Tri-unification

A separate $SU(5)$ for each fermion family

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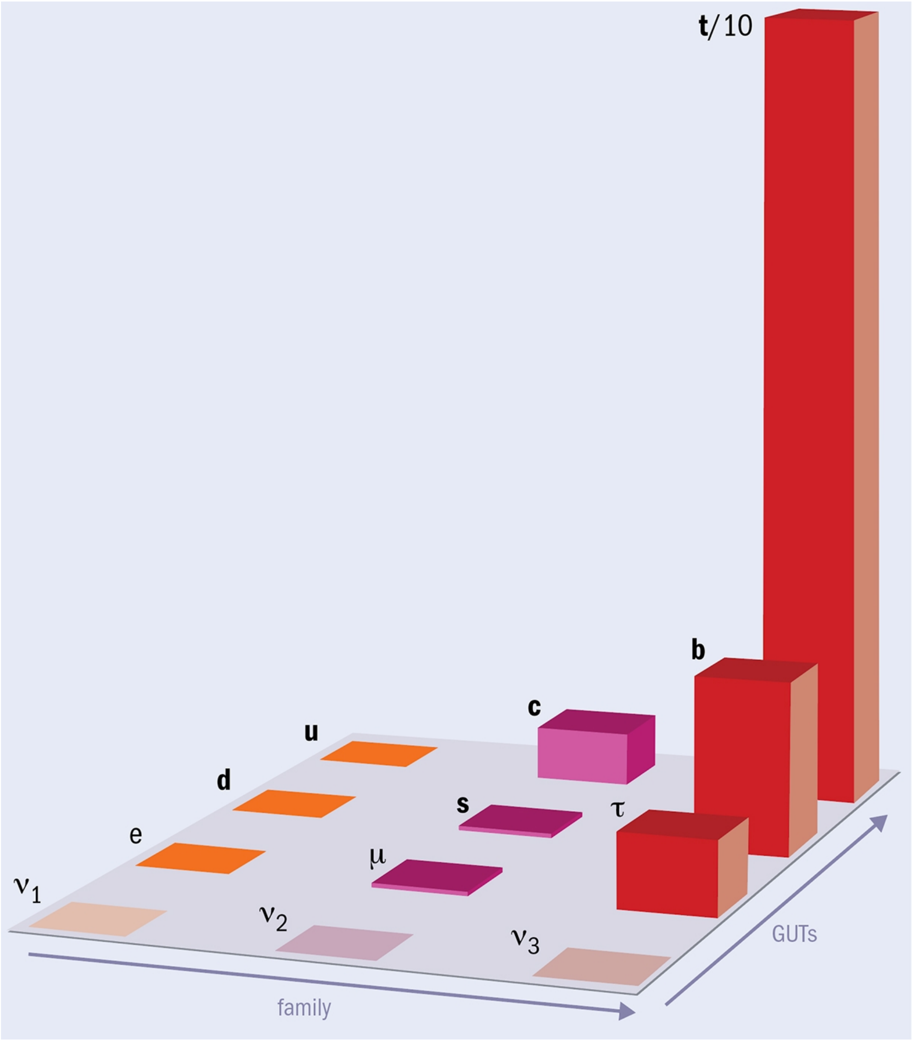
Based on work in collaboration with
Mario Fernández Navarro and **Stephen F. King**

2311.05683 & **2404.12442**

CATCH 22+2

Dublin

The flavor puzzle



- Why are there 3 fermion families?
- Why are fermion masses so hierarchical?
- Why do quarks mix so little... and leptons so much?

Talk by Gabriela Lichtenstein

Deconstructing flavor

General idea:

SM embedded in a larger gauge symmetry with a separate factor for each family

$$G = G_{\text{universal}} \times G_1 \times G_2 \times G_3$$

The SM Higgs is a 3rd family particle: singlet of G_1 and G_2, but not of G_3

⇒ Only the 3rd family masses at renormalizable level

⇒ Explain the SM flavor structure with $\mathcal{O}(1)$ Yukawa couplings

Examples:

- Tri-hypercharge: $SU(3)_c \times SU(2)_L \times U(1)_Y^3$ [Fernández Navarro, King, AV]
- $SU(3)_c \times SU(2)_L^3 \times U(1)_Y$ [Li, Ma, Muller, Nandi, Chiang, Deshpande, He, Jiang...]
- (Pati-Salam)^3 [Bordone, Cornella, Fuentes-Martin, Isidori, Pagès, Stefanek...]
- Grand unified models [Rajpoot, Barbieri, Dvali, Strumia, Babu, Barr, Gogoladze...]

+ others

Tri-unification

$$\boldsymbol{SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3}$$

★ $\mathbb{Z}_3$ cyclic symmetry

- Enforces a common gauge coupling (at unification scale)
- Invariance under cyclic permutations: (**A**,**B**,**C**) + (**B**,**C**,**A**) + (**C**,**A**,**B**)

★ Symmetry breaking:

$$\begin{aligned} SU(5)^3 &\to \mathrm{SM}_1 \times \mathrm{SM}_2 \times \mathrm{SM}_3 \\ &\to G_{\mathrm{universal}} \times G_1 \times G_2 \times G_3 \\ &\to G_{\mathrm{universal}} \times G_{1+2} \times G_3 \\ &\to \mathrm{SM}_{1+2+3} \end{aligned}$$

Minimal field content

In addition:
bi-representations, messenger fields, ...

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

Note: These are only 4 irreps of the full symmetry group

Minimal field content

<u>In addition</u>:
bi-representations, messenger fields, ...

SM fermions →

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1}$$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

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$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1}$$

Scalars in the adjoint →

Breaking of $SU(5)^3$ and SM^3

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

<u>Note</u>: These are only 4 irreps of the full symmetry group

Minimal field content

In addition:
bi-representations, messenger fields, ...

