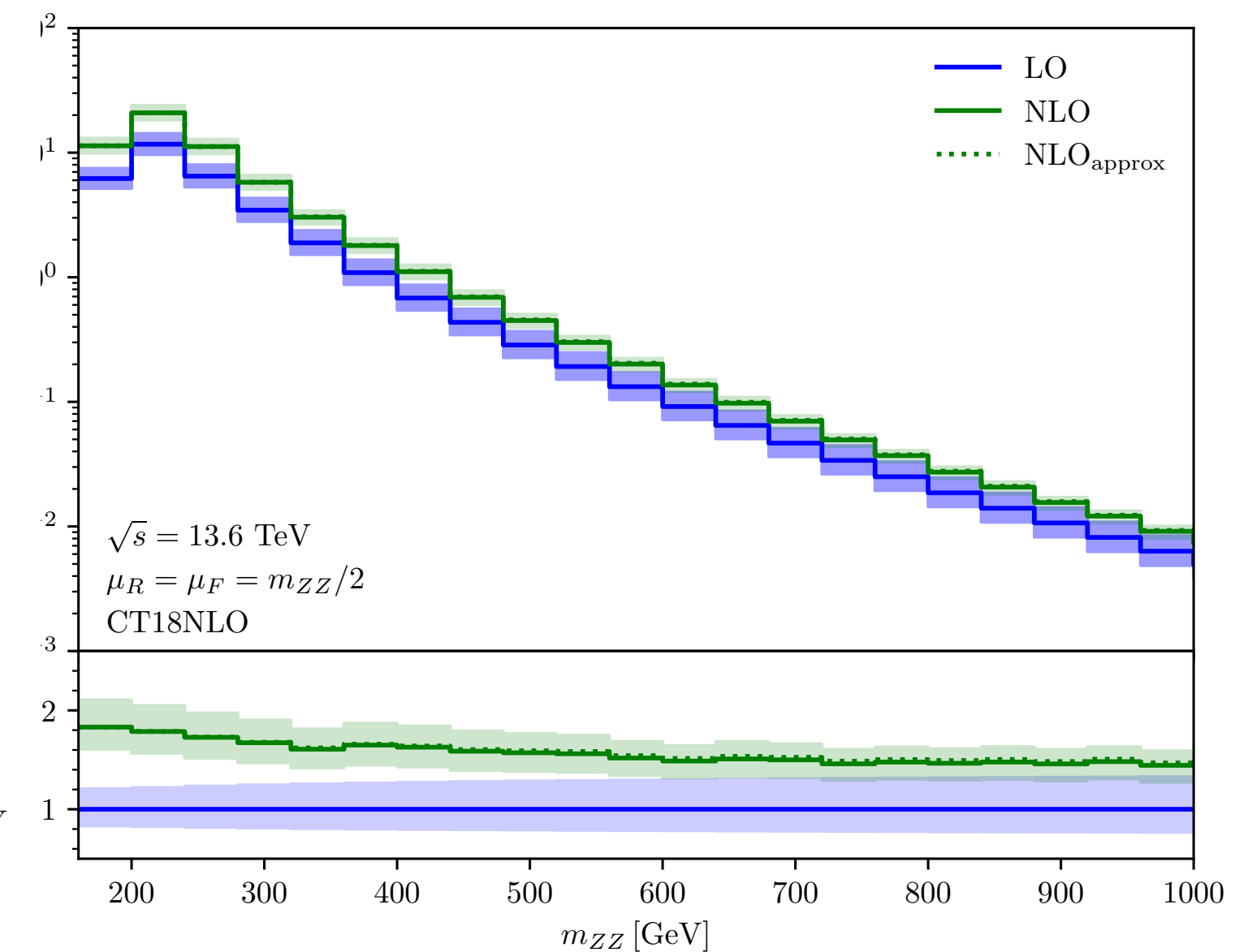
It is well-appreciated by now that one can extract the Higgs boson width from ZZ production using peculiar properties of the Higgs-boson off-shell contributions. Need to control the irreducible background; top-quark loop is (was) a challenge.



$$\Gamma_H \sim \frac{\sigma_{\text{off}}}{\sigma_{\text{on}}} \quad \Gamma_H = 3.2^{+2.8}_{-2.2} \text{ MeV}$$



Both the signal and the irreducible background are part of the same process.



