Probing Parity-Violation in the Stochastic Gravitational Wave background in astrometry

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Content

- Brief review of nano Hertz Stochastic gravitational wave background
- Test Gravity in PTA

  Massive gravity Phys.Rev.D 104 (2021) 8, 084052 Q.Liang, M.Trodden

  Modification of dispersion relation 2304.02640 Q.Liang, M-X Lin, M.Trodden
- Parity-Violation signal in astrometry system

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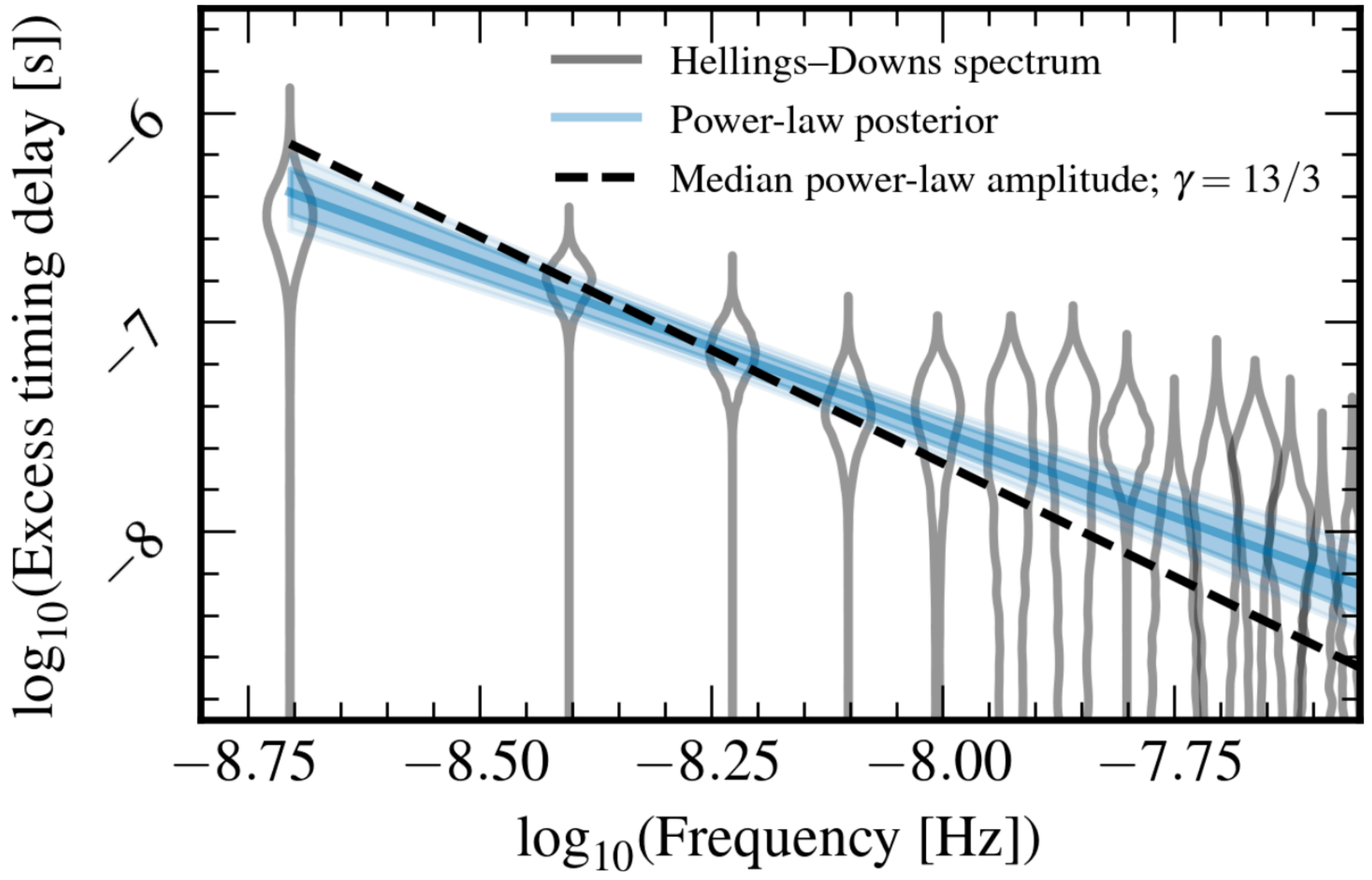
- Discussion

Nano Hertz SGWB

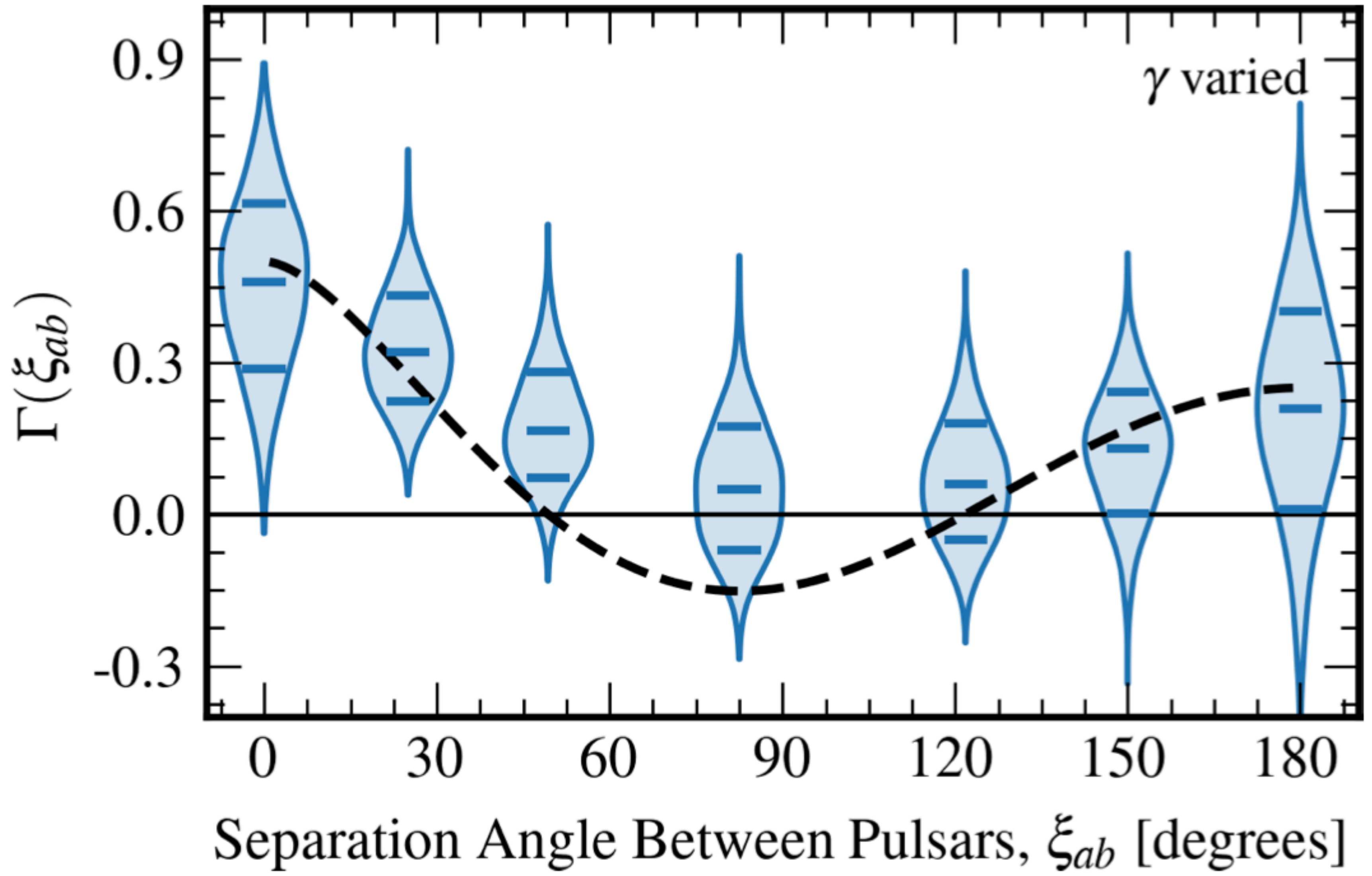
- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;

Nano Hertz SGWB

- Astrophysical Source: Supermassive Black Hole Binary;
- Cosmological Source: Primordial Gravitational Wave; Phase transition;
- PTA (pulsar timing array) collaboration claimed a detection in last July!



Power spectrum



Angular correlation

Credit: NANOGrav 15 yr result

What can we learn from this SGWB?

- Astro: origin of supermassive black hole formation & population rate… 2401.04161 2312.06756 2306.17021,2306.16222 2305.05955
- Early universe: different inflation scenarios, primordial gravitational waves… 2212.05594 2311.03391 2311.02065 2311.00741
- Defect: cosmic string, phase transition,…2306.17205 2304.04793 2304.02636
- Beyond Standard Model physics: dark matter, baryon number violation, string compactification … 2306.05389 2305.11775 2304.10084 …
- Modified Gravity: 2304.02640 Q.Liang, M-X, Lin, M. Trodden 2108.05344 Q.Liang, M. Trodden

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Power spectrum

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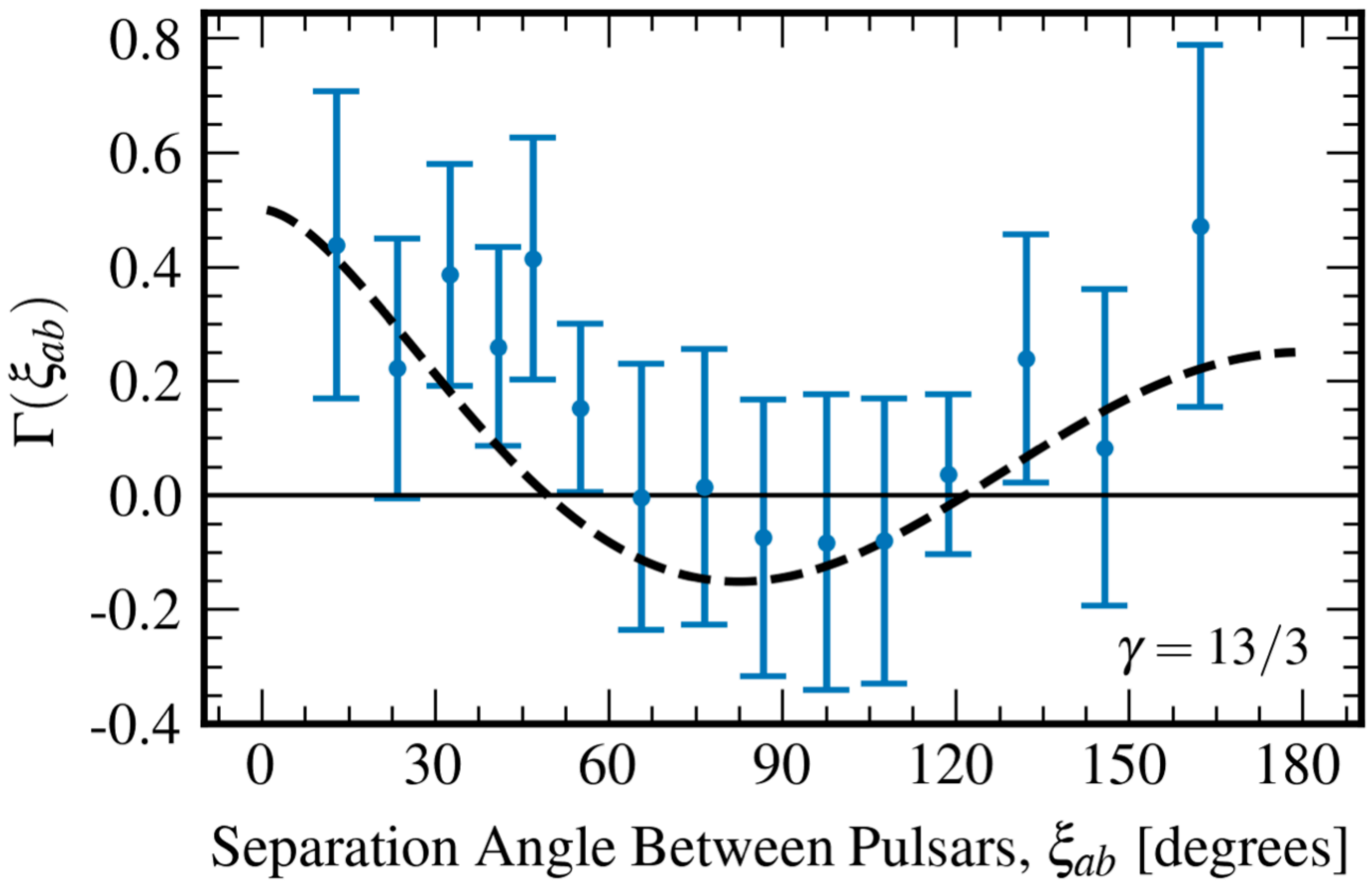
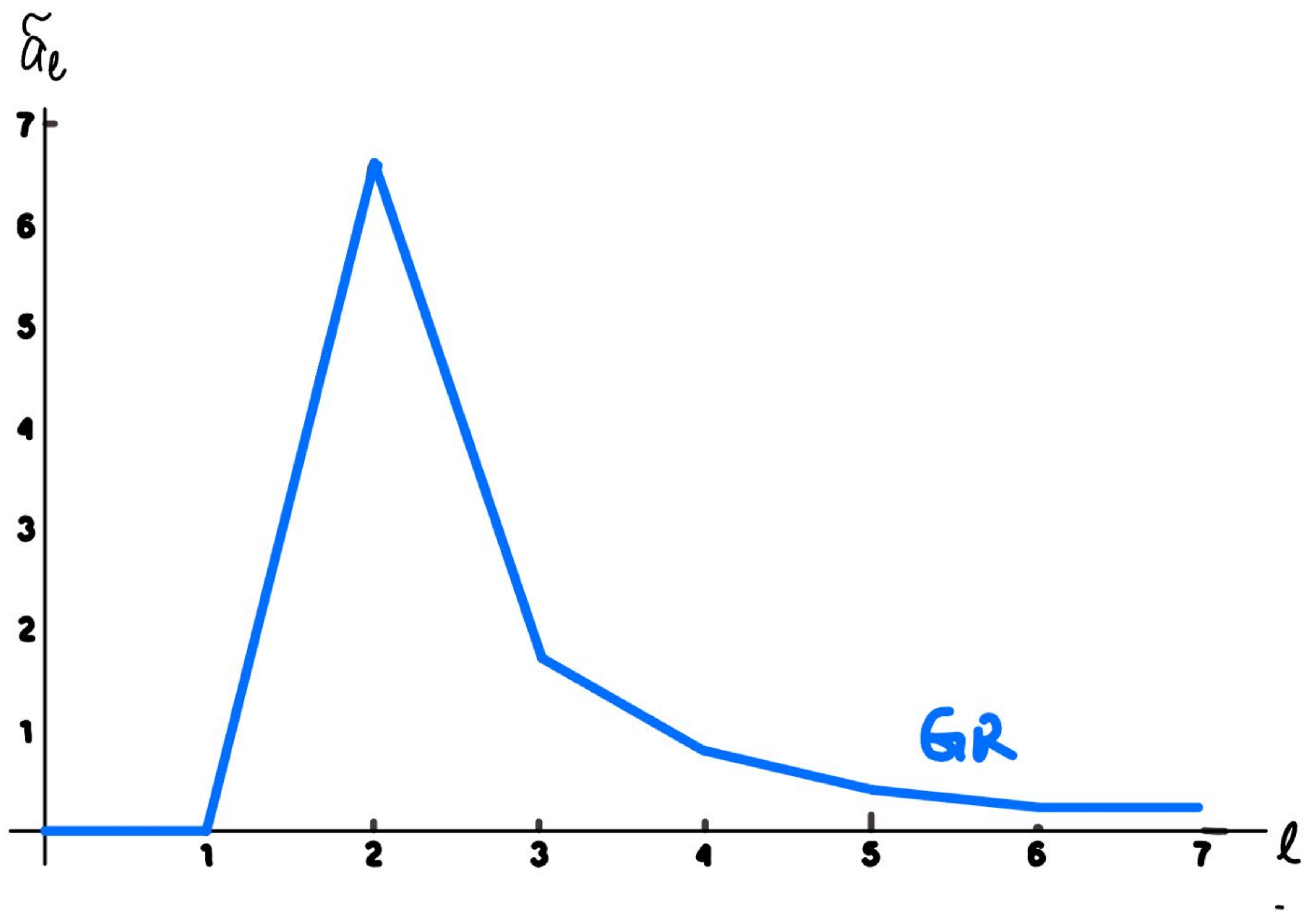
Angular correlation