SM fermions →

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1}$$

Scalars in the adjoint →

Breaking of $SU(5)^3$ and SM³

Higgs doublets →

Only H_3 in most non-universal theories
$\mathbb{Z}_3$ breaking to get a light H_3

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$
H_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$

Note: These are only 4 irreps of the full symmetry group

A complete model

$$\boldsymbol{G_{\text{universal}} = SU(3)_c \times SU(2)_L}$$

$$\boldsymbol{G_1 \times G_2 \times G_3 = U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}}$$

Tri-hypercharge

Symmetry breaking:

$$SU(5)^3 \xrightarrow{v_{\text{GUT}}} \text{SM}_1 \times \text{SM}_2 \times \text{SM}_3$$

$$\xrightarrow{v_{\text{SM}^3}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$$

$$\xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$$

$$\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}$$

A complete model

★ **Higgs fields:**

Up Higgses → H_1^u, H_2^u, H_3^u

'Down' Higgses → $H_1^{\bar{5}}, H_2^{\bar{5}}, H_3^{\bar{5}}$

d-quarks and charged leptons splitting via Georgi-Jarlskog factor → $H_1^{45}, H_2^{45}, H_3^{45}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
H_1^u	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H_2^u	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H_3^u	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$
$H_1^{\bar{\mathbf{5}}}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{\bar{\mathbf{5}}}$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$H_3^{\bar{\mathbf{5}}}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
$H_1^{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$
$H_2^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$	$\mathbf{1}$
$H_3^{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{45}$

Three pairs of doublets below $M_{\rm GUT}$

$$H_1^{u,d} \quad H_2^{u,d} \quad H_3^{u,d}$$

⇒

$$M_{H_3^{u,d}} \ll M_{H_2^{u,d}} \ll M_{H_1^{u,d}}$$

$$\tan\beta = \frac{v_3^u}{v_3^d} \approx 20$$

A complete model

Bi-representations:

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
Φ_{12}^F	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
Φ_{13}^F	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Φ_{23}^F	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$
Φ_{12}^T	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
Φ_{13}^T	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
Φ_{23}^T	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$

"Hyperons"

$$\phi_{q12} \sim (\mathbf{1},\mathbf{1})_{(-1/6,1/6,0)} \qquad \phi_{q23} \sim (\mathbf{1},\mathbf{1})_{(0,-1/6,1/6)} \qquad \phi_{\ell 23} \sim (\mathbf{1},\mathbf{1})_{(0,1/2,-1/2)}$$

$$\phi_{q13} \sim (\mathbf{1},\mathbf{1})_{(-1/6,0,1/6)} \qquad \phi_{\ell 13} \sim (\mathbf{1},\mathbf{1})_{(1/2,0,-1/2)}$$

- Spontaneous breaking of individual hypercharges
- Generate the SM flavor structure

Embedding

For instance:

$$\phi_{q12} \sim (\mathbf{1},\mathbf{1})_{(-1/6,1/6,0)} \subset \Phi_{12}^T \sim (\overline{\mathbf{10}},\mathbf{10},\mathbf{1})$$

A complete model

★ **Messenger fields:**

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
χ_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
χ_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
χ_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$

(Vector-like fermions)

- Provide UV-complete theory by mediating the effective operators behind the SM flavor structure
- The heavy Higgses also act as messengers

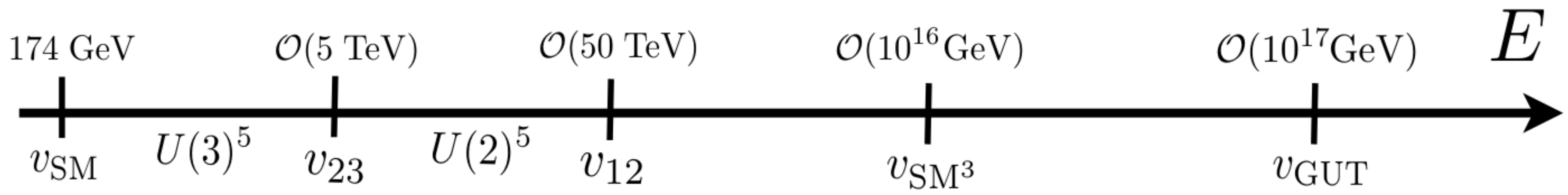
Embedding
For instance:

$$Q_1 \sim (\mathbf{3}, \mathbf{2})_{(1/6,0,0)} \subset \chi_1 \sim (\mathbf{10}, \mathbf{1}, \mathbf{1})$$