**Class A<sub>h</sub>:** Both Z bosons couple directly to the same heavy top-quark loop. For these one- and two-loop contributions, we use the recent calculation [31] by some of us for which a combination of syzygy techniques [31, 36–40], finite field methods [41, 42], multivariate partial fractioning [43–47], and constructions of finite integrals were employed, and the resulting finite master integrals were evaluated numerically with pySecDec [48–50].

For a competitive measurement of the strong coupling at the LHC, one needs to find a quantity which

- 1) is proportional to the strong coupling constant;
- 2) **can be predicted theoretically with a percent precision (NNLO and higher);**
- 3) is independent (nearly independent) of poorly-known parton distribution functions;
- 4) refers to low(er) region of hard momentum region;
- 5) does not suffer from unknown non-perturbative effects.

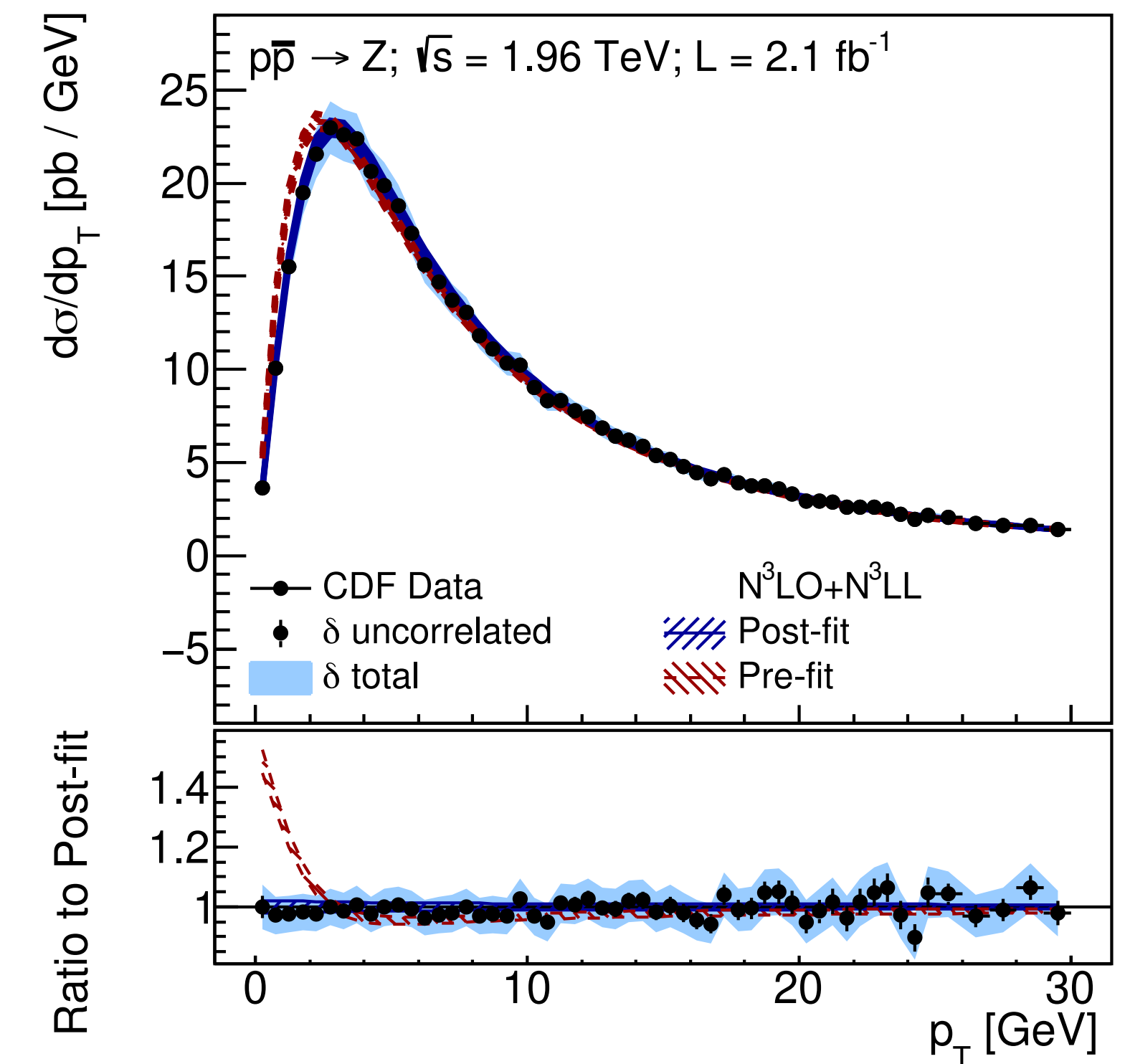
Inclusive Z transverse momentum distribution seems to fit the bill.

