Phenomenology of the Weinberg 3HDM potential

CP violation

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also borrows from R. Plantey et al, EPJC, 2023

Weinberg 3HDM

S. Weinberg, 1976 (Notation of Ivanov and Nishi):

3 SU(2) doublets:

Z_2 X Z_2 symmetry

$$V = V_2 + V_4, \quad \text{with} \quad V_4 = V_0 + V_{\rm ph},$$

$$\begin{aligned} V_2 &= -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)], \\ V_0 &= \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{22}(\phi_2^\dagger\phi_2)^2 + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ &\quad + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ &\quad + \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

$$V_{\rm ph} = \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{ h.c.}$$

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Observation:

Disregarding the phase-dependent part: Z_2 X Z_2 symmetry

$$V = V_2 + V_4, \quad \text{with} \quad V_4 = V_0 + \cancel{V_{\text{ph}}},$$

$$\begin{aligned} V_2 &= -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)], \\ V_0 &= \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{22}(\phi_2^\dagger\phi_2)^2 + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ &\quad + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ &\quad + \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

the potential has two U(1) symmetries

(will return to this point)

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There are different ways of solving the minimization equations:

Solution 1 (CPC):

$$v_i = 0, \quad v_j = 0, \quad m_{kk} = \lambda_{kk} v_k^2.$$

Solution 2 (CPV):

$$v_i = 0, \quad \lambda_i = 0,$$
$$m_{jj} = \tfrac{1}{2} v_k^2 \left(\lambda'_{jk} + \lambda_{jk}\right) + \lambda_{jj} v_j^2, \quad m_{kk} = \tfrac{1}{2} v_j^2 \left(\lambda'_{jk} + \lambda_{jk}\right) + \lambda_{kk} v_k^2.$$

This solution has a $\mathbb{Z}_2$ symmetry preserved by the vacuum. violates CP

Solution 3 (CPC):

$$v_i = 0, \quad \sin(\theta_k - \theta_j) = 0,$$
$$m_{jj} = \tfrac{1}{2} v_k^2 \left(\lambda'_{jk} + \lambda_{jk} + 2\lambda_i\right) + \lambda_{jj} v_j^2, \quad m_{kk} = \tfrac{1}{2} v_j^2 \left(\lambda'_{jk} + \lambda_{jk} + 2\lambda_i\right) + \lambda_{kk} v_k^2$$

...

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...

Solution 12 (CPV):

$$m_{11} = \frac{1}{2}\left(v_2^2\,(\lambda'_{12} + \lambda_{12}) + v_3^2\,(\lambda'_{13} + \lambda_{13}) + \frac{2\lambda_1 v_2^2 v_3^2 \sin^2 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_2) \sin 2(\theta_1-\theta_3)} + 2\lambda_{11}v_1^2\right)$$

$$m_{22} = \frac{1}{2}\,(v_1^2\,(\lambda'_{12} + \lambda_{12}) + v_3^2\,(\lambda'_{23} + \lambda_{23}) + 2\lambda_{22}v_2^2) + \lambda_1 v_3^2 \frac{\sin 2(\theta_1-\theta_3)}{\sin 2(\theta_1-\theta_2)},$$

$$m_{33} = \frac{1}{2}\,(v_1^2\,(\lambda'_{13} + \lambda_{13}) + v_2^2\,(\lambda'_{23} + \lambda_{23}) + 2\lambda_{33}v_3^2) + \lambda_1 v_2^2 \frac{\sin 2(\theta_1-\theta_2)}{\sin 2(\theta_1-\theta_3)},$$

$$\lambda_3 = \frac{\lambda_1 v_3^2 \sin 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_2)}, \qquad \lambda_2 = -\frac{\lambda_1 v_2^2 \sin 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_3)}.$$ violates CP

The latter is the solution identified by Branco and studied in our earlier work

R. Plantey et al, EPJC, 2023

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General case:

Minimization conditions: 3 moduli and 2 phases

May express m_{ii} in terms of λs (3 conditions)

May relate λ_2 and λ_3 to λ_1 (2 conditions)

Consequence:

Mass-squared matrices are homogeneous in λs,
i.e., masses are bounded
by the perturbativity constraint on λs

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Our work:

Real potential, all three vevs non-zero, complex,
i.e., spontaneous CP violation.

Note that in the limit when phase-dependent
terms λ_1, λ_2, λ_3 vanish,
the potential has two U(1) symmetries

These break when vevs are non-zero,
$\Rightarrow$ 2 Goldstone bosons

Turn on U(1)-violating terms,
"would-be Goldstone bosons" acquire mass

However, in order to have an "almost SM-like"
$WWH_{\rm SM}$ coupling, must be close to U(1)$\times$ U(1) symmetry,
$\Rightarrow$ two light states that have "large CP-odd content"

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Remainder: 2 intertwined stories

The potential tends to yield one or two states below 125 GeV

that have a significant CP-odd admixture

CP-violation in specific processes

3HDM Gauge couplings

Cubic gauge-gauge-scalar part:

VVS (vector-vector-scalar)

$$\mathcal{L}_{VVh} = \left(g m_W W_\mu^+ W^{\mu-} + \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu \right) \sum_{i=1}^{5} \frac{e_i}{v} h_i$$

$$m_W = \tfrac{1}{2} g v$$

For the cubic gauge-scalar-scalar terms, we find

VSS

VSS+

$$\mathcal{L}_{Vhh} = -\frac{g}{2\cos\theta_W} \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{\lambda_{ij}}{v} (h_i \overleftrightarrow{\partial}_\mu h_j) Z^\mu + \frac{g}{2} \sum_{i=1}^{5} \sum_{k=1}^{2} \left[\frac{f_{ki}}{v} (h_k^+ \overleftrightarrow{\partial}_\mu h_i) W^{\mu-} + \text{h.c.} \right]$$

$$+ \left(ieA^\mu + \frac{ig\cos 2\theta_W}{2\cos\theta_W} Z^\mu \right) \sum_{k=1}^{2} (h_k^+ \overleftrightarrow{\partial}_\mu h_k^-)$$

In the 2HDM, all related

3HDM Gauge couplings

Cubic gauge-gauge-scalar part:

VVS

$$\mathcal{L}_{VVh} = \left(g m_W W_\mu^+ W^{\mu-} + \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu \right) \sum_{i=1}^{5} \frac{e_i}{v} h_i$$

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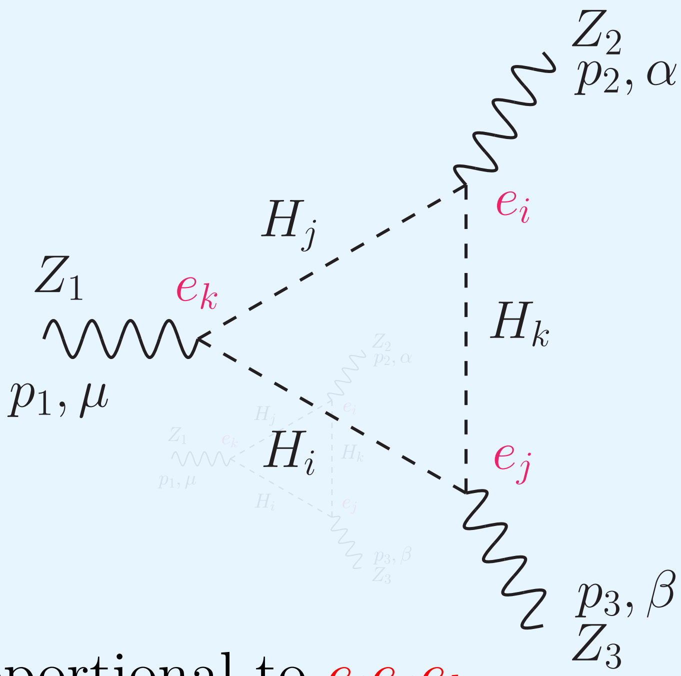
$$+ \left(ieA^\mu + \frac{ig\cos 2\theta_W}{2\cos\theta_W} Z^\mu \right) \sum_{k=1}^{2} (h_k^+ \overleftrightarrow{\partial}_\mu h_k^-)$$

In the 2HDM, all related

CP violation and alignment

In a 2HDM



contributions proportional to $e_i e_j e_k$

But these are constrained by unitarity: $e_i^2 + e_j^2 + e_k^2 = v^2$

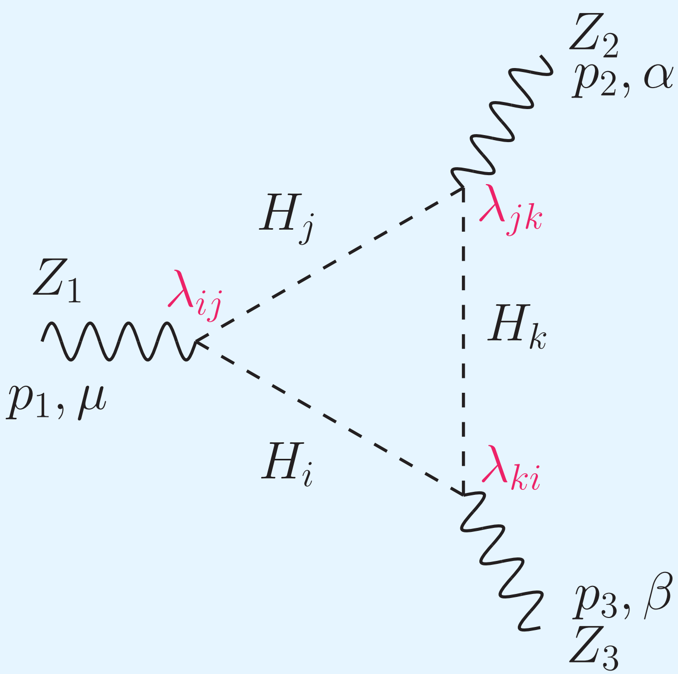
Alignment requires $\max(v_1, v_2, v_3) = v$

Thus, $e_i e_j e_k \to 0$

Alignment prohibits such contribution to CPV in a 2HDM

CP violation and alignment

In a 3HDM



contributions proportional to $\lambda_{ij}\lambda_{jk}\lambda_{ki}$

This does not vanish in the alignment limit!

