Université Claude Bernard Lyon-1 / University of Johannesburg

Probing cosmic censorship in Reissner-Nordström de Sitter black holes

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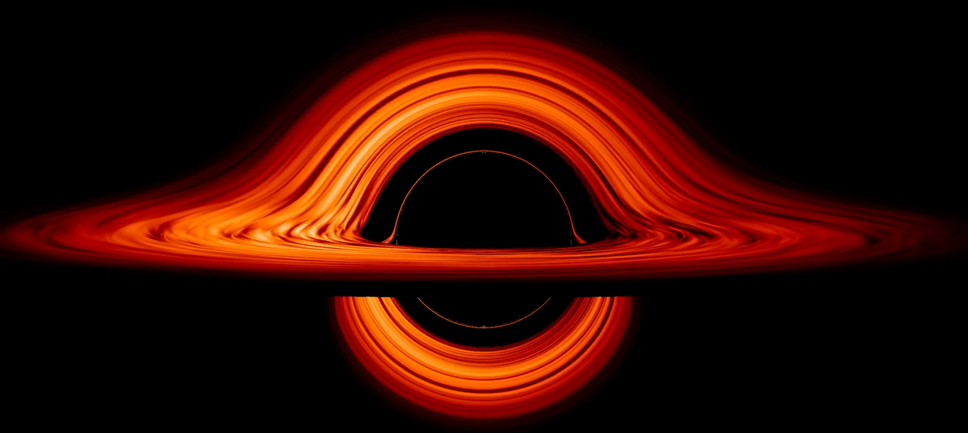
Overview

NASA's *Anatomy of a Black Hole*



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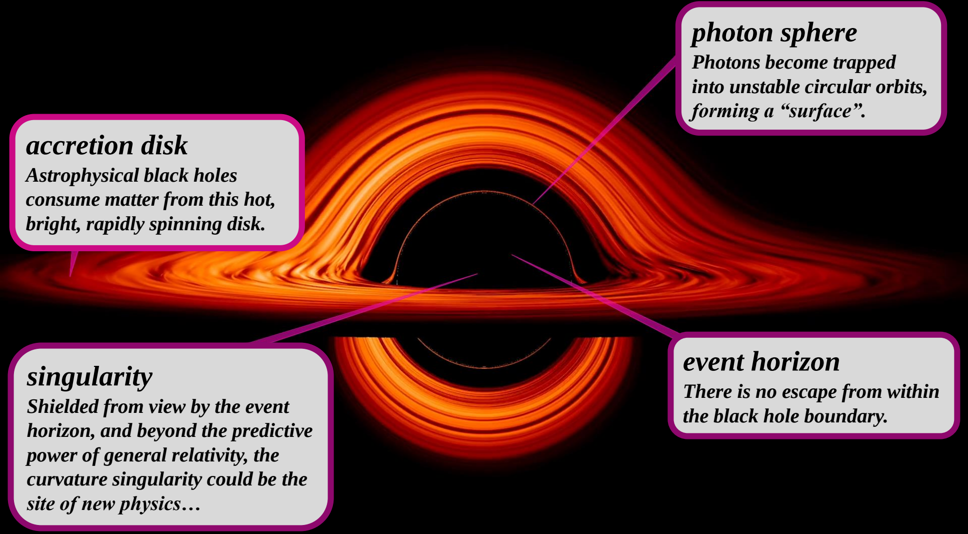

Preliminaries: what is cosmic censorship?

Singularity: Consider $(\mathcal{M}, g)$ to be a 4D time-orientable Lorentzian manifold. Then a singularity is denoted by a future-directed future-inextendible time-like curve $C \subset \mathcal{M}$.

