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Reduction of Couplings in the 2HDM

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Reduction of Couplings Basics

An RGI expression among couplings

$$\mathcal{F}(g_1, ..., g_A) = 0$$

must satisfy the pde

$$\mu \frac{d\mathcal{F}}{d\mu} = \sum_{a=1}^{A} \beta_a \frac{\partial \mathcal{F}}{\partial g_a} = 0$$

Assumption: there are $A - 1$ independent $\mathcal{F}s$ among A couplings.

Finding them is equivalent to solve the ode

$$\beta_g \left(\frac{dg_a}{dg} \right) = \beta_a, \qquad a = 1, ..., A - 1$$

where g is considered the *primary coupling*.

The above equations are called *reduction equations (RE)*.

Zimmermann (1985)

However, the general solutions of the REs have integration constants.

$\rightarrow$ We just traded an integration constant for each coupling $\rightarrow$ we have not reduced the freedom of the parameter space.

$\rightarrow$ Assume power series solutions to the REs (which preserve perturbative renormalizability):

$$g_a = \sum_n \rho_a^{(n)}\, g^{2n(+1)}$$

- Examining in one-loop sufficient for uniqueness to all loops

Oehme, Sibold, Zimmermann (1984); (1985)

For some models this *complete reduction* can prove to be too restrictive $\rightarrow$ use fewer $\mathcal{F}s$ as RGI constraints (*partial reduction*).

RoC Applications

- Standard Model
  - → $m_t \sim 98$ GeV
  - → $m_h \sim 65$ GeV — *Kubo, Sibold, Zimmermann (1984); (1985)*
- Non-Supersymmetric SM Extensions
  - Two Higgs Doublet Models
  - Three Higgs Doublet Models
  - SM + Vector-like Quarks
  - SM + Asymptotically Safe Gravity
- Supersymmetric SM Extensions
  - Reduced MSSM — *Mondragon, Tracas, Zoupanos (2014)*
- Finite Unified Theories
  - Reduced Minimal $N = 1$ $SU(5)$ — *Kubo, Mondragon, Zoupanos (1994)*
  - All-loop Finite $N = 1$ $SU(5)$ — *Heinemeyer, Mondragon, Zoupanos (2008)*
  - Two-loop Finite $N = 1$ $SU(3)^3$ — *Ma, Mondragon, Zoupanos (2004)*

Heinemeyer, Mondragon, GP, Tracas, Zoupanos (2020)

Heinemeyer, Kalinowski, Kotlarski, Mondragon, GP, Tracas, Zoupanos (2021)

The 2HDM

$$V_h = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)$$
$$+ \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right)$$
$$+ \left[\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right]$$

$\rightarrow$ We choose all parameters to be real

$\rightarrow$ We choose to work with the Type-II scenario ($u_R^i \rightarrow \Phi_2,\ \ d_R^i, e_R^i \rightarrow \Phi_1$):

- $\lambda_6 = \lambda_7$ to avoid tree-level FCNCs

and we need

- $\lambda_4 < 0$ to conserve the electric charge and
- $\lambda_1 > 0,\ \lambda_2 > 0$ and $\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0$ for the potential to be bounded from below.

A first Attempt

→ Reduction on dimensionless parameters, m_{11}^2, m_{22}^2 and m_{12}^2 remain free, will be fixed to give m_A

→ Partial *1-loop* reduction on the g_3 - y_t - λ_i space, g_3 is the primary coupling

→ g_2, g_1 *switched off*, will be added as corrections to the reduction process

$$\beta_3 \equiv \mathcal{D}g_3 = -7g_3^3 \quad , \quad \beta_2 \equiv \mathcal{D}g_2 = -3g_2^3 \quad , \quad \beta_1 \equiv \mathcal{D}g_1 = 7g_1^3$$

$$\beta_t = \beta_{t_0} + \beta_{t_c} = \left(\frac{9}{2}y_t^2 - 8g_3^2\right)y_t + \left(-\frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right)y_t$$

$$\beta_{\lambda_i} = \beta_{\lambda_{i0}} + \beta_{\lambda_{ic}}$$

$$\beta_{\lambda_1} = 12\lambda_1^2 + 4\lambda_3^3 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_6^2 + \frac{3}{4}(3g_2^4 + g_1^4 + 2g_2^2g_1^2) - 3\lambda_1(3g_2^2 + g_1^2)$$

$$\beta_{\lambda_2} = 12\lambda_2^2 + 4\lambda_3^3 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_7^2 + 12\lambda_2\lambda_t^2 - 12\lambda_t^4 + \frac{3}{4}(3g_2^4 + g_1^4 + 2g_2^2g_1^2) - \dots$$

$$\beta_{\lambda_3} = \dots$$

where $\mathcal{D} = 16\pi^2\mu(d/d\mu)$, $\lambda_t = y_t \sin\beta$ and y_b, y_τ are considered negligible

Reducing the parameters w.r.t. g_3, the power series solutions are:

$$y_t = p_t g_3 \quad , \quad \lambda_i = p_i g_3^2$$

Substituting the solutions into the REs:

$$\beta_3 \frac{dy_t}{dg_3} = \beta_{t_0} \quad , \quad \beta_3 \frac{d\lambda_i}{dg_3} = \beta_{\lambda_{i0}}$$

we get sets of p_i, p_t that depend on $\sin\beta$ and are RGI.

