Binary dynamic with classical spin

Gang Chen
Niels Bohr Institute

Bjerrum-Bohr, GC, Skowronek (2302.00498)
Bjerrum-Bohr, GC, Skowronek (2309.11249)
Brandhuber, Brown, GC, Gowdy,Travaglini (2310.04405)
GC, Wang (2406.09086)
GC, Kim, Wang (2406.17658)

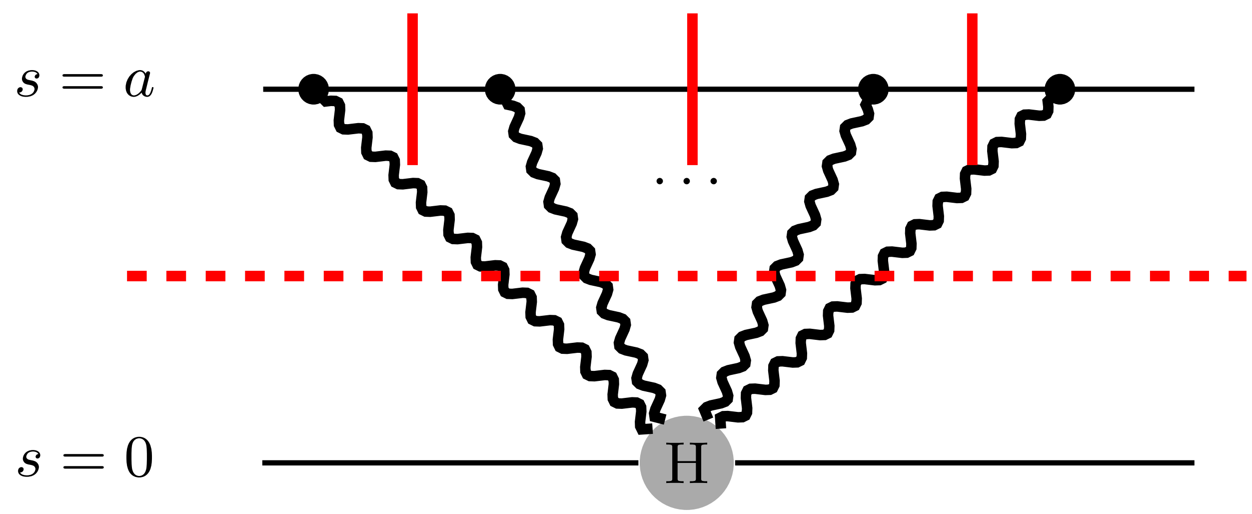
@ICHEP

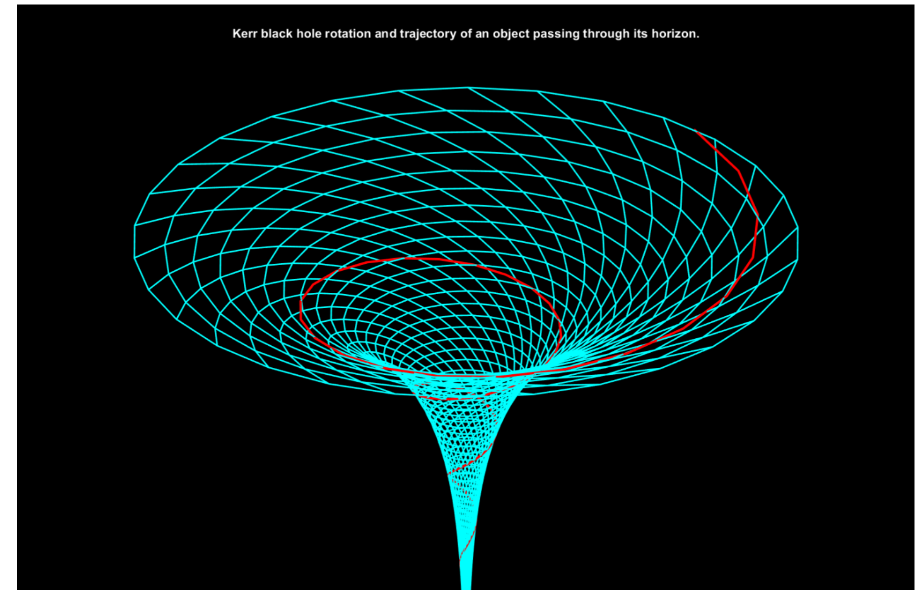
Motivation

- Understand consistent massive high-spin interactions in gauge theory and gravity theory
- Construct the generalised Compton amplitude
- Dynamics in binary Kerr black hole system