Weak Cosmic Censorship: Consider this strongly causal space-time $(\mathcal{M}, g)$ that is asymptotically-flat at null infinity. Then $(\mathcal{M}, g)$ contains no naked singularities

Strong Cosmic Censorship: For generic vacuum data sets, the maximal Cauchy development $(\mathcal{M}, g_{ab})$ is inextendible as a suitably regular Lorentzian manifold.
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$\Rightarrow$ ensures that for a *physically-reasonable space-time*, a future-directed time-like curve is *inextendible* past a singularity

- ⋆ no formal universal proof, many revisions
- ⋆ *Sbierski (2018):* proved inextendability of the metric beyond $r = 0$ singularity for *Schwarzschild* $\Rightarrow$ SCC preservation

Stationary, charged, spherically-symmetric black hole:

$$g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Einstein–Maxwell field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 16\pi\left[F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right]$$

$$\nabla^\nu F_{\mu\nu} = 0\ ,\ \nabla_{[\mu}F_{\nu\rho]=0}$$

in de Sitter space-time

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L_{dS}^2}$$

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horizons: $r_-,\; r_+,\; r_c;\; r_0$

c.f. RN : $r_\pm = M^2 \pm \sqrt{M^2 - Q^2}$

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length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

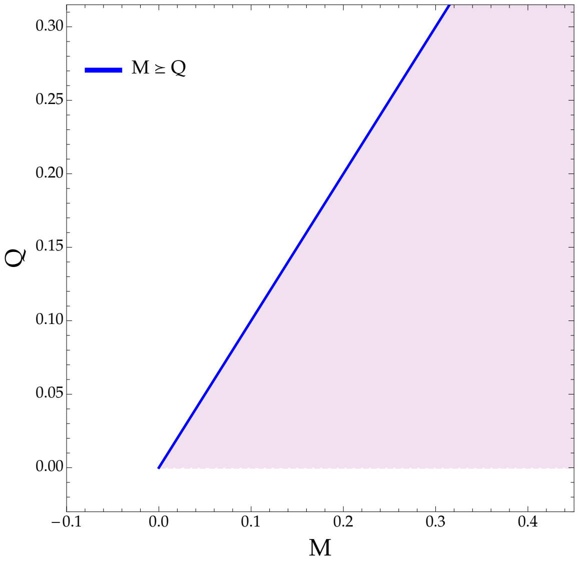
$L_{dS} = (3/\Lambda)^{1/2} = 1$

$[G] = M^{-1}L^3T^{-2}$, $[c] = LT^{-1}$

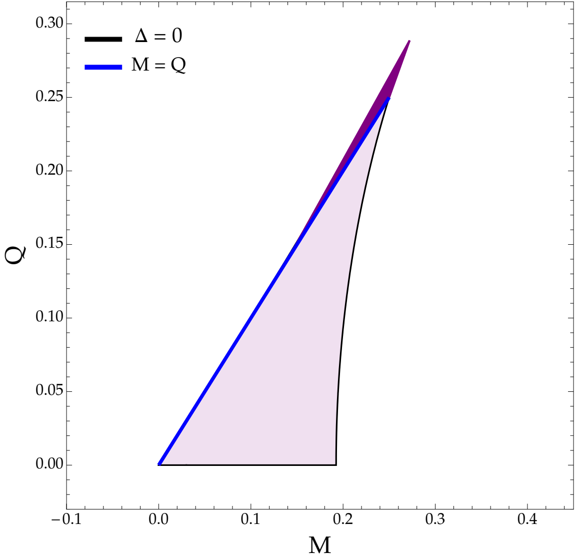
$[m_{BH}] = M$

$$r_\pm \neq M^2 \pm \sqrt{M^2 - Q^2}$$

Consequences of $\Lambda > 0$: RN phase space

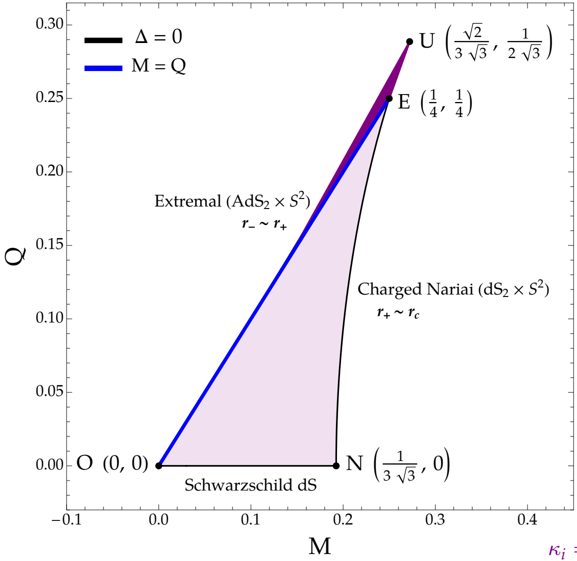


$$\frac{Q^2}{M^2} \lesssim 1$$



$$\frac{Q^2}{M^2} \lesssim 1 + \frac{1}{3}(M^2\Lambda) + \dots$$

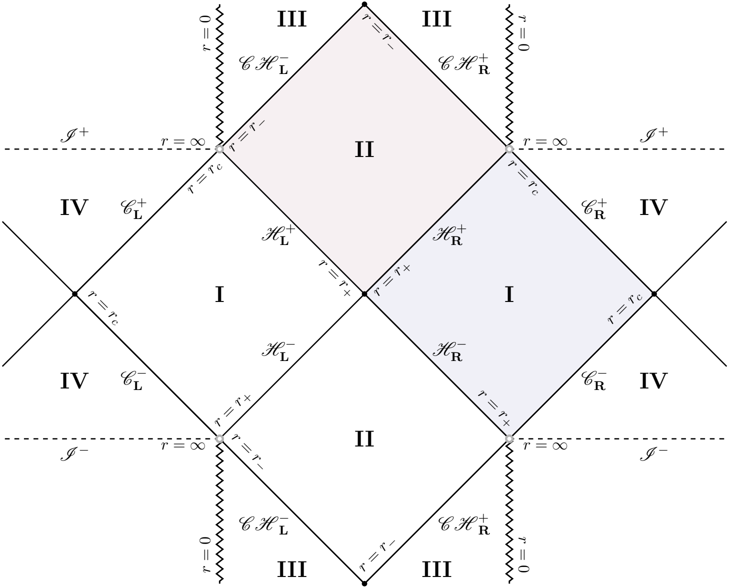
Consequences of $\Lambda > 0$: RNdS phase space



Thermal equilibrium:
$M = Q, r_+ \approx r_c$

$$\kappa_i = \frac{1}{2}\frac{d}{dr}f(r)\Big|_{r=r_i} \qquad T_i = \frac{\kappa_i}{2\pi}\frac{\hbar c}{k_B}$$

What about the black hole structure?



III
III
$r=0$
$r=r_-$
$r=0$
$\mathscr{C}\mathscr{H}^-_{\mathbf{L}}$
$\mathscr{C}\mathscr{H}^+_{\mathbf{R}}$
$\mathscr{I}^+$
$r=\infty$
$r=r_-$
II
$r=\infty$
$\mathscr{I}^+$
$r=r_c$
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IV
$\mathscr{C}^+_{\mathbf{L}}$
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I
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IV
$\mathscr{C}^-_{\mathbf{L}}$
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III
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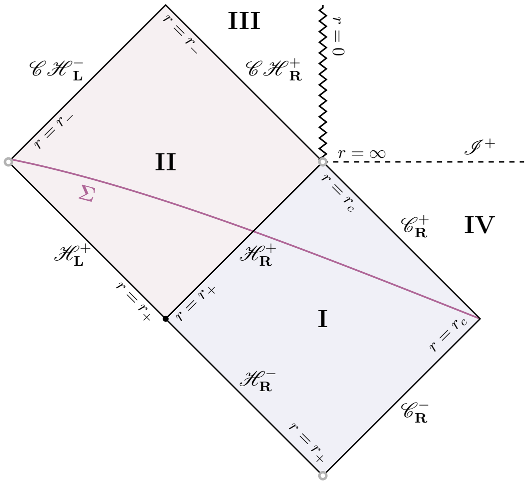
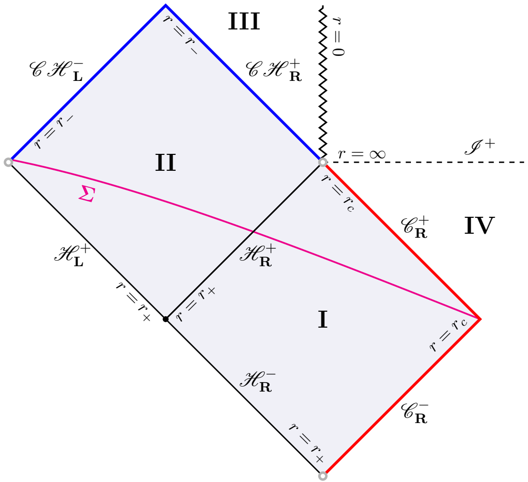
Strong cosmic censorship hypothesis in RNdS