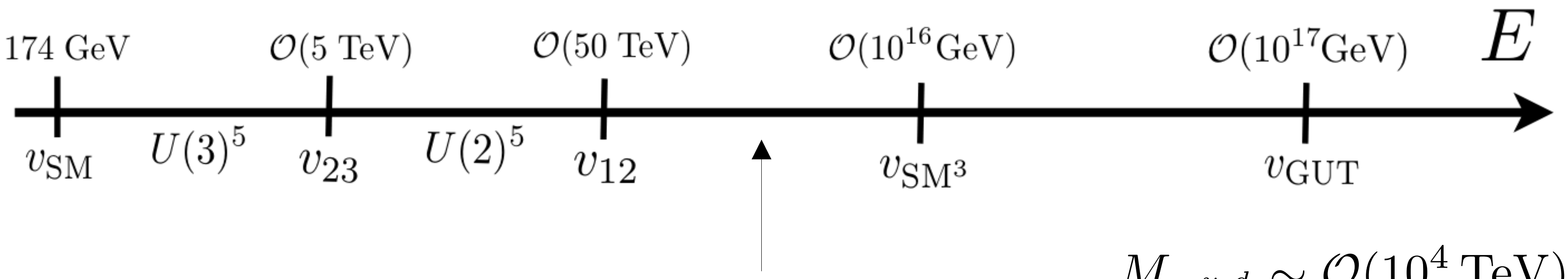
GUTs checklist

- Low-energy theory
- Gauge coupling unification
- Proton decay

Low-energy theory



Low-energy theory



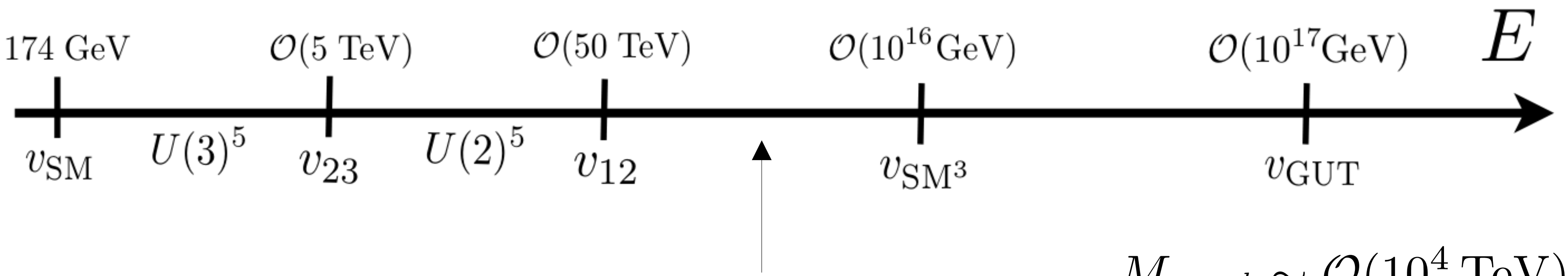
Heavy $Q_i, H^{u,d}_{2,3}$

$$M_{H^{u,d}_1} \sim \mathcal{O}(10^4\ \mathrm{TeV})$$

$$M_{H^{u,d}_2} \sim \mathcal{O}(100\ \mathrm{TeV})$$

$$M_{Q_i} \sim \mathcal{O}(100\ \mathrm{TeV})$$

Low-energy theory



Heavy $Q_i, H_{2,3}^{u,d}$

$$M_{H_1^{u,d}} \sim \mathcal{O}(10^4\ \text{TeV})$$
$$M_{H_2^{u,d}} \sim \mathcal{O}(100\ \text{TeV})$$
$$M_{Q_i} \sim \mathcal{O}(100\ \text{TeV})$$

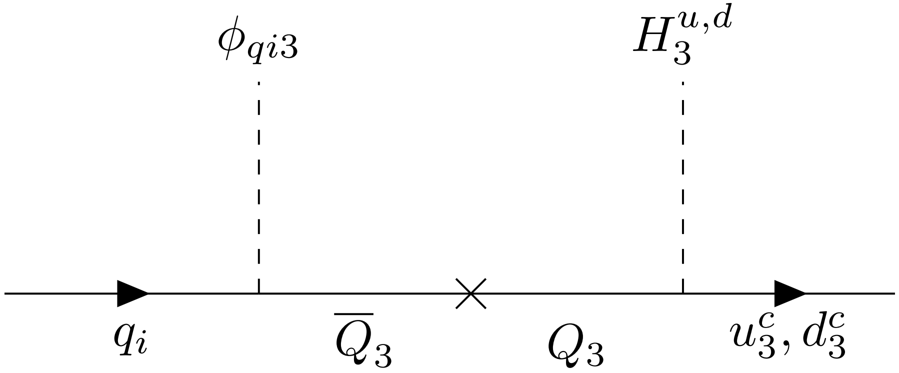
Tri-hypercharge at low (intermediate) energies

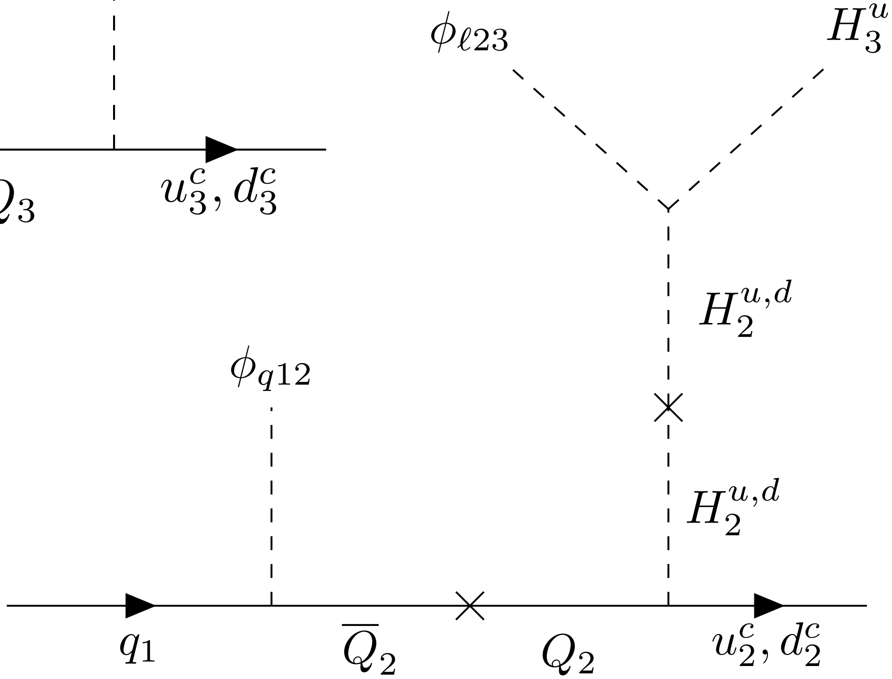
[Fernández Navarro, King, 2023]
[Fernández Navarro, King, AV, 2024]

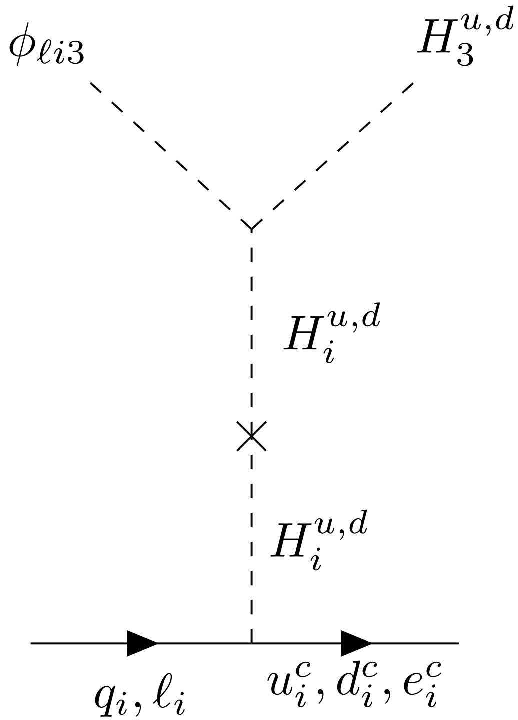
$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

Generation of SM flavor structure

(with $\mathcal{O}(1)$ Yukawa couplings)







Low-energy theory

Generation of SM flavor structure

(with $\mathcal{O}(1)$ Yukawa couplings)

