Searching new physics via features of the stochastic gravitational wave background

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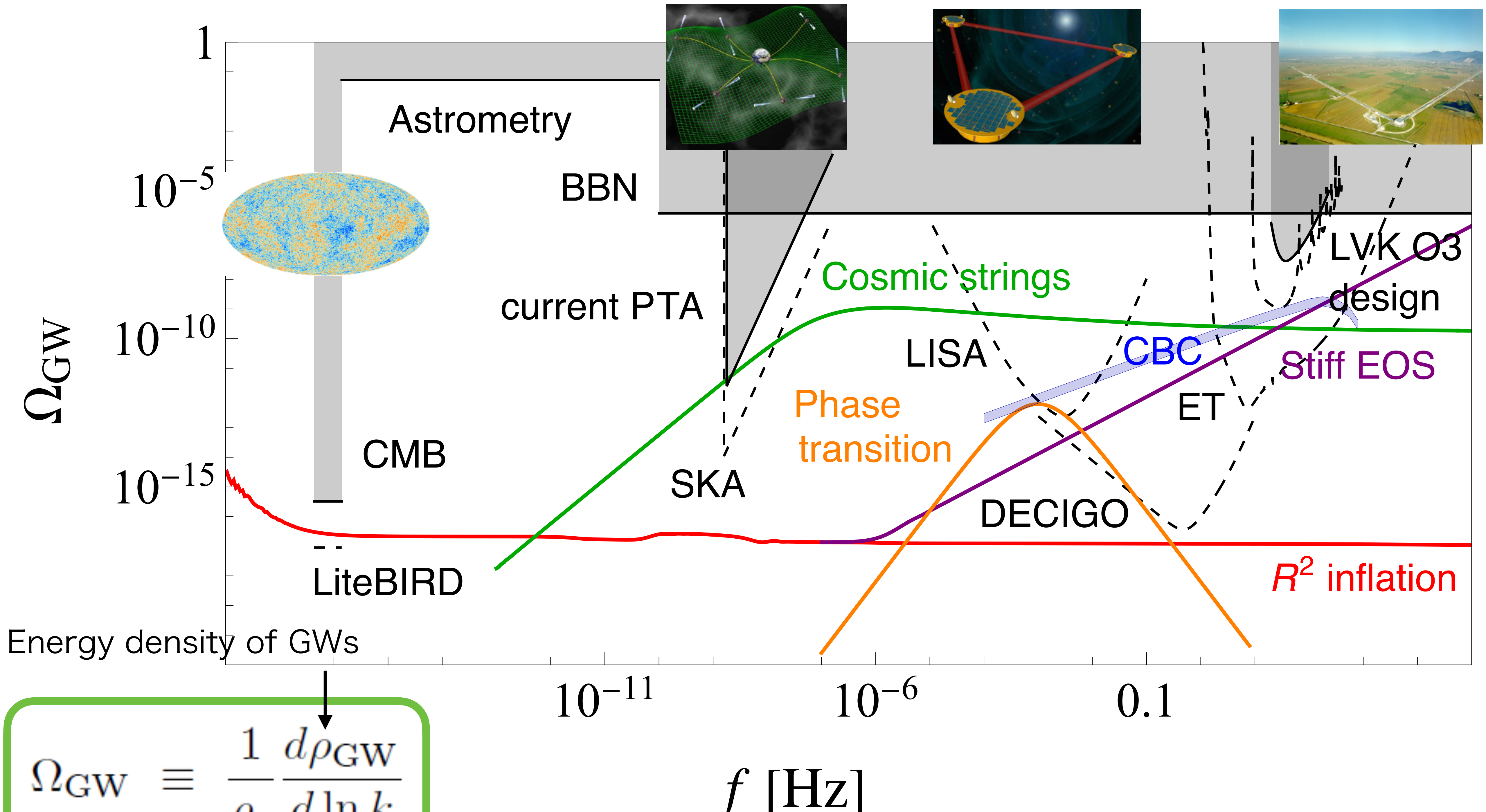
Stochastic GWs as a probe of the early universe!