$$\frac{d\sigma_Z}{\sigma_Z dp_\perp} \sim \frac{\alpha_s(p_\perp)}{2\pi p_\perp} \ln \frac{M_Z}{p_\perp}$$

ATLAS followed up on the proposal and obtained a very precise value of the strong coupling constant which is very well-compatible with the world average.

$$\alpha_s(m_z) = 0.1183 \pm 0.0009$$

ATLAS, 8 TeV data



Camarada, Ferrera, Schott

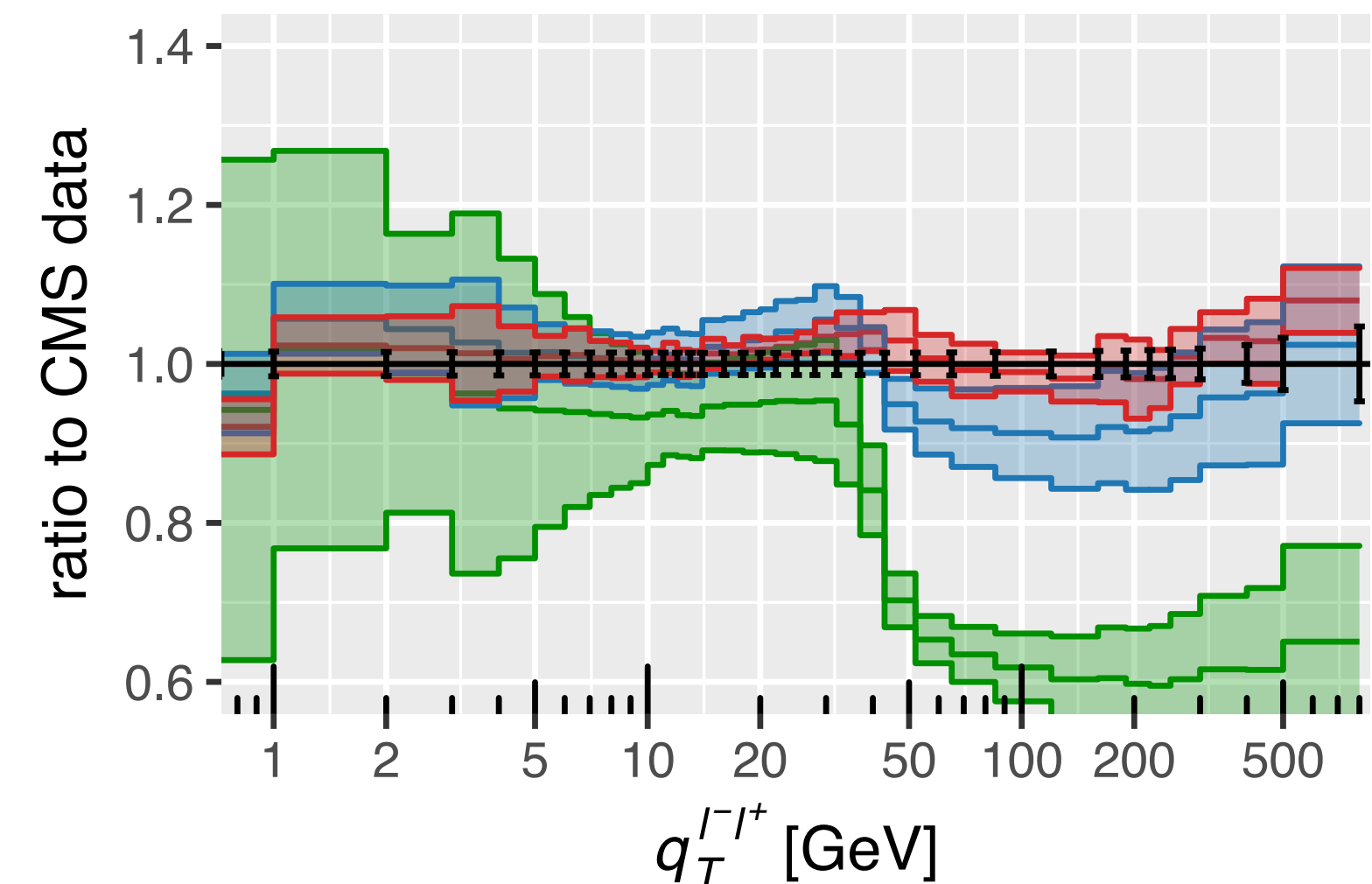
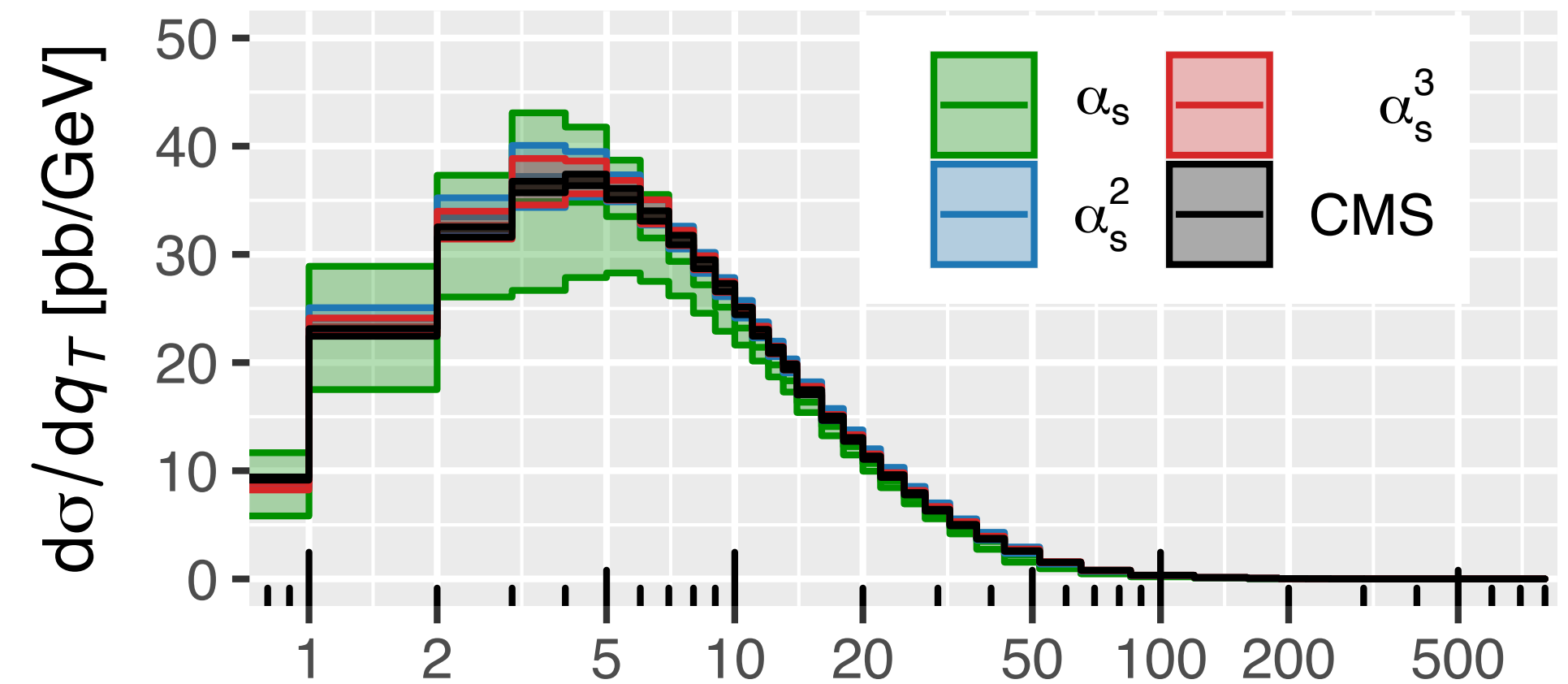
A percent-level prediction for Z transverse momentum distribution requires us to employ some of the most sophisticated theoretical computations ever performed in pQCD.

