
# Phenomenological aspects of flavour and CP symmetries

Claudia Hagedorn
IFIC - UV/CSIC

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VNIVERSITAT DE VALÈNCIA
IFIC INSTITUT DE FÍSICA CORPUSCULAR
CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

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  - Replication of fermion generations
  - Fermion masses
  - Quark and lepton mixing
  - Baryon asymmetry of the Universe (BAU)

$$Y_B = \frac{n_B - n_{\overline{B}}}{s}\Big|_0 = 8.75 \times 10^{-11}$$

Planck ('18)

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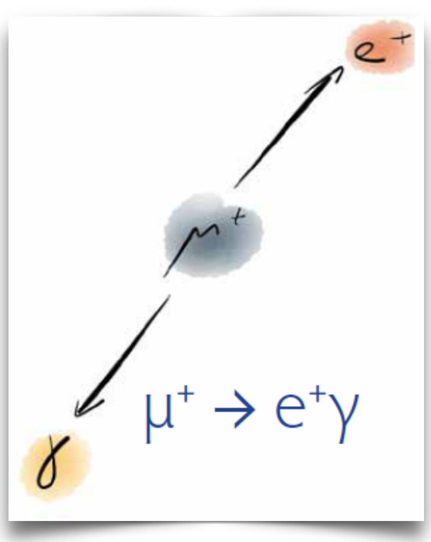
- Additionally, beyond SM (BSM) theories can have a rich phenomenology.
  - Processes forbidden/highly suppressed in SM can be in reach
  - Flavour and CP violation needs to be kept under control
  - Possible correlations among different signals




$\mathrm{BR}(\mu \to e\gamma) < 3.1 \cdot 10^{-13}$

MEG II at PSI ('23)

# Flavour and CP symmetries

**Properties of this new symmetry $G_f$ ?**

$G_f$ could be …

- … abelian or non-abelian
- … continuous or discrete
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to non-trivial subgroups
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

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# Flavour and CP symmetries

There are many options …

- Dihedral symmetries $D_n$ as well as $D'_n$
- Symmetric and alternating groups, $S_n$ and $A_n$
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
- Adding CP symmetries
- Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP
- …

Altarelli, Antusch, Branco, Calibbi, Centelles Chulia, Chen, Chu, Dasgupta, de Medeiros Varzielas, Ding, Dziewit, Everett, Feruglio, Gavela, Gehrlein, Girardi, Gluza, Gonzalez Felipe, Grimus, CH, He, Hirsch, Joaquim, Jurciukonis, Karmakar, King,, Lavoura, Lima, Luhn, Medina, Melis, Meloni, Merlo, Meroni, Mohapatra, Nilles, Nishi, Pas, Pascoli, Petcov, Rebelo,Rodejohann, Schumacher, Serodio, Shimizu, Smirnov, Spinrath, Srivastava, Stuart, Tanimoto, Titov, Valle, Vergeest, Vicente, Vien, Vives, Xu, Yamamoto, Ziegler, …

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# Flavour and CP symmetries

**Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$**

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(3\,n^2)$

Luhn/Nasri/Ramond ('07)

$$a^3 = e \;,\quad c^n = e \;,\quad d^n = e \;,$$
$$c\,d = d\,c \;,\quad a\,c\,a^{-1} = c^{-1}d^{-1} \;,\quad a\,d\,a^{-1} = c$$

$$g = a^\alpha c^\gamma d^\delta \quad \text{with} \quad \alpha = 0, 1, 2 \,,\;\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group $A_4$.

# Flavour and CP symmetries

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$\Delta(6\,n^2)$ Add to relations of $\Delta(3\,n^2)$ Escobar / Luhn ('08)

$$b^2 = e \;,\; (a\,b)^2 = e \;,\; b\,c\,b^{-1} = d^{-1} \;,\; b\,d\,b^{-1} = c^{-1}$$

$$g = a^{\alpha} b^{\beta} c^{\gamma} d^{\delta} \quad \text{with} \quad \alpha = 0, 1, 2 \;,\; \beta = 0, 1 \;,\; 0 \leq \gamma, \delta \leq n - 1$$

A well-known member is the permutation group $S_4$.

# Flavour and CP symmetries

**Add CP as further symmetry**

Grimus/Rebelo ('95),

Ecker/Grimus/Neufeld ('84,'87,'88)

- Motivation:
  For more than one generation of certain particle species, define CP that also acts on generations of particles, e.g.
  $$\Phi_i(x) \quad \to \quad X_{ij}\, \Phi_j^\dagger(x_P) \;\; \text{with} \;\; (x_P)_\mu = x^\mu$$
  with
  $$X X^\dagger = X X^\star = 1$$

- CP is involution and corresponds to automorphism of flavour symmetry

Feruglio/CH/Ziegler ('12)
Holthausen/Lindner/Schmidt ('12), Chen et al. ('14)

# Flavour and CP symmetries

**Breaking of symmetries**

Feruglio / CH / Ziegler ('12)

Idea: Keep some residual symmetry among charged leptons
and neutrinos, $G_e$ and $G_\nu$ , with $G_e \neq G_\nu$
Mismatch of symmetries corresponds to lepton mixing