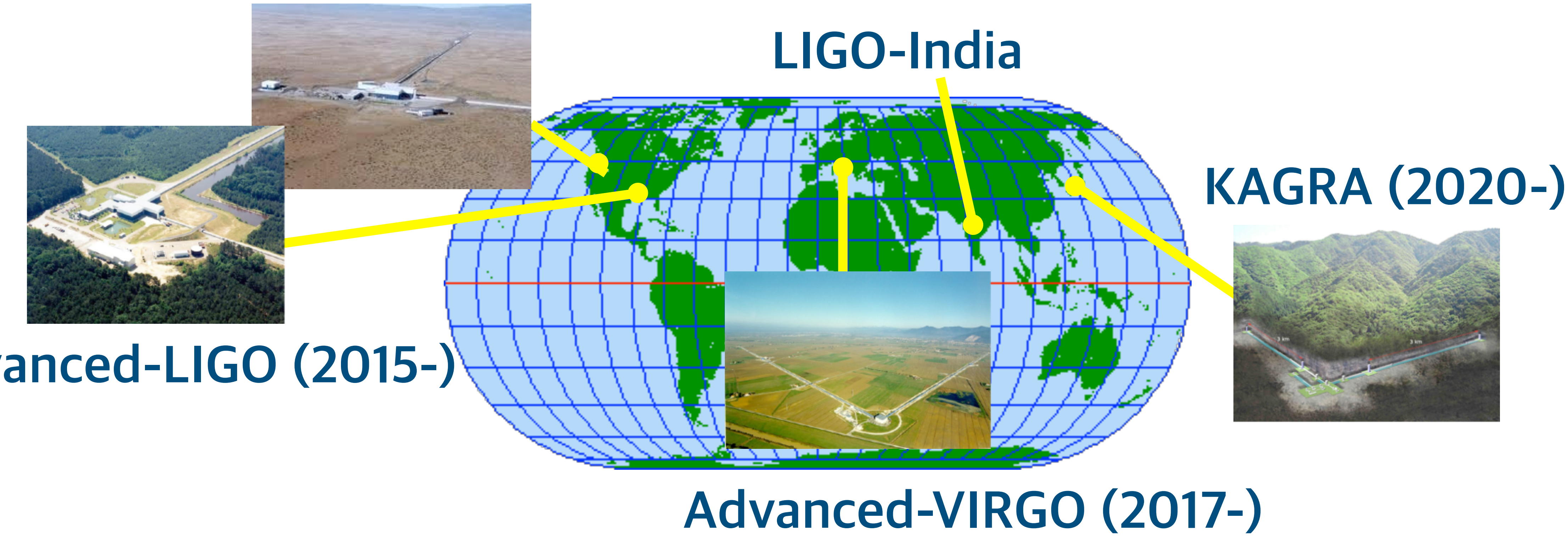
Energy density of GWs

$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k}$$

Critical density of the Universe

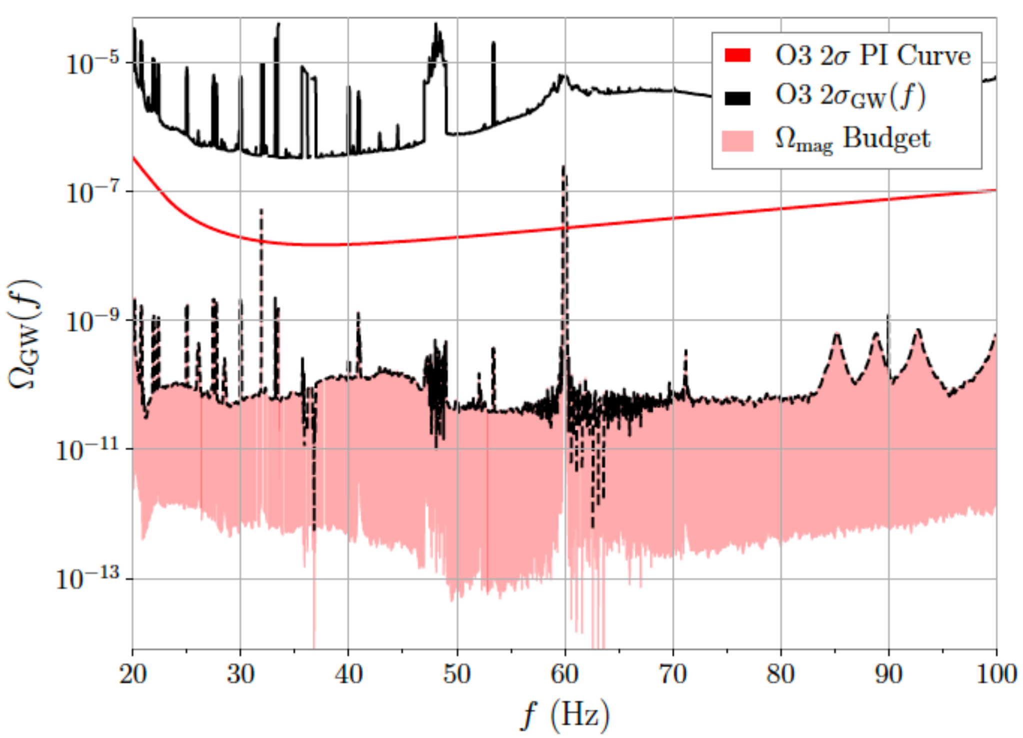