- 1) N3LO QCD predictions for the inclusive Z-boson production cross section and rapidity distribution;
- 2) NNLO QCD predictions for Z+jet production;
- 3) state-of-the-art transverse momentum resummation, that describe Z-boson transverse momentum distribution at small  $p_T$ ;
- 4) electroweak corrections to Z+jet production;
- 5) advanced knowledge of parton distribution functions;
- 6) models for non-perturbative smearing at small transverse momenta.

Duhr, Mistlberger, X. Chen, Gehrmann, Gehrmann-de Ridder, Glover, Zhu, Yang, Huss, Vita, Ebert, Luou, Boughezal, Focke, Liu, Petriello, Ellis, Giele, Campbell et al.

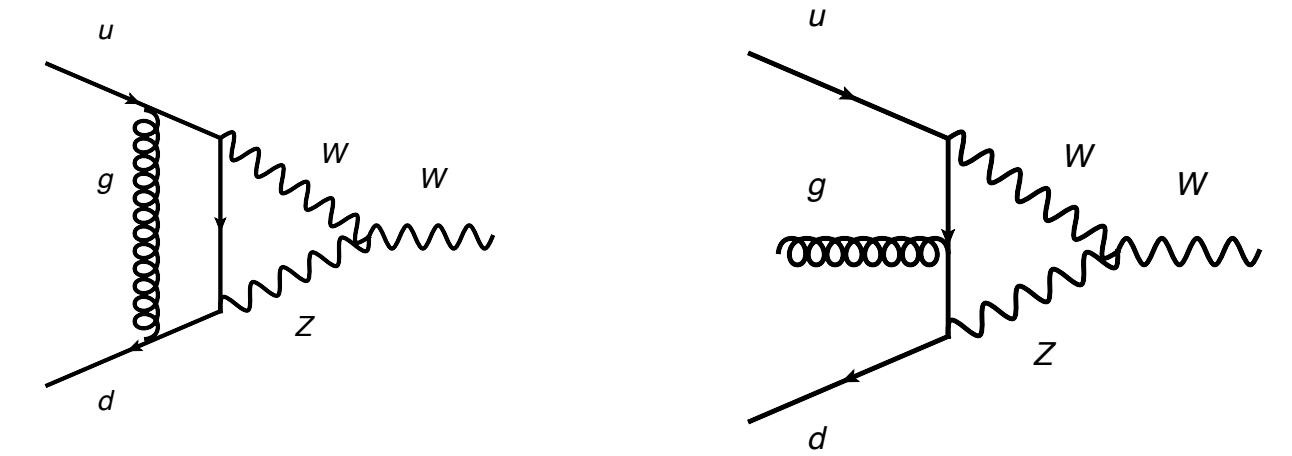
$$\alpha_s(m_z) = 0.1183 \pm 0.0009$$

ATLAS, 8 TeV data



To minimize the impact of QCD theory on the determination of the  $W$ -mass, models for vector boson production are tuned using  $Z$ -production data and then used to describe the  $W$  case. It becomes important to carefully study all effects that distinguish between  $Z$  and  $W$  production and electroweak and mixed electroweak-QCD corrections is an important example of such effects.

$$m_W^{\text{meas}} = \frac{\langle p_{\perp}^{l,W} \rangle^{\text{meas}}}{\langle p_{\perp}^{l,Z} \rangle^{\text{meas}}} m_Z C_{\text{th}}. \quad C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_{\perp}^{l,Z} \rangle^{\text{th}}}{\langle p_{\perp}^{l,W} \rangle^{\text{th}}}$$