Credit: NANOGrav 15 yr result

Overlap reduction function

- First detection (3 sigma) of Hellings-Downs curve! (NANOGrav 15 yrs)





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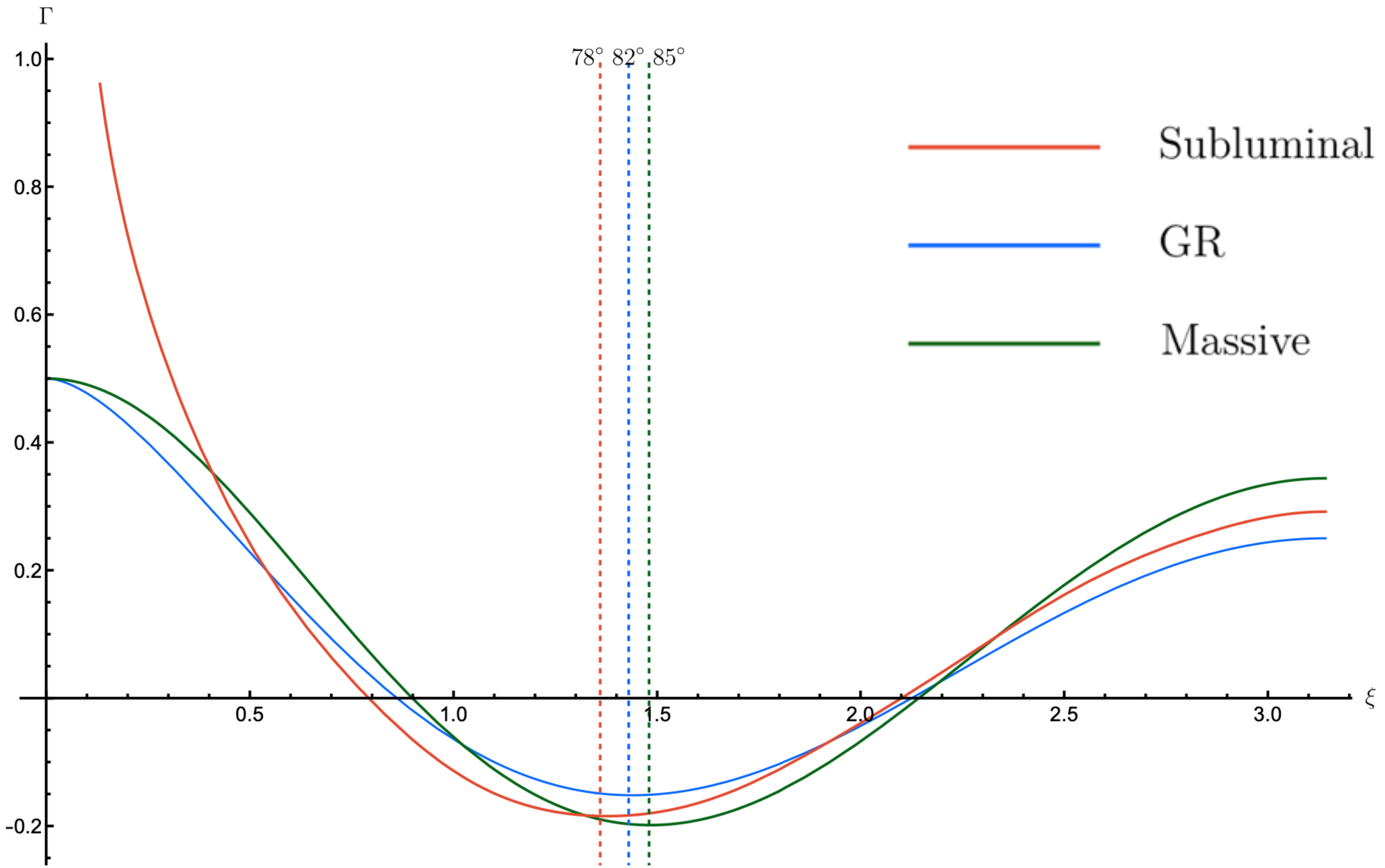
Modify the dispersion relation with plane wave assumption



$\omega = c_s k$

$v_{ph} < 1$ To left

$v_{ph} > 1$ To right

$\omega(k) = \sqrt{k^2 + m^2},$

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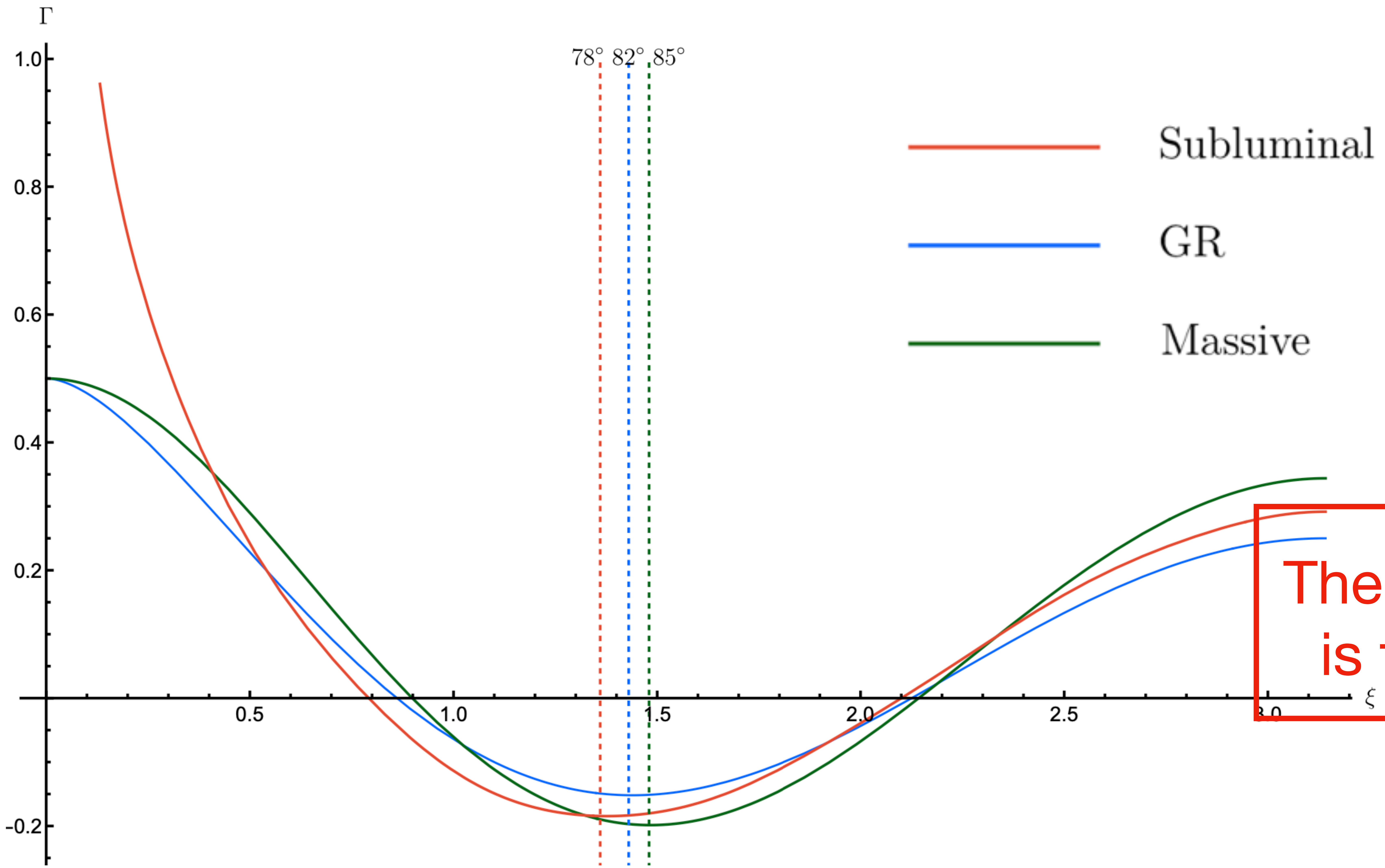

$$\omega = c_s k$$

$v_{ph} < 1$ To left

$v_{ph} > 1$ To right

$$\omega(k) = \sqrt{k^2 + m^2}$$

The important parameter is the phase velocity!!!

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Can PTA be sensitive to GW with parity information? i.e polarized GW, or Chern-Simon modified gravity?

No! PTA is not sensitive to parity information!

- With an isotropic background assumption, PTA cannot tell a left-moving GW coming from up to a right-moving GW from bottom.
- This is because PTA can only measure a scalar quantity: residue arrival time for each pulsar;

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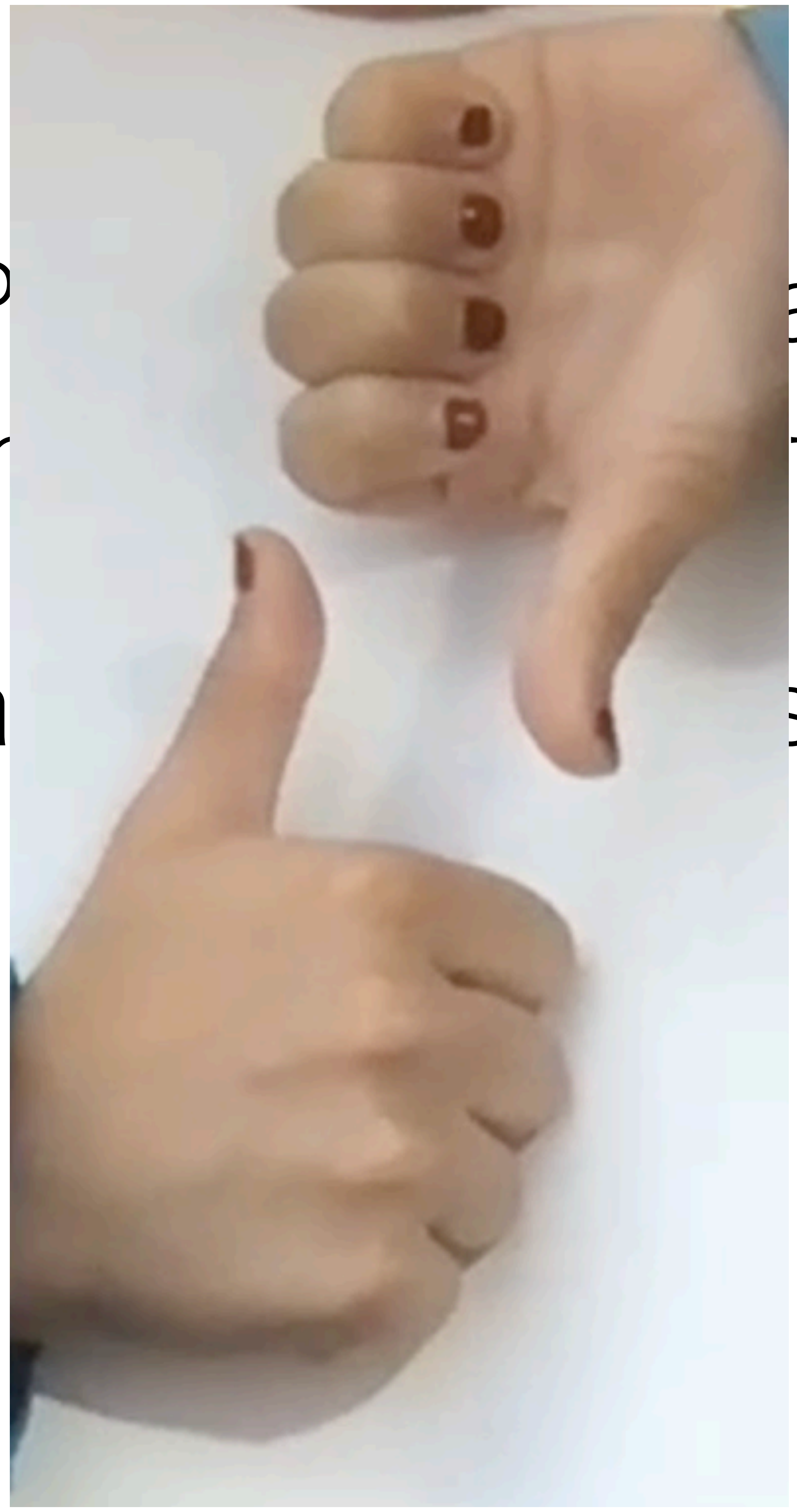