Kerr metric from three point amplitude

Arkani-Hamed, Huang, and Huang; Guevara, Ochirov, Vines
Chung, Huang, Kim, Lee





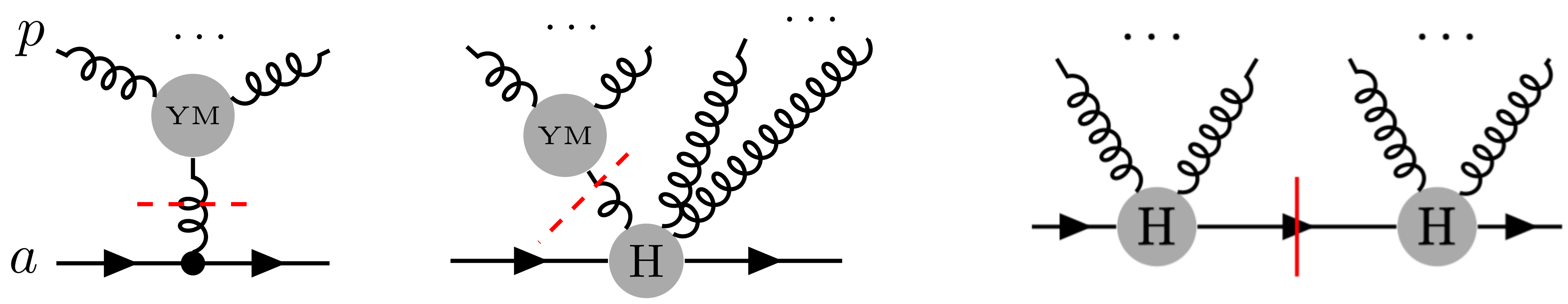
$$m\, v^\mu \exp\left(i\, \frac{p_1 \cdot S \cdot \varepsilon_1}{m\, v \cdot \varepsilon_1} \right) = m\, \cosh(p_1 \cdot a) v^\mu - i \left(\frac{\sinh(p_1 \cdot a)}{p_1 \cdot a} \right) (p_1 \cdot S)^\mu \equiv w_1 .$$

$$0 < |a| < 1$$

Primary Entire Function $G_1(x_1)$

Generalised Compton amplitude in QCD

Factorisation constraints



General primary entire function (homogeneous polynomials in the denominator)

$$G_r(x_1; x_2, \cdots, x_r) = \frac{1}{x_r}\Big(G_{r-1}(x_1{+}x_r; x_2, \cdots, x_{r-1}) - G_{r-1}(x_1; x_2, \cdots, x_{r-1})\cosh(x_r)\Big)$$

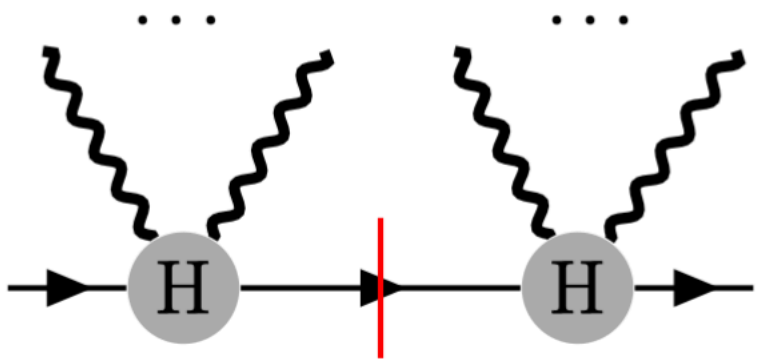
$$x_i = p_i \cdot a$$

$$G_2(x_1; x_2) \equiv \frac{1}{x_2}\Big(G_1(x_{12}) - \cosh(x_2) G_1(x_1)\Big)$$

Amplitude is unique(checked upto five point)

A new constructible theory

Generalised Compton amplitude in Gravity



Factorisation (locality)

1. Hyper-classical order ✓
2. Classical order ✗

- No solution for amplitude (using primary entire functions)
- No other primary entire function

Locality needs descendant entire functions

$$G^{r_1,\cdots,r_j}(x_1,\ldots,x_j) \equiv \Big(\prod_{i=1}^{j} \partial_{x_i}^{r_i}\Big) G_j(x_1;x_2,\ldots,x_j)$$

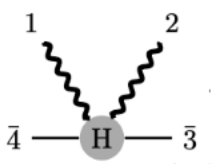
Example: In-homogeneous

$$\partial_{x_1} G_2(x_1;x_2) = \frac{\sinh(x_1)\sinh(x_2)}{x_2(x_1+x_2)} - \frac{\cosh(x_1)\cosh(x_2)}{x_1(x_1+x_2)} + \frac{\sinh(x_1)\cosh(x_2)}{x_1^2(x_1+x_2)} + \frac{\sinh(x_1)\cosh(x_2)}{x_1(x_1+x_2)^2} - \frac{\sinh(x_2)\cosh(x_1)}{x_2(x_1+x_2)^2}$$