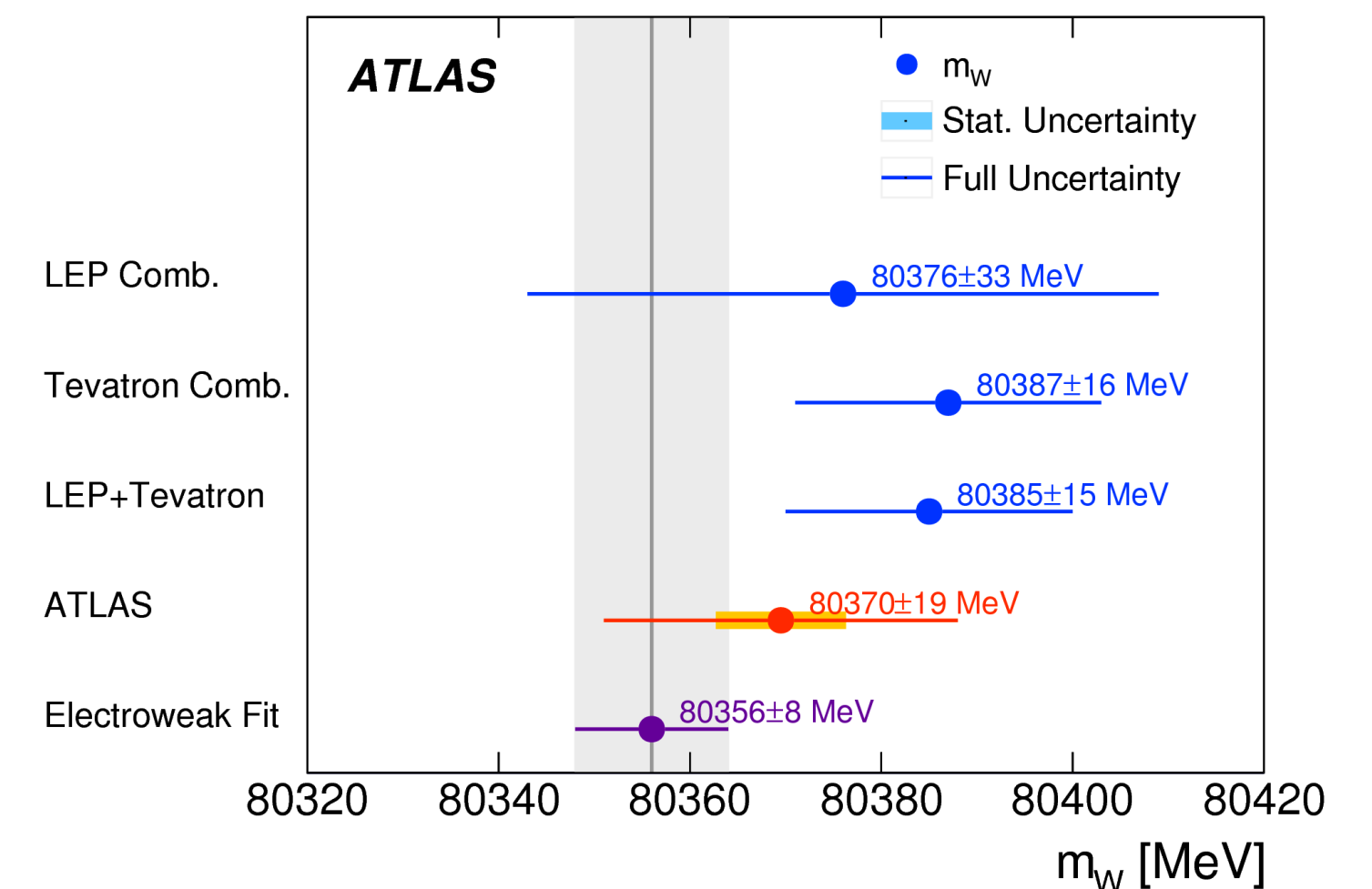
A better theory changes the theoretical correction factor and leads to changes in the extracted value of the  $W$  mass.

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_{\perp}^{l,Z} \rangle^{\text{th}}}{\langle p_{\perp}^{l,Z} \rangle^{\text{th}}} - \frac{\delta \langle p_{\perp}^{l,W} \rangle^{\text{th}}}{\langle p_{\perp}^{l,W} \rangle^{\text{th}}}$$

**No fiducial cuts:**  $\Delta m_W = m_W - m_W^{EW} = 7 \text{ MeV}$

- QCD-electroweak effects are more important than the electroweak ones;
- Compensation mechanism between  $W$  and  $Z$  distribution is important; in first moments taken separately are close to 50 MeV;
- PDF uncertainty has a very minor impact on these shifts;