$G_f$ and CP

$\swarrow \qquad \searrow$

| charged leptons | neutrinos |
|---|---|
| $G_e$ | $G_\nu = Z_2 \times$ CP |
| $U_e$ | $U_\nu = \Omega_\nu R(\theta) K_\nu$ |

$\searrow \qquad \swarrow$

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Apply to symmetries $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

CH / Meroni / Molinaro ('14)

# Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric]

## Case 1)

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta_L$$
$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta_L}$$
$$\sin^2\theta_{23} = \frac{1}{2}\left(1+\frac{\sqrt{3}\sin 2\theta_L}{2+\cos 2\theta_L}\right)$$

$$\sin\delta = 0$$
$$\sin\beta = 0$$

*s* fixed by CP symmetry

$$|\sin\alpha| = \left|\sin\left(\frac{6\,\pi\,s}{n}\right)\right|$$

# Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric]

## Case 1)

$$\begin{aligned} \sin^2\theta_{13} &\approx 0.0220\,(0.0222) \\ \sin^2\theta_{12} &\approx 0.341 \\ \sin^2\theta_{23} &\approx 0.605\,(0.606) \end{aligned}$$

$$\begin{aligned} \sin\delta &= 0 \\ \sin\beta &= 0 \end{aligned}$$

*s* fixed by CP symmetry

$$|\sin\alpha| = \left|\sin\left(\frac{6\,\pi\,s}{n}\right)\right|$$

# Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric]

## Case 2)

$n = 14$

| $u$ | $u=-1$ | $u=0$ | $u=+1$ |
|---|---|---|---|
| $\theta_L$ | 0.146 (0.148) | 0.184 | 0.146 (0.148) |
| $\sin^2\theta_{12}$ | 0.341 | 0.341 | 0.341 |
| $\sin^2\theta_{13}$ | 0.0222 (0.0224) | 0.0222 (0.0224) | 0.0222 (0.0224) |
| $\sin^2\theta_{23}$ | 0.437 | 0.5 | 0.563 |
| $\Delta\chi^2$ | 9.25 (11.2) | 10.8 (12.5) | 8.27 (8.62) |

$u = 2\,s - t$

$v = 3\,t$

fixed by CP symmetry

$\sin\delta = -1$ for $u = 0$

$\sin\delta \approx -0.811\,(-0.813)$ for $u = \pm 1$

several choices for $v$ admitted

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

- Consider a scenario of **type I seesaw with 3 RH neutrinos**, i.e. 3 generations of LH lepton doublets and 3 generations of gauge singlets $\nu_{R\,i}$

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \mathrm{h.c.}$$

- Light neutrino masses

$$m_\nu = -\,m_D\,M_R^{-1}\,m_D^T \quad \text{with} \quad m_D = Y_D\,\langle H\rangle$$

Minkowski ('77), Glashow ('80), Gell-Mann / Ramond / Slansky ('79), Mohapatra / Senjanovic ('80), Yanagida ('80), Schechter / Valle ('80)

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

- We take

$$\alpha_R \sim 1 \qquad\qquad l_{L\alpha} \sim 3\,,\nu_{R\,i} \sim 3'$$

[detail: use additional $Z_3$ to distinguish $e, \mu, \tau$]

see also Dev/CH/Molinaro ('18); Chauhan/Dev ('22)

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[detail: use additional $Z_3$ to distinguish $e, \mu, \tau$]

**Charged lepton mass matrix**

residual symmetry $G_e$

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

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[detail: use additional $Z_3$ to distinguish $e, \mu, \tau$]

**Neutral lepton sector** residual symmetry $G_\nu$

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \text{h.c.}$$

**No symmetry breaking**

$$M_R = M_R^0 = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

**RH neutrino masses are degenerate**

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

- We take

$\alpha_R \sim 1$

[detail: use additional $Z_3$ to distinguish $e, \mu, \tau$]

$l_{L\alpha} \sim 3 \,, \nu_{R\,i} \sim 3'$

**Neutral lepton sector**

residual symmetry $G_\nu$

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \mathrm{h.c.}$$

**Symmetry breaking**

$$Y_D = \Omega(\mathbf{3})\,R_{ij}(\theta_L)\,\mathrm{diag}(y_1, y_2, y_3)\,P_{kl}^{ij}\,R_{kl}(-\theta_R)\,\Omega(\mathbf{3}')^\dagger$$

CH/Molinaro ('16)

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

**Neutral lepton sector**

$$Y_D = \Omega(\mathbf{3}) R_{ij}(\theta_L) \, \mathrm{diag}(y_1, y_2, y_3) P_{kl}^{ij} R_{kl}(-\theta_R) \Omega(\mathbf{3}')^\dagger$$

**In total five free real parameters corresponding to three light neutrino masses, one free parameter for lepton mixing and one free parameter related to RH neutrinos**

**Possible small symmetry breaking for RH neutrino masses**

$$\delta M_R = \kappa \, M \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