$\lambda \approx 0.224$

Wolfenstein parameter

$$\frac{\langle \phi_{q23} \rangle}{M_{Q_i}} \approx \lambda^2 \qquad \frac{\langle \phi_{q13} \rangle}{M_{Q_i}} \approx \lambda^3 \qquad \frac{\langle \phi_{\ell 23} \rangle}{M_{H_2^{u,d}}} \approx \lambda^3 \qquad \frac{\langle \phi_{q12} \rangle}{M_{Q_i}} \approx \lambda \qquad \frac{\langle \phi_{\ell 23} \rangle}{M_{H_1^{u,d}}} \approx \lambda^6$$

$$\mathcal{M}_u = \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^7 & \lambda^3 & \lambda^2 \\ \lambda^9 & \lambda^5 & 1 \end{pmatrix} v_{\rm SM} \qquad \mathcal{M}_d = \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ \lambda^7 & \lambda^3 & \lambda^2 \\ \lambda^9 & \lambda^5 & 1 \end{pmatrix} \lambda^2 \, v_{\rm SM} \qquad \mathcal{M}_e = \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \lambda^2 \, v_{\rm SM}$$

- Correct mass hierarchies
- Small quark mixing
- Lepton mixing from neutrinos *(in backup)*

Gauge coupling unification

Discontinuities due to matching conditions

For instance

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}$$

at $v_{23} = 5\,\mathrm{TeV}$

Unification achieved with 'light' (~ 100 TeV) color octets

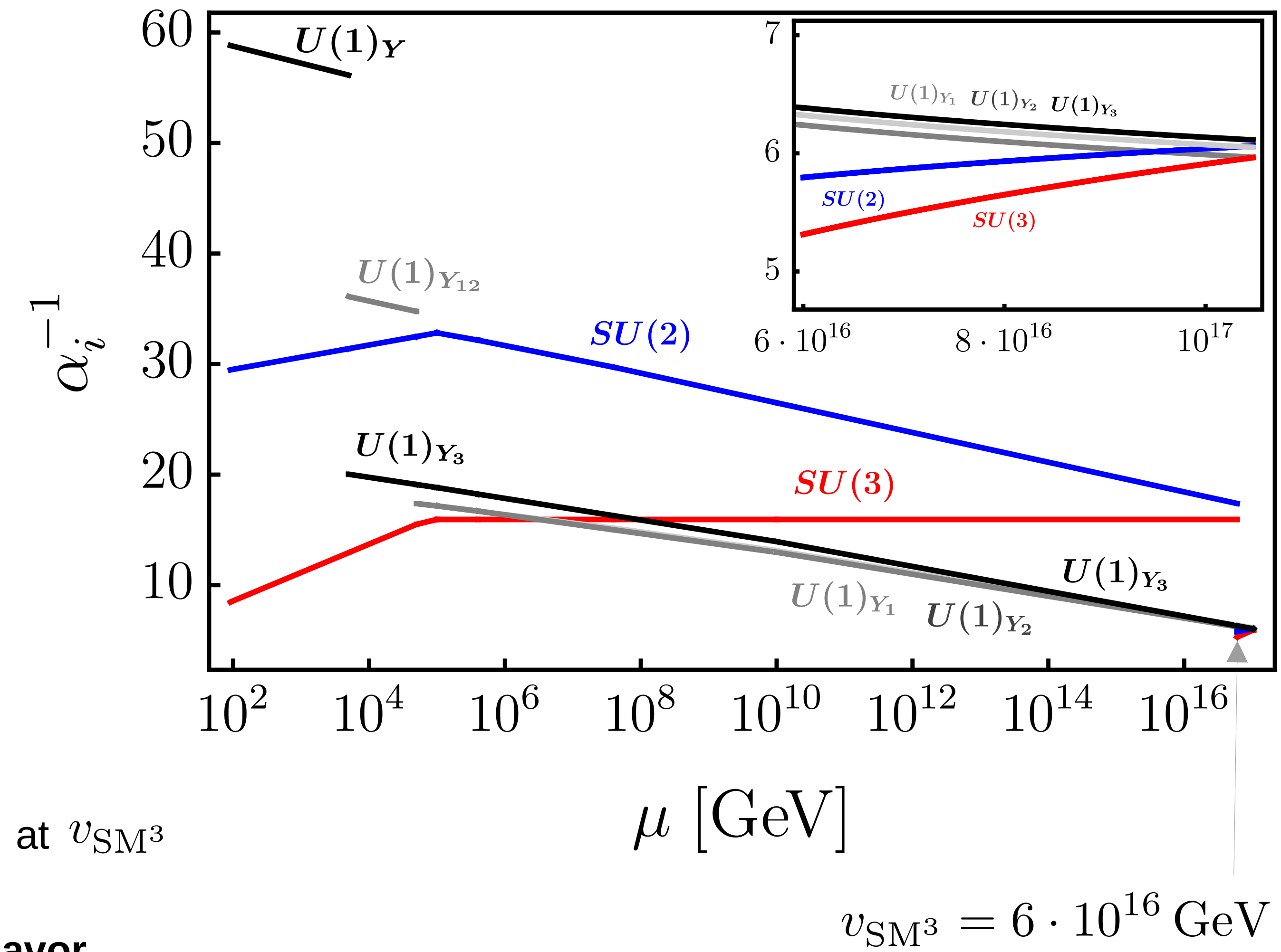
$$\Theta_i \sim (\mathbf{8}, \mathbf{1}, 0)_i$$

(from cyclic **24**)

