g-2 from lattice QCD: the most recent result

Z. Fodor

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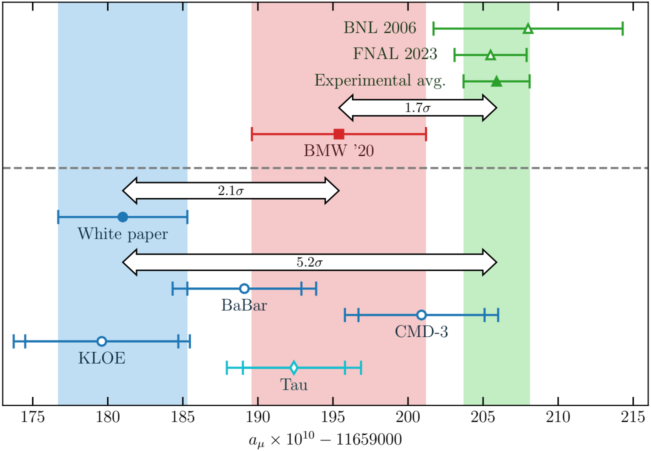
for the BMW+DMZ Coll.: Nature 593 (7857), 51, 2021
& arXiv: 2308.04221, 2407.10913

ICHEP'24, July 20, 2024, Prague

Tensions in (g-2)$_\mu$: take-home message before '24



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

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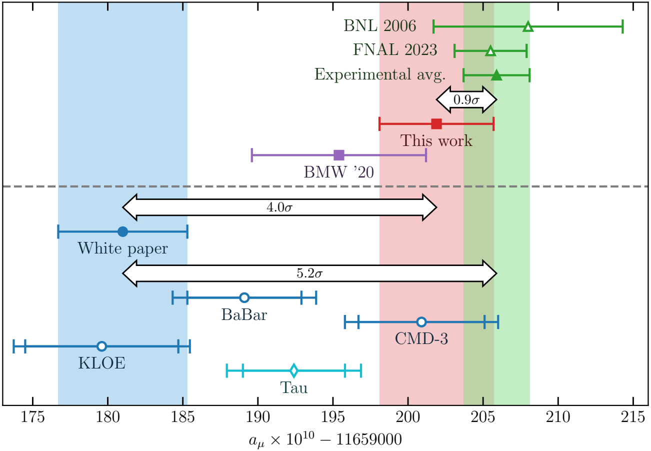

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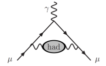
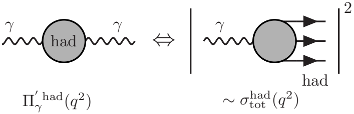
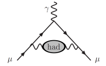
Outline

- Data driven method
- Lattice result
– scale determination
– noise reduction tail contribution
– window observables
- Summary

HVP  ⇒ using the data-driven/R-ratio method

- Optical theorem



HVP 

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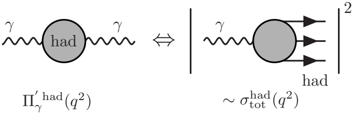

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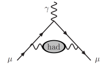
HVP  ⇒ using the data-driven/R-ratio method

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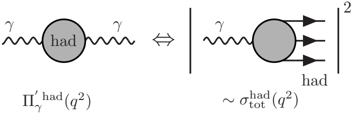

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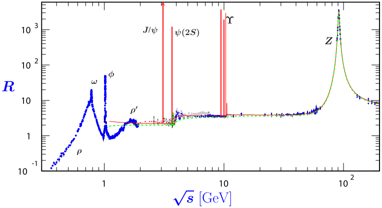
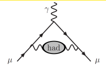


HVP  ⇒ using the data-driven/R-ratio method

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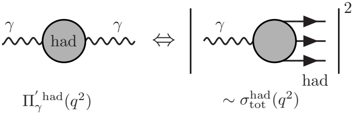