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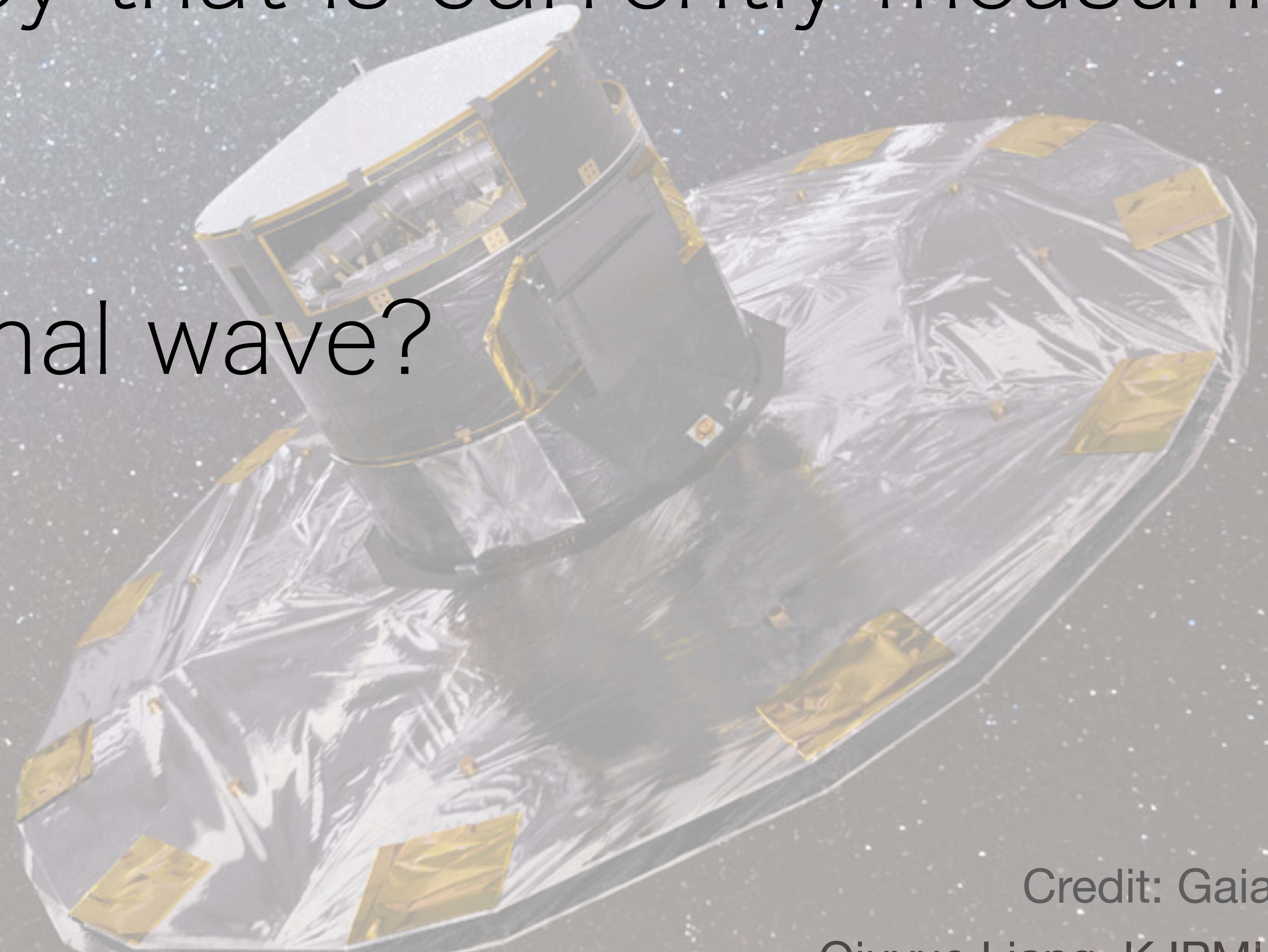
Astrometry!

What is astrometry?

- Astrometry is a precise measurement of the positions and movements of stars and other celestial bodies
- Gaia is one of the many astrometry survey that is currently measuring about1800 million stars in our milky way
- Exoplanet; Dark Matter profile; Gravitational wave?



Credit: Gaia

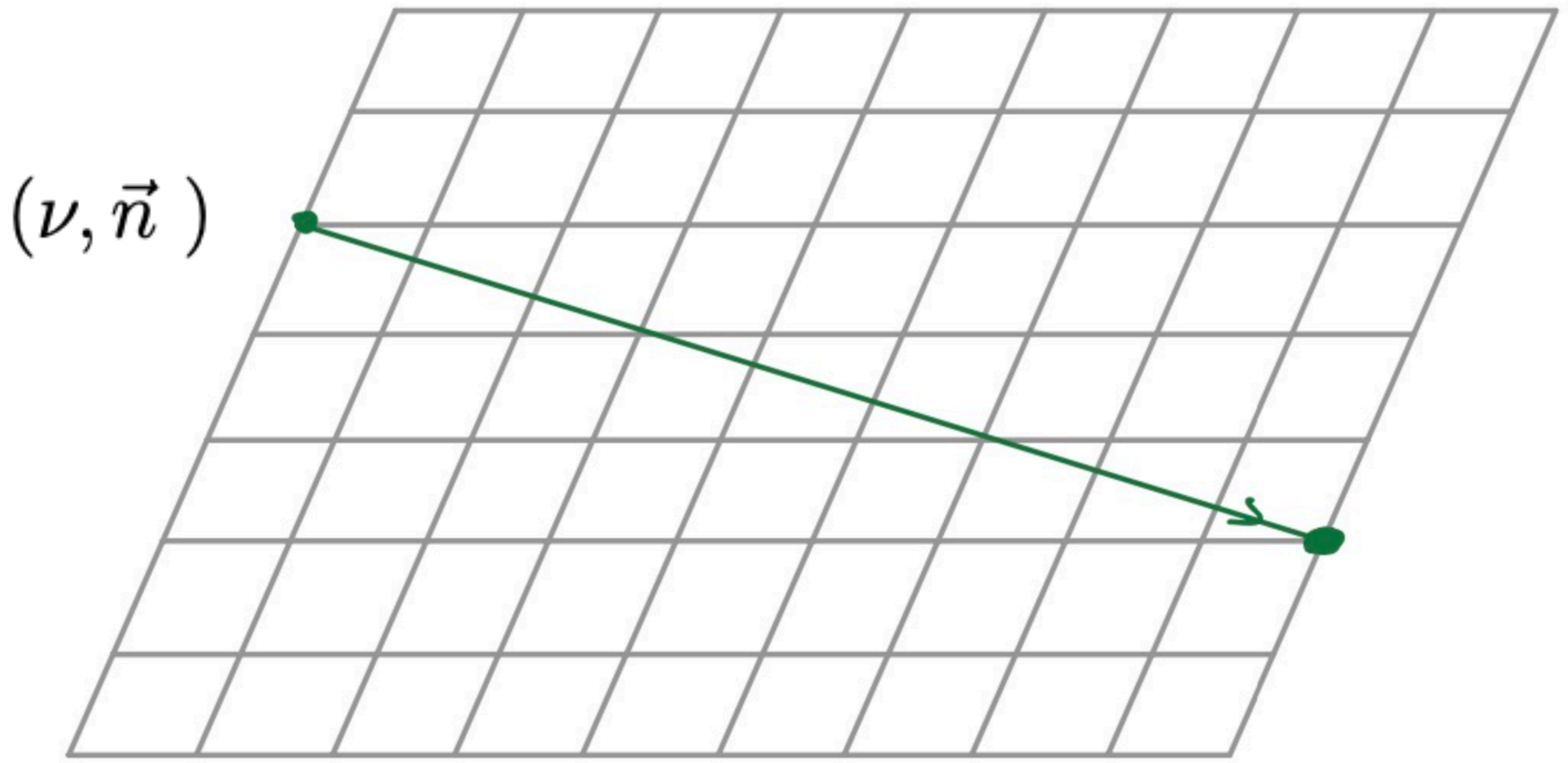
Using Astrometry to detect SGWB

$$\delta\nu(t,\hat{n}) = h_{IJ}(\vec{k})F^{IJ}(\hat{n},\hat{k})e^{-ikt}$$

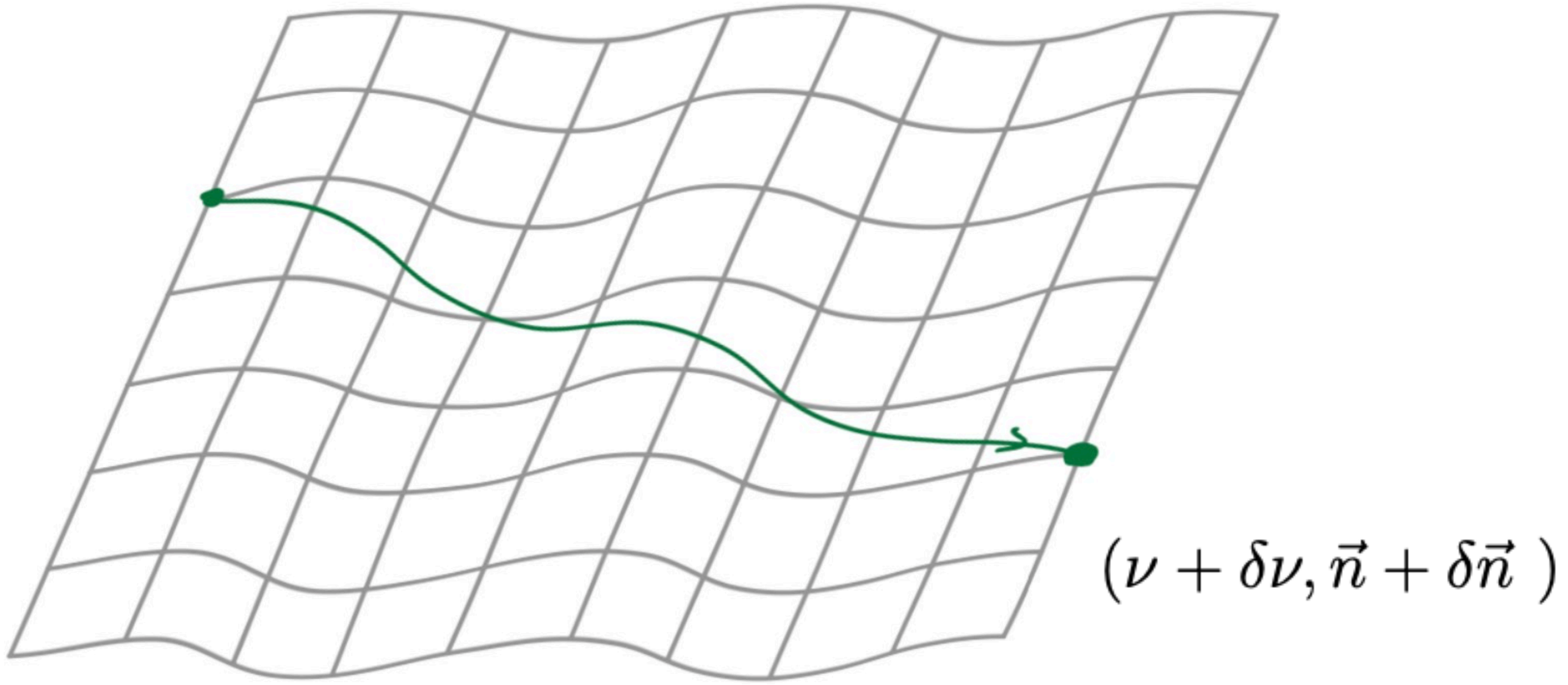
$$F^{IJ}(\hat{n},\hat{k}) = \frac{\hat{n}^I\hat{n}^J}{2(1+\hat{k}\cdot\hat{n})}$$

$$\delta n^I(t,\hat{n}) = e^I{}_\mu \delta n^\mu(t,\hat{n}) = \mathcal{R}^{IJK}(\hat{n},\hat{k})h_{JK}(\vec{k})e^{-ikt}$$

$$\mathcal{R}^{IJK}(\hat{n},\hat{k}) = \frac{\hat{n}^I+\hat{k}^I}{2(1+\hat{k}\cdot\hat{n})}\hat{n}^J\hat{n}^K - \frac{1}{2}\delta^{IJ}\hat{n}^K$$



Light path in flat space



Light path in perturbed space

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Correlation functions

- Assuming the isotropic background, one can separate the two-point correlation function in power spectrum and the overlap reduction function

$$\langle \delta\nu(t,\hat{n})\delta\nu\,(t,\hat{n}')\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} F^{KL}(\hat{n},\hat{k})F^{MN}\left(\hat{n}',\hat{k}\right) \times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k}) \left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

PTA — Overlap reduction function — Power spectrum Pss'

$$\langle \delta n^I(t,\hat{n})\;\delta n^J\,(t,\hat{n}')\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathcal{R}^{IKL}(\hat{n},\hat{k})\mathcal{R}^{JMN}\left(\hat{n}',\hat{k}\right) \times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k}) \left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

Astrometry — Overlap reduction tensor H^{IJ}_{SS'} — Power spectrum Pss'

Correlation functions

- For parity-even power spectrum, $P_{++} = P_{\times\times}\ , \quad P_{+\times} = P_{\times +} = 0$, we have
- PTA: Hellings-Downs curve $\Gamma_{++}(\xi) = \frac{1}{8}\left(3 + \cos\xi + 6(1-\cos\xi)\log\frac{1-\cos\xi}{2}\right)$
- Astrometry: $H^{IJ}_{++}(\vec{n}, \vec{n}') = \alpha(\xi)\,(A_I A_J - B_I C_J)$ $\qquad \cos\xi = \hat{n}\cdot\hat{n}'$

$$\vec{A} = \vec{n}\times\vec{n}', \quad \vec{B} = \vec{n}\times\vec{A}, \quad \vec{C} = -\vec{n}'\times\vec{A}$$

suppose we choose $\vec{n} = \hat{z}$, then the non-vanishing component in the overlap reduction tensor is the 11 component, and 22 component.

Sensitive gravitational strain

- When a light passes through a SGWB, the deflection angle is proportional to the characteristic GW strain and therefore relates to the GW amplitude through $\delta_{\rm rms}(f) \sim h_{\rm rms}(f) \sim \frac{H_0}{f}\sqrt{\Omega_{\rm gw}(f)}$.