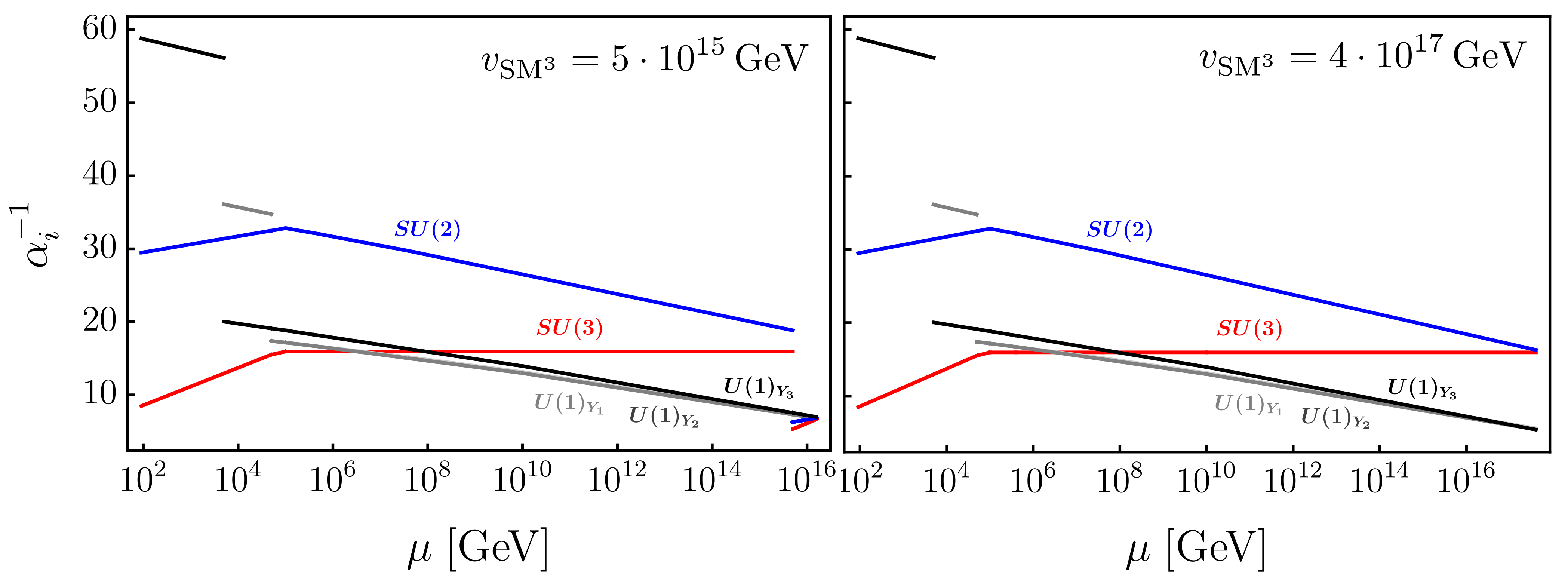
Approximate $\mathbb{Z}_3$ at low energies (exact at the unification scale)

$$g_{s,1} \approx g_{s,2} \approx g_{s,3} \approx \sqrt{3} g_s$$ at v_{SM^3}

Unification fully compatible with flavor



Gauge coupling unification



How low can we deconstruct $SU(3)_c \times SU(2)_L$?

Very low unification scale: proton decay!

No intermediate SM³ scale

Very high unification scale

Proton decay

Proton decay mediated by gauge leptoquarks in $SU(5)^3$

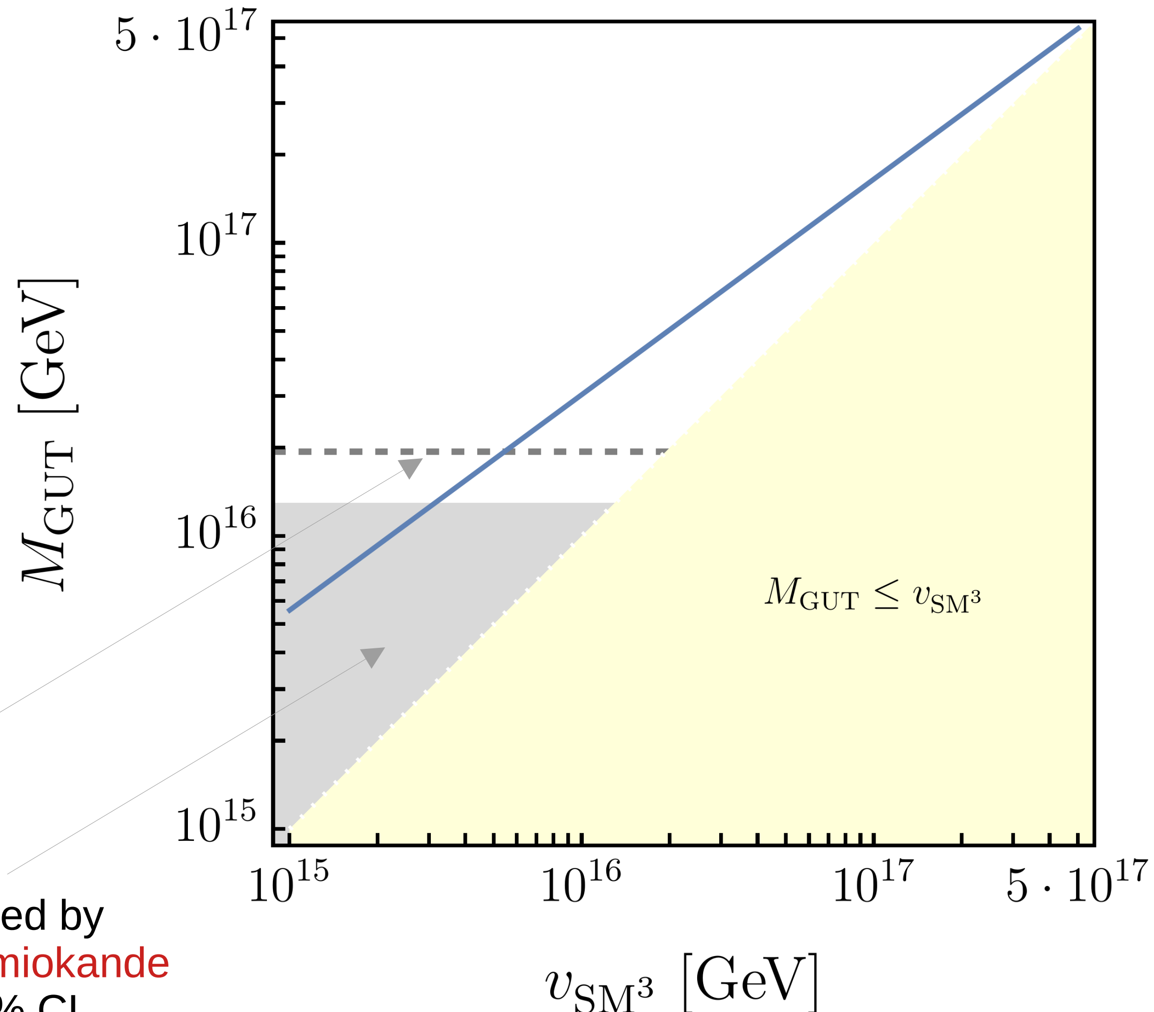
#LQs = 3 x #LQs in standard $SU(5)$

However, 2nd and 3rd family LQs couple to 1st family fermions only via mixing

$$\tau_p \approx \tau_p^{\text{app}} = \frac{m_X^4}{\alpha_{\text{GUT}}^2 m_p^5}$$