→ a unique test of high energy theories

Worldwide GW detector network

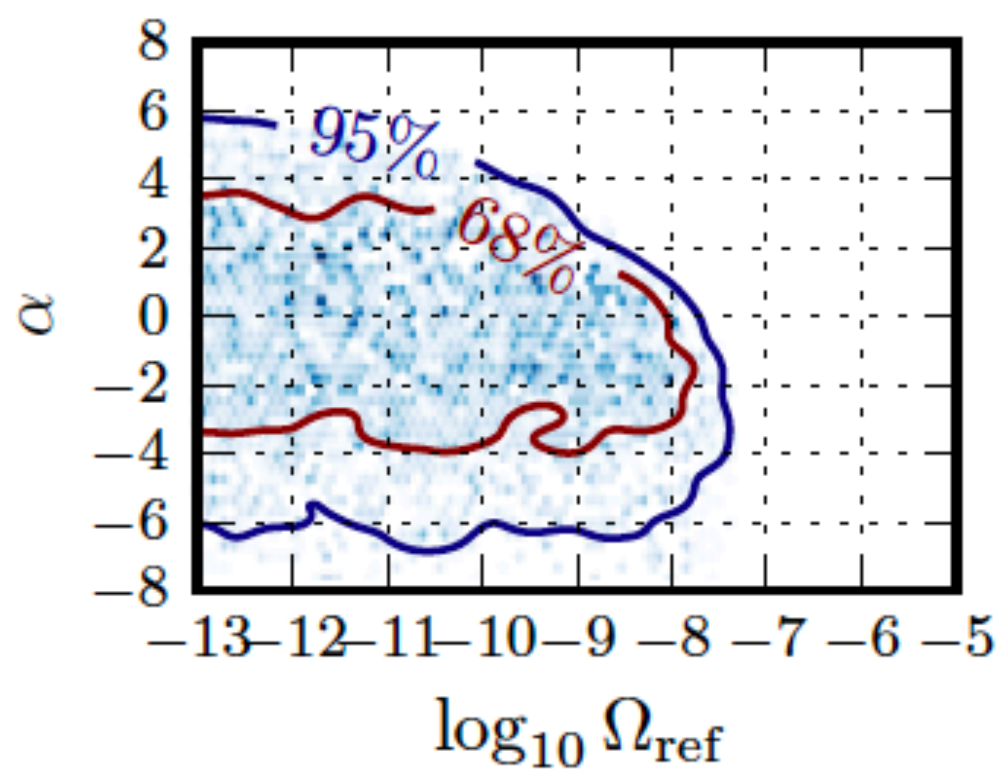


Most recent public data: O3 (April 2019 - March 2020)

