All-loop heavy-heavy-light-light correlators in $\mathcal{N} = 4$ super Yang-Mills theory

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Large Charge 't Hooft Limit

$\mathcal{N} = 4$ Super Yang-Mills (SYM) theory is very useful to study

- Many observables can be determined analytically

We will consider the four-point correlator of $1/2$-BPS superconformal primaries with $SU(N)$ gauge group

Large N 't Hooft limit: $N \to \infty$, with $g_{YM}^2 N$ fixed ['t Hooft, 74]

- Selects a specific class of conformal Feynman integrals (planar)
- Loop integrands determined up to 10 loops [Bourjaily, Heslop, Tran, 16]
- Evaluating the integrals may still be difficult

We will instead consider the **large-charge** 't Hooft limit [Bourget, Rodriguez-Gomez, Russo, 18] of $\langle \mathcal{H}\mathcal{H}\mathcal{O}_2\mathcal{O}_2 \rangle$:
Charge $\Delta_{\mathcal{H}} \to \infty$, with $\lambda = \Delta_{\mathcal{H}} g_{YM}^2$ fixed, and generic N

- We will find this is determined to **all loops** by the ladder integrals

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Single- and multi-trace operators:

$$\mathcal{O}_p(x,Y) = \frac{1}{p} Y_{I_1}\cdots Y_{I_p} \mathrm{Tr}\left(\Phi^{I_1}(x)\cdots\Phi^{I_p}(x)\right) ,$$
$$\mathcal{O}_{p_1,\cdots,p_n}(x,Y) = \frac{p_1\cdots p_n}{p}\mathcal{O}_{p_1}(x,Y)\cdots\mathcal{O}_{p_n}(x,Y) ,$$

where Φ^I are the 6 scalar fields, x is spacetime position, and Y_I is the $SO(6)$ R-symmetry null vector.

$\mathcal{H}$ is a combination of multi-trace operators (as $p_i \le N$), with dimension, or charge, $\Delta_{\mathcal{H}}$.

$$\langle\mathcal{H}(x_1,Y_1)\mathcal{H}(x_2,Y_2)\mathcal{O}_2(x_3,Y_3)\mathcal{O}_2(x_4,Y_4)\rangle = \mathcal{G}_{\text{free}}(x_i,Y_i) + \mathcal{I}(x_i,Y_i)\mathcal{T}_{\mathcal{H}}(u,v)$$

We will consider $\mathcal{T}_{\mathcal{H}}(u,v)$ in the limit $\Delta_{\mathcal{H}}\to\infty$, with $\lambda = \Delta_{\mathcal{H}} g_{YM}^2$ fixed.

We construct the L-loop integrands by inserting chiral Lagrangians,

[Eden, Heslop, Korchemsky, Sokatchev, 12]

$$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\rangle|_{L-\text{loop}}$$
$$= \frac{(-1)^L}{L!} \int d^4x_5 \dots d^4x_{L+4} \langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5) \dots \mathcal{L}(x_{L+4})\rangle|_{\text{tree}} \, .$$

We can therefore express $\mathcal{T}_{\mathcal{H}}(u, v)$ as

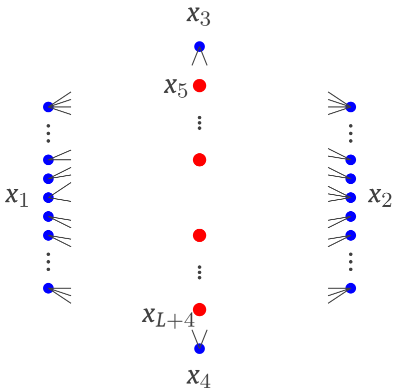
$$\mathcal{T}_{\mathcal{H}}(u, v) = \sum_{L=1}^{\infty} \left(-\frac{g_{YM}^2}{4\pi^2}\right)^L \frac{x_{12}^2 x_{13}^4 x_{14}^2 x_{23}^2 x_{24}^4 x_{34}^2}{(\pi^2)^L} \int d^4x_5 \dots d^4x_{L+4} \sum_{\alpha} d_{\mathcal{H},N;L}^{(\alpha)} f_{\alpha}^{(L)}(x_i) \, ,$$

where $f_{\alpha}^{(L)}(x_i)$ are f-graph functions, and $d_{\mathcal{H},N;L}^{(\alpha)}$ are their corresponding colour factors.