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$\mathscr{I}^+$
Σ
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$\mathscr{C}^+_{\mathbf{R}}$
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I
$r = r_c$
$\mathscr{H}^-_{\mathbf{R}}$
$\mathscr{C}^-_{\mathbf{R}}$
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Two competing behaviours ($\Lambda > 0 \Rightarrow$ exp *vs* exp):

Asymptotic decay at r_+, $\psi \sim e^{\Im m\{\omega_{n=0}\}t}$ [exponential decay]

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A blueshift amplification phenomenon characterises the dynamics of infalling fields as they accumulate along the inner Cauchy horizon

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$$-\frac{\Im m\{\omega_{n=0}\}}{|\kappa_-|} < \frac{1}{2} \qquad [\textit{Hintz \& Vasy}]$$

where $\Im m\{\omega\} < 0$ *is a necessary condition for black hole stability*

What are black hole quasinormal modes?

Black hole quasinormal modes

Quasinormal mode and frequency

$$\Psi(x^\mu) = \sum_{n=0}^{\infty} \sum_{\ell,m} \frac{\psi_{sn\ell}(r)}{r} e^{-i\omega t} Y_{\ell m}(\theta, \phi)$$

- s: spin of perturbing field
- m: azimuthal number for spherical harmonic decomposition in θ, ϕ
- ℓ: angular/multipolar number for spherical harmonic decomposition in θ, ϕ
- n: overtone number labels ω by a monotonically increasing $|\mathfrak{Im}\{\omega\}|$

$$\omega_{sn\ell} = \omega_R - i\omega_I$$

- $\mathfrak{Re}\{\omega\}$: physical oscillation frequency $\rightarrow$ $\mathfrak{Re}\{\omega\} \propto \ell$
- $\mathfrak{Im}\{\omega\}$: damping $\rightarrow$ dissipative, "quasi" $\rightarrow$ $\lim_{\ell\to\infty} |\mathfrak{Im}\{\omega\}| = constant$

Quasinormal mode eigenvalue problem

Black hole wave equation for massive charged scalar field:

$$\frac{d^2}{dr_*^2}\varphi + \left[\left(\omega - \frac{qQ}{r}\right)^2 - V(r)\right]\varphi = 0\,, \qquad \frac{dr}{dr_*} = f(r)$$

$$e.g.\;\; V_{s=0} = f(r)\left(\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} + \mu^2\right)$$

$$\mu = \frac{m_s}{\hbar}\,,\;\; q \propto e \;\;\; ([\mu] = L^{-1},\; [q] = 1)$$

Subjected to **QNM boundary conditions**:

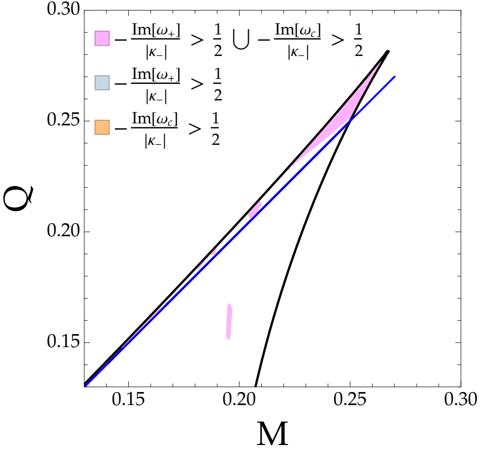
purely ingoing: $\varphi(r) \sim e^{-i\left(\omega - \frac{qQ}{r_+}\right)r} \qquad r \to r_+$

purely outgoing: $\varphi(r) \sim e^{+i\left(\omega - \frac{qQ}{r_c}\right)r} \qquad r \to r_c$

Waves escape domain of study at the boundaries $\Rightarrow$ dissipative

Regions in violation of SCC

e.g. for $\ell = 1$, $\mu = 1$ *&* $q = 0.1$



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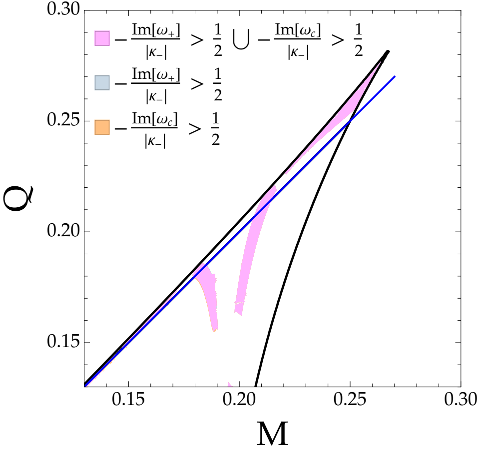