Upper bound on a stochastic GW background



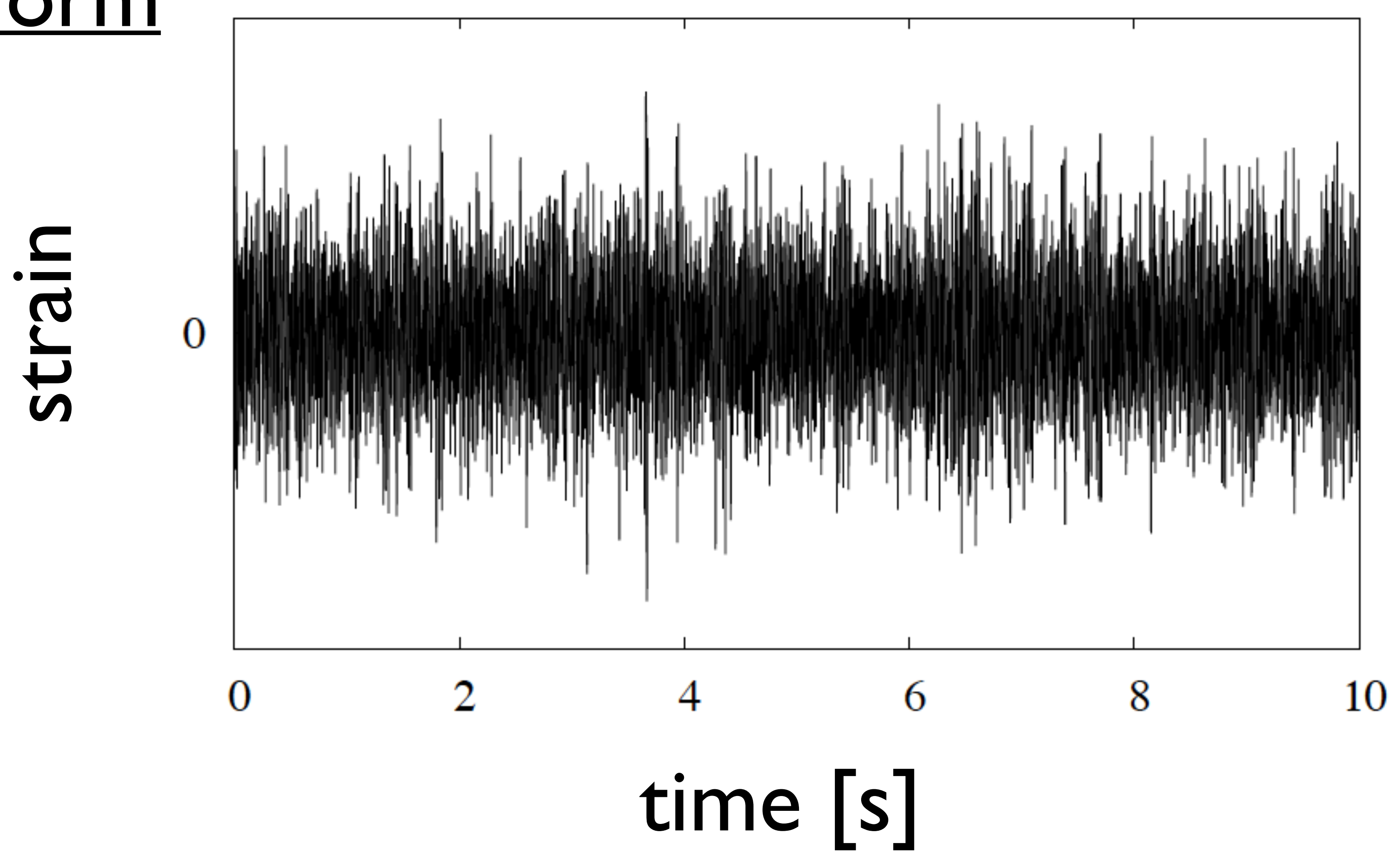
$\Omega_{GW} < 5.8 \times 10^{-9}$ (95%CL) for a flat spectrum



LVK collaboration, PRD 104, 022004 (2021)

Stochastic GW background

Waveform



Continuous and random GW signal
coming from all directions → very similar to noise

How to detect a stochastic background

Cross Correlation

detector1

$s_1(t) = h(t) + n_1(t)$

detector2

$s_2(t) = h(t) + n_2(t)$

s: observed signal
h: gravitational waves
n: noise

$$\langle S \rangle = \int_{-T/2}^{T/2} dt \ \langle s_1(t) s_2(t) \rangle$$

$$= \int_{-T/2}^{T/2} dt \ \langle h^2(t) + \underline{h(t) n_2(t) + n_1(t) h(t) + n_1(t) n_2(t)} \rangle$$

no correlations → 0

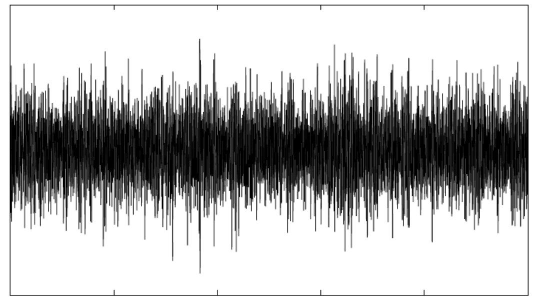
$$= \int_{-T/2}^{T/2} dt \ \underline{\langle h^2(t) \rangle}$$