Technical digression

Comments on the H_iH_jZ coupling

Coupling comes from kinetic part of Lagrangian, conserves CP
Z is odd, $\Rightarrow$ only non-zero coupling if H_iH_j has some CP-odd content
H_i and H_j need not be eigenstates of CP,
but must have different "mix" of even and odd

$$\lambda_{ij} = ZH_iH_j \quad \text{coupling}$$

In units of $g/(2\cos\theta_W)$, in a CP-conserving 2HDM:

$$\begin{aligned} \lambda_{HA} &= 1, \quad ZHA \text{ coupling (even and odd)} & \leq 1 \\ \lambda_{hA} &= 0, \quad ZHh \text{ coupling (both even)} & \leq 1 \\ \lambda_{hA} &= 0, \quad ZhA \text{ coupling (even and odd)} & \leq 1 \end{aligned}$$

Scan

Scan philosophy

- Parameter ranges: $v_i \in \{0, v\}$, subject to $v_1^2 + v_2^2 + v_3^2 = v^2$, $|\lambda_{ij}|, |\lambda_i| \in \{0, 4\pi\}$, $\theta_{2,3} \in \{-\pi, \pi\}$
- Diagonalize mass-squared matrix, determine h_i
- Identify SM candidate by $h_i WW$ coupling (only one possible solution for h_i)
- Rescale all λs such that $h_{\rm SM}$ at 125.25 GeV (mixing matrix unchanged)

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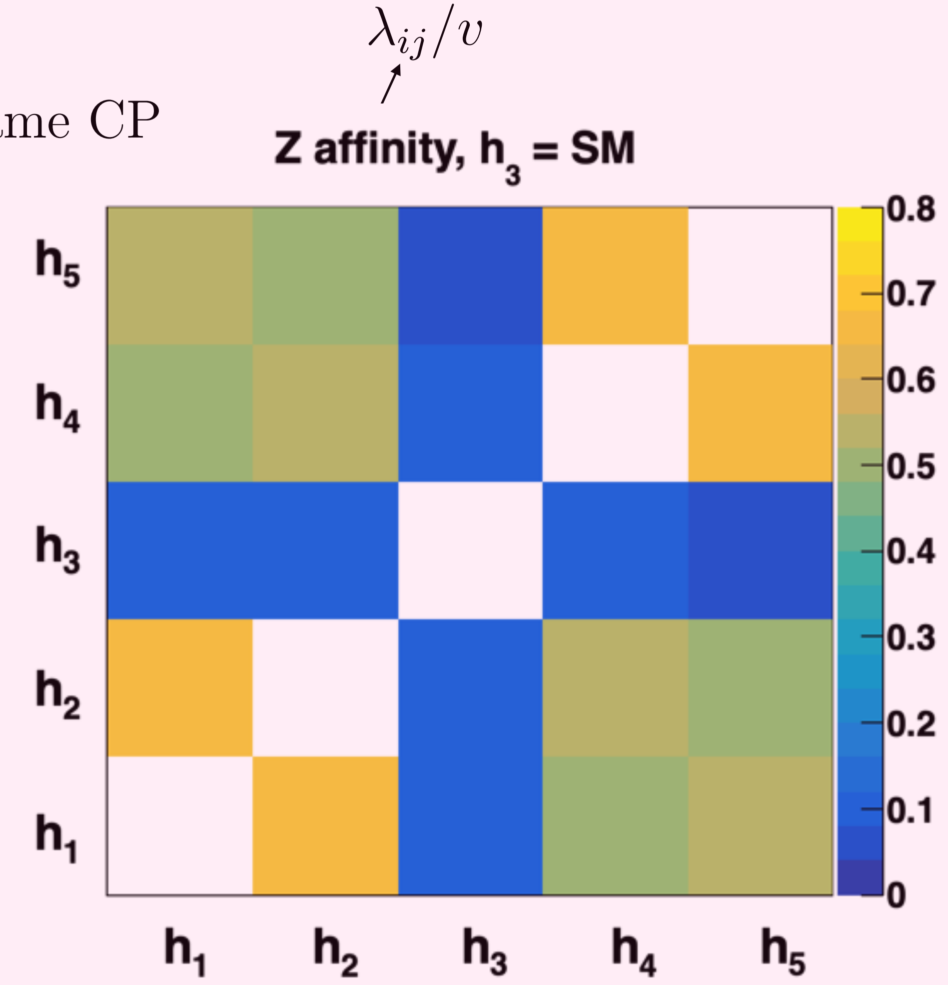
λ_{ij} is a measure of Zh_ih_j interaction
to be non-zero, h_ih_j can not have the same CP

