Amplitude Zeros from the Double Copy

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Motivation

- Modern amplitudes methods make use of symmetries and physical principles
- Can compute complicated observables from simpler building blocks
- Constraints on building blocks include
  - Lorentz invariance of kinematic variables
  - Momentum conservation
  - Mass dimension of the amplitude
  - Gauge invariance

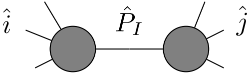
Motivation

- Additional constraints from analytical structure of amplitudes

$$A_n = \frac{n(p_1, p_2, ..., p_n)}{d(p_1, p_2, ..., p_n)} \quad (1)$$

- Constraints from pole structure well known and effective
  - Locality tells us A_n have poles that go like $\lim_{p \longrightarrow 0} \frac{1}{p^2}$
  - Unitarity tells us A_n factorizes as $Res_{p^2=0} A_n = A_L \times A_R$

$$A_n = \sum_I \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I) = \sum_{diagrams} \quad (1)$$



- What if poles are not enough? Zeros provide additional data

Hidden Zeros

- Some examples of zeros
  - Adler type zeros from shift symmetry, such as NLSM, Dirac-Born-Infeld, Special Galileon[1]

$$\lim_{t\to 0} \mathcal{A}_n = \mathcal{O}(t^{\sigma})\,. \tag{2}$$

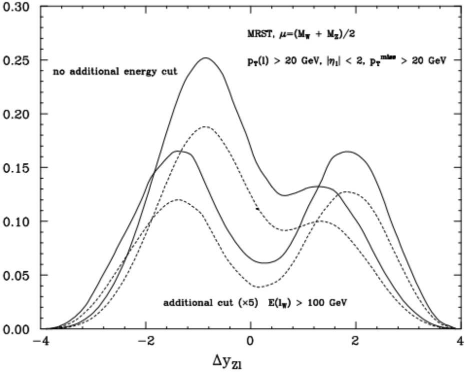
  - Standard model radiation zero, $q_1\bar{q}_1 \to W^{\pm}\gamma$ that vanishes for certain angles and approximate $q_1\bar{q}_1 \to W^{\pm}Z$ zero[2]

[1] Clifford Cheung et al. "A Periodic Table of Effective Field Theories". In: *Journal of High Energy Physics* 2017.2 (Feb. 2017). ISSN: 1029-8479.

[2] R. W. Brown and K. O. Mikaelian. "W^+W^- and Z^0Z^0 pair production in e^+e^-, pp, and $\bar{p}p$ colliding beams". In: *Phys. Rev. D* 19 (3 Feb. 1979), pp. 922–934.

Hidden Zeros

- Standard Model zeros in the $q_1\bar{q}_1 \to W^{\pm}Z$ cross section graph[3]



[3] L. Dixon, Z. Kunszt, and A. Signer. "Vector boson pair production in hadronic collisions at Order α_s: Lepton correlations and anomalous couplings". In: *Physical Review D* 60.11 (Nov. 1999). ISSN: 1089-4918.

Hidden Zeros

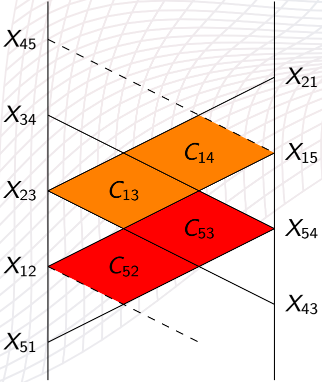
- Recently studied class of zeros of NLSM, Yang-Mills, and $Tr(\phi^3)$ called Hidden zeros[4]
- Zero location can be read off from kinematic mesh
- For n-point, the partial tree amplitude goes to 0 at

$$C_m = \left\{ \begin{matrix} s_{1m+2} = s_{1m+3} = \ldots = s_{1n-1} = 0 \\ s_{2m+2} = s_{2m+3} = \ldots = s_{2n-1} = 0 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ s_{mm+2} = s_{mm+3} = \ldots = s_{mn-1} = 0 \end{matrix} \right\} . \qquad (3)$$

[4] Nima Arkani-Hamed et al. *Hidden zeros for particle/string amplitudes and the unity of colored scalars, pions and gluons*. 2024. arXiv: 2312.16282 [hep-th].

Hidden Zeros

For the 5-point $A[12345]$ partial amplitude kinematic mesh looks like



(4)

Hidden Zeros

- Hidden zeros are not manifest
- For example, the 6-point NLSM partial amplitude is

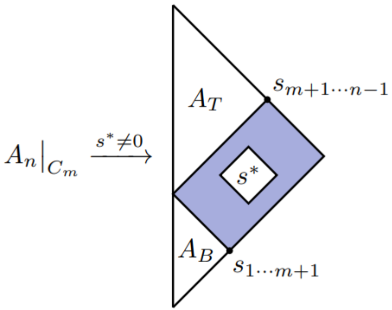
$$A_6^{NLSM}(123456) = \frac{s_{13}s_{46}}{s_{123}} + \frac{s_{35}s_{26}}{s_{26}} + \frac{s_{15}s_{24}}{s_{156}} - s_{135} \qquad (5)$$

- This has two sets of hidden zeros
  - $s_{13} = s_{14} = s_{15} = 0$
  - $s_{14} = s_{15} = s_{24} = s_{25} = 0$

Factorization Near Zeros

Amplitudes also factorize near the hidden zeros

$$A_n\big|_{C_m} \xrightarrow{s^*\neq 0} A_4 \times A_{m+2,B} \times A_{n-m,T}$$



Our Work

- Hidden zero theories all play some role in the double copy
- Is there a connection?
- Interested in the following questions
  - What does BCJ imply about hidden zeros?
  - Do hidden zeros double copy?
  - What does this say about factorization?