GW signal

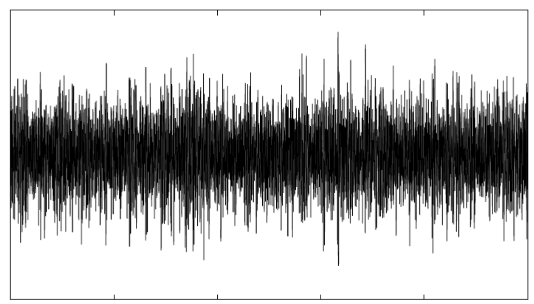
(for detectors at the same location)

What we do in the LVK stochastic search

Renzini et al., ApJ 952, 25 (2023)



strain data 1



strain data 2

strain data in discrete Fourier space

$$\tilde{s}_f \equiv \sum_{t_k=0}^{T-\delta t} s(t_k)\, e^{-i2\pi m t_k/T}$$

for every T = 192 s

cross-correlated spectrum

$$C_{IJ,f} = \frac{2}{T}\tilde{s}^*_{I,f}\tilde{s}_{J,f}$$

IJ: detector combinations

estimator

$$\hat{\Omega}_{\mathrm{GW},f} = \frac{\mathrm{Re}[C_{IJ,f}]}{\gamma_{IJ}(f)\, S_0(f)}$$

$$S_0(f) = \frac{3H_0^2}{10\pi^2}\frac{1}{f^3}$$

c.f. $g_{ab} = \eta_{ab} + h_{ab}$

$$\langle h_{ab}(t)h^{ab}(t)\rangle = 2\int_{-\infty}^{\infty} \mathrm{d}f\, S_h(f)$$

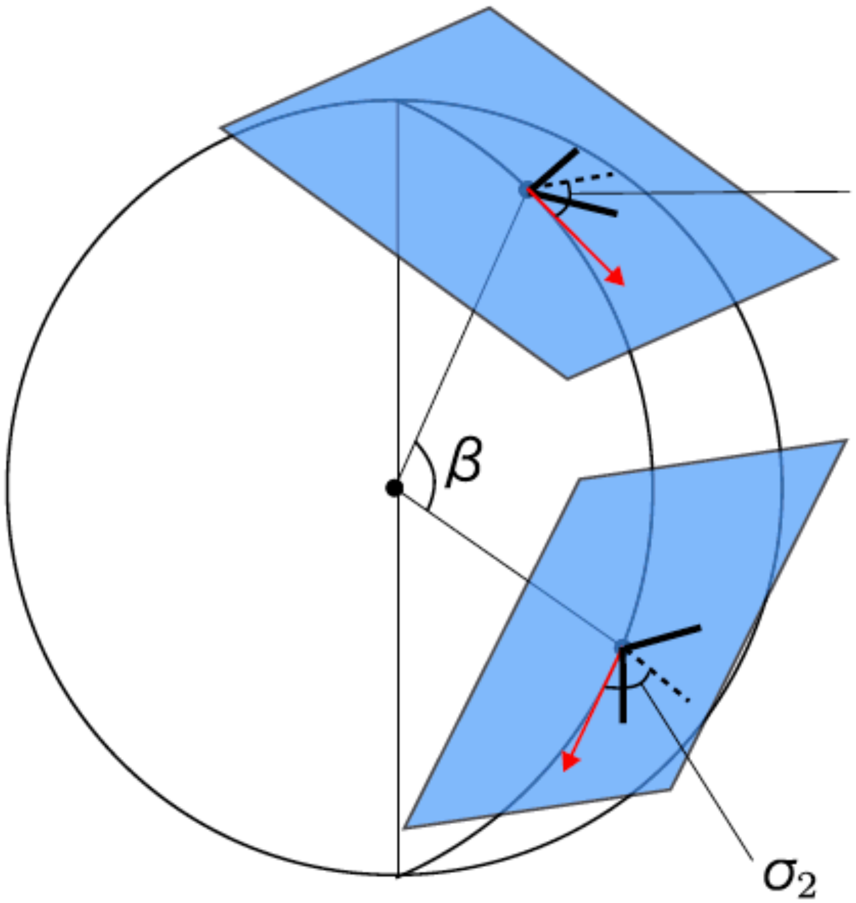
$$\Omega_{\mathrm{gw}}(f) = (4\pi^2/3H_0^2)f^3 S_h(f)$$