Constructing the integrand

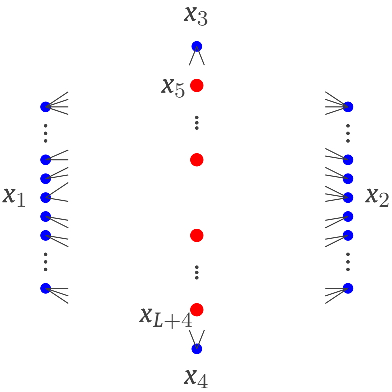
2 Derivation

$$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}:$$



Number of traces in $\mathcal{H} \sim \Delta_{\mathcal{H}}$

$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



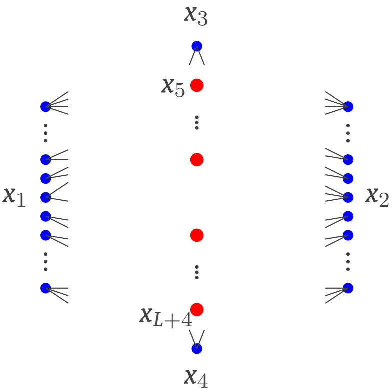
Number of traces in $\mathcal{H} \sim \Delta_{\mathcal{H}}$

Maximising the number of legs
from chiral Lagrangians
to the heavy operators **maximises the charge scaling**

- The integrand has a factor $\binom{\Delta_{\mathcal{H}}}{L} \sim (\Delta_{\mathcal{H}})^L$
- These integrands dominate in the limit $\Delta_{\mathcal{H}} \to \infty$

Constructing the integrand

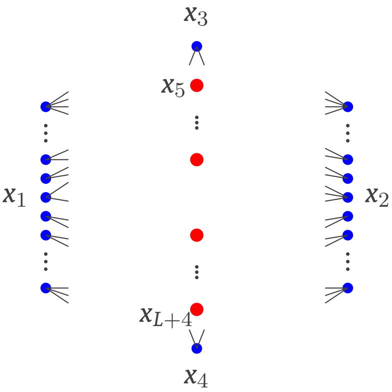
$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



Maximise number of connecting legs

Constructing the integrand

$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



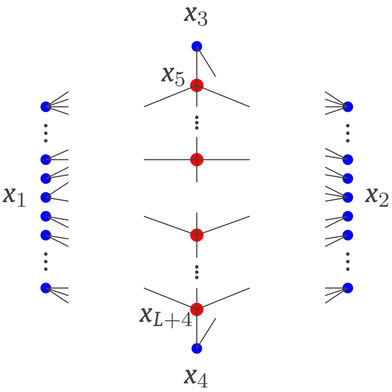
Maximise number of connecting legs

The chiral Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}\Big\{ -\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} + \sqrt{2}\lambda^{\alpha A}[\phi_{AB}, \lambda^B_\alpha] - \frac{1}{8}[\phi^{AB}, \phi^{CD}][\phi_{AB}, \phi_{CD}]\Big\}$$

The four-scalar vertex will maximise the number of legs

$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



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Constructing the integrand

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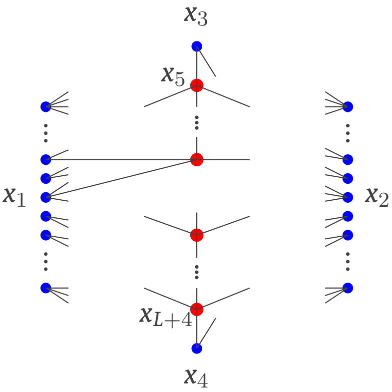

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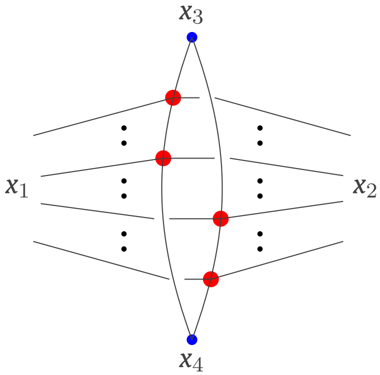
The four-scalar vertex will maximise the number of legs

These types of diagrams will **not** contribute.
(Supersymmetric non-renormalisation theorems)

[Baggio, de Boer, Papadodimas; 12]