Conclusions

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- In non-extremised regions, $V(r)$ is a potential barrier on $r_+ \leq r \leq r_c$ for $\ell > 0 \Rightarrow$ QNMs throughout the phase space
- Schwarz: beyond $\mu^2 > V(r^{peak})$, QNFs long-lived
  $\Rightarrow$ RNdS (non-ext.): $V(r^{peak})$ rises with μ on $r_+ < r < r_c$
- But below μ_{crit}: anomalous behaviour ($\Im m\{\omega\} \downarrow$ with ℓ)
  (weak ℓ dependence; before which QNMs are anomalous)

Conclusions

- $\Lambda > 0$ & cosmic censorship influence parameter space
- In non-extremised regions, $V(r)$ is a potential barrier on $r_+ \leq r \leq r_c$ for $\ell > 0 \Rightarrow$ QNMs throughout the phase space
- Schwarz: beyond $\mu^2 > V(r^{peak})$, QNFs long-lived
  $\Rightarrow$ lose QNM character
  RNdS (non-ext.): $V(r^{peak})$ rises with μ on $r_+ < r < r_c$
- But below μ_{crit}: anomalous behaviour ($\mathfrak{Im}\{\omega\} \downarrow$ with ℓ)
  (weak ℓ dependence; before which QNMs are anomalous)
- SCC violated for "cold" RNdS black holes, on the OU line, particularly near $M \sim Q \sim 0.089$ – quantum effects?

Thank you

Backup slides

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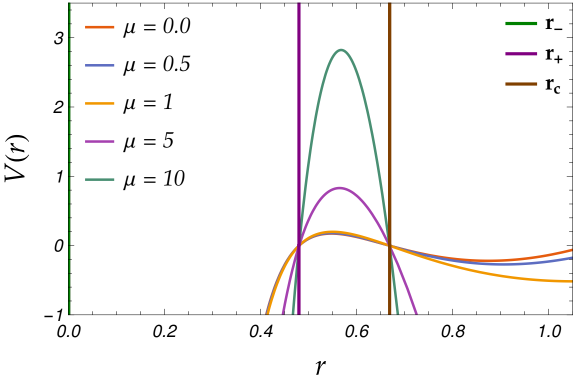
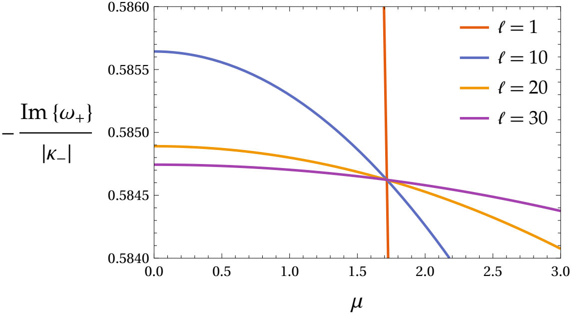
- ⋆ Influence of field mass
- ⋆ Weak Gravity Conjecture & Festina-Lente bound
- ⋆ Families of QNMs within RNdS, no anomalous behaviour $q \neq 0$
- ⋆ RNdS: Penrose diagram, thermodynamics
- ⋆ Strong Cosmic Censorship: background, RN vs RNdS
- ⋆ Strong Cosmic Censorship: QNMs in RNdS
- ⋆ Kaluza-Klein modes in 5D
- ⋆ Landscape of extra dimensional models

Effective potential for $M = 0.185$, $Q = 0.016$, $\Lambda = 3$, $n = 0$, $\ell = 1$, $q = 0.1$, *and* $\uparrow \mu$:



Increasing μ increases the height of the potential barrier for $r_+ < r < r_c$

anomalous behaviour observed for $\beta > 1/2$



Recall: $\mu \uparrow \Rightarrow |\mathfrak{Im}\{\omega\}| \downarrow$ & $\ell \uparrow \Rightarrow |\mathfrak{Im}\{\omega\}| \uparrow$