overlap reduction function

determined by detector response

Overlap reduction function

Detectors are located at **different site** and **facing different direction**



① Time difference

② Antenna pattern

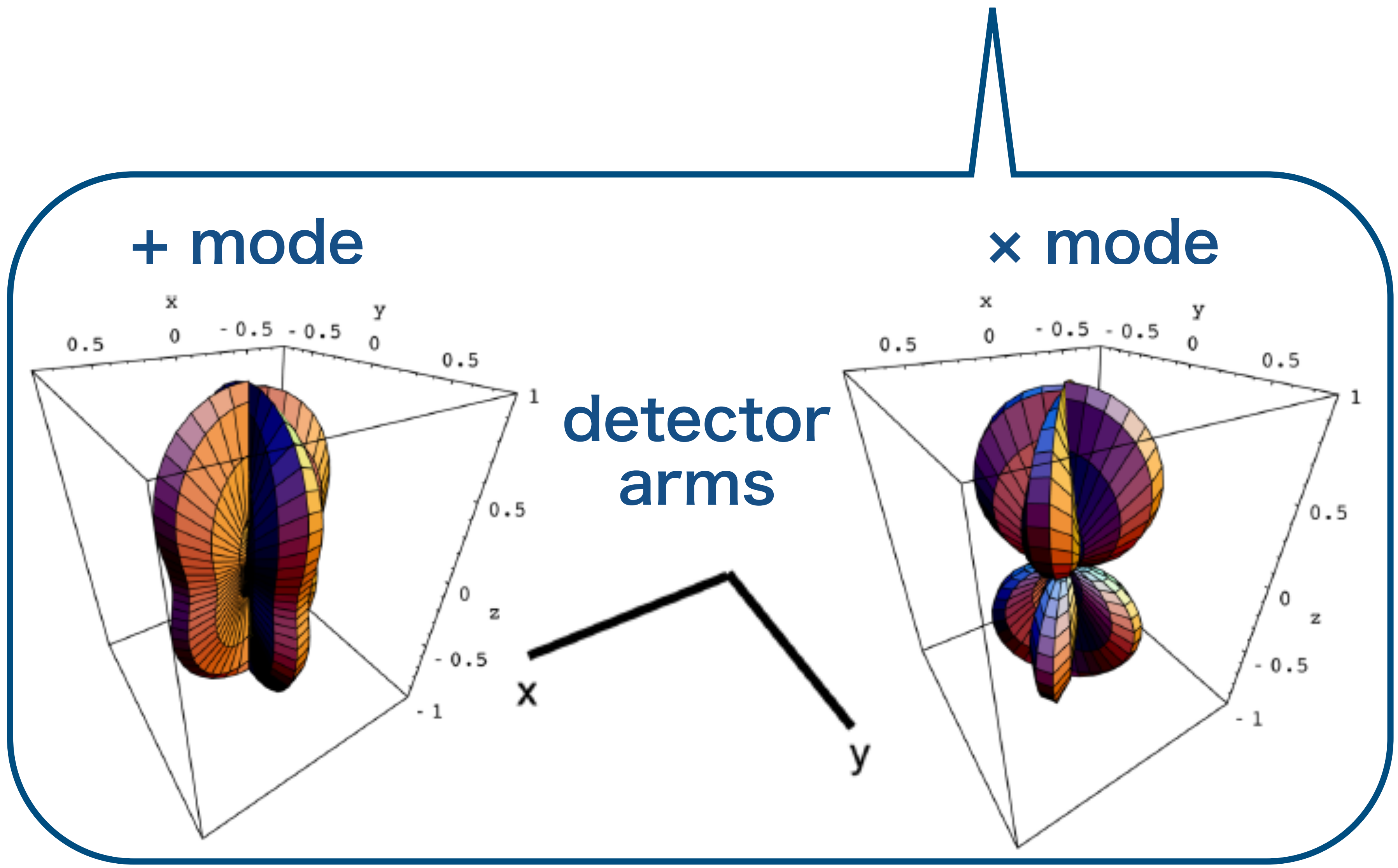


Figure from A. Nishizawa et al. PRD 79, 082002 (2009)