Constructing the integrand

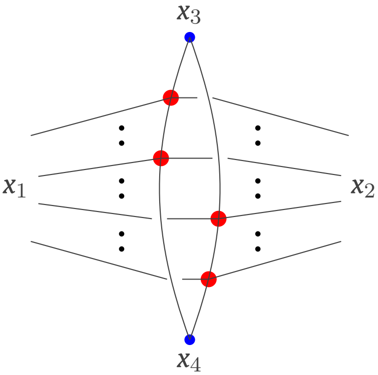
$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\dots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



The allowed diagrams are all associated with a single f-graph function:

$$f^{(L)}_{\text{ladder}}(x_i) = \frac{1}{2(L+2)} \frac{(x_{12}^2)^{L-2}}{\prod_{i=3}^{L+4} x_{i,i+1}^2 x_{1i}^2 x_{2i}^2} + S_{L+2}\,.$$

$\langle \mathcal{H}(x_1, Y_1)\mathcal{H}(x_2, Y_2)\mathcal{O}_2(x_3, Y_3)\mathcal{O}_2(x_4, Y_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{L+4})\rangle|_{\text{tree}}$:



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Integrating over the internal coordinates leads to the ladder integrals $\Phi^{(\ell)}(u, v)$:

$$\frac{x^2_{12} x^4_{13} x^2_{14} x^2_{23} x^4_{24} x^2_{34}}{(\pi^2)^L} \int d^4 x_5 \ldots d^4 x_{L+4} f^{(L)}_{\text{ladder}}(x_i) = \frac{1}{u} \sum_{\ell=0}^{L} \Phi^{(\ell)}(u, v) \Phi^{(L-\ell)}(u, v)$$

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All-loop expression

Therefore $\mathcal{T}_{\mathcal{H}}(u,v)$ is given by

$$\mathcal{T}_{\mathcal{H}}(u,v) = \sum_{L=1}^{\infty} d_{\mathcal{H},N;L} \frac{(-a)^L}{u} \sum_{\ell=0}^{L} \Phi^{(\ell)}(u,v)\Phi^{(L-\ell)}(u,v)\,.$$

$a = \frac{\lambda}{4\pi^2} = \frac{\Delta_{\mathcal{H}} g_{YM}^2}{4\pi^2}$, and the ladder integrals are known to all loops [Usyukina, Davydychev; 93].

The colour factors can be explicitly computed by contracting the colour tensors for specific cases of $\mathcal{H}$.

The *integrated correlators* $I_2[\mathcal{T}_{\mathcal{H}}(u,v)]$ precisely match with results from supersymmetric localisation. [Pestun; 12] [Binder, Chester, Pufu, Wang; 19] [Paul, Perlmutter, Raj; 23] [AB, Wen, Xie; 23]

"Canonical" heavy operators: $d_{\mathcal{H},N;L} = \beta c^L$

- e.g. $SU(2)$, $\mathcal{H} = (\mathcal{O}_2)^p$, $d_{\mathcal{H},2,L} = 4(\frac{1}{2})^L$ [Caetano, Komatsu, Wang; 23]
- $\mathcal{T}_{\mathcal{H}}(u,v)$ can be resummed [Broadhurst, Davydychev; 10]:

$$\mathcal{T}_{\mathcal{H}}(u,v) = \frac{\beta}{u}\left[\left(\sum_{\ell=0}^{\infty}(-c\,a)^{\ell}\,\Phi^{(\ell)}(u,v)\right)^2 - 1\right],$$

- Exponential decay at strong coupling

Conclusion and Outlook

The dynamical part of $\langle \mathcal{H}\mathcal{H}\mathcal{O}_2\mathcal{O}_2 \rangle$ in the limit $\Delta_{\mathcal{H}} \to \infty$, with $\lambda = \Delta_{\mathcal{H}} g^2_{YM}$ fixed, is given by the all-loop expression

$$\mathcal{T}_{\mathcal{H}}(u,v) = \sum_{L=1}^{\infty} d_{\mathcal{H},N;L} \frac{(-a)^L}{u} \sum_{\ell=0}^{L} \Phi^{(\ell)}(u,v)\Phi^{(L-\ell)}(u,v)$$

Future directions:

- General classification of canonical operators
- Resummation properties of generic heavy operators
- Beyond large-charge planar limit