Excludes SM³ intermediate scale below

$$\sim 3 \cdot 10^{15} \, \text{GeV}$$



Future Hyper-Kamiokande @ 90% CL

Excluded by Super-Kamiokande @ 90% CL

Proton decay

Proton decay mediated by gauge leptoquarks in $SU(5)^3$

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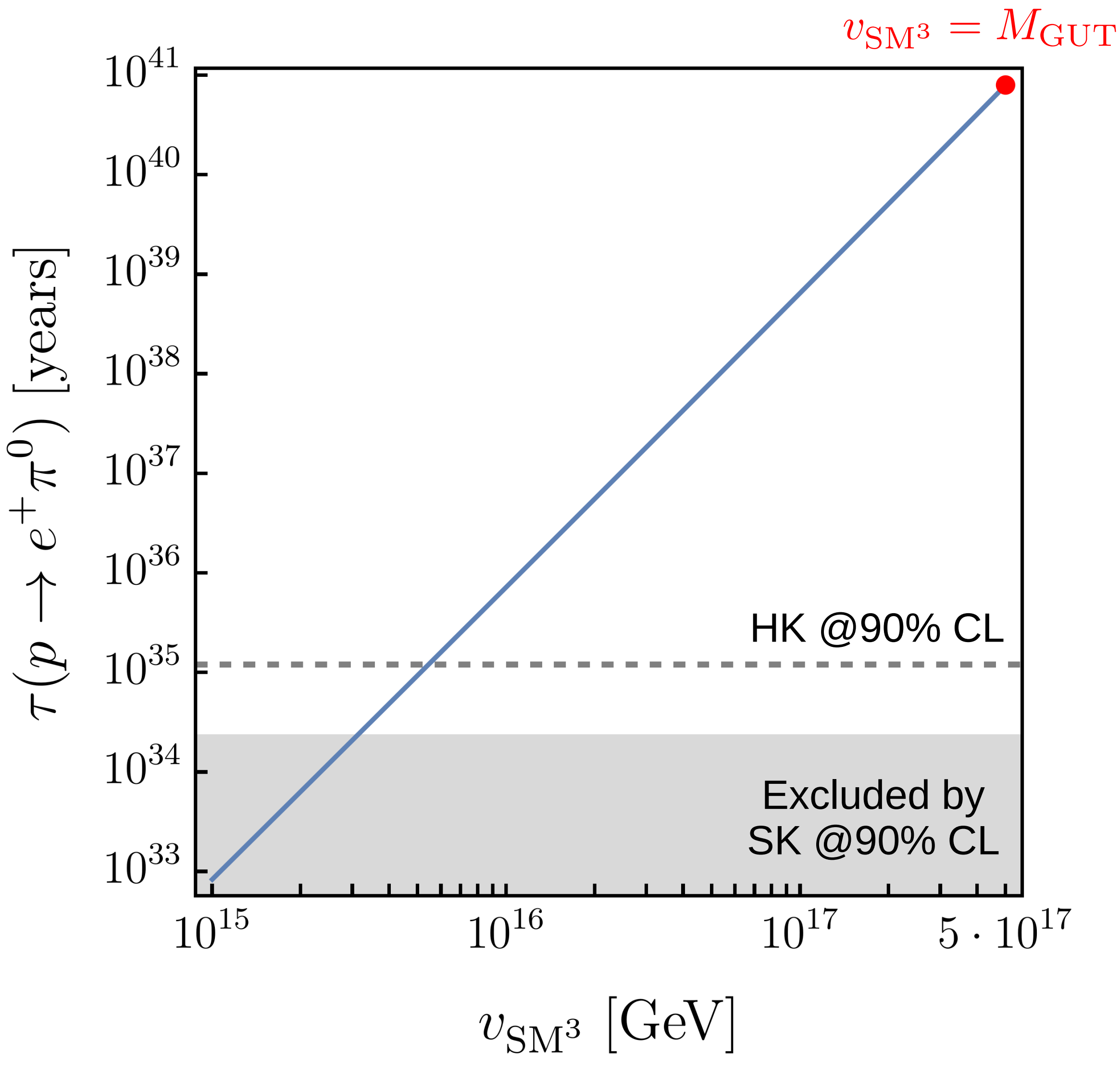