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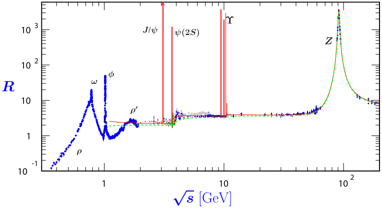

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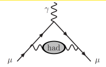
HVP  ⇒ using the data-driven/R-ratio method

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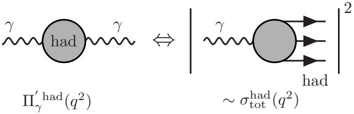

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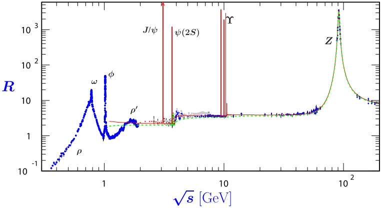

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Systematic uncertainty: ≈4 times larger than the statistical error (e.g. Davier et al.)

$a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

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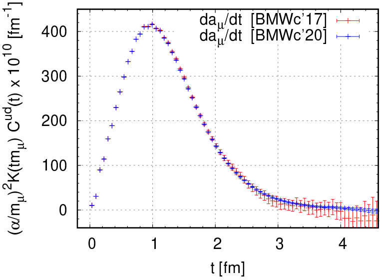
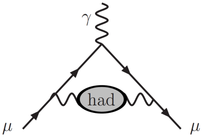
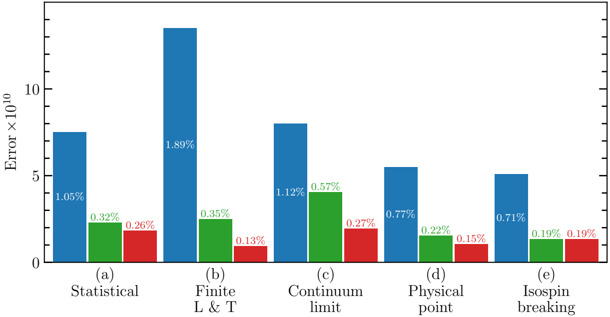


$K(t)$ describes the leptonic part of diagram



Error reduction 2017 (blue) – 2020 (green) - 2024 (red)



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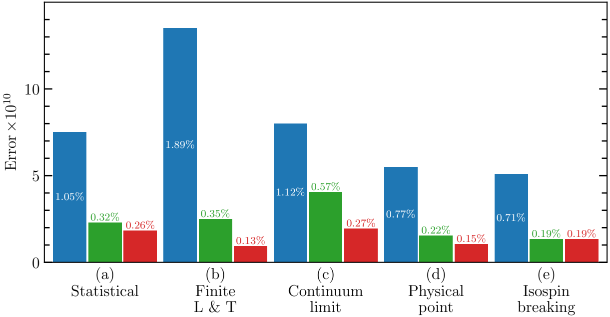

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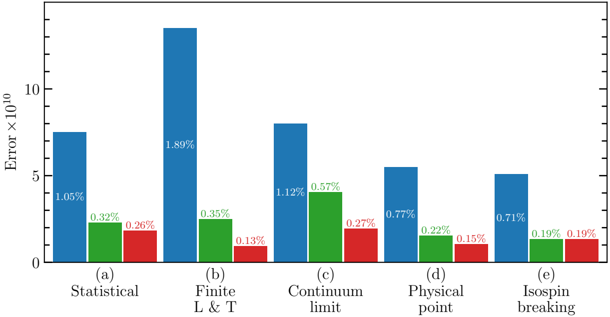

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• In 2020 the dominant uncertainty: continuum extrapolation
$\Longrightarrow$ new, even finer lattice with 0.048 fm lattice spacing