h_3 aligned



2209.06499

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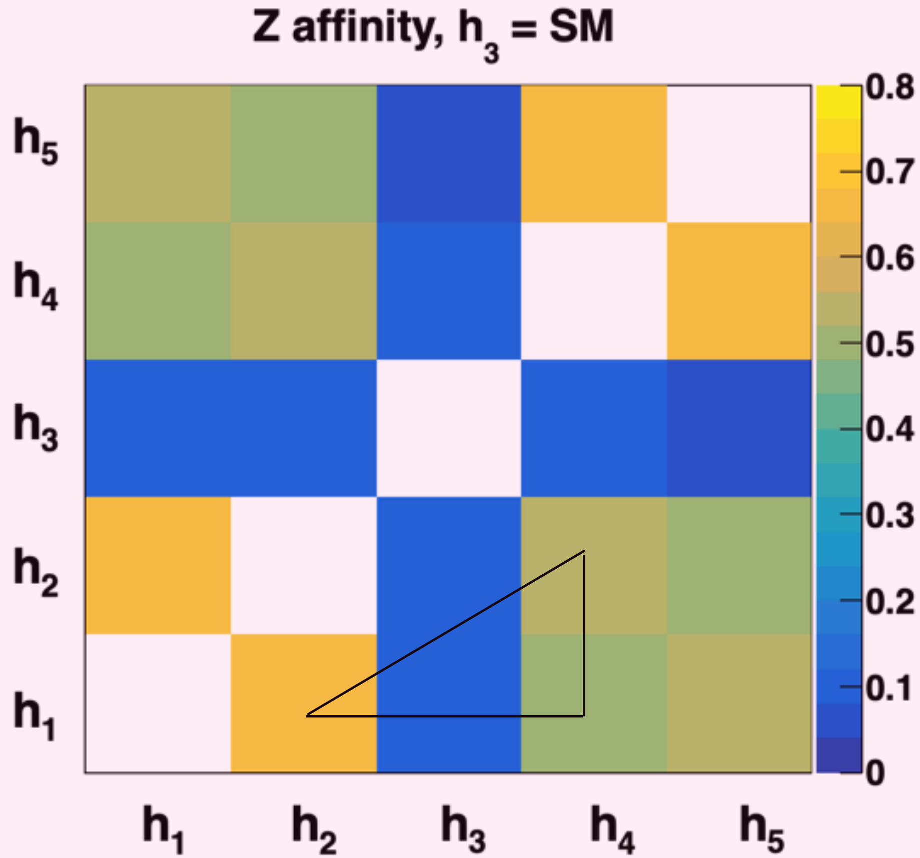
λ_{ij}/v



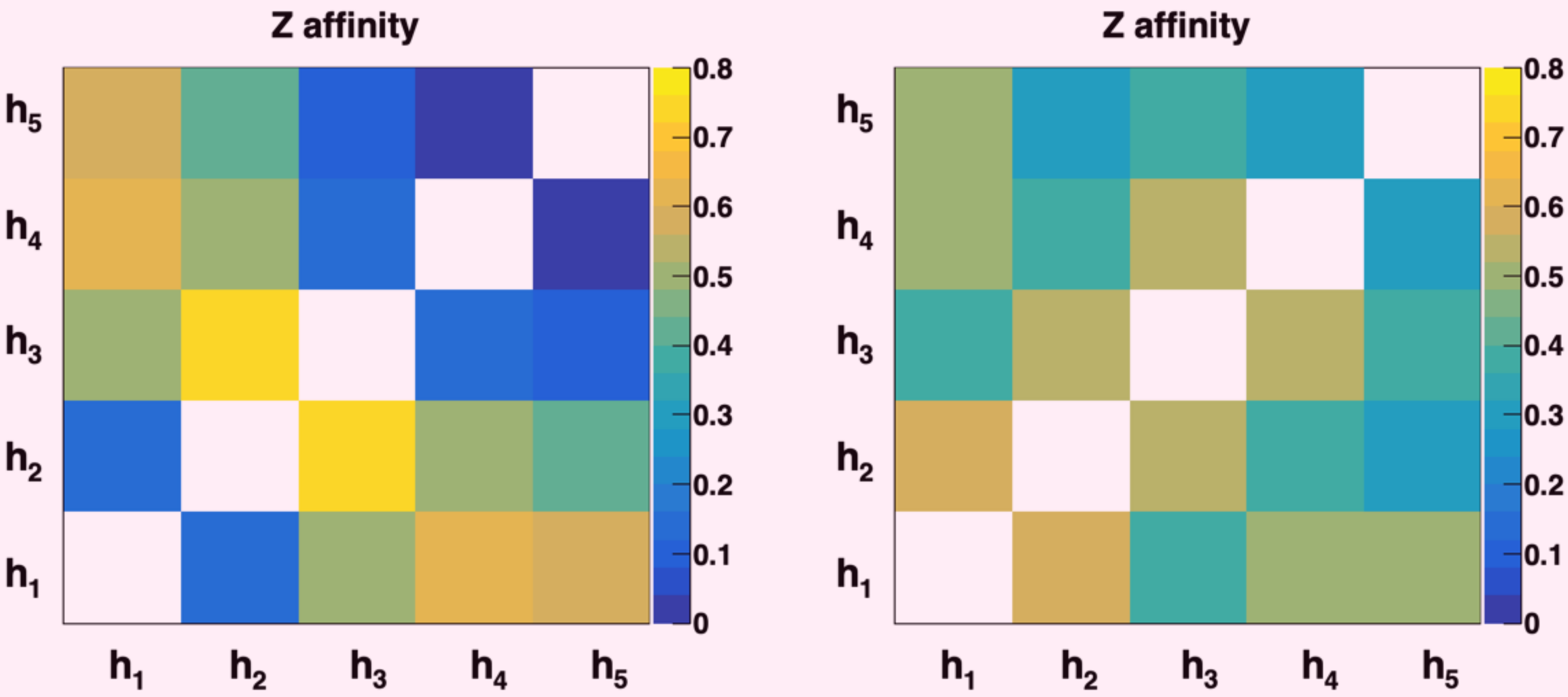
h_3 aligned

for example:

$$\lambda_{12}\lambda_{24}\lambda_{41} \neq 0$$

2209.06499

Weinberg



$$\max(|\lambda_1|, |\lambda_2|, |\lambda_3|) = 0.01$$

$$h_1,\ h_2 \sim \text{odd}, \quad h_3,\ h_4,\ h_5 \sim \text{even}$$

Figure from parameter scan in 2209.06499

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CP-violating invariants

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + \tfrac{1}{2} Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

$$\hat{V}_{ab} = \frac{v_a e^{i\theta_a}}{v} \frac{v_b e^{-i\theta_b}}{v}$$

$$\begin{aligned}
J_1 &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadf}Z_{edfg}Z_{gbhh}\},\\
J_2 &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadf}Z_{edfg}Z_{ghhb}\},\\
J_3 &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadf}Z_{egfd}Z_{gbhh}\},\\
J_4 &= \mathrm{Im}\{V_{ac}V_{bd}Z_{cedg}Z_{eafh}Z_{gbhf}\},\\
J_5 &= \mathrm{Im}\{V_{ac}V_{bd}Z_{cedg}Z_{ehfa}Z_{gfhb}\},\\
J_6 &= \mathrm{Im}\{V_{ac}V_{bd}Z_{cedf}Z_{eafg}Z_{gbhh}\},\\
J_7 &= \mathrm{Im}\{V_{ad}V_{be}V_{cf}Z_{daeh}Z_{fbgi}Z_{hcig}\},\\
J_8 &= \mathrm{Im}\{V_{ad}V_{be}V_{cf}Z_{daeh}Z_{figb}Z_{hgic}\},\\
J_9 &= \mathrm{Im}\{V_{ad}V_{be}V_{cf}Z_{daeg}Z_{fbgh}Z_{hcii}\},\\
J_{10} &= \mathrm{Im}\{V_{ad}V_{be}V_{cf}Z_{daeg}Z_{fhgi}Z_{hbic}\},\\
J_{11} &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadg}Z_{edff}Z_{gihh}Z_{ibjj}\},\\
J_{12} &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadg}Z_{effd}Z_{ghhi}Z_{ijjb}\},\\
J_{13} &= \mathrm{Im}\{V_{ac}V_{be}Z_{cadf}Z_{edfg}Z_{gihj}Z_{ibjh}\},\\
J_{14} &= \mathrm{Im}\{V_{ac}V_{bd}Z_{cedf}Z_{eafg}Z_{gihj}Z_{ibjh}\},\\
v^6 J_{15} &= \mathrm{Im}\{V_{ac}V_{bd}Y_{cf}Y_{dg}Y_{ea}Z_{fbge}\}.
\end{aligned}$$

If all J_i vanish, then CP is conserved

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Measures of CP-violation:

$$A_{\text{sum}} = \log_{10} \sum_{i=1}^{15} J_i^2, \qquad A_{\text{max}} = \log_{10}(\max_i J_i^2).$$

(grey) (blue)

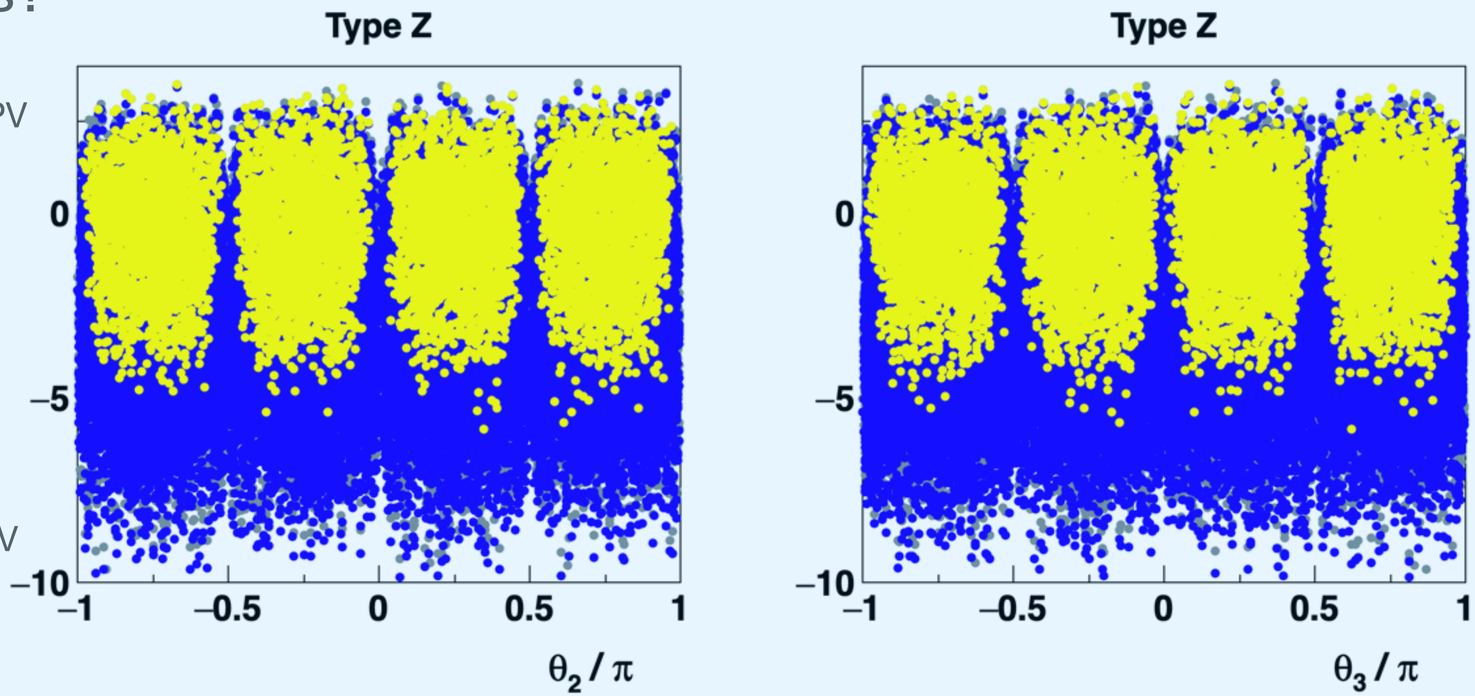
Related to phases of vevs?