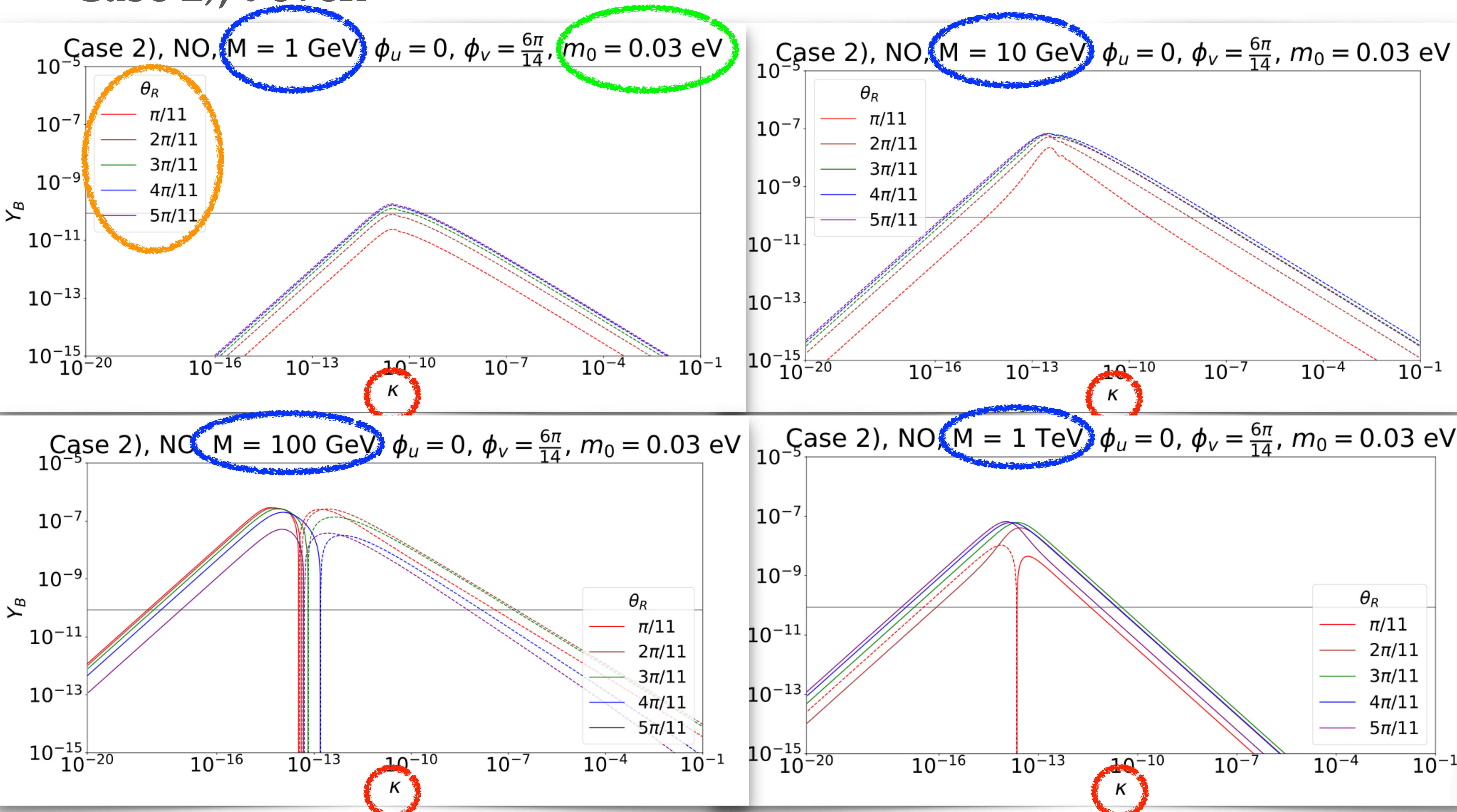
$$M_1 = M\,(1 + 2\,\kappa) \quad \text{and} \quad M_2 = M_3 = M\,(1 - \kappa)$$

Often needed for generating correct amount of BAU.