Proton decay

Proton decay mediated by gauge leptoquarks in $SU(5)^3$

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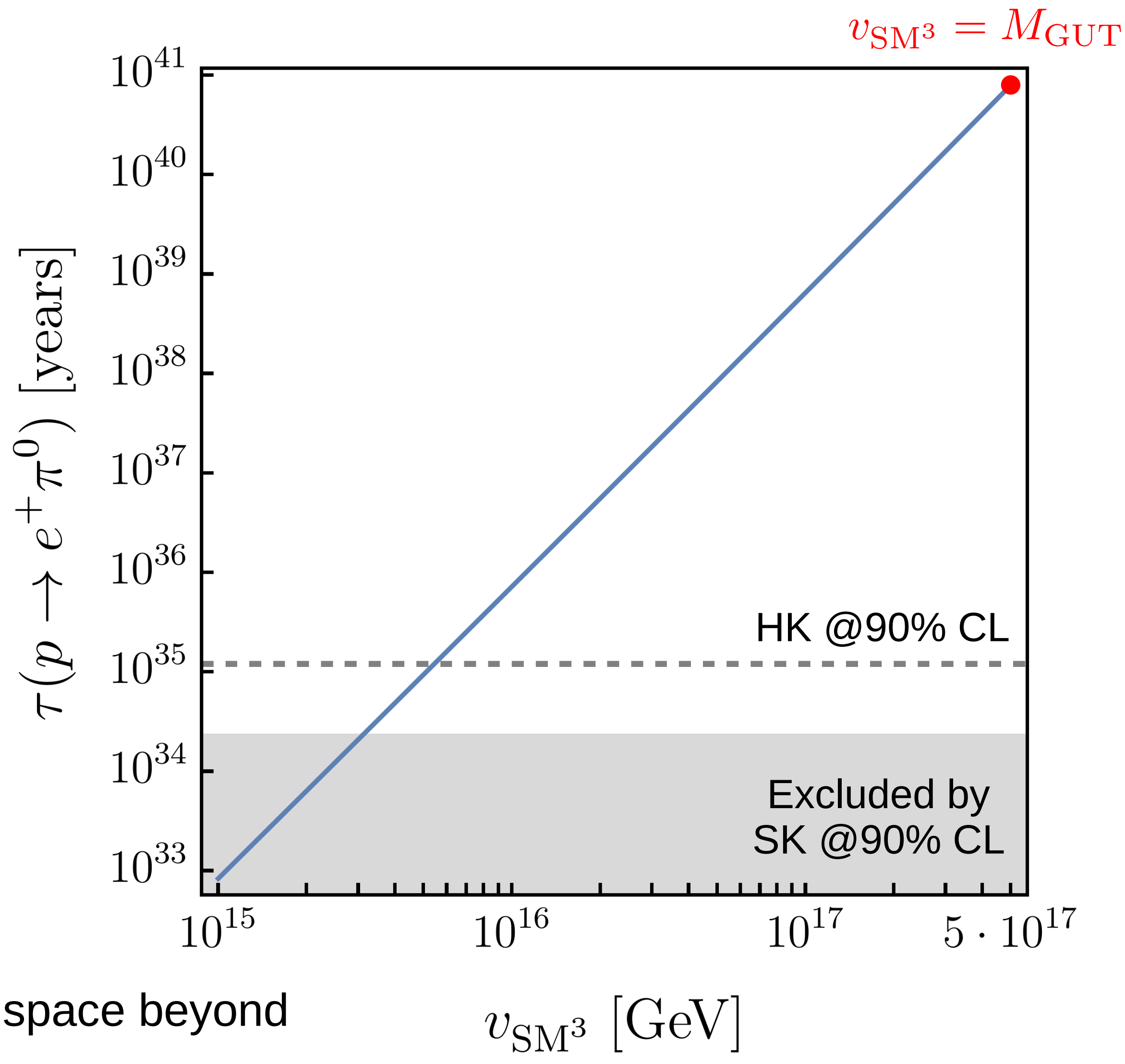


Model probed in proton decay searches

Although large regions of the parameter space beyond any foreseen experiment

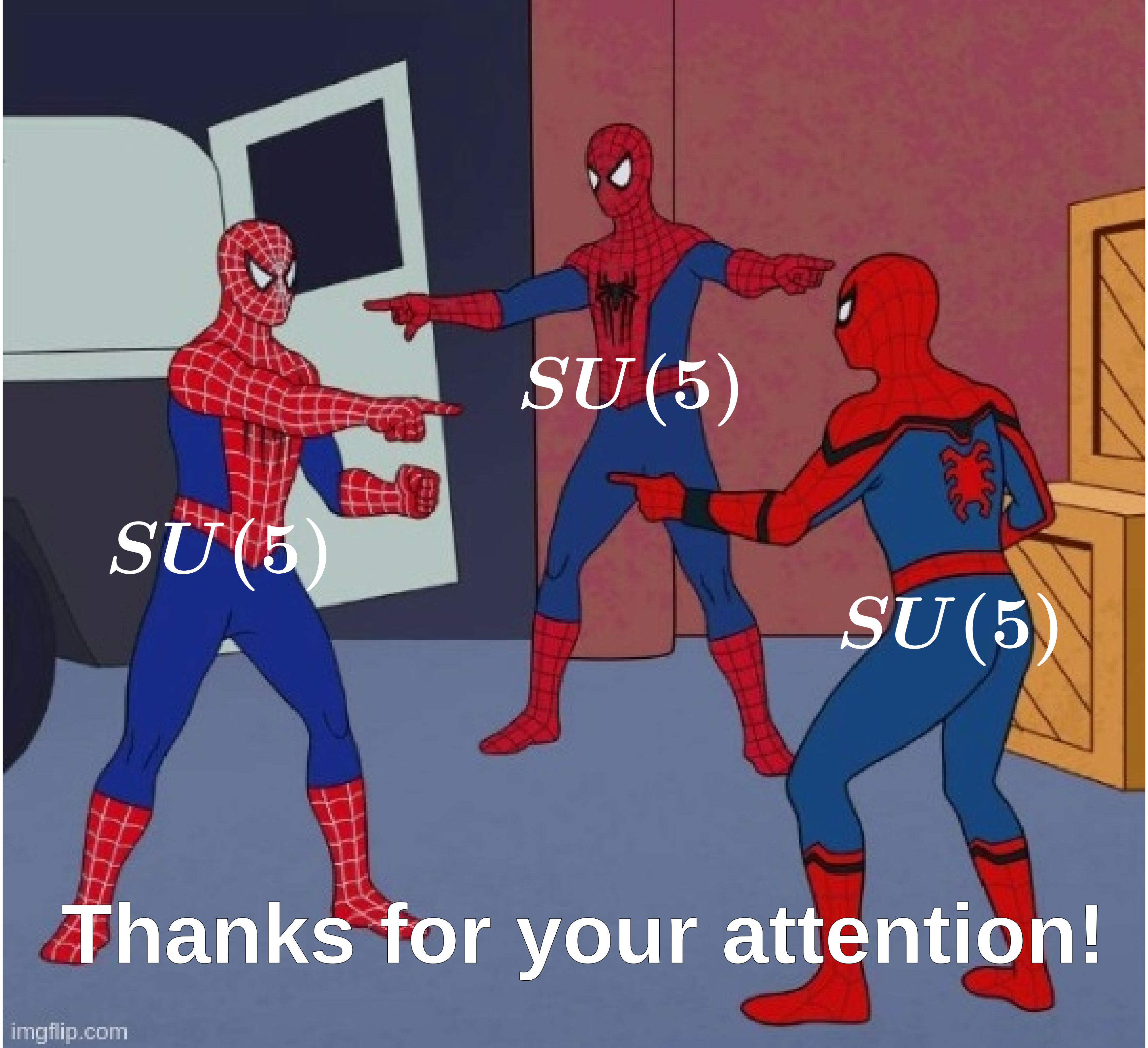
Take-home messages

Flavor deconstruction is a successful way to generate the flavor structure of the SM