BCJ and Zeros

- 6-point NLSM amplitude obeys the BCJ relations[5]

$$
\begin{aligned}
\mathcal{A}_6[123456] = \frac{1}{s_{12}s_{123}s_{56}} \Big[& \mathcal{A}_6[162345]\; s_{15}(s_{12}+s_{23})(s_{14}-s_{56}) \\
& - \mathcal{A}_6[162354]\; s_{14}(s_{12}+s_{23})(s_{25}+s_{35}) \\
& + \mathcal{A}_6[162435]\; s_{13}s_{15}s_{24} \\
& + \mathcal{A}_6[162453]\; s_{13}s_{24}(s_{15}+s_{35}) \\
& - \mathcal{A}_6[162534]\; s_{14}s_{25}(s_{12}+s_{23}) \\
& + \mathcal{A}_6[162543]\; s_{13}s_{25}(s_{56}-s_{24}) \Big].
\end{aligned}
\tag{6}
$$

[5] Z. Bern, J. J. M. Carrasco, and H. Johansson. "New relations for gauge-theory amplitudes". In: *Physical Review D* 78.8 (Oct. 2008). ISSN: 1550-2368. DOI: 10.1103/physrevd.78.085011.

BCJ and Zeros

- Hidden zeros $s_{13} = s_{14} = s_{15} = 0$ are manifest here

$$
\begin{aligned}
\mathcal{A}_6[123456] = \frac{1}{s_{12}s_{123}s_{56}} \Big[& \mathcal{A}_6[162345]\, s_{15}(s_{12}+s_{23})(s_{14}-s_{56}) \\
& - \mathcal{A}_6[162354]\, s_{14}(s_{12}+s_{23})(s_{25}+s_{35}) \\
& + \mathcal{A}_6[162435]\, s_{13}s_{15}\, s_{24} \\
& + \mathcal{A}_6[162453]\, s_{13}\, s_{24}(s_{15}+s_{35}) \\
& - \mathcal{A}_6[162534]\, s_{14}\, s_{25}(s_{12}+s_{23}) \\
& + \mathcal{A}_6[162543]\, s_{13}\, s_{25}(s_{56}-s_{24}) \Big] .
\end{aligned} \tag{7}
$$

BCJ and Zeros General Case

$$\mathcal{A}_n[123\cdots n] = (-1)^n \sum_{\sigma(3\ldots n-1)} \mathcal{A}_n[1n2\sigma] \times \prod_{k=3}^{n-1} \frac{\mathcal{F}_k[2\sigma 1]}{s_{kk+1\ldots n}}, \quad (8)$$

$$\mathcal{F}_k[\rho] = \begin{cases} \sum_{l=t_k}^{n-1} \mathcal{S}_{k,\rho_l} + \theta(t_{k-1}, t_k) s_{kk+1\ldots n} & \text{if } t_k > t_{k+1}, \\ -\sum_{l=1}^{t_k} \mathcal{S}_{k,\rho_l} - \theta(t_k, t_{k-1}) s_{kk+1\ldots n} & \text{if } t_k < t_{k+1} \end{cases} \quad (9)$$

We can show that there is at least one k where

$$\mathcal{F}_k[2\sigma 1]\Big|_{C_m} = 0$$

And hence BCJ implies zeros, $A_n[12\ldots n]\Big|_{C_m} = 0$

Higher Derivative Theories

- Soft behavior puts constraints on high derivative corrections[6]
- What about hidden zeros?

Table: Number of six-point amplitudes from various constraints in NLSM

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14
Alder zero	1	2	10	29	78	203	461
Hidden zeros	1	1	5	13	41	112	282
BCJ satisfying	1	0	1	1	2	4	7

- Zeros less constraining than BCJ relations

$$\text{(BCJ satisfying)} \subset \text{(Hidden zeros)} \subset \text{(Adler zero)} \tag{10}$$

[6] Henriette Elvang et al. "Soft bootstrap and supersymmetry". In: *Journal of High Energy Physics* 2019.1 (Jan. 2019). ISSN: 1029-8479.

The Zeros from Double Copy

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$$(\text{KLT amp}) = (\text{BCJ amp}) \otimes (\text{BCJ amp}) \tag{11}$$

- We can write the special Galileon amplitude as a double copy of zero satisfying NLSM

$$M_n = \sum_{\sigma\gamma} S[\sigma|\gamma] A_n(1\gamma m{+}1n) A_n(1m{+}1n\sigma)\,, \tag{12}$$

- We can always find a basis where the kernel vanishes at C_m
- Zeros carry over to special Galileon
- Double copied sGal amplitude also exhibits factorization

Hidden Zeros of Special Galileon EFT

- Can we produce all hidden zero satisfying sGal + high derivative terms from KLT?

Table: Number of six-point amplitudes from various constraints in special Galileon theory

$\mathcal{O}(p^{\#})$	10	12	14	16	18	20	22
$\mathcal{O}(t^3)$ soft behavior	1	0	1	3	10	23	49
Hidden zeros	1	0	1	1	4	6	14
Generated from KLT	1	0	1	1	3	5	10

- Zeros not enough but maybe zeros + factorization? Left for future work

$$(\text{From KLT}) \subset (\text{Hidden zeros}) \subset (\mathcal{O}(t^3)\,\text{behavior})$$

Outlook

- BCJ compatibility implies zeros when there are no 2-particle poles
- Zeros also double copy
- Do gravity amplitudes have these zeros?
- Which 4-dimensional theories have these zeros?
- Novel constraints from factorization and zeros on EFT's
- Can we tie zeros in double copied theories into underlying geometry?

The End

Thanks for Listening!