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- For N sources, the angular velocity has a correlated signal of order

$$f\delta_{\mathrm{rms}} \sim \omega_{\mathrm{rms}} \sim \frac{\Delta\theta}{T\sqrt{N}}$$

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Frequency

Angular resolution 10 microarcssec

$$f\delta_{\rm rms} \sim \omega_{\rm rms} \sim \frac{\Delta\theta}{T\sqrt{N}}$$

Root mean square

angular velocity

Number of sources ~ 10^6

Time interval ~ 3 yr

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Sensitive gravitational strain

- When a light passes through a SGWB, the deflection angle is proportional to the characteristic GW strain and therefore relates to the GW amplitude through $\delta_{\rm rms}(f) \sim h_{\rm rms}(f) \sim \frac{H_0}{f}\sqrt{\Omega_{\rm gw}(f)}$.
- For N sources, the angular velocity has a correlated signal of order

$$f\delta_{\rm rms} \sim \omega_{\rm rms} \sim \frac{\Delta\theta}{T\sqrt{N}} \qquad \Omega_{\rm gw}(f) \lesssim \frac{\Delta\theta^2}{NT^2H_0^2} \qquad \Omega_{\rm gw} \sim 10^{-3} - 10^{-6}$$

- Future Theia telescope can give $h_{\rm rms} \sim 10^{-14}$ which is capable to reach the sensitivity of PTA detection!

Correlation functions

- Assuming the isotropic background, one can separate the two-point correlation function in power spectrum and the overlap reduction function

$$\langle \delta\nu(t,\hat{n})\delta\nu\left(t,\hat{n}'\right)\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} F^{KL}(\hat{n},\hat{k})F^{MN}\left(\hat{n}',\hat{k}\right)\times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k})\left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

PTA — Overlap reduction function — Power spectrum Pss'

$$\langle \delta n^I(t,\hat{n})\ \delta n^J\left(t,\hat{n}'\right)\rangle = \sum_{S,S'}\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathcal{R}^{IKL}(\hat{n},\hat{k})\mathcal{R}^{JMN}\left(\hat{n}',\hat{k}\right)\times e^S_{KL}(\hat{k})e^{S'}_{MN}(-\hat{k})\left\langle h_S(t,\vec{k})h_{S'}\left(t,\vec{k}'\right)\right\rangle$$

Astrometry — Overlap reduction tensor H^{IJ}_{SS'} — Power spectrum Pss'

Parity violation

- For non-vanishing $P_{+\times}, P_{\times+}$, PTA won't response! $\Gamma_{+\times}(\xi) = \Gamma_{\times+}(\xi) = 0$
- Astrometry, on the other hand, has non-vanishing overlap reduction tensor for this signal

$$H^{IJ}_{+\times}(\hat{n}, \hat{n}') = \alpha(\Theta) A^I B^J_2 + \beta(\Theta) B^I_1 A^J$$
$$H^{IJ}_{\times+}(\hat{n}, \hat{n}') = \alpha_2(\Theta) A^I B^J_2 + \beta_2(\Theta) B^I_1 A^J$$

$$\vec{A} = \hat{n} \times \hat{n}', \quad \vec{B}_1 = \hat{n} \times \vec{A}, \quad \vec{B}_2 = \hat{n}' \times \vec{A}.$$

- For $\vec{n} = \hat{z}$, the non vanishing components would be $H^{12}_{+\times}$, $H^{21}_{+\times}$, $H^{23}_{+\times}$

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Non-vanishing EB correlation

- Like CMB analysis, one can decompose the deflection vector to a spherical harmonic and work on the angular power spectrum

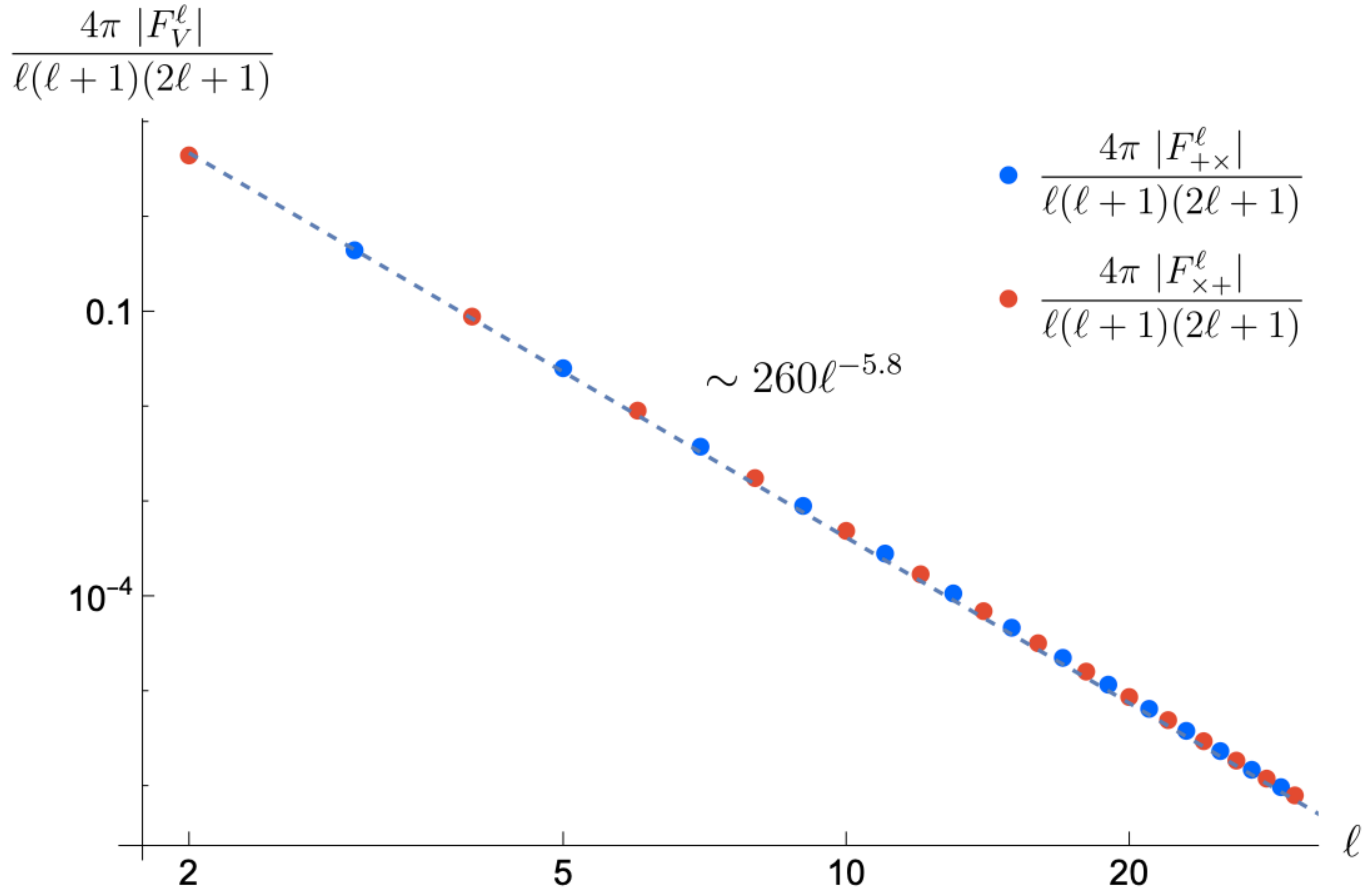
$$\delta n(t,\hat{n}) = \sum_{\ell m}\left[\delta n_{E\ell m}(t)\vec{Y}^{E}_{\ell m}(\hat{n}) + \delta n_{B\ell m}(t)\vec{Y}^{B}_{\ell m}(\hat{n})\right]$$

$$\langle \delta n_{E\ell m}(t)\delta n_{B\ell' m'}(t)^{*}\rangle = \int \mathrm{d}^2\Omega_{\hat{n}}\ \mathrm{d}^2\Omega_{\hat{n}'} Y^{E*}_{\ell m I}(\hat{n}) Y^{B}_{\ell' m' J}(\hat{n}') \left\langle \delta n^{I}(t,\hat{n})\delta n^{J}(t,\hat{n}')\right\rangle$$

$$= \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)}\frac{4\pi}{2\ell+1}\left(\mathcal{A}_{+\times}F^{\ell}_{+\times} + \mathcal{A}_{\times +}F^{\ell}_{\times +}\right)$$

Non-vanishing EB correlation

$$\langle \delta n_{E\ell m}(t) \delta n_{B\ell' m'}(t)^* \rangle$$
$$= \frac{\delta_{\ell\ell'}\delta_{mm'}}{\ell(\ell+1)} \frac{4\pi}{2\ell+1} \left(\mathcal{A}_{+\times} F^{\ell}_{+\times} + \mathcal{A}_{\times+} F^{\ell}_{\times+} \right)$$



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Sensitivity for EB correlation

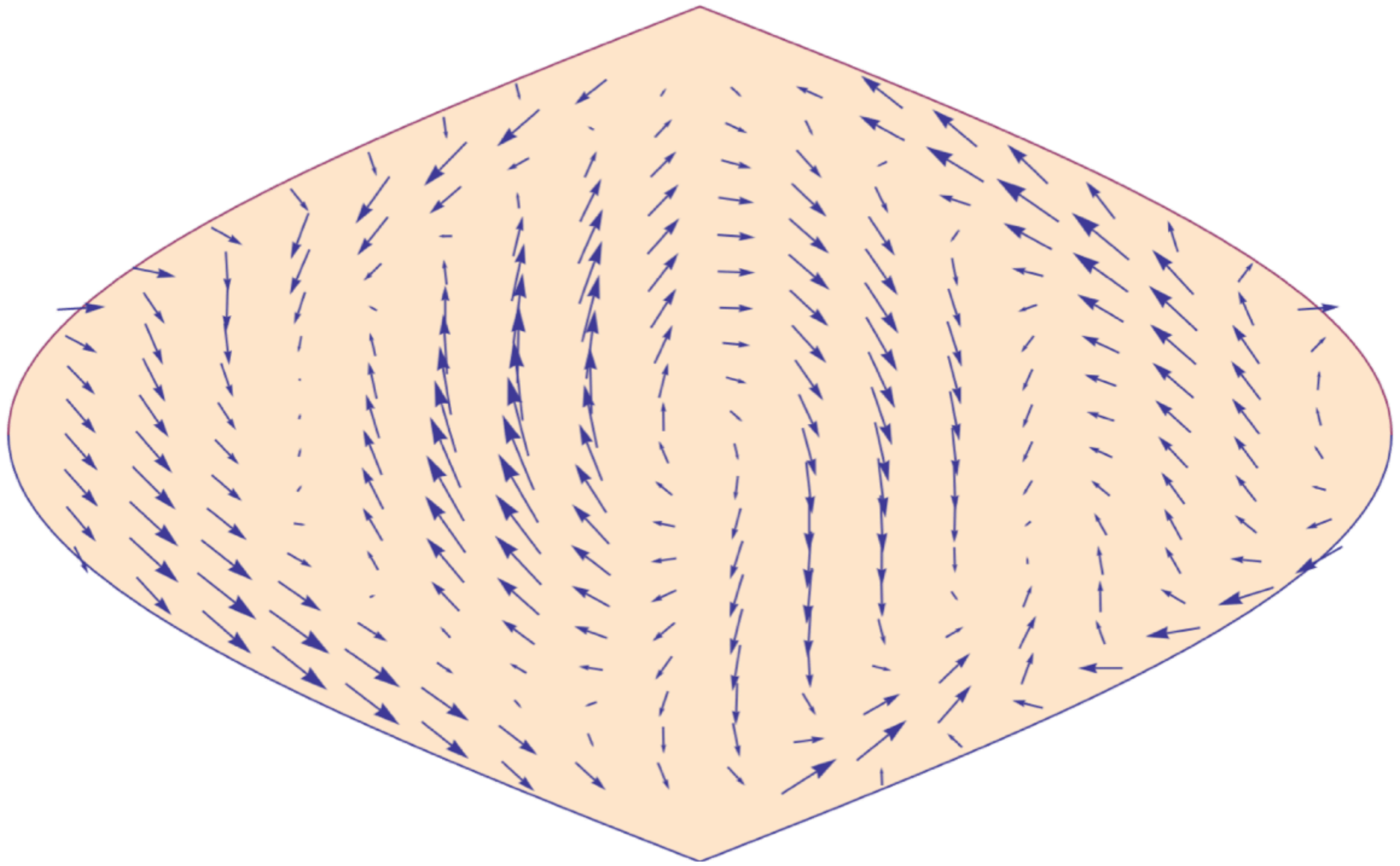
- Since in parity even theory, there is a vanishing EB correlation, the non-vanishing EB correlation is a **null test**.
- For stars in our galaxy, the sensitivity of EB correlation is again set by the angular resolution, the observation time, and the number of sources, since we have Milky way streams that breaks the parity symmetry.
- For quasars that are far away in other galaxies, if we can find an absolute inertial frame to measure their deflection vector, we might also use quasars to do astrometry, and assume a significantly smaller noise for EB correlation.

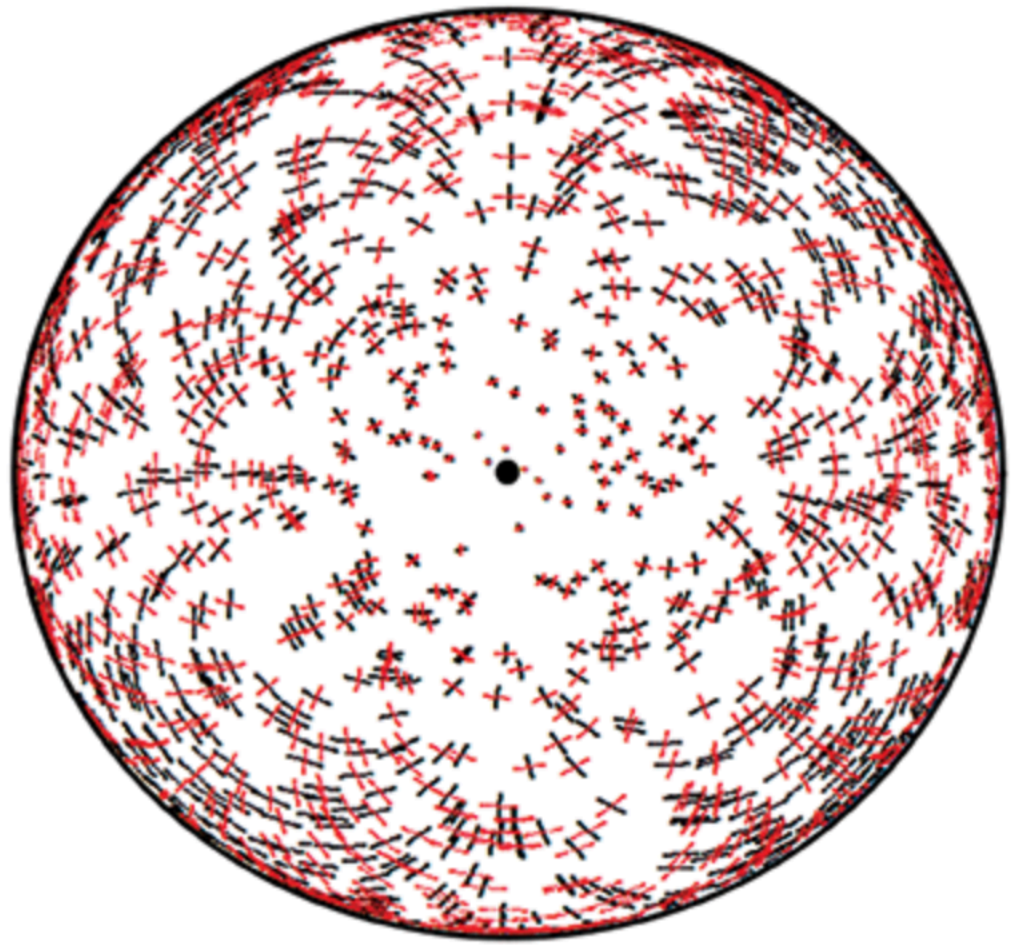
Conclusion and Discussion

- The phase velocity is the actual parameter entering the overlap reduction function in PTA system
- The overlap reduction tensor for astrometry system has non-vanishing EB correlation from the parity-violation signal
- Future work involving detectability of specific models need to be done!
- The current telescope might not be able to detect SGWB yet, but future galaxy survey should provide a parallel probe to PTA

Thanks for your attention!