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

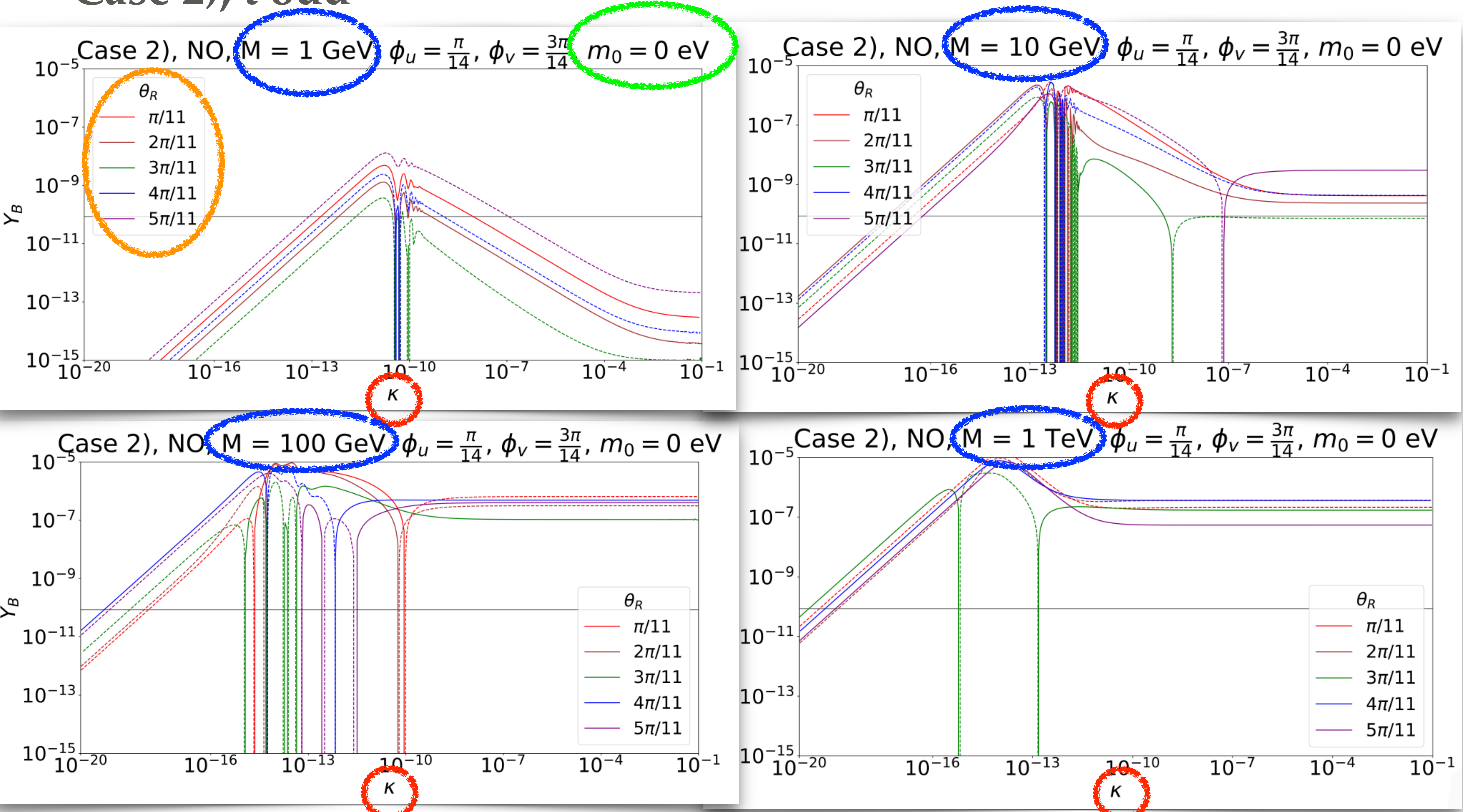
## Case 2), t even



# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

## Case 2), t odd



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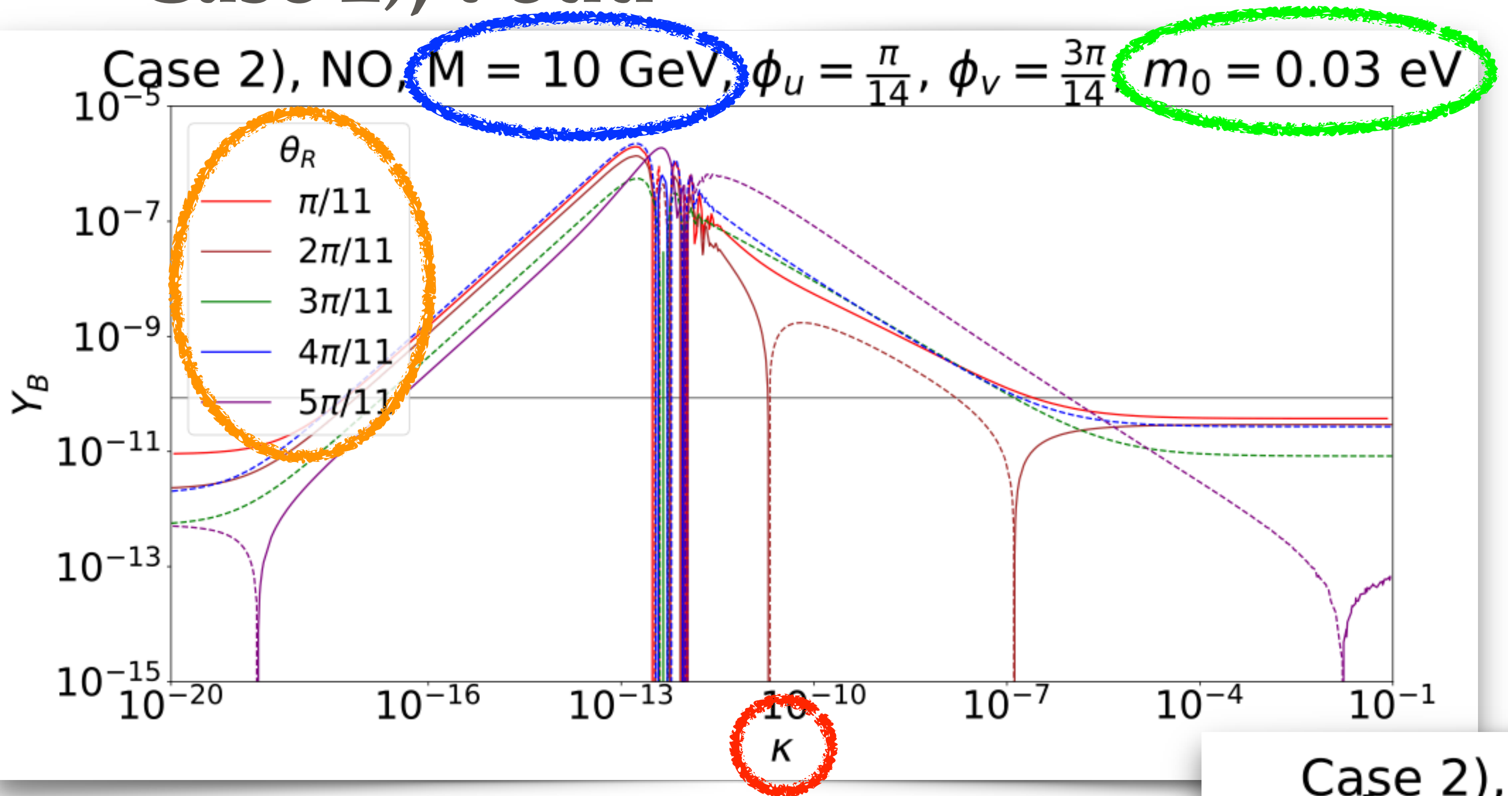