Example:Gravitational Compton amplitude

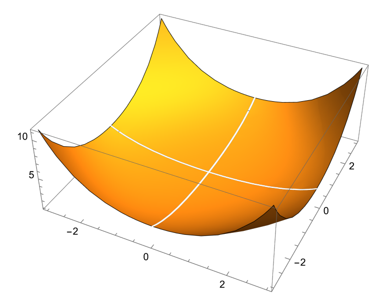
Compton amplitude

$$M(1,2,3,4) = -\frac{\mathcal{N}_a(1,2,3,4)\,\mathcal{N}_0(1,2,4,3)}{2(p_1\cdot p_2)} + \frac{\mathcal{N}_{\rm r}(1,2,3,4)}{4(\overline{p}_4\cdot p_1)(\overline{p}_4\cdot p_2)} + \mathcal{N}_{\rm c}(1,2,3,4),$$

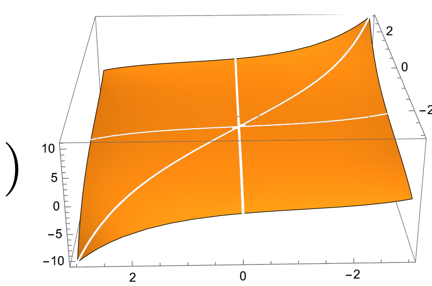


(1) (2) (3)

(1)Double copy part: $G_1(x_1)G_1(x_2)$



$G_2(x_1;x_2)$



(2)Spin flip effect: $(\partial_{x_1}-\partial_{x_2})\Big(G_1(x_1)G_1(x_2)\Big)$ $(\partial_{x_1}-\partial_{x_2})\Big(G_2(x_1,x_2)\Big)$

Fixed from factorisation behaviour in Chen, Chung, Huang,Kim(2022)

(3)Contact term: $(\partial_{x_1}-\partial_{x_2})^2\Big(G_1(x_1)G_1(x_2)\Big)$ $(\partial_{x_1}-\partial_{x_2})^2\Big(G_2(x_1,x_2)\Big)$

Bautista,Guevara, Kavanagh,Vines 2022

Fixed from far-zone of Teukolsky solution at spin fifth order

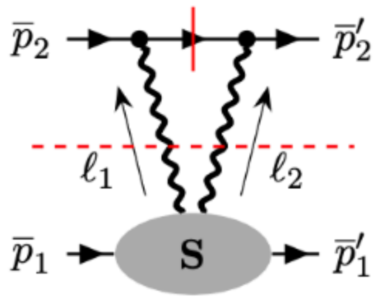
Bautista,Bonelli,Iossa, Tanzini,Zhou2023

Result consistent with Teukolsky solution up to spin eighth order

Binary Dynamics

2PM bending angle from amplitudes

Spinning waveform
see Gab's talk



$$\mathcal{M}^{(1)}_{a_1 a_2} = \frac{1}{2}\sum_{h_i=\pm}\frac{(32\pi G)^2}{(4\pi)^{D/2}}\int\frac{d^D\ell_1}{\pi^{D/2}}\frac{\delta(m_2\, v_2\cdot\ell_1)}{\ell_1^2\ell_2^2} \times \Big(\mathcal{M}_3^{-h_1}(-\ell_1, v_2)\mathcal{M}_3^{-h_2}(-\ell_2, v_2)\mathcal{M}_4^{h_1 h_2}(\ell_1, \ell_2, v_1)\Big).$$

UV divergent

$e^{\ell\cdot a}$ Λ^{∞}

Regularisation

$a_j^{\mu} = i\tilde{a}_j^{\mu}$

Revised UV divergent

Same as spinless fields

- Regular under dimensional regularisation and spin rotation
- The 2PM bending angle is divergent, if adding contact terms like $(\partial_{x_1}+\partial_{x_2})^2 G_2(x_1; x_2)$ $(\partial_{x_1}-\partial_{x_2})^3 G_2(x_1, x_2)$

Binary Dynamics

Integral in 2PM eikonal phase

$$\mathcal{I}^{(\alpha)}[\mathbf{y}] = \int dt e^{it} \tilde{\mathcal{I}}^{(\alpha)}[t, \mathbf{y}]\,,$$