Typical deflection pattern (would be cool if I have a deflection pattern with the parity violation signals ···)

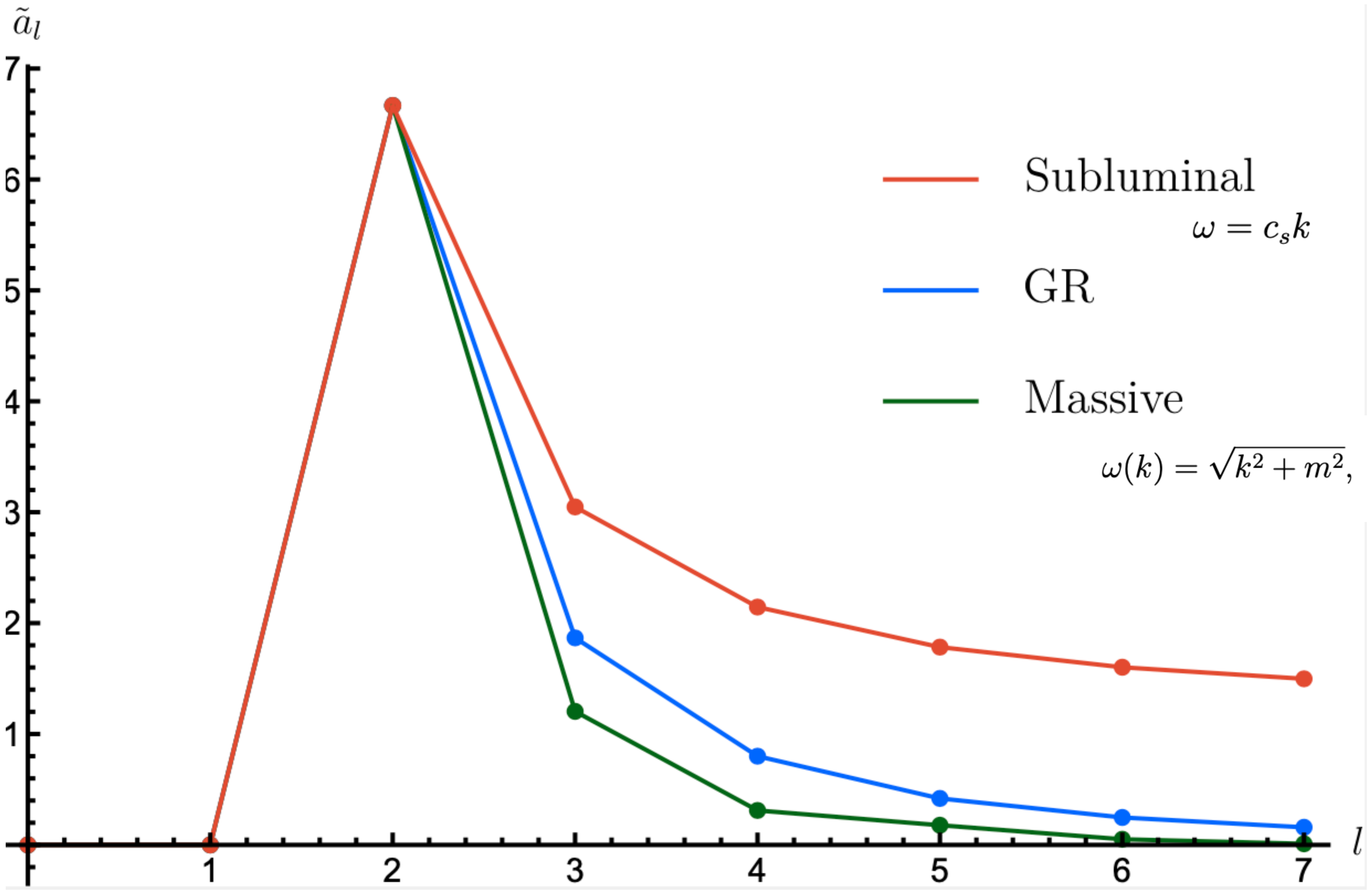




https://cosmology.lbl.gov/talks/Book_11.pdf

The minimum angle

- In GR, the minimal angle is determined by the coefficients of harmonic analysis
- If we only have the quadrupole, then the minimal angle is 90 degree.
- The coefficients of the higher multipole modes will make the minimal angle less than 90 degree.
- When the coefficients of higher modes got enhanced or suppressed in modified theories, the minimal angle will shift.



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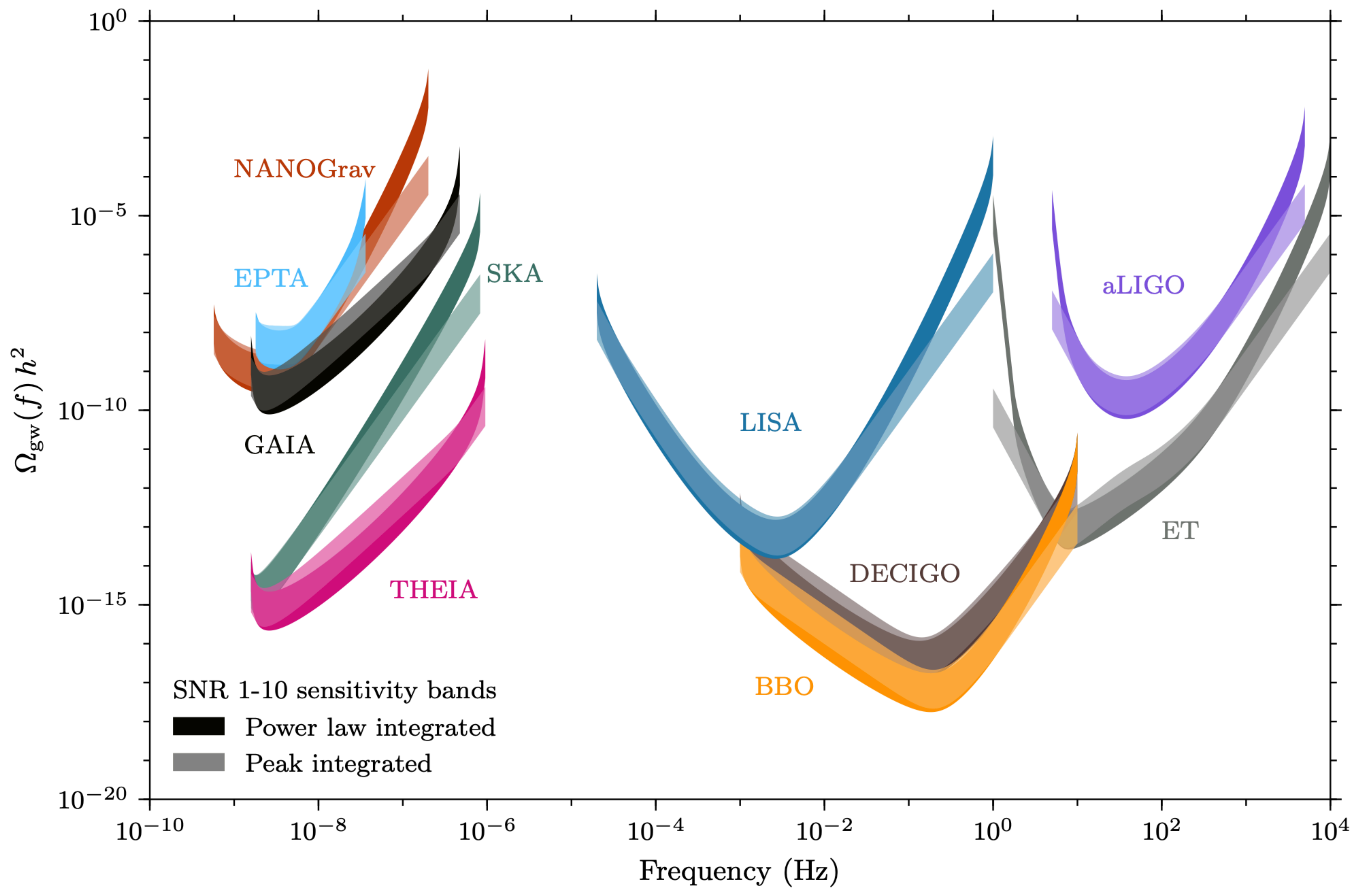
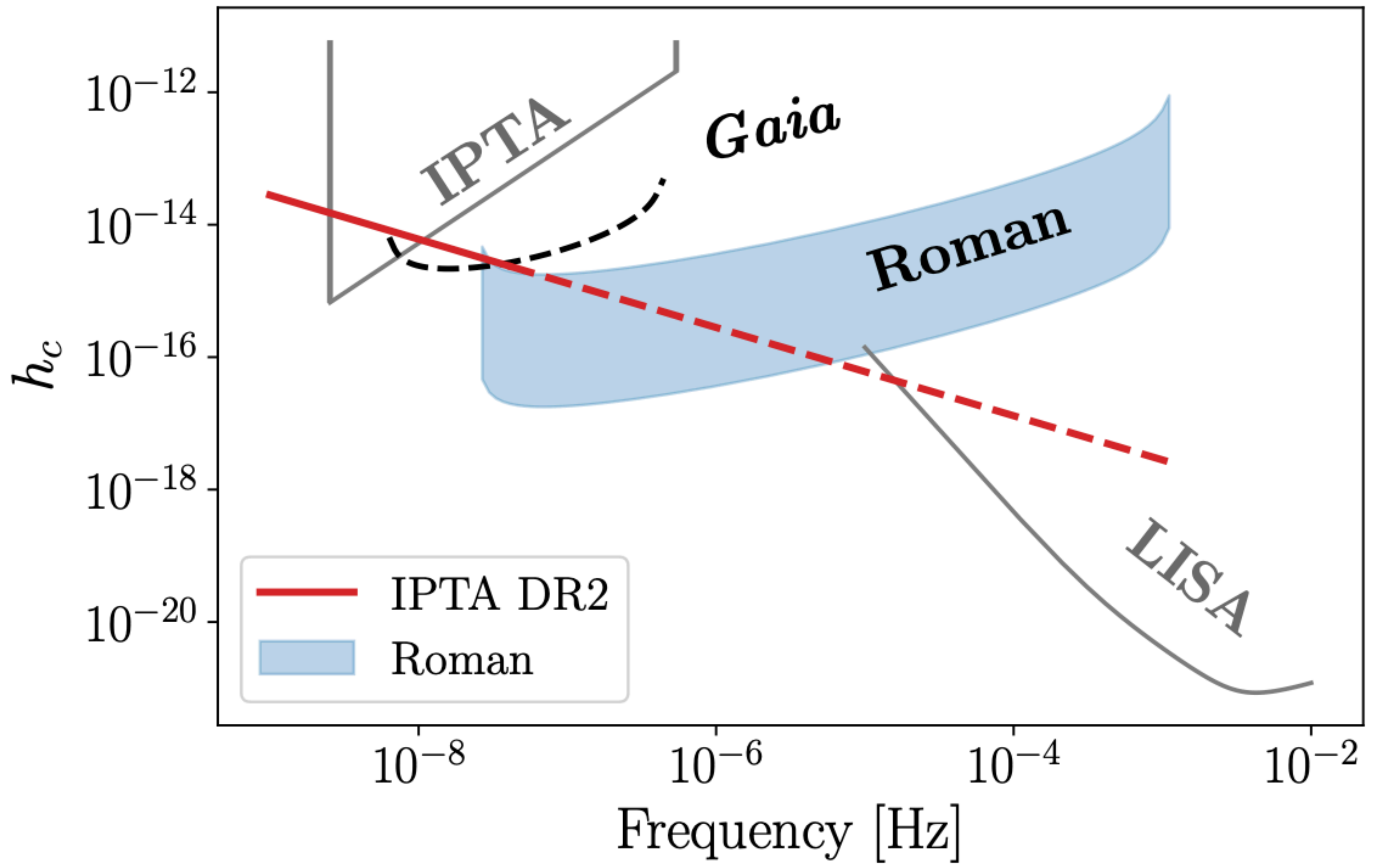
Estimated sensitivity curve



Credit: 2104.04778



Credit: 2205.07962

Massive Gravity

- Receiving function:

$$F^{(i)}(\hat{\mathbf{\Omega}}) \equiv -\frac{\hat{p}^\mu \hat{p}^\nu}{2\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}\right)}\epsilon^{(i)}_{\mu\nu} + \frac{\hat{p}^\mu}{2}\epsilon^{(i)}_{0\mu} \ ,$$

$$\boxed{\frac{|\boldsymbol{k}|}{k_0} = \frac{1}{v_{ph}}}$$

- Overlap reduction function for each mode:

$$\Gamma(\xi) = \beta_I \sum_i \int_{S^2} d^2\hat{\mathbf{\Omega}} \left(e^{i2\pi f L_1\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}_1\right)} - 1\right)\left(e^{-i2\pi f L_2\left(1+\frac{|\boldsymbol{k}|}{k_0}\hat{\mathbf{\Omega}}\cdot\hat{\boldsymbol{p}}_2\right)} - 1\right) F_1^{(i)}(\hat{\mathbf{\Omega}}) F_2^{(i)}(\hat{\mathbf{\Omega}})$$