Tri-unification is a flavor deconstructed grand unified theory based on the symmetry

$$\boldsymbol{SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3}$$

- ☆ Explains mass hierarchies and mixings
- ☆ Unifies the gauge couplings at high energies
- ☆ Can be probed by proton decay experiments


$SU(5)$
$SU(5)$
$SU(5)$
Thanks for your attention!
imgflip.com

Backup slides

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
F_2	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
F_3	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
T_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
T_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
T_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
χ_1	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$
χ_2	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{1}$
χ_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{10}$
Ξ_0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Ξ_{12}	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
Ξ_{13}	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Ξ_{23}	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$
$\Sigma_{\rm atm}$	$\mathbf{1}$	$\mathbf{10}$	$\overline{\mathbf{10}}$
$\Sigma_{\rm sol}$	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
$\Sigma_{\rm cyclic}$	$\mathbf{10}$	$\overline{\mathbf{10}}$	$\mathbf{1}$
Ω_1	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
Ω_2	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$
Ω_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$

Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
H^u_1	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$
H^u_2	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$
H^u_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5}$
$H^{\bar{\mathbf{5}}}_1$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$
$H^{\bar{\mathbf{5}}}_2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
$H^{\bar{\mathbf{5}}}_3$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{5}}$
$H^{\mathbf{45}}_1$	$\mathbf{45}$	$\mathbf{1}$	$\mathbf{1}$
$H^{\mathbf{45}}_2$	$\mathbf{1}$	$\mathbf{45}$	$\mathbf{1}$
$H^{\mathbf{45}}_3$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{45}$
Φ^F_{12}	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
Φ^F_{13}	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Φ^F_{23}	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$
Φ^T_{12}	$\overline{\mathbf{10}}$	$\mathbf{10}$	$\mathbf{1}$
Φ^T_{13}	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
Φ^T_{23}	$\mathbf{1}$	$\overline{\mathbf{10}}$	$\mathbf{10}$
$\Phi^{\mathbf{45}}_{12}$	$\mathbf{1}$	$\overline{\mathbf{45}}$	$\mathbf{45}$
$\Phi^{\mathbf{45}}_{13}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$
$\Phi^{\mathbf{45}}_{12}$	$\overline{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$
Φ^{TFT}	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{10}$
Φ^{FTT}	$\mathbf{5}$	$\mathbf{10}$	$\mathbf{10}$
Φ^{TTF}	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$

Neutrino masses

Field	$\boldsymbol{SU(5)_1}$	$\boldsymbol{SU(5)_2}$	$\boldsymbol{SU(5)_3}$
Ξ_0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Ξ_{12}	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$
Ξ_{13}	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$
Ξ_{23}	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$
$\Sigma_{\rm atm}$	$\mathbf{1}$	$\mathbf{10}$	$\overline{\mathbf{10}}$
$\Sigma_{\rm sol}$	$\mathbf{10}$	$\mathbf{1}$	$\overline{\mathbf{10}}$
$\Sigma_{\rm cyclic}$	$\mathbf{10}$	$\overline{\mathbf{10}}$	$\mathbf{1}$
$\Phi_{12}^{\mathbf{45}}$	$\mathbf{1}$	$\overline{\mathbf{45}}$	$\mathbf{45}$
$\Phi_{13}^{\mathbf{45}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{45}$
$\Phi_{12}^{\mathbf{45}}$	$\overline{\mathbf{45}}$	$\mathbf{45}$	$\mathbf{1}$
Φ^{TFT}	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{10}$
Φ^{FTT}	$\mathbf{5}$	$\mathbf{10}$	$\mathbf{10}$
Φ^{TTF}	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$

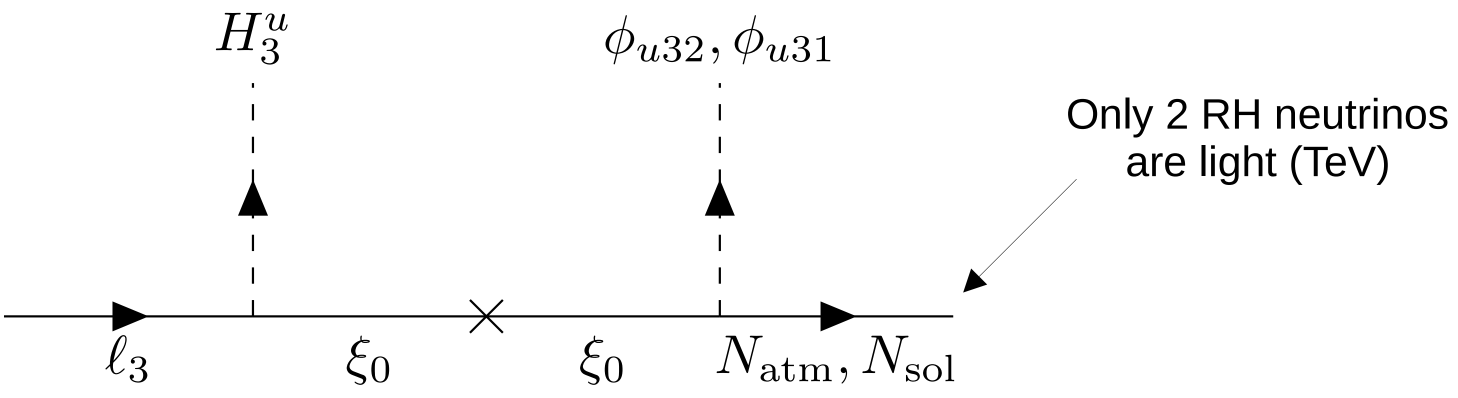
New messenger fields → (Ξ rows)

Right-handed neutrinos → (Σ rows)

New bi-representations (new hyperons) → (Φ rows)

More economical implementation in [Fernández Navarro, King, AV, 2024]

Neutrino masses



$$m_\nu \simeq m_D M_N^{-1} m_D^{\rm T} \approx \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\rm SM}^2}{M_\xi^2}$$

Type-I seesaw mechanism

Correct lepton mixing + a massless neutrino

Phenomenology

[Fernández Navarro, King, AV, 2024]

All new particles are too heavy to be probed directly

FCNCs from heavy Z' bosons

VL fermions may induce large (but acceptable) contributions to meson mixing via boxes

General (model-independent) bounds:

- Dilepton tails at the LHC
- Modification of EW precision observables