Yellow: masses > 45 GeV

Yellow: no mass below 45 GeV

more CPV

less CPV



Weinberg 3HDM constraints

Basic:

- perturbativity, unitarity, bounded from below
- coupling WWh_{SM} within 3σ
- $h_{\text{SM}}\tau\bar{\tau}$ CP constraint (LHC)

Further:

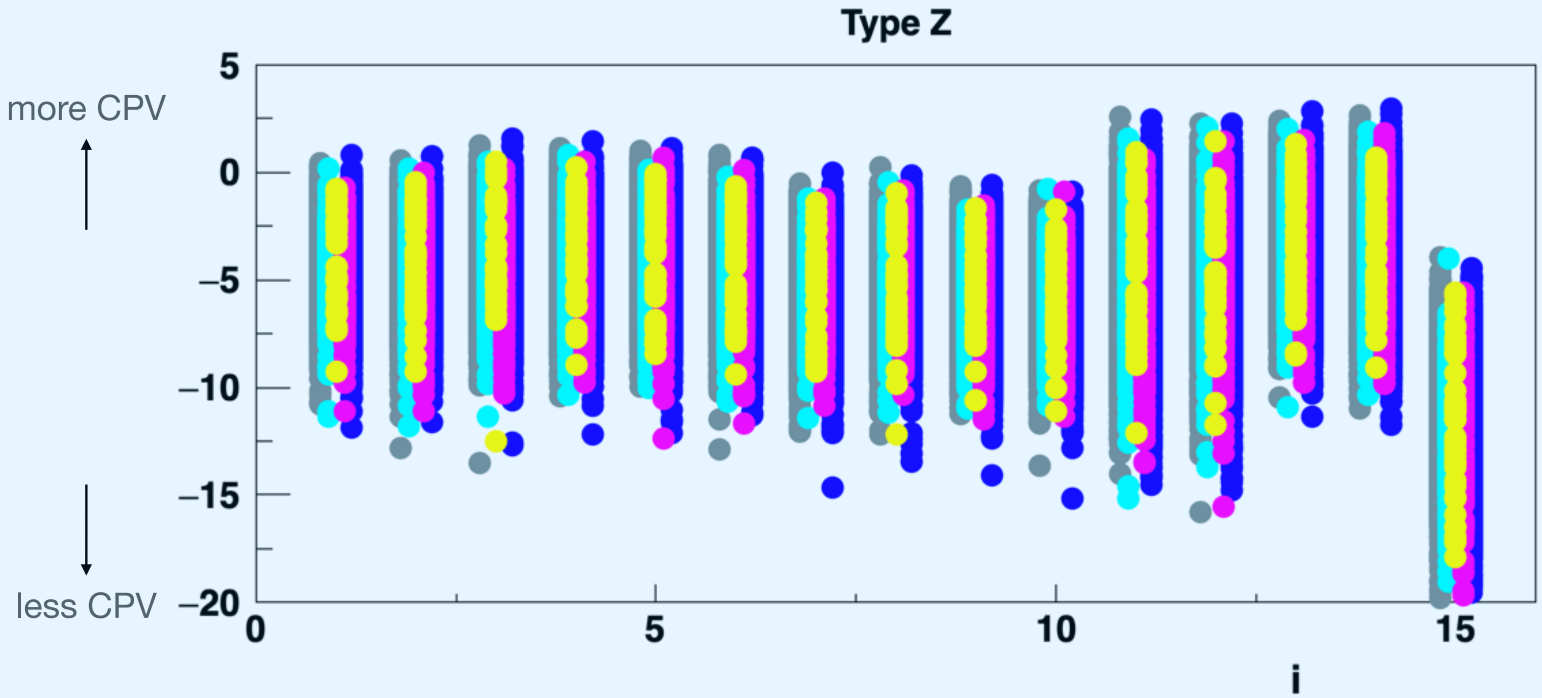
- S, T, U (electroweak precision observables)
- $h_{\text{SM}} \to \gamma\gamma$ signal strength, modified by charged scalars in loop
- $\bar{B} \to X_s\gamma$, *two* charged scalars, complex Wilson coefficients
- electron EDM, violates CP
- pruning low-mass states

Weinberg 3HDM

15 CPV invariants:

scatter plot of points surviving constraints

different colours:
different range of eEDM



conclusion?
systematics?