- Combined effect on the 2-point correlation function

- $$\langle \tilde{z}^2 \rangle \propto \left(\frac{\Omega_T}{\beta_T}\Gamma_T + \frac{\Omega_V}{\beta_V}\Gamma_V + \frac{\Omega_S}{\beta_S}\Gamma_S\right) = \frac{\Omega_T}{\beta_T}\boxed{\Gamma_T\left(1+\frac{\Gamma_V}{\Gamma_T}\frac{\Omega_V}{\Omega_T}\frac{\beta_T}{\beta_V}+\frac{\Gamma_S}{\Gamma_T}\frac{\Omega_S}{\Omega_T}\frac{\beta_T}{\beta_S}\right)} .$$

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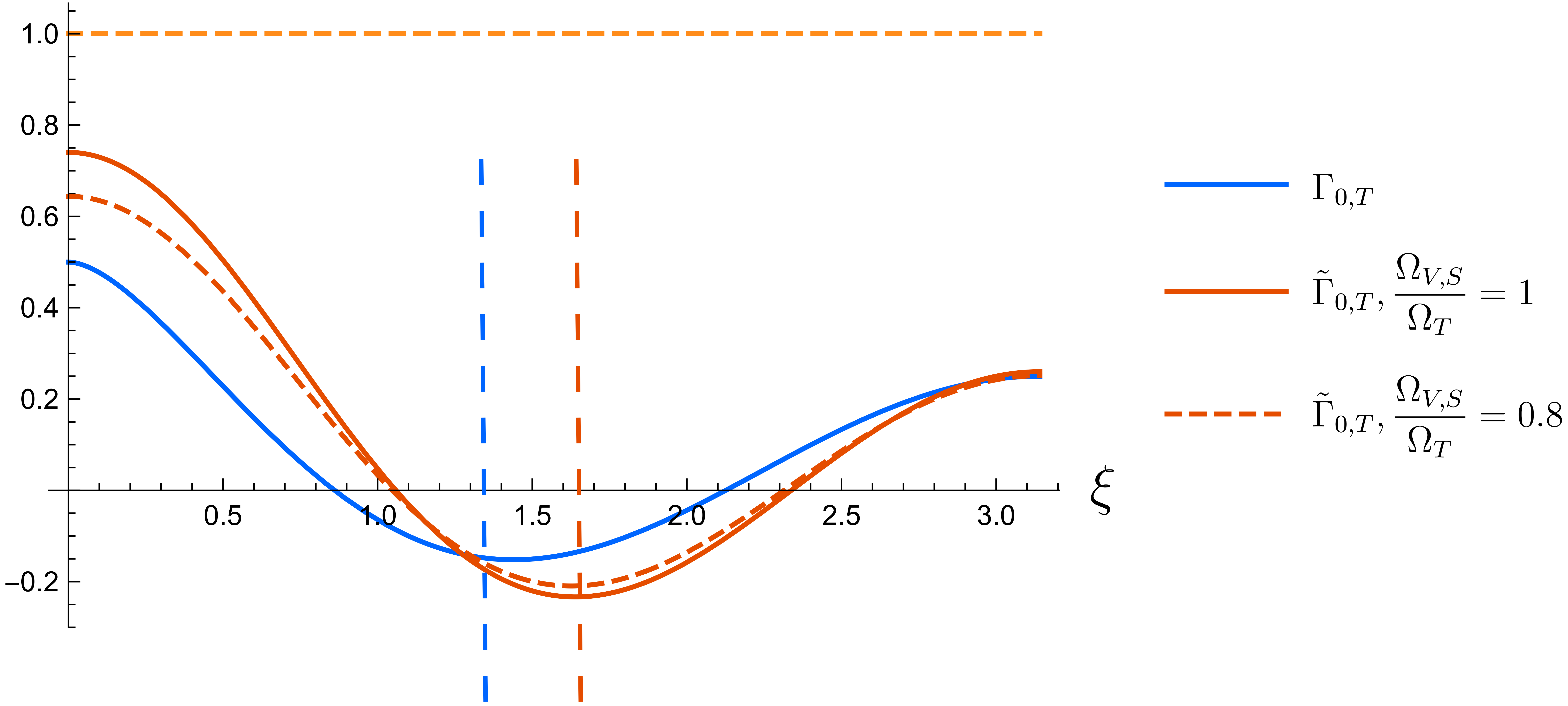
Combined effective overlap reduction function



Notice the shift of the minimum angle! This might be a distinguishable feature!

Hellings-Downs curve

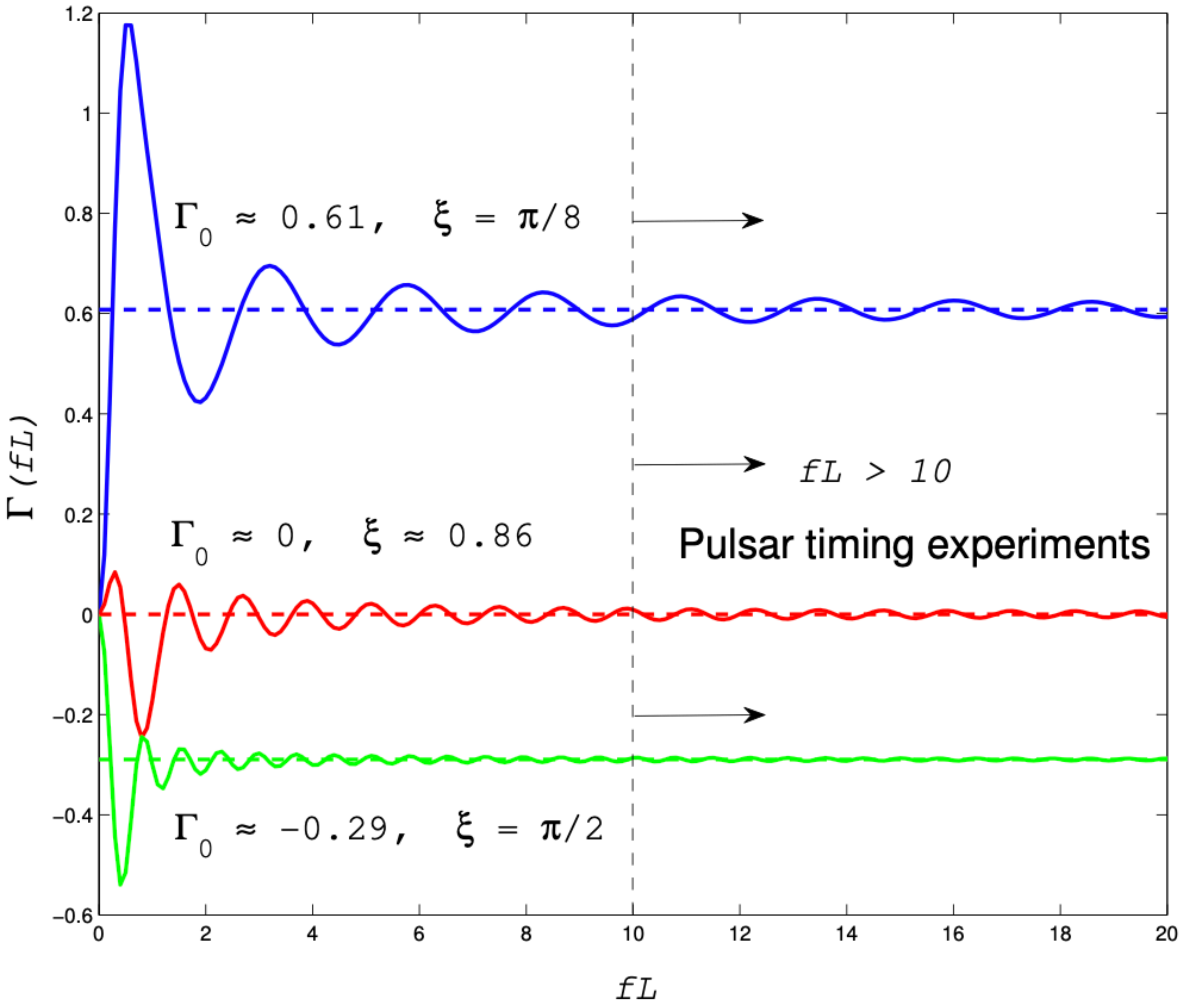
- Overlap reduction function:

$$\Gamma(|f|) = \beta \sum_A \int_{S^2} d\hat{\Omega} \left(e^{i2\pi f L_1(1+\hat{\Omega}\cdot\hat{p}_1)} - 1\right) \times$$

- Exponential factor!

- Hellings-Downs curve:

$$\Gamma_0 \equiv \frac{3}{4\pi} \sum_A \int_{S^2} d\hat{\Omega}\, F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega})$$
$$= 3\left\{\frac{1}{3} + \frac{1-\cos\xi}{2}\left[\ln\left(\frac{1-\cos\xi}{2}\right) - \frac{1}{6}\right]\right\},$$



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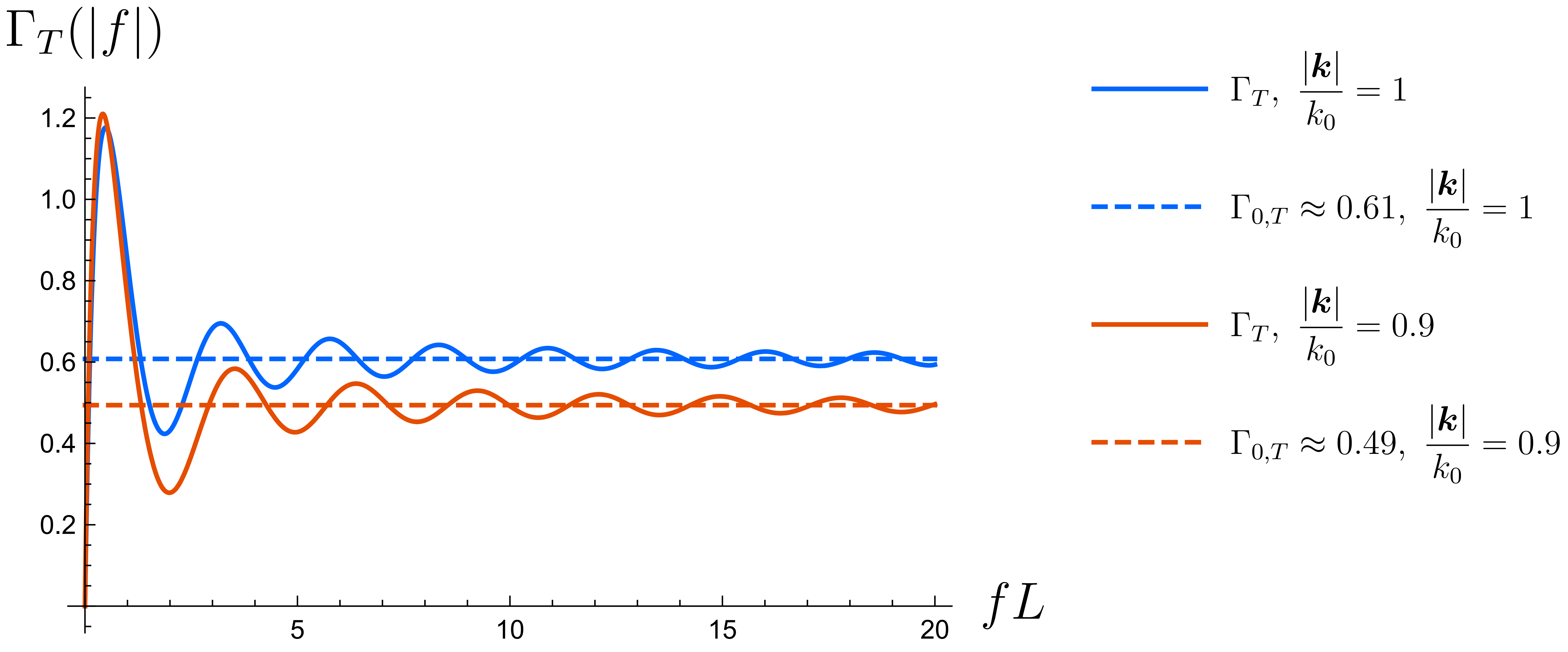
- supermassive black hole binary systems (γ ~ 13/3); primordial gravitational waves (γ ~ 5); and networks of cosmic strings (γ ~ 16/3)