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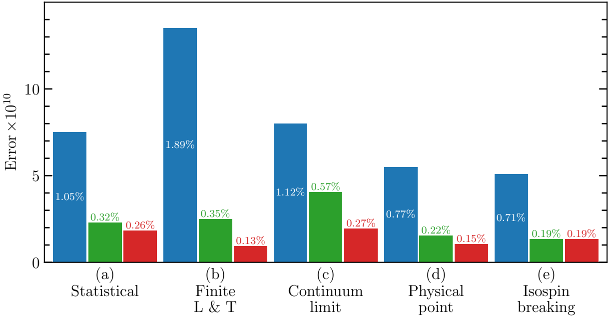

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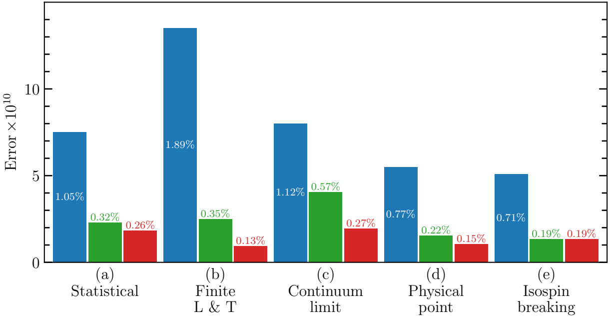

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most of the noise comes from the tail: $t > 3$ fm
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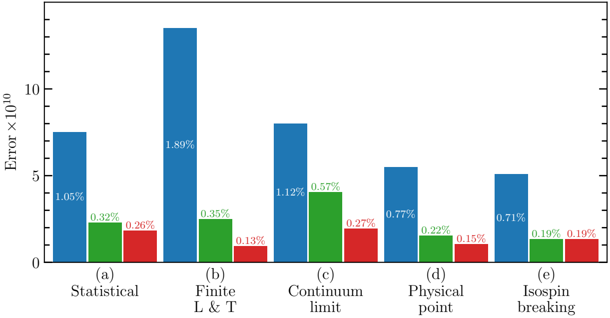

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- Scale for physical point & dedicated isospin breaking analysis

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2. w_0 scale setting: gauge flow in a fictitious fifth dimension
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   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17245(22)(46)[51] \text{ fm}$$

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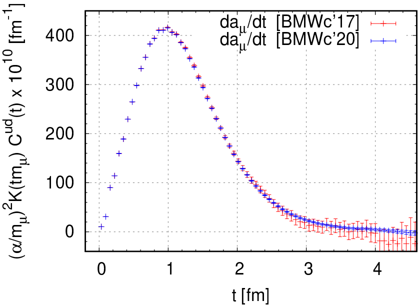

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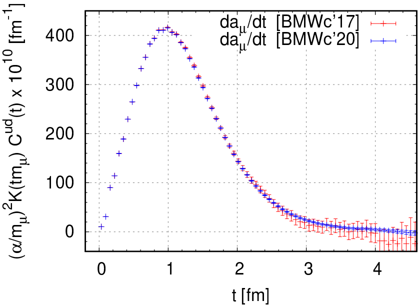

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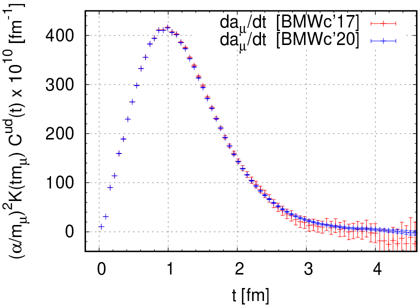

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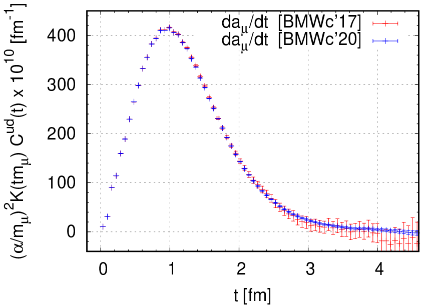

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- 2024: tail contribution for $t > 2.8$ fm from data driven approach