$\hookrightarrow$ *not so for* $\mu < \mu_{crit} \sim 1.7$

QNFs in eikonal regime for $M = 1$

$$\textit{Schwarz.}: \qquad \Delta\Re e\{\omega\} = \Omega = \overline{\Lambda} = 1/\sqrt{27} \quad \pm 10^{-4}$$

$$\lim_{\ell\to\infty} |\omega_{n=0,\ell}| = 2\Delta\Re e\{\omega\} = \overline{\Lambda}/2$$

$$\textit{SdS}: \qquad \Delta\Re e\{\omega\} = \left(\frac{27}{1-9\Lambda}\right)^{-1/2} \pm 10^{-4}$$

$$\lim_{\ell\to\infty} |\omega_{n=0,\ell}| = 2\Delta\Re e\{\omega\}$$

$$\textit{RN} \qquad \Delta\Re e\{\omega\} = \left(\frac{\left(3+\sqrt{9-8Q^2}\right)^3}{2\left(1+\sqrt{9-8Q^2}\right)}\right)^{-1/2} \pm 10^{-4}$$

$$\lim_{\ell\to\infty} |\omega_{n=0,\ell}| = 2\Delta\Re e\{\omega\}$$

Weak Gravity Conjecture & Festina-Lente bound

From Andreas Maximilianus'
Scriptorum Seu Togae & Belli Notationum Selecta

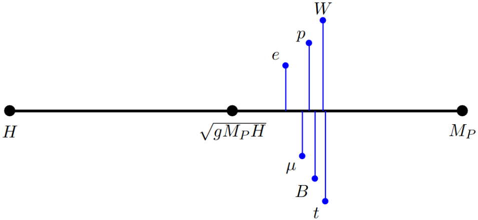
N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, JHEP 06 (2007) 060
M. Montero, T Van Riet, G. Venken, JHEP 01 (2020) 039

WGC: $\dfrac{m}{g_1 q} < \sqrt{2} M_p$ for *some* charged state

FL: $\dfrac{m^4}{2 g_1^2 q^2} \gtrsim 3 M_P^2 H^2$ for *every* charged state

$$\sqrt{6} g_1 q M_P H < m^2 < 2 g_1^2 q^2 M_P^2$$

All SM fields satisfy the full bound!



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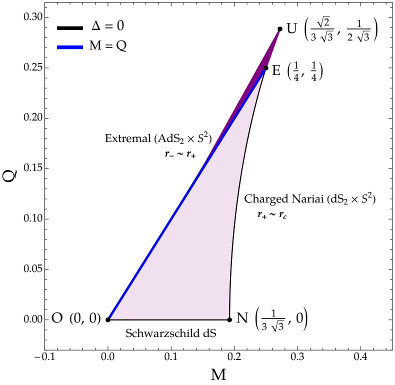

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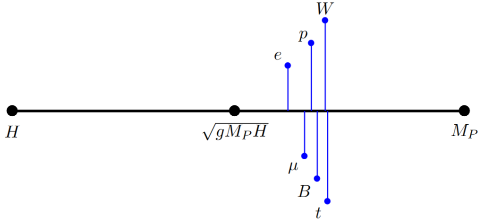

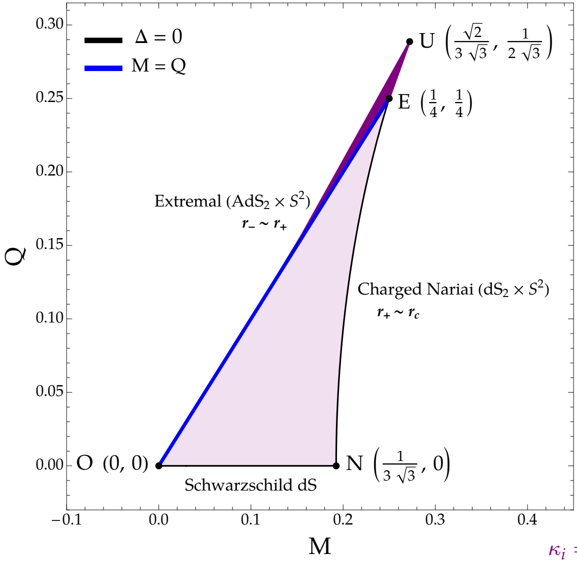

Thermal equilibrium:
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Quasinormal frequencies: 2 main branches