Tensor

$$\xi = \frac{\pi}{8}$$



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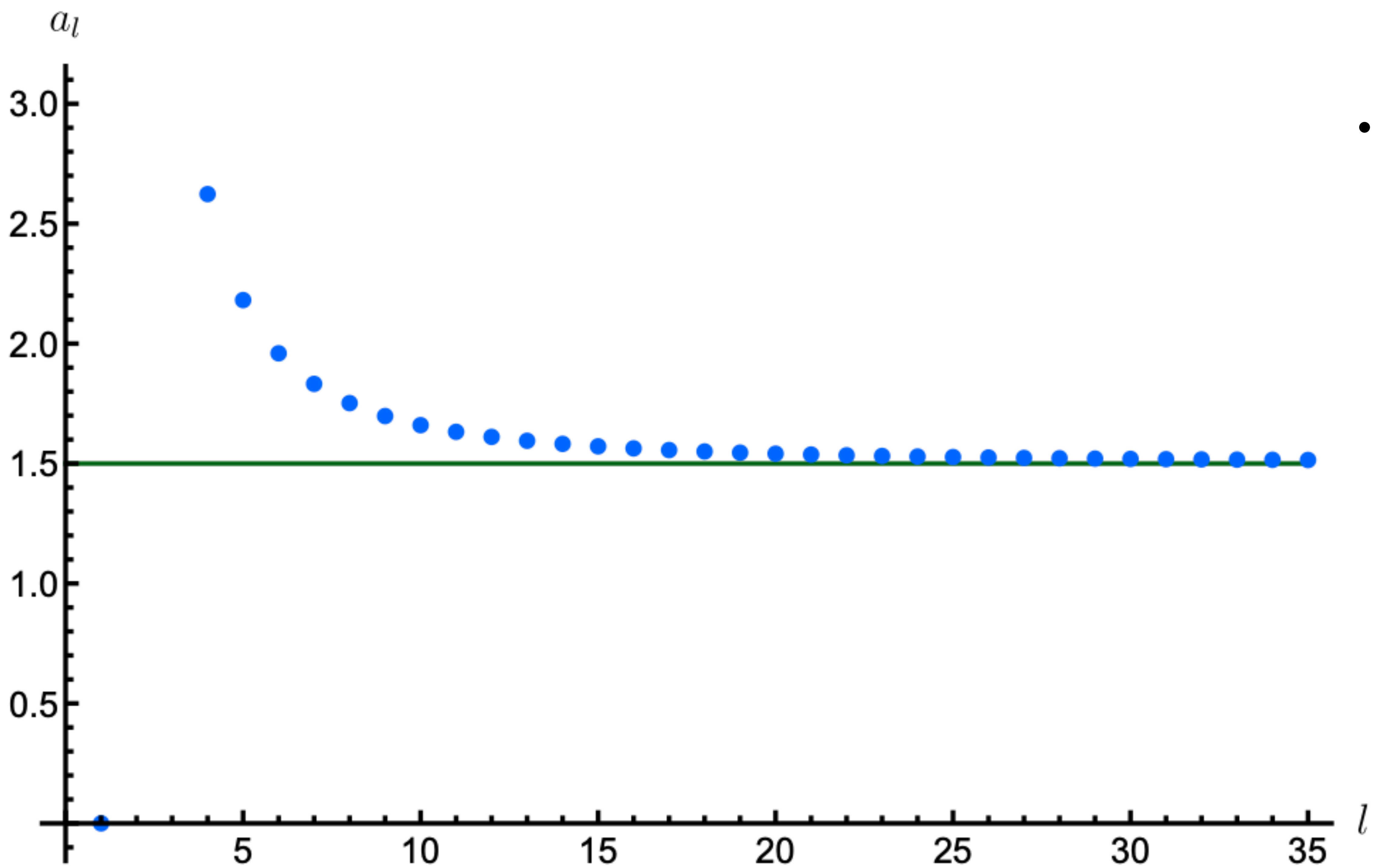
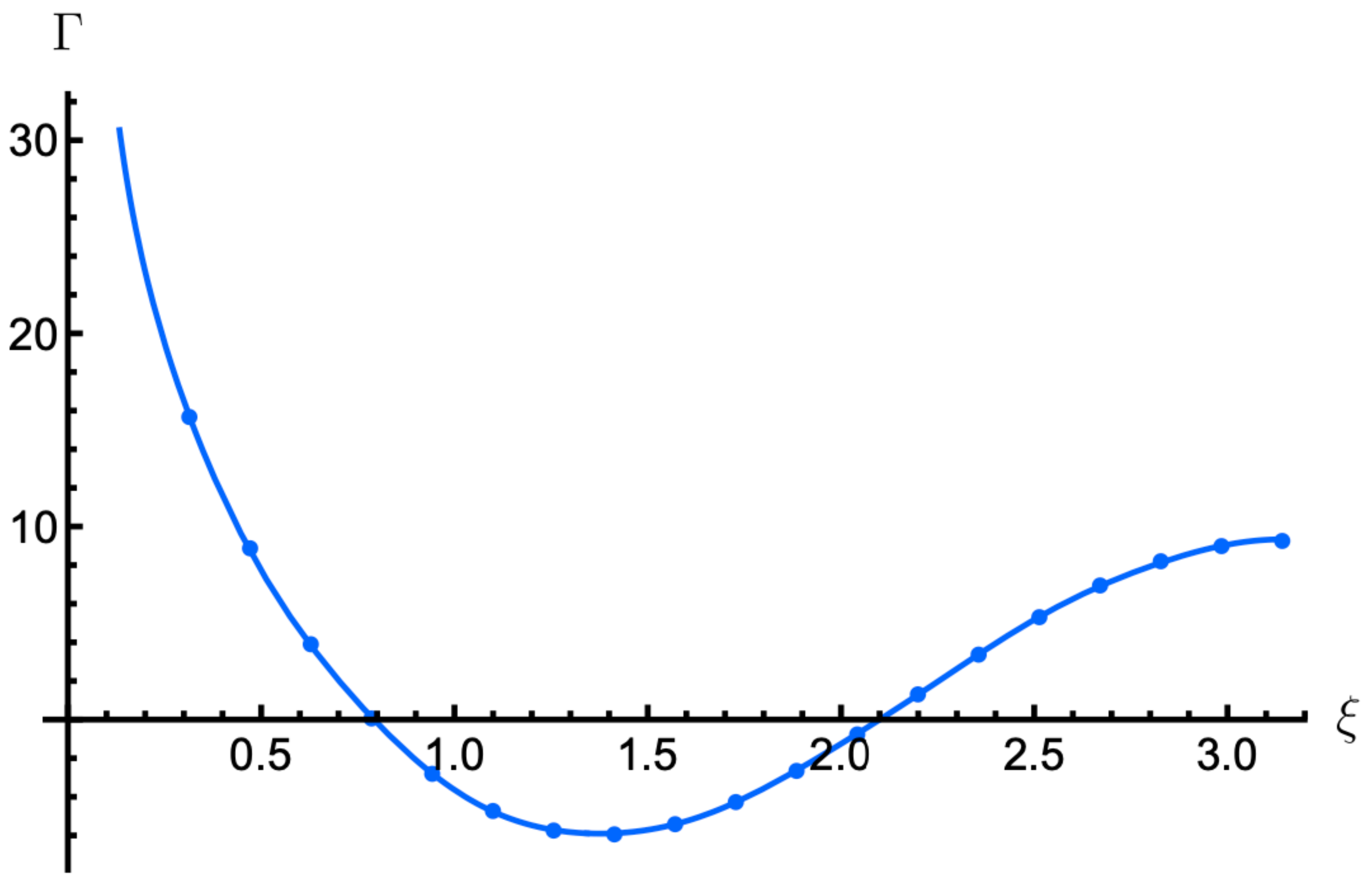
Divergence



- Since the coefficients approach a constant at large multipole mode, they contribute equivalently at $\xi = 0$ These infinite many modes lead to the divergence in the overlap reduction function

Divergence



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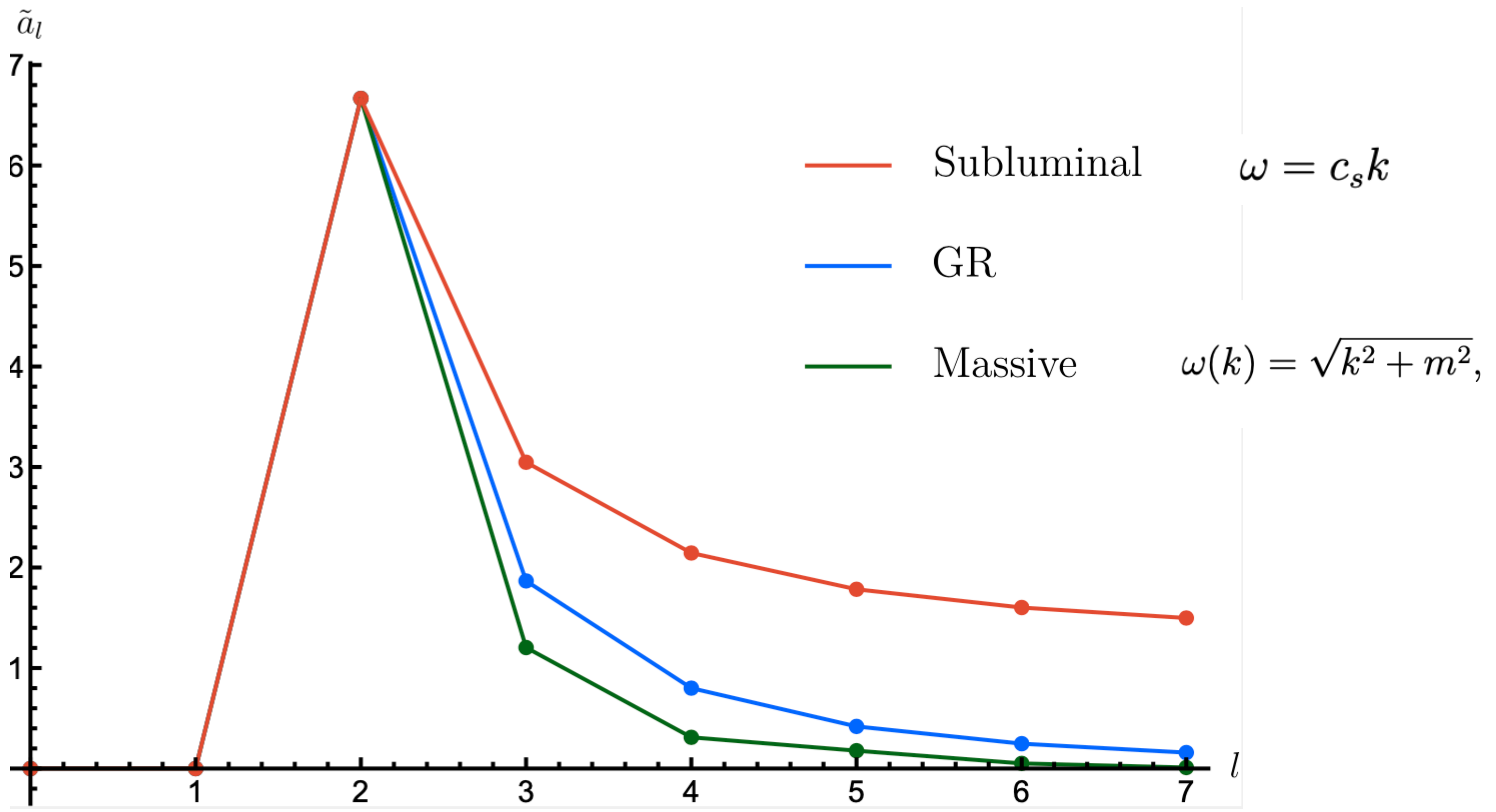
These infinite many modes lead to the divergence in the overlap reduction function

Dispersion relation



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Spherical harmonic analysis

$$Y^E_{(\ell m)ab} = N_l \left(Y_{(\ell m);ab} - \frac{1}{2} g_{ab} Y_{(\ell m);c}{}^{c} \right) \quad , \quad Y^B_{(\ell m)ab} = \frac{N_\ell}{2} \left(Y_{(\ell m);ac} \epsilon^c_b + Y_{(\ell m);bc} \epsilon^c_a \right)$$

$$h_{ab}(f, \hat{\Omega}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{l} \left[a^E_{(\ell m)}(f) Y^E_{(\ell m)ab}(\hat{\Omega}) + a^B_{(\ell m)}(f) Y^B_{(\ell m)ab}(\hat{\Omega}) \right]$$

Spherical harmonic analysis

These two bases are related by

$$Y^E_{(\ell m)ab}(\hat{\Omega}) = \frac{N_\ell}{2}\left[W_{(\ell m)}(\hat{\Omega})e^+_{ab}(\hat{\Omega}) + X_{(\ell m)}(\hat{\Omega})e^\times_{ab}(\hat{\Omega})\right],$$
$$Y^B_{(\ell m)ab}(\hat{\Omega}) = \frac{N_\ell}{2}\left[W_{(\ell m)}(\hat{\Omega})e^\times_{ab}(\hat{\Omega}) - X_{(\ell m)}(\hat{\Omega})e^+_{ab}(\hat{\Omega})\right],$$

with the coefficients being related by

$$h^+(f,\hat{\Omega}) = \sum_{(\ell m)} \frac{N_\ell}{2}\left[a^E_{(\ell m)}(f)W_{(\ell m)}(\hat{\Omega}) - a^B_{(\ell m)}(f)X_{(\ell m)}(\hat{\Omega})\right],$$
$$h^\times(f,\hat{\Omega}) = \sum_{(\ell m)} \frac{N_\ell}{2}\left[a^E_{(\ell m)}(f)X_{(\ell m)}(\hat{\Omega}) + a^B_{(\ell m)}(f)W_{(\ell m)}(\hat{\Omega})\right].$$

Correlation function

- One can separate the two-point correlation function in power spectrum Ω_{gw} and the overlap reduction function $\Gamma(|f|)$ assuming the isotropic SGWB
$$\langle \tilde{z}_1^*(f)\tilde{z}_2(f')\rangle = \frac{3H_0^2}{32\pi^3}\frac{1}{\beta}\delta(f-f')|f|^{-3}\Omega_{\text{gw}}(|f|)\Gamma(|f|),$$

- The collaborations claim a strong evidence for a power-law like power spectrum: $\Omega_{\text{gw}} \sim f^{-\gamma}$, the PPTA collaboration finds $\gamma \in (1.5, 5.5)$ and NANOGrav collaboration finds $\gamma \in (3.76, 6.78)$, EPTA: $\gamma \in (3.11, 4.65)$, IPTA: $\gamma \in (3.1, 4.9)$

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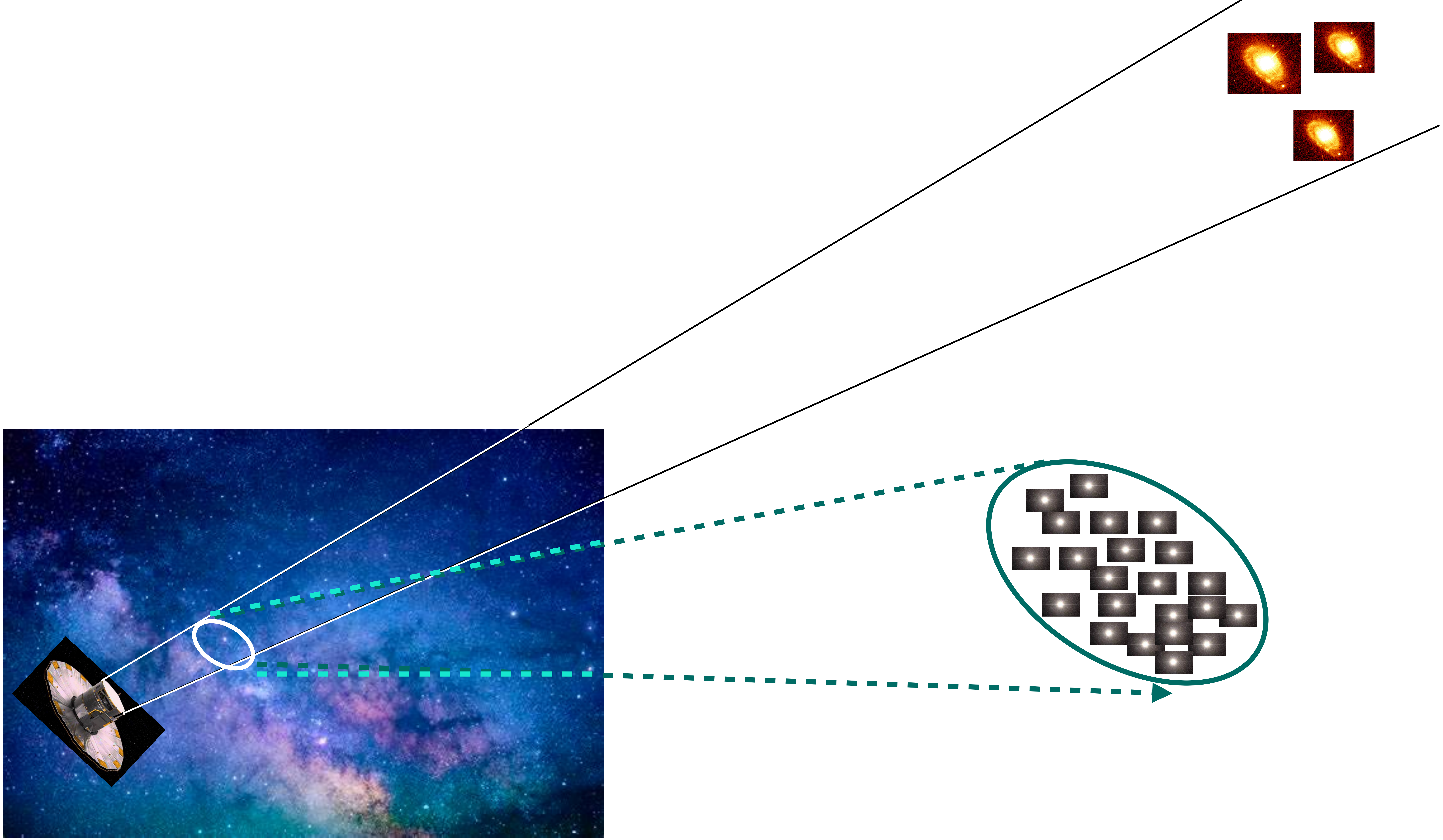