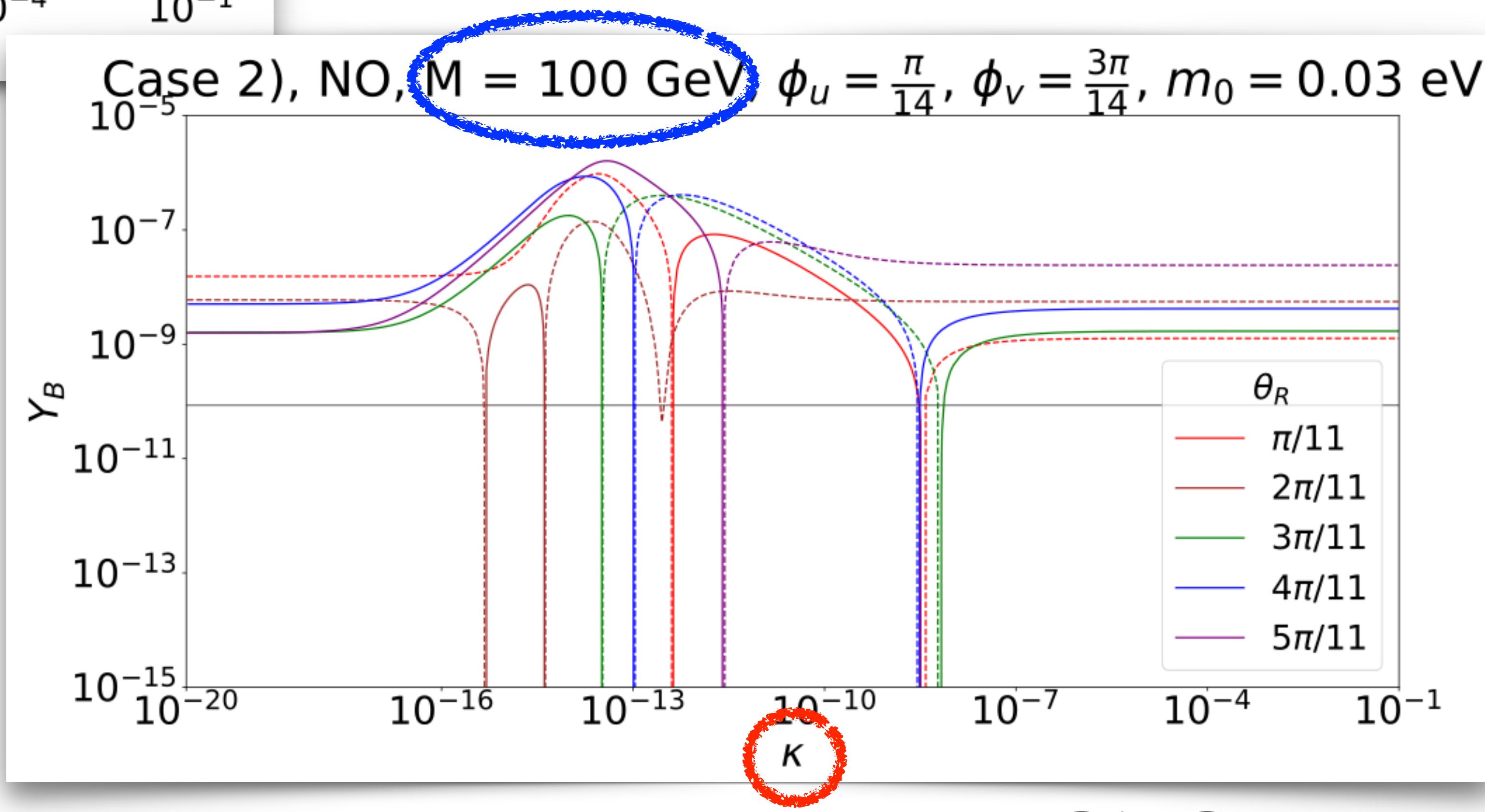

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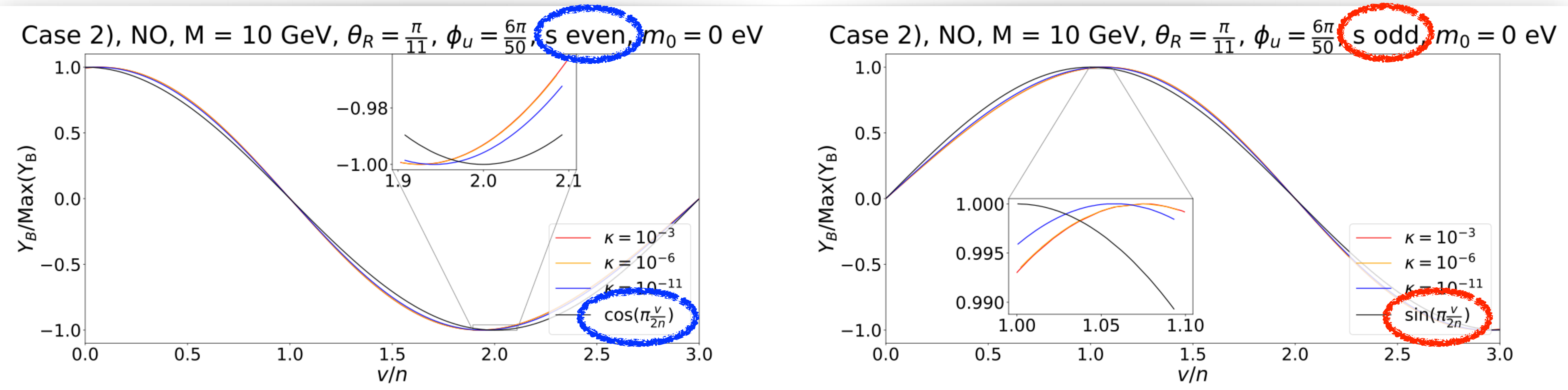


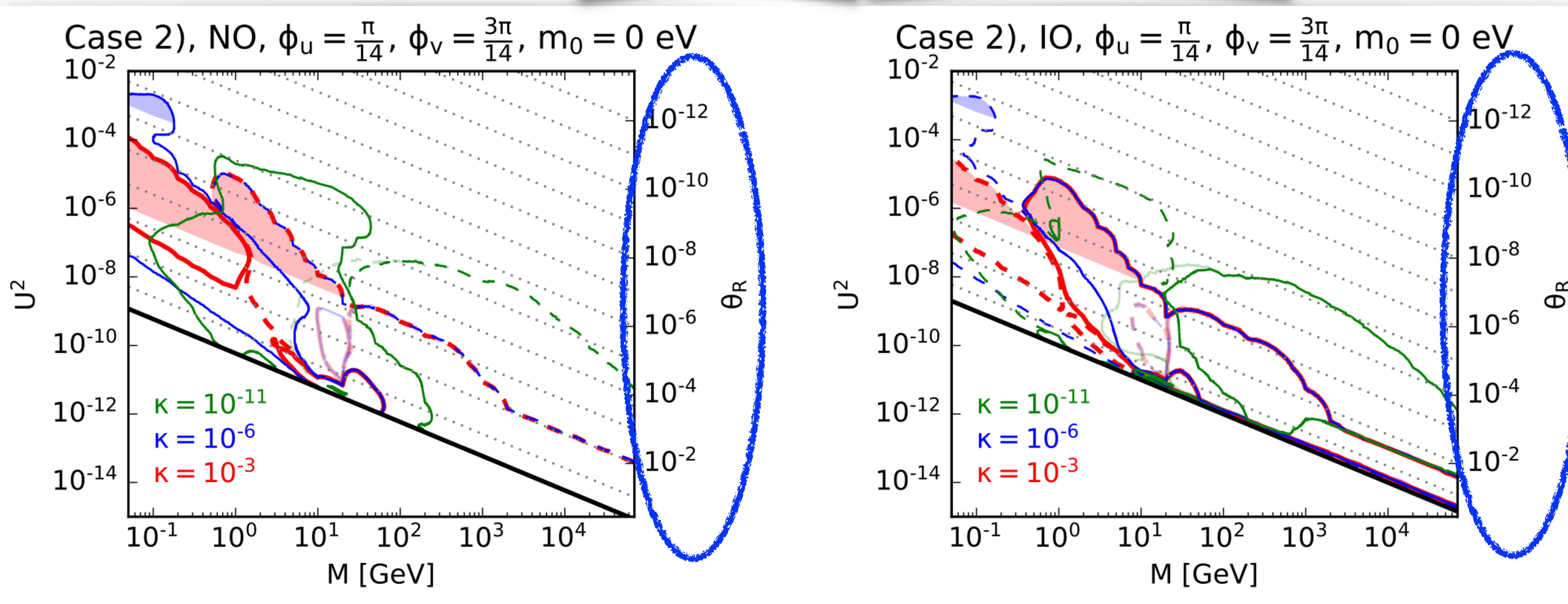

Majorana phase $\alpha$ fulfils $\quad |\sin\alpha| \approx |\sin\left(\frac{\pi\, v}{n}\right)|$

[Remember $\quad \sin\left(\frac{\pi\, v}{n}\right) = 2\cos\left(\frac{\pi\, v}{2\, n}\right)\sin\left(\frac{\pi\, v}{2\, n}\right)$]

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

**Case 2), t odd** $m_1 = 0$ $m_2 = \frac{y_2^2 \langle H \rangle^2}{M}$ $m_3 = \frac{y_3^2 \langle H \rangle^2}{M} |\sin 2\theta_R|$ (strong NO)




Special values of $\theta_R$ are not (always) a tuning, but related to enhanced residual symmetry, i.e. check $Y_D^\dagger Y_D$.