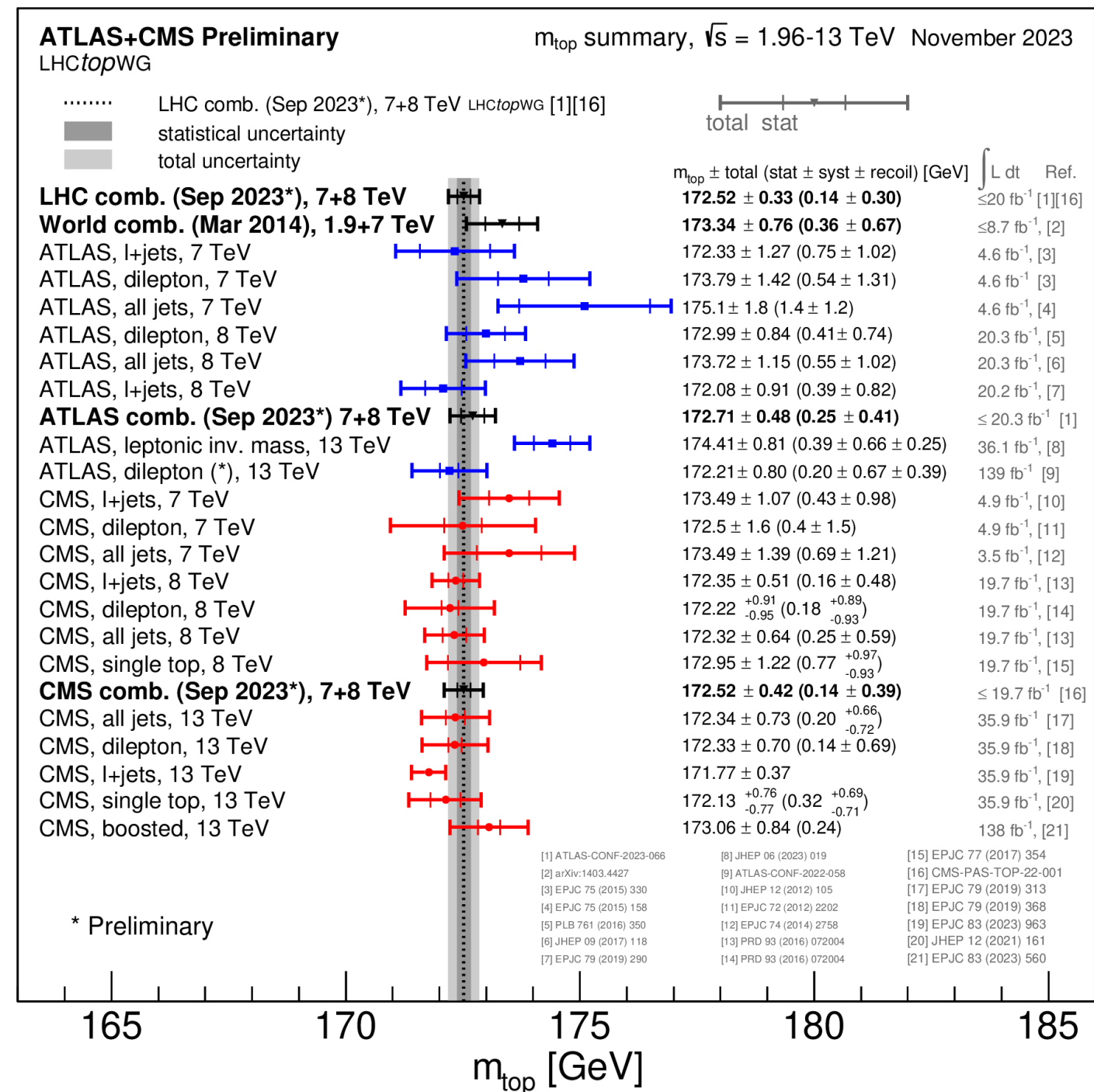
**ATLAS cuts:**  $\Delta m_W = m_W - m_W^{EW} = 17 \text{ MeV}$