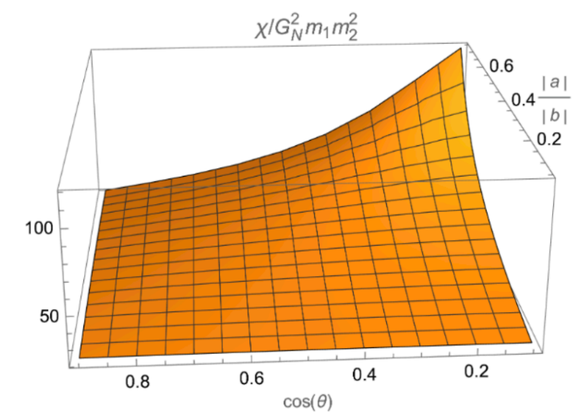
$$\mathcal{I}^{(\alpha)}[\mathbf{y}] := \int \prod_{j=1}^{L} d^D K_j \frac{e^{i(\sum_{j=1}^{L} \alpha_j \cdot K_j)} \left(\prod_{k=r+1}^{n} \delta^{\lambda_k - 1}(\mathcal{D}_k) \right)}{\mathcal{D}_1^{\lambda_1} \cdots \mathcal{D}_r^{\lambda_r}}\,. \rightarrow \int \prod_{j=1}^{L} d^D K_j \frac{\delta(\sum_{j=1}^{L} \alpha_j \cdot K_j - t) \left(\prod_{k=r+1}^{s} \delta^{\lambda_k - 1}(\mathcal{D}_k) \right)}{\mathcal{D}_1^{\lambda_1} \cdots \mathcal{D}_r^{\lambda_r}}\,.$$

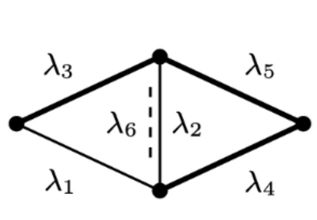
Separation of variables

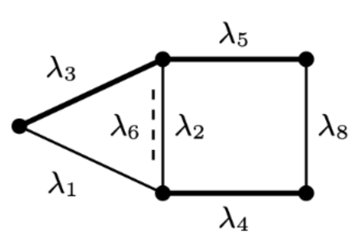
$$\tilde{\mathcal{I}}_i^{(\alpha)}[t, \mathbf{y}] = \sum_{j=1}^{n} f_{ij}(t)\, \widehat{\mathcal{I}}_j^{(\alpha)}[\mathbf{y}]\,,$$

Closed form of eikonal phase (exact finite spin dependent)

$$\sum_{\alpha=1}^{320} \left(c_{1,\alpha}(\mathbf{y}) \widehat{\mathcal{I}}_1^{(\alpha)}[\mathbf{y}] + c_{2,\alpha}(\mathbf{y}) \widehat{\mathcal{I}}_2^{(\alpha)}[\mathbf{y}] + c_{3,\alpha}(\mathbf{y}) \widehat{\mathcal{I}}_3^{(\alpha)}[\mathbf{y}] \right)$$







$$\widehat{\mathcal{I}}_1[\mathbf{y}] = \frac{C}{\sqrt{-\hat{y}_3}\sqrt{y_1^2 - 1}} \frac{K(\hat{y}_2')}{\pi}\,,$$

$$\widehat{\mathcal{I}}_2[\mathbf{y}] = -\frac{C\sqrt{-\hat{y}_3}}{\sqrt{y_1^2 - 1}} \frac{(\hat{y}_2' - 1)K(\hat{y}_2') + E(\hat{y}_2')}{\pi}$$

$$\widehat{\mathcal{I}}_3[\mathbf{y}] = \frac{C\sqrt{-\hat{y}_3}}{\sqrt{y_1^2 - 1}} \Bigg[\frac{\sqrt{\hat{y}_2' + \hat{y}_4^2 - 2\hat{y}_4}}{\pi\sqrt{\hat{y}_2' - 2\hat{y}_4}} \Bigg(\frac{\pi - 2K(\hat{y}_2')}{2} - (E(\hat{y}_2') - K(\hat{y}_2'))\int_0^{\hat{y}_4} \frac{dz}{\mathcal{Y}} - K(\hat{y}_2')\int_0^{\hat{y}_4} \frac{z\,dz}{\mathcal{Y}} \Bigg) + \frac{E(\hat{y}_2') + (1 - \hat{y}_4)\, K(\hat{y}_2')}{\pi} \Bigg]\,,$$

Incomplete elliptic function

$$\mathcal{Y}^2 = (\hat{y}_2' - 2z)(z^2 - 2z + \hat{y}_2')$$

Conclusion

- Classical spin theory for Kerr black hole
  A NICE EFT
- Prediction on physical observables
  can be detected in future(LISA et al)

Short term

- Develop spinning HEFT theory and
  Construct general Compton amplitude
- 3PM bending angle (resummed spin)
- One loop waveform (scalar and resummed spin)
- Two loop waveform (scalar)

Long term

- Infinity spin amplitude and Hawking Radiation

"Thanks!"