## Summary and Outlook

- Flavour and CP symmetries are useful for understanding fermion mixing and potentially also fermion masses
- They also have considerable effects on other observables in extensions of the SM
- Example with interesting applications:
  Low-scale seesaw mechanism with **strongly degenerate** RH neutrino masses and generation of BAU possibly **correlated with** low energy CP phases
  Study of phenomenology of heavy neutral leptons

- One can think about embedding in larger framework
- Obviously flavour and CP symmetries are applied to many more extensions of the SM

Many thanks for your attention!

# Back-up slides

# Flavour and CP symmetries

**Breaking of symmetries**

Feruglio/CH/Ziegler ('12)

$G_f$ and CP

↙ ↘

charged leptons — neutrinos

$G_e$ — $G_\nu = Z_2 \times$ CP

$U_e$ — $U_\nu = \Omega_\nu R(\theta) K_\nu$

↘ ↙

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Minimal choice $Z_3$

3 different masses

Choice $Z_2$ and CP

Majorana masses

Free parameter

Size of masses not explained

CH/Meroni/Molinaro ('14)

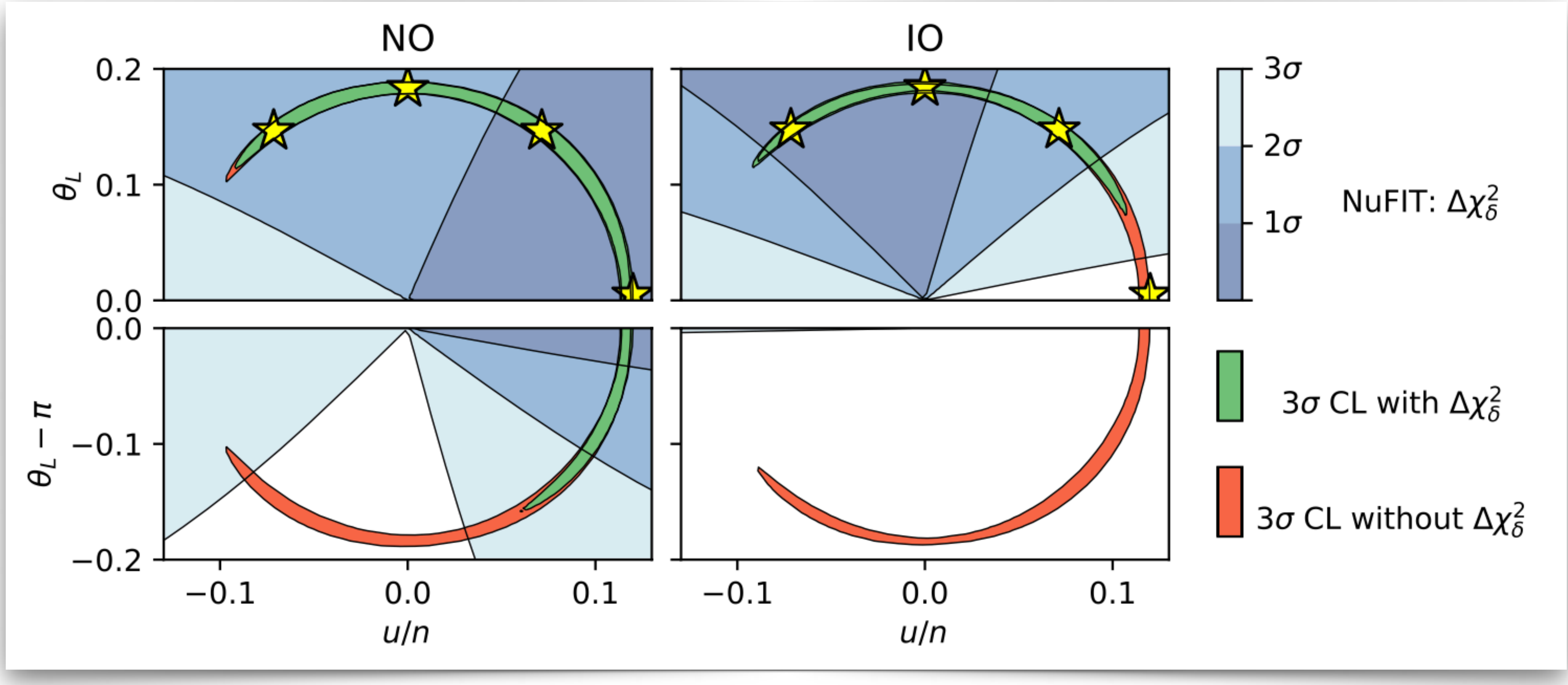
Result: four different types of mixing patterns with different properties

**Case 1) Case 2) Case 3 a) Case 3 b.1)**

# Flavour and CP symmetries

## Case 2)

[M. Drewes, Y. Georis, CH, J. Klaric]