$$\rightarrow \; m_t \sim \sin\beta \; 105 \text{ GeV}$$

Switching on g_2 and g_1, we have solutions of the form:

$$y_t = p_t g_3 + q_t g_2 + r_t g_1 \quad , \quad \lambda_i = p_i g_3^2 + q_i g_2^2 + r_i g_1^2$$

where p_t, p_i are known from the above procedure and now the full REs will be

$$\beta_3 \frac{dy_t}{dg_3} = \beta_t \quad , \quad \beta_3 \frac{d\lambda_i}{dg_3} = \beta_{\lambda_i}$$

Solving them requires the conditions

$$\mathcal{D}(q_a g_2) \sim 0 \quad , \quad \mathcal{D}(r_a g_1) \sim 0 \quad , \quad a = t, 1, ..., 7$$

They hold for $\mu \geq 10^7$ GeV and are not RGI $\quad \rightarrow \quad$ X

Realistic Approach - Reduction at a Boundary Scale

Main Idea:

$\rightarrow$ Solve the REs at a specific scale M_{bdry}

$\rightarrow$ Above M_{bdry} a covering theory is assumed that makes the solutions RGI

$\rightarrow$ Use solutions as BCs to run the usual 2HDM RGEs from M_{bdry} to M_{EW}

About the boundary scale:

- Solutions wrt g_2, g_1 demand $M_{bdry} \geq 10^7$ GeV
- g_2, g_1 are treated as corrections, should not be comparable to g_3 at the boundary scale. This demands $M_{bdry} < 10^8$ GeV

$$\rightarrow M_{bdry} \sim 10^7 \text{ GeV}$$

This is the scale that New Physics appear.

We can now reduce our system using only the following input:

$$g_i(M_{EW}) \quad , \quad m_A$$

First we focus on the g_3 - y_t reduction, as it can be performed independently from the λ_i's and we switch off $g_{1,2}$. We find

$$y_t = 0.471 g_3$$

Now, switching on the two remaining gauge couplings, we solve the full REs at M_{bdry}, using their respective values $g_{1,2}(M_{bdry})$:

$$y_t = 0.471 g_3 - 0.119 g_2 + 1.228 g_1$$

Choosing the appropriate value for $\tan\beta$:

$$\tan\beta = 2.2 \pm 0.5$$

we obtain a pole top mass that satisfies the most recent experimental limits:

$$m_t = (172.69 \pm 0.30)\ \text{GeV} \quad \checkmark$$

allowing for a 1 GeV theoretical uncertainty due to higher order contributions and the absence of y_b, y_τ

Now we can repeat the procedure for the full system g_3 - y_t - λ_i.

We choose $m_{11}^2, m_{22}^2, m_{12}^2$ values appropriately, in order for the CP odd Higgs scalar mass to be:

$$m_A = 800 \text{ GeV}$$

Out of all the possible reduction solutions, we look for those that satisfy the light Higgs mass experimental limits:

$$m_h^{\text{exp}} = (125.25 \pm 0.17) \text{ GeV}$$

where we estimate our theoretical calculations to have a 5 GeV uncertainty due to threshold corrections and higher order contributions

There are 4 viable sets of solutions ✓

$$m_h^{(1)} = 127.28 \text{ GeV} \quad , \quad m_h^{(2)} = 120.87 \text{ GeV}$$
$$m_h^{(3)} = 121.20 \text{ GeV} \quad , \quad m_h^{(4)} = 122.81 \text{ GeV}$$

#	p_t	p_1	p_2	p_3	p_4	p_5	p_6	p_7
SET1	0.471	-1.377	-1.167	0	0	0	0	0
SET2	0.471	-1.377	-1.167	0	0	0	0	0
SET3	0.471	-1.109	-0.773	-0.955	0	0	0	0
SET4	0.471	-1.109	-0.773	-0.955	0	0	0	0
#	q_t	q_1	q_2	q_3	q_4	q_5	q_6	q_7
SET1	-0.119	4.606	4.198	-0.087	-0.060	-0.060	0	0
SET2	-0.119	4.598	4.189	0.124	-0.595	0	0	0
SET3	-0.119	3.652	2.819	3.317	0	0	0	0
SET4	-0.119	3.652	2.819	3.317	0	0	0	0
#	r_t	r_1	r_2	r_3	r_4	r_5	r_6	r_7
SET1	1.228	10.022	0.498	-1.033	-3.275	0	0	0
SET2	1.228	10.197	0.240	-0.415	-1.490	-1.490	0	0
SET3	1.228	7.929	1.245	5.518	-9.425	0	0	0
SET4	1.228	-3.196	0.312	8.017	-8.394	0	0	0

Conclusions & Outlook

- Partial RoC on the Type-II 2HDM at a boundary scale M_{bdry}
- Input parameters: $g_i(M_{EW})$, m_A
- Top quark mass obtained within experimental limits
- Light Higgs boson mass obtained within experimental limits
- Prediction for $\tan\beta \sim 2.2$
- Predicted scale of New Physics: $M_{bdry} \sim 10^7$ GeV

Next:

- 2-loop analysis
  - 2HDM with complex parameters
  - realistic description of spontaneous CP violation with minimal input
  - natural selection of one of the six symmetries of the Higgs potential
  - apply on *3HDM* - more predictive reduction