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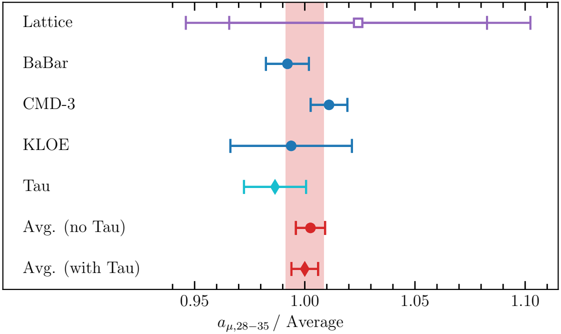

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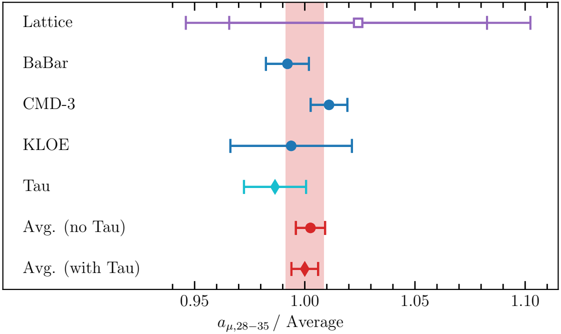

- $t > 2.8$ fm contributes only less than 5%
- data driven error: an order of magnitude better than lattice
- low energy part (below ρ) all agree
- we can be generous with errors: even 5 times larger wouldn't change

Window observable

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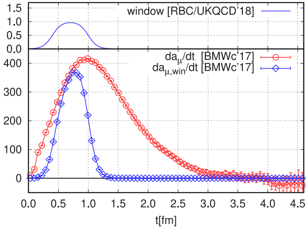

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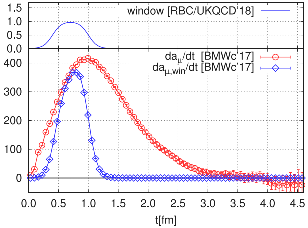

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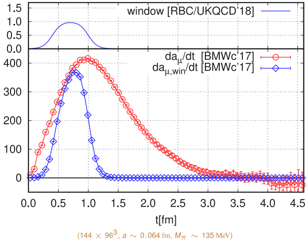

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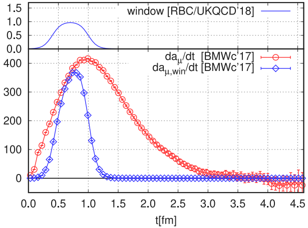

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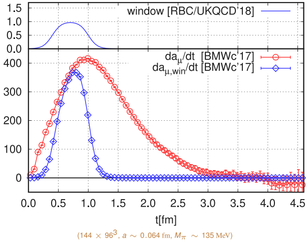

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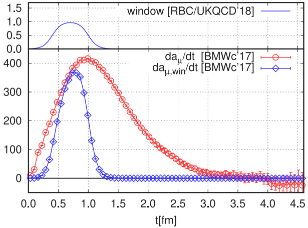

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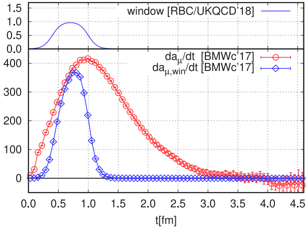