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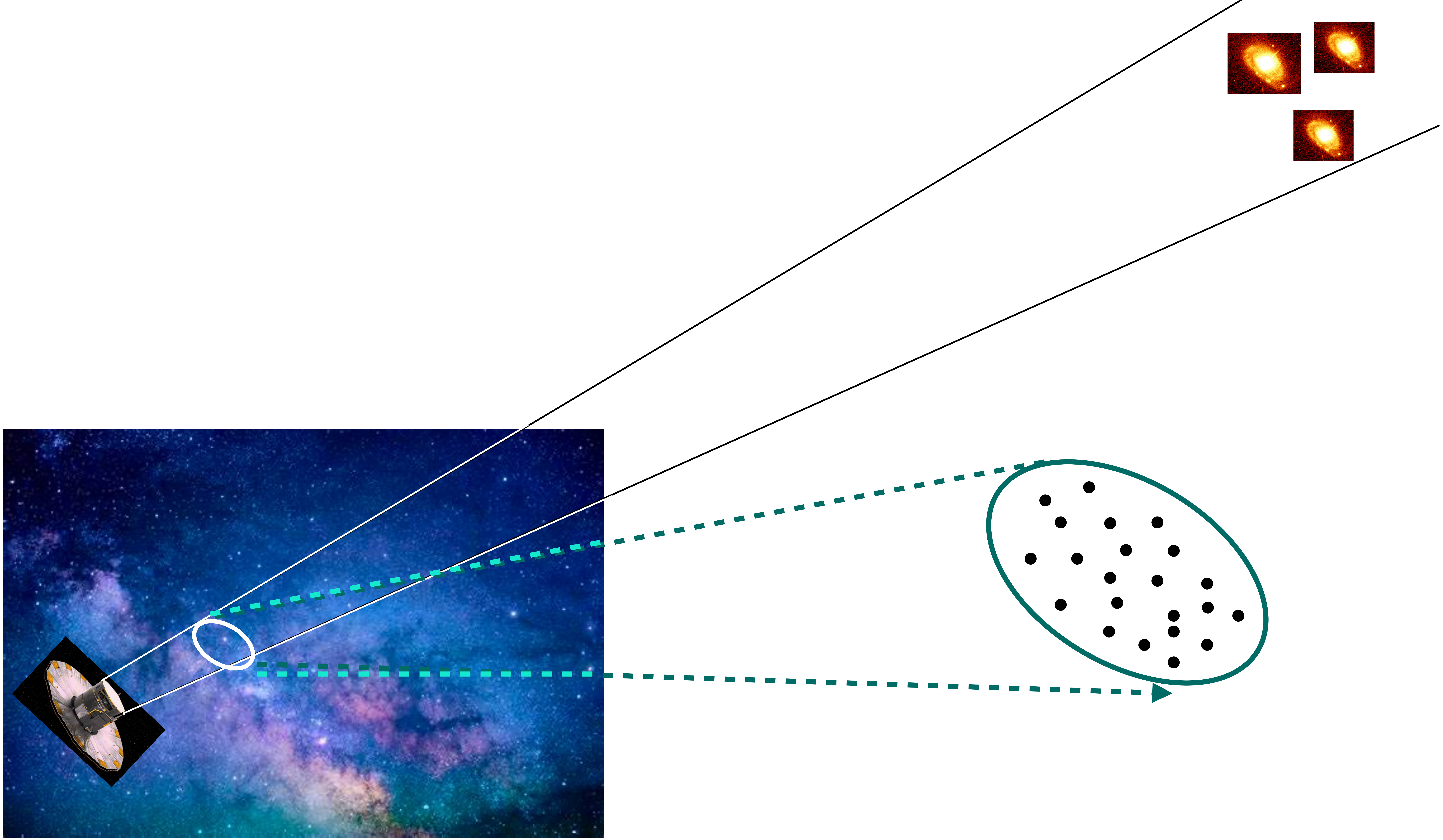

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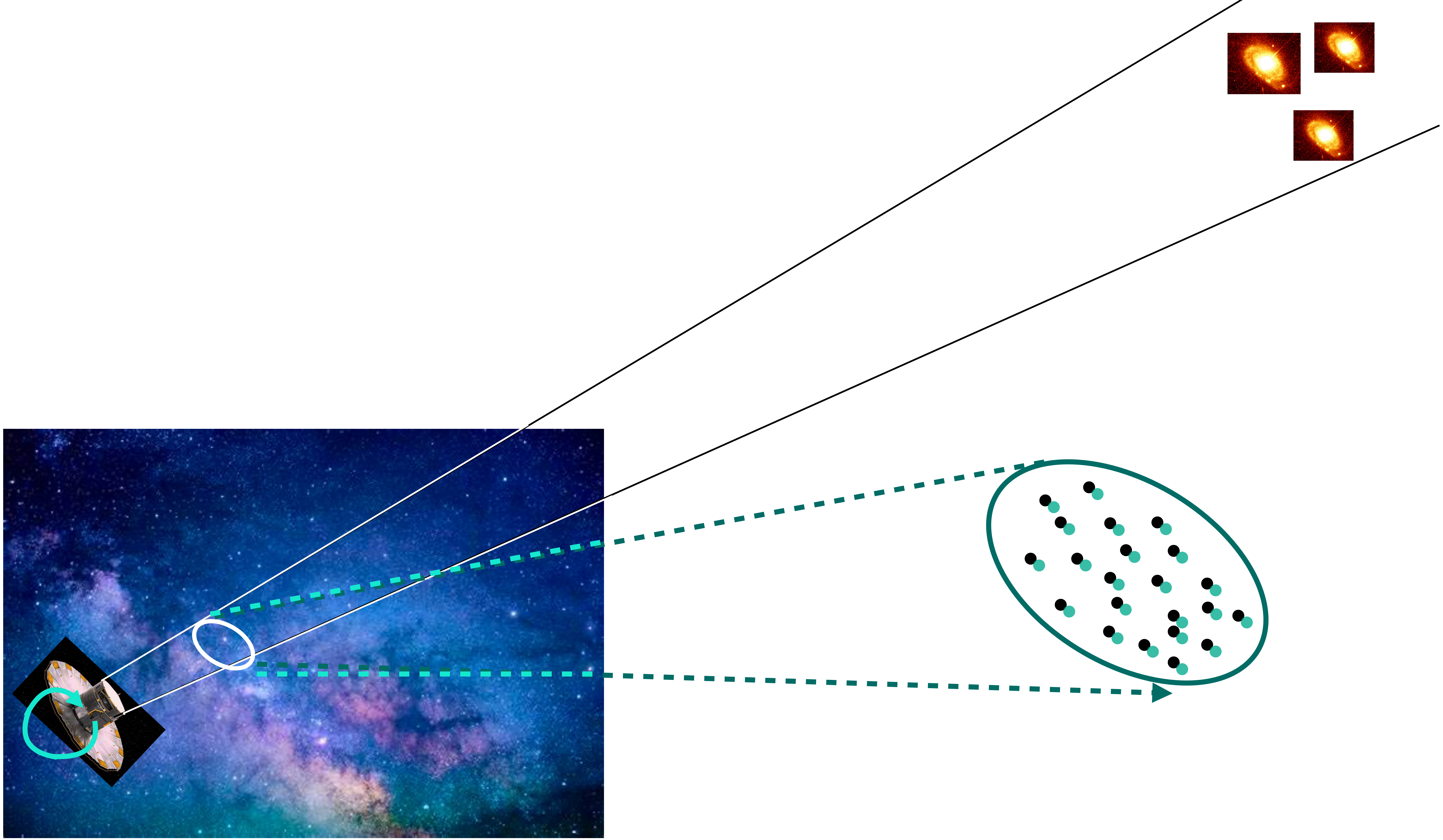

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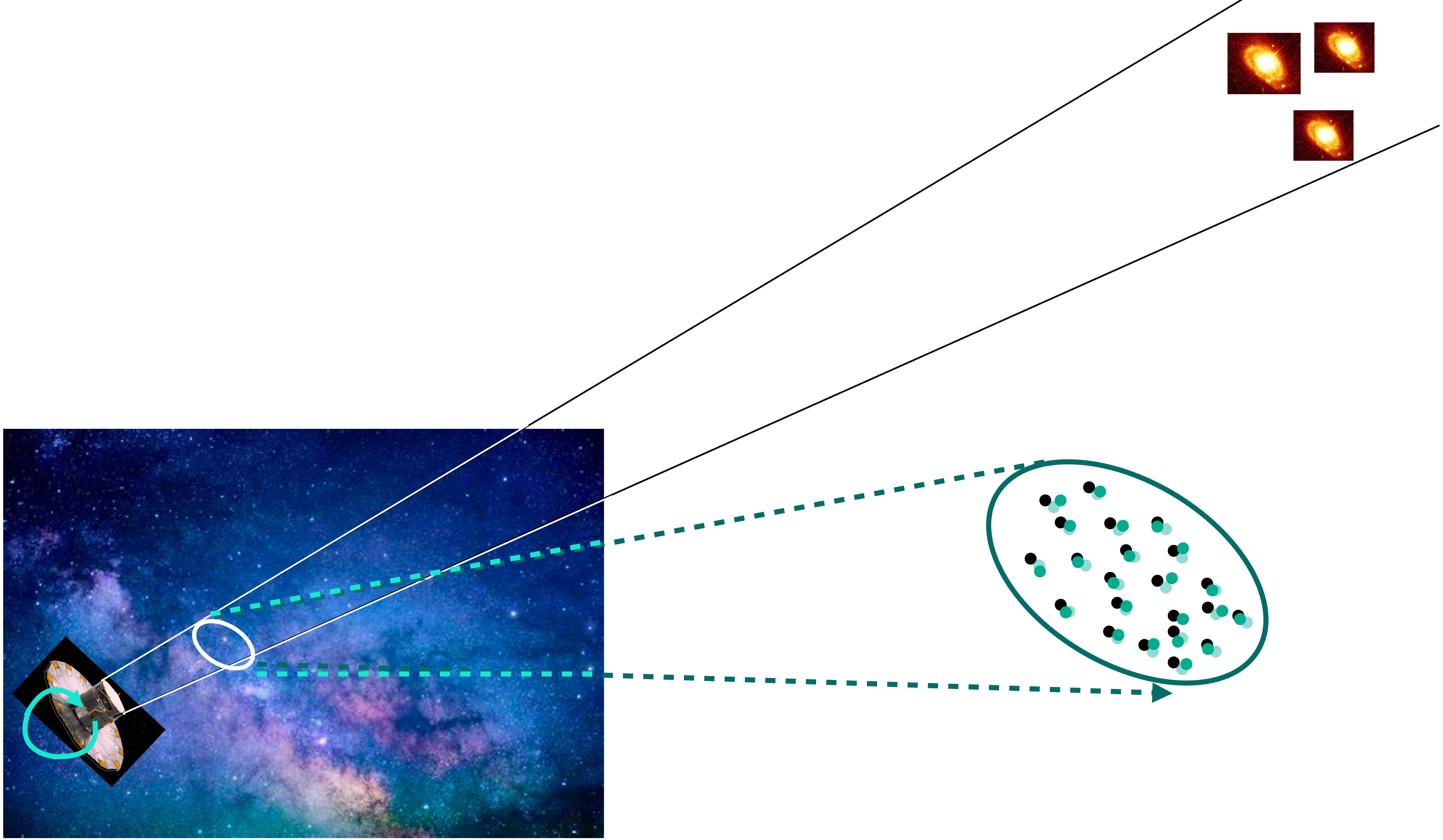

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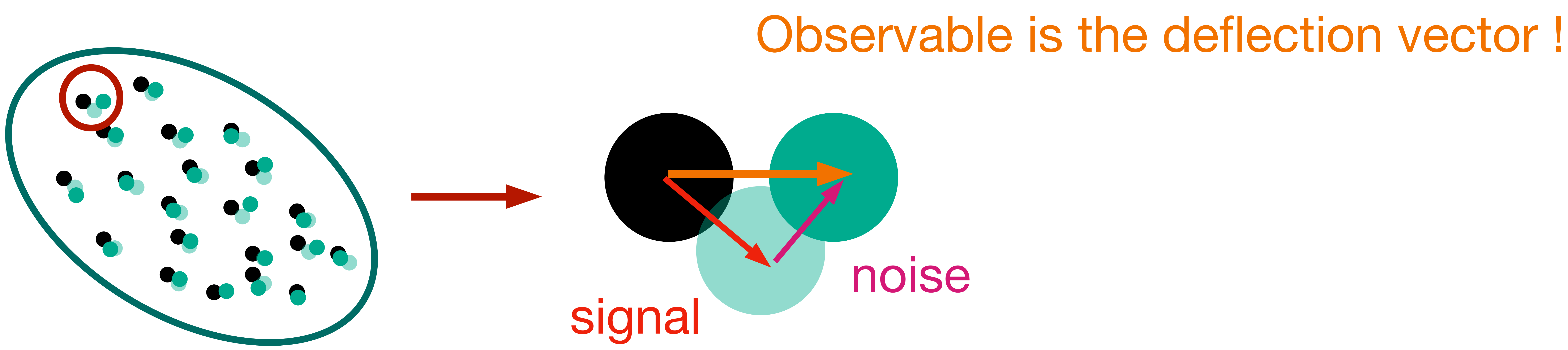

- Noise term includes stellar motion and other white noise which need to be deducted from the observational signal.
- By analyzing the correlation with large numbers of stars, one can extract information of the underlying gravitational wave background.