$v$ relevant mainly for Majorana phase $\alpha$

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

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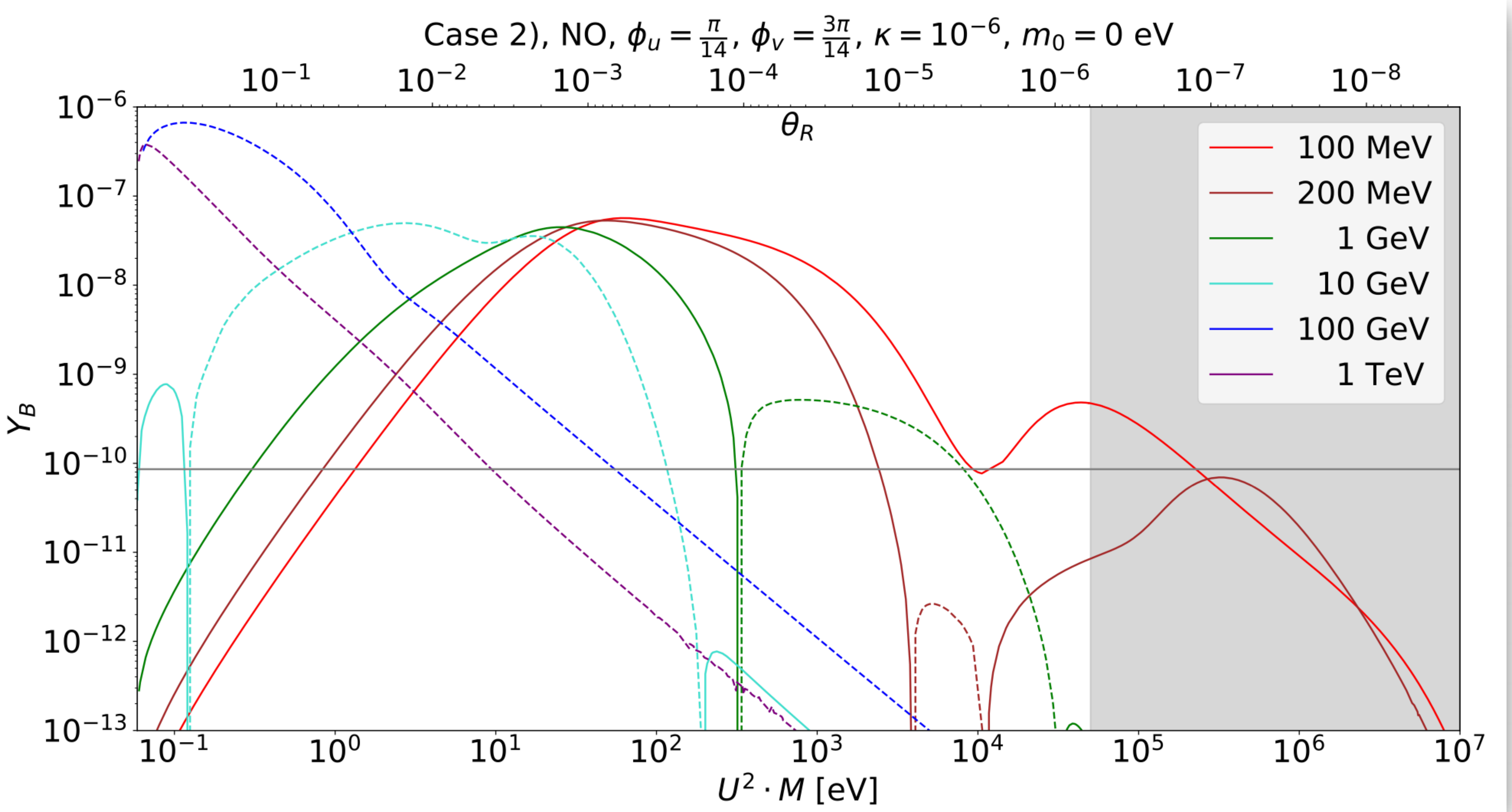

# Example: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric]

## Overview over results

| Type of mixing pattern | BAU non-zero for $\kappa = 0$? | BAU non-zero for large $\kappa$? | Large total mixing angle $U^2$ possible? |
|---|---|---|---|
| Case 1) | No, see Fig. 4 | Yes, see Fig. 4 | Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 9 |
| Case 2), $t$ even | No, see Fig. 12 | No, see Fig. 12 | No |
| Case 2), $t$ odd | Yes, for $m_0 \neq 0$ see Fig. 17, plot (a) | Yes, see Fig. 16 | Yes, for $\sin 2\,\theta_R \approx 0$ see Fig. 19 |
| Case 3 b.1), $m$ and $s$ even | No, see Fig. 20 | No, see Fig. 20 | No |
| Case 3 b.1), $m$ even, $s$ odd | Yes, see Fig. 22 except for strong IO | No, see Fig. 22 | Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 25 |
| Case 3 b.1), $m$ odd, $s$ even | Yes, see Fig. 26 except for strong IO | Yes, see Fig. 26 | Yes, for $\cos 2\,\theta_R \approx 0$ |
| Case 3 b.1), $m$ and $s$ odd | No | No | No |