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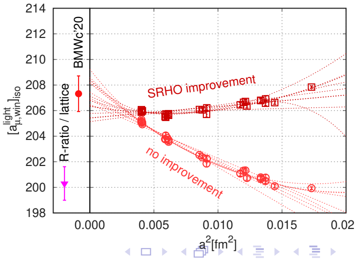
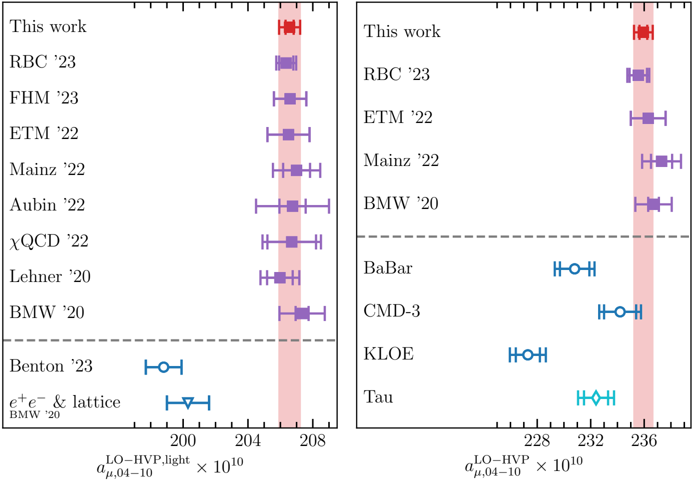
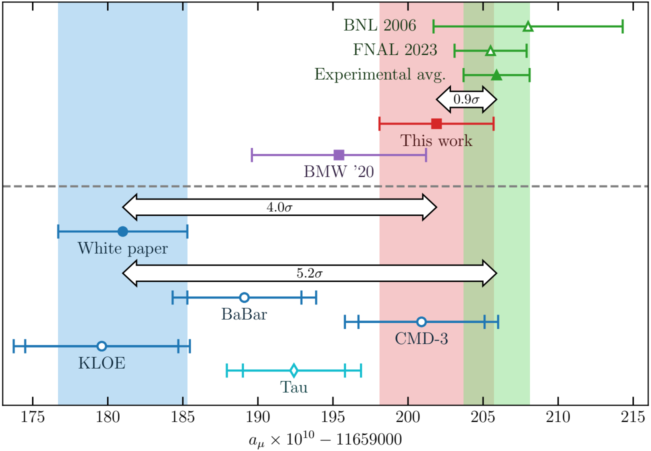


Window observable tension: other groups



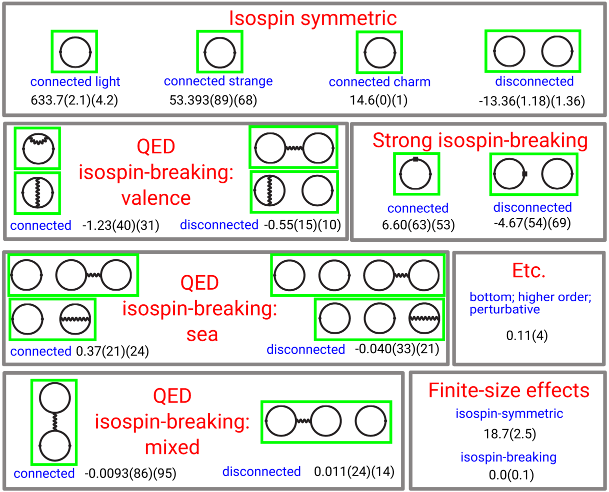
Huge tension with our result and with the average even more

Summary & take-home message



Hadronic vacuum polarization final result: $714.1(2.2)(2.5)[3.3] \cdot 10^{-10}$

Final result 2020



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

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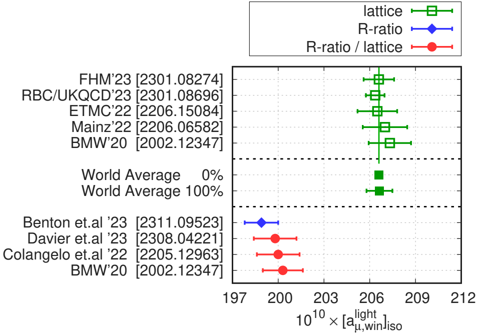

Tension: take-home message #2 lattice/e^+e^- window

$\approx$4–5 σ tensions for distance & energy regions (reduced, see Zhang)

Lattice window: 0.4-1.0 fm
approx. 30% of the total

e^+e^- window 0.60–0.88 GeV
more than 50% of the total