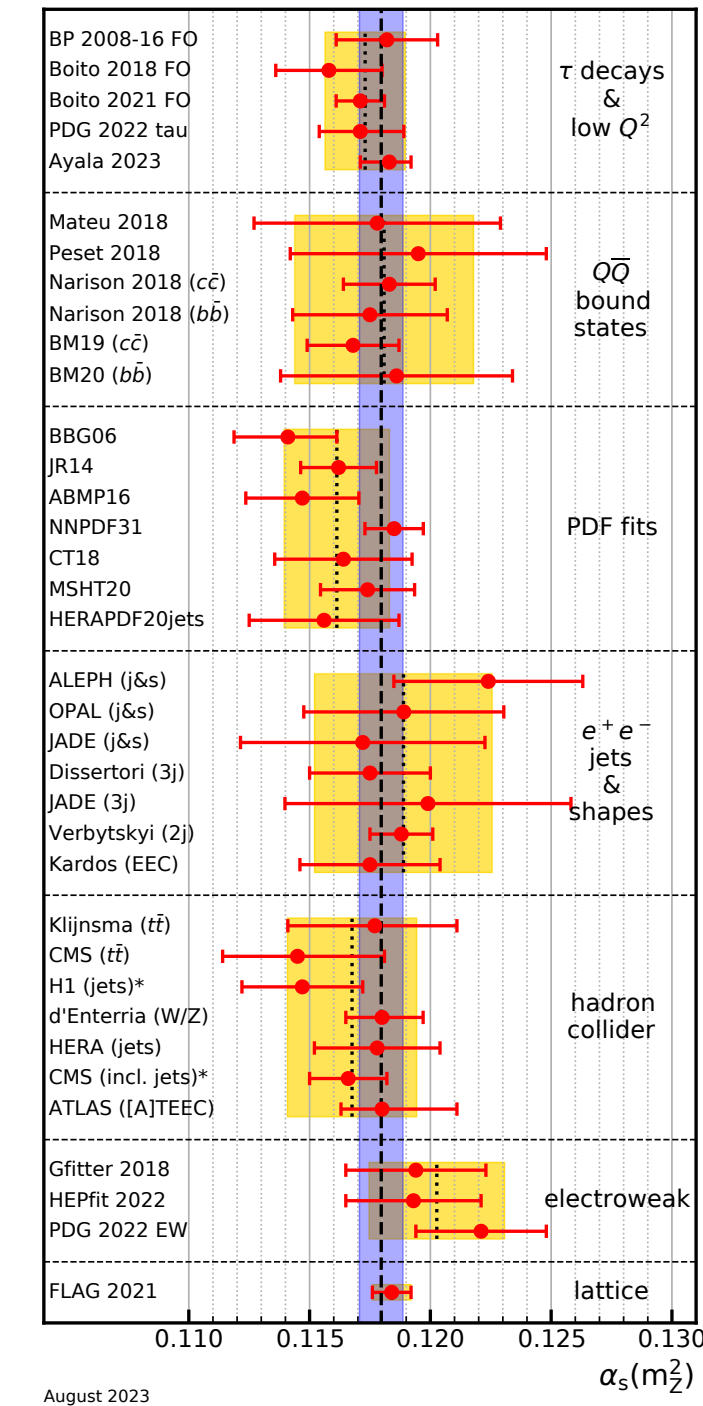
# THE CONCEPTUAL PROBLEM OF NON-PERTURBATIVE POWER CORRECTIONS

Modelling non-perturbative effects with parton showers is not satisfactory for high-precision observables and is known to cause significant confusion.



$$m_t = 172.52 \pm 0.33 \text{ GeV}$$

$$d\sigma_{\text{hard}} = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij}(x_1, x_2, \{p_{\text{fin}}\}) O_J(\{p_{\text{fin}}\}) (1 + \mathcal{O}(\Lambda_{\text{QCD}}^n/Q^n))$$



$$\alpha_s(M_Z) = 0.118 \pm 0.001$$

Can one learn something relevant about these effects from perturbation theory given all the advances that we have had in this field?

A recent discussion of inter-dependences between the perturbative evolution of parton showers and hadronization models through a shower infra-red cut-off is an interesting example of this.

Hoang, Jin, Plätzer, Samitz

Furthermore, since Feynman integrals run over all momenta, including the soft ones, one can use Feynman diagrams to estimate the sensitivity of cross sections and observables to these problematic integration regions.

The famous Kinoshita-Lee-Nauenberg infra-red cancellation, as well as the idea of [renormalons](#) and its connection to QCD with a (fake) gluon mass can be interpreted in this way.

Calculation of linear  $\mathcal{O}(\Lambda_{\text{QCD}})$  non-perturbative corrections in the context of renormalon models can be simplified using Low-Burnett-Kroll next-to-soft-emission theorem and some tricks from the perturbative toolbox.

The approach based on renormalons has its limitations but it also leads to important insights into non-perturbative effects that are listed below:

- 1) one cannot determine the pole mass of the top quark from top production cross section with a precision better than  $\mathcal{O}(\Lambda_{\text{QCD}})$ ;
- 2) even basic kinematic distributions in top-production processes receive linear power corrections independent of the top mass parameter used; these power corrections are not described by parton showers;
- 3) polarization effects in top quark production processes are affected by linear power corrections (in the narrow width approximation);
- 4) in electron-positron collisions, non-perturbative corrections to shape variables in 3-jet and 2-jet regions are different, in variance with the standard assumption that are made when fitting the strong coupling constant  $\alpha_s$ .

## SUMMARY

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Perturbative QCD is a well-developed theory whose role, in the context of the LHC physics, is to facilitate interpretation of experimental results in terms of parameters that appear in the Lagrangian of the SM or its extensions.

Continuous methodological progress in perturbative QCD allows us to describe collider processes of ever increasing complexity with higher and higher precision.

State-of-the-art calculations at next-to-leading and next-to-next-to-leading orders in perturbative QCD remain very challenging, but are becoming more and more manageable. The focus is slowly shifting towards the next perturbative order, N3LO.

These impressive successes of the perturbative approach to hadron collisions, emphasize the need of a systematic understanding of non-perturbative power corrections at hadron colliders. Without it, further meaningful improvements in ultra-precise determinations of physical parameters (the top quarks mass, the strong coupling constant etc.) may not be possible, in spite of being statistically achievable.