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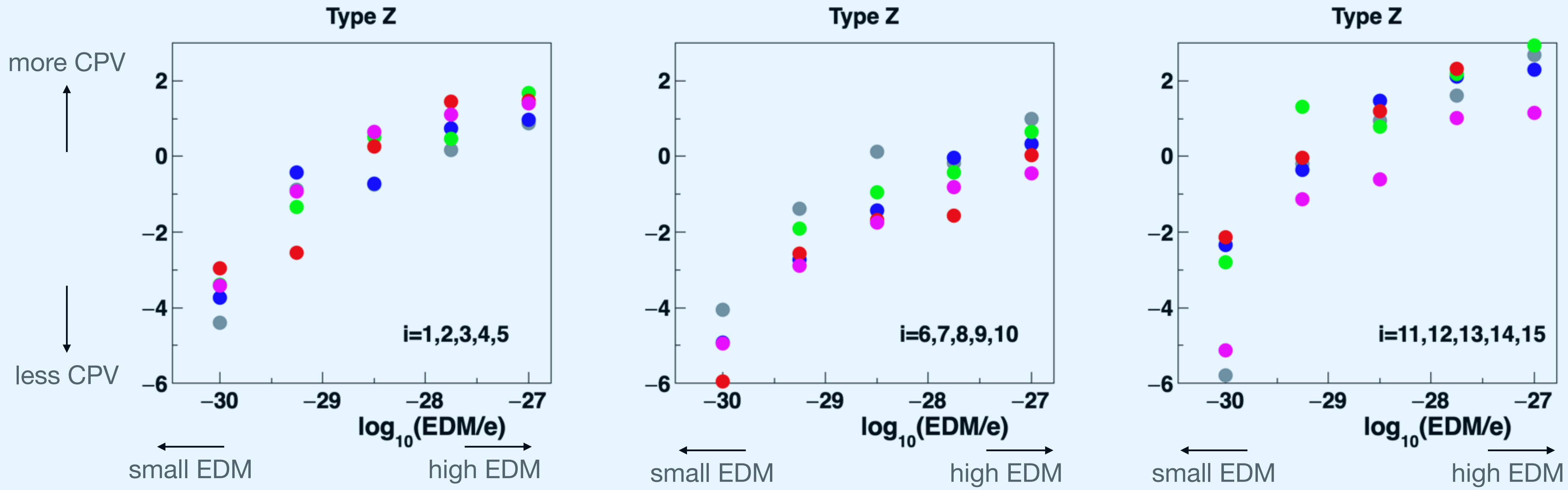
maxima of 15 CPV invariants vs eEDM



grey: $i = 1, 6, 11$; blue: $i = 2, 7, 12$; green: $i = 3, 8, 13$;
red: $i = 4, 9, 14$; purple: $i = 5, 10, 15$.

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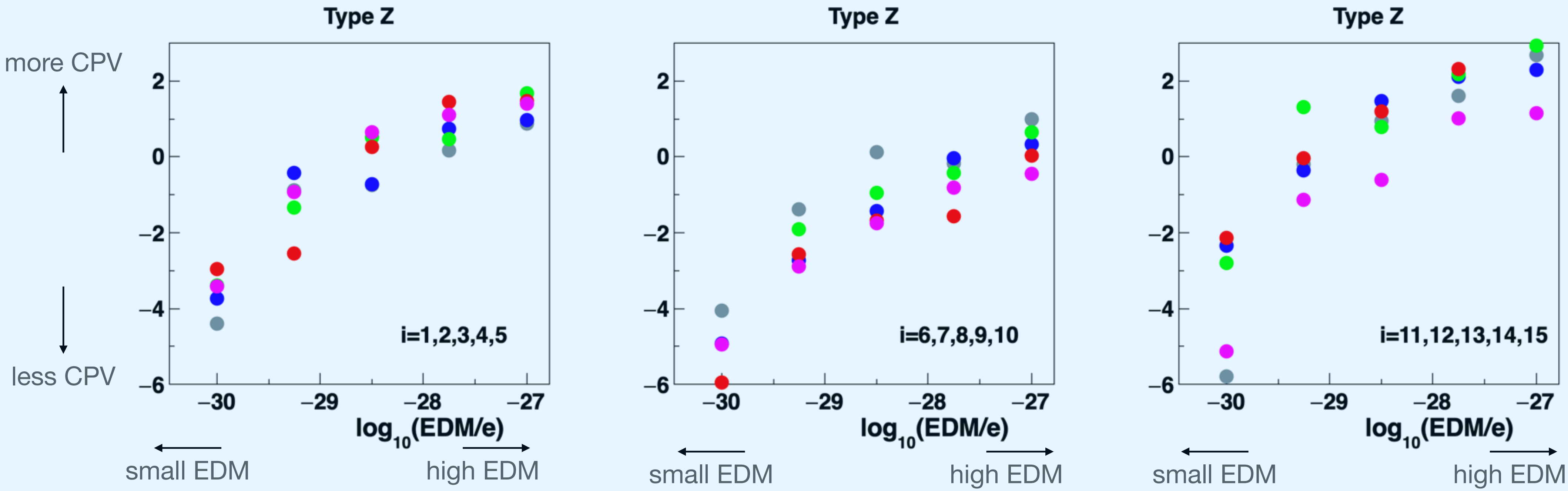
QUESTION: Why do all 15 invariants correlate with eEDM?