Schwarzschild modes (I) & de Sitter modes (II):

$(I.i)$ photon-sphere modes: beneath line $M = Q$, approaching NU
large $\mathfrak{Im}\{\omega\}$, $\mathfrak{Re}\{\omega\} \sim \mathcal{O}(0.01)$ for $Q < 0.1$

$$\mathfrak{Im}\{\omega_{PS}\} \approx -i\left(n+\frac{1}{2}\right)\kappa_+ \quad \text{on } NU$$

$(I.ii)$ near-extremal modes: branch OU $(r_- \sim r_+)$

$$\omega_{NE} \approx -i(\ell+n+1)\kappa_- = -i(\ell+n+1)\kappa_+ \ ;$$

(II) dS modes: along branch OU $(\kappa_c \sim 1/L_{dS})$

$$\omega_{dS_{n=0}} \approx -i\ell\kappa_c \ , \quad \omega_{dS_{n\neq 0}} \approx -i(\ell+n+1)\kappa_c \ ;$$

For $L = \sqrt{\ell(\ell+1)}$, as a series expansion $\omega = \sum_{k=-1} \omega_k L^{-k}$

$$\omega_k = \sqrt{V(r_*^{max}) - 2iU}\,, \quad U \equiv U(V^{(2)}, V^{(3)}, V^{(4)}, V^{(5)}, V^{(6)})$$

$$V^j = \frac{d^j V(r_*^{max})}{dr^j} = f(r)\frac{d}{dr}\left[f(r)\frac{d}{dr}\left[\dots\left[f(r)\frac{dV(r)}{dr}\right]\dots\right]\right]_{r\to r_*^{max}}\,.$$

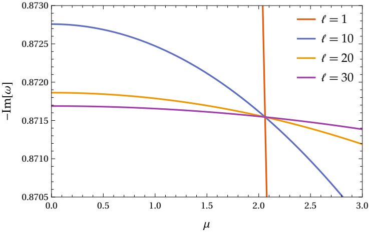
$$r_*^{max} \approx r_0 + r_1 L^{-2} + \dots\,,$$
$$V(r_*^{max}) \approx V_0 + V_1 L^{-2} + \dots$$

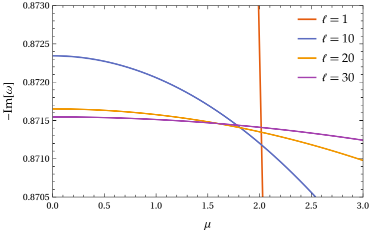
$q \neq 0 \Rightarrow$ two sets of solutions:

ω_+ (black-hole family) & ω_c (cosmological-horizon family)

For a fixed q, $|\mathfrak{Re}\{\omega_+\}| < |\mathfrak{Re}\{\omega_c\}|$ *&* $|\mathfrak{Im}\{\omega_+\}| < |\mathfrak{Im}\{\omega_c\}|$

e.g. for $M = Q = 0.104$,





Increasing the charge from $q = 0$ to $q = 0.1$ displaces the intersection
$\hookrightarrow$ *μ_{crit} can only be found in the $\mu M \gg qQ$ regime.*

Three physical horizons,

$$r_\pm = \pm a \mp b \;, \quad r_c = +a + b \;, \quad r_0 = -a - b \;,$$

$$a = \frac{1}{2\sqrt{3}}\sqrt{\frac{(1+X)^2 - 12Q^2}{X}}, \quad b = \frac{1}{2}\sqrt{\frac{4}{3} - \frac{1-12Q^2}{3X} - \frac{X}{3} + \frac{2M}{a}} \;,$$

$$X = \left(-1 + 54M^2 - 36Q^2 - 2\sqrt{27}\sqrt{\underline{\Delta}}\right)^{1/3} \;,$$

and constraints,

$$\frac{Q^2}{M^2} \lesssim 1 + \frac{1}{3}(M^2\Lambda) + \frac{4}{9}(M^2\Lambda)^2 + \frac{8}{9}(M^2\Lambda)^3 + \mathcal{O}(M^8\Lambda^4).$$

$$M^2\Lambda \leq \frac{1}{18}\left[1 + 12Q^2\Lambda + (1 - 4Q^2\Lambda)^{3/2}\right]$$