maxima of 15 CPV invariants vs eEDM



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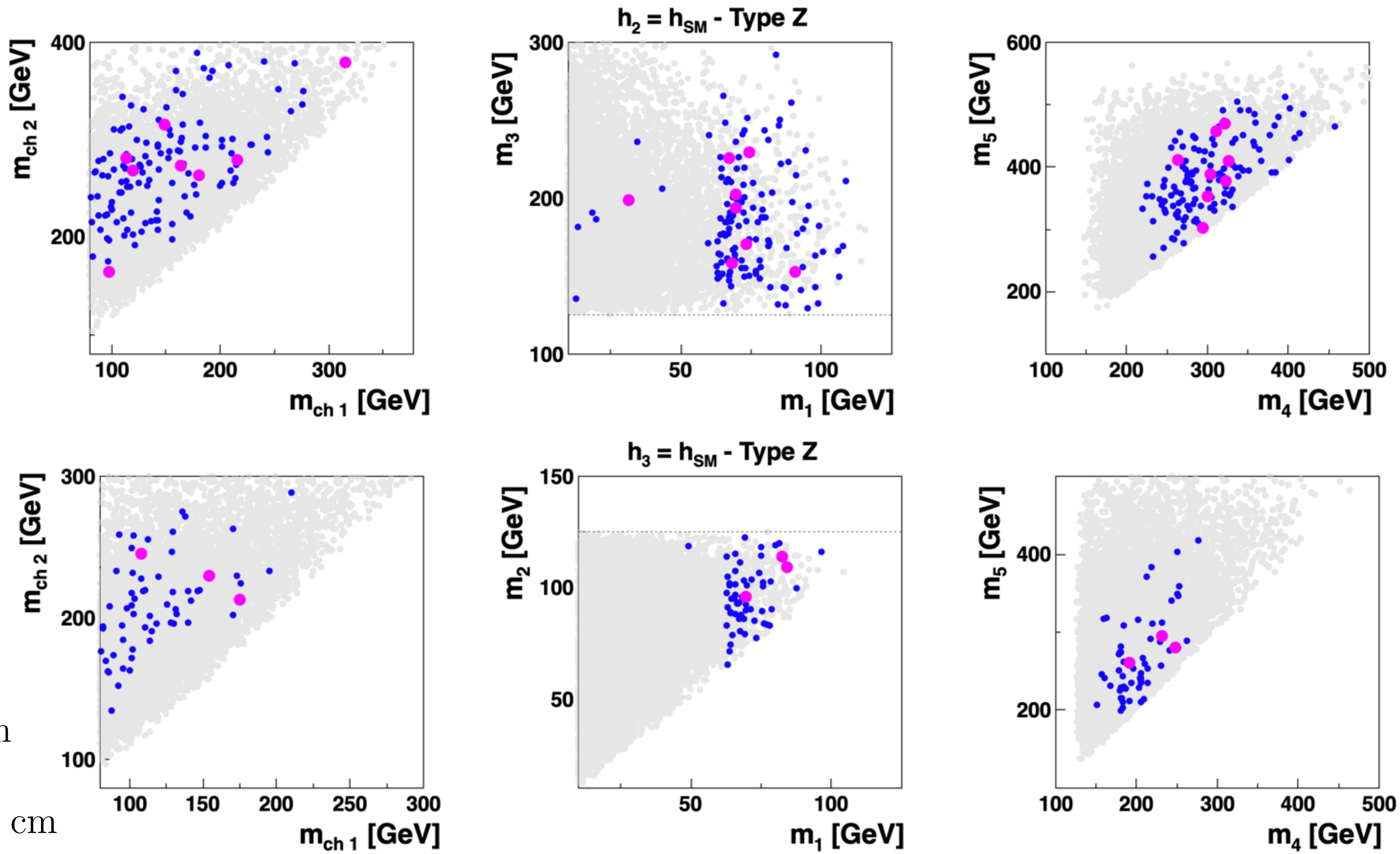
MASSES:



blue:
$\text{eEDM} < 10^{-27} e \cdot \text{cm}$
purple:
$\text{eEDM} < 5 \cdot 10^{-29} e \cdot \text{cm}$

Summary

- Additional states are light

- *If* an extra light state below $h_{\text{SM}} = 125$ GeV is discovered, and
  If this has a considerable coupling to $h_{\text{SM}}Z$,
  then a model based on the Weinberg potential
  would be in a stronger position than the 2HDM

- light states (h_1, h_2) would predominantly decay to $b\bar{b}$,
  to $b\bar{b}\tau\bar{\tau}$ or $b\bar{b}$ + invisible (from $h_2 \to h_1 Z$)

- Weinberg potential can accommodate neutral scalar at 95 GeV
  (and charged one at 130 GeV)

Summary

- but does not contribute (significantly) to $(g-2)_\mu$

sorry