Overlap reduction function

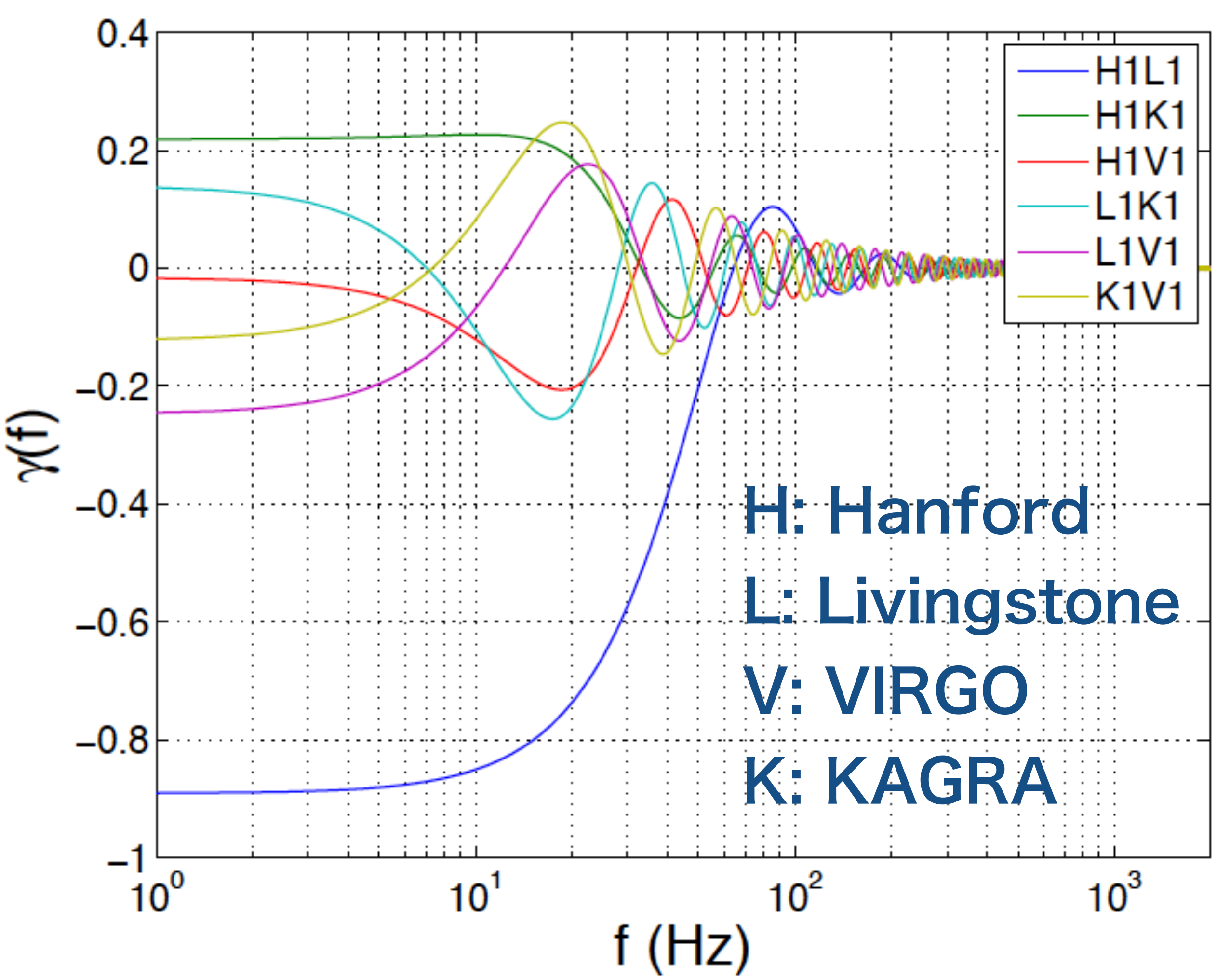
For a stochastic GW background, we construct

$$\gamma_{IJ}^{T}(f) \equiv \frac{5}{2}\int_{S^2}\frac{d\hat{\Omega}}{4\pi}e^{2\pi i f\hat{\Omega}\cdot\Delta\vec{X}/c}(F_I^{+}F_J^{+}+F_I^{\times}F_J^{\times})$$

integration over the whole sky

① Time difference ② Antenna pattern

I and J denote different detectors



→ **represents reduction in sensitivity due to the separation/orientation of the detectors**

Figure from E. Thrane & J. D. Romano, PRD 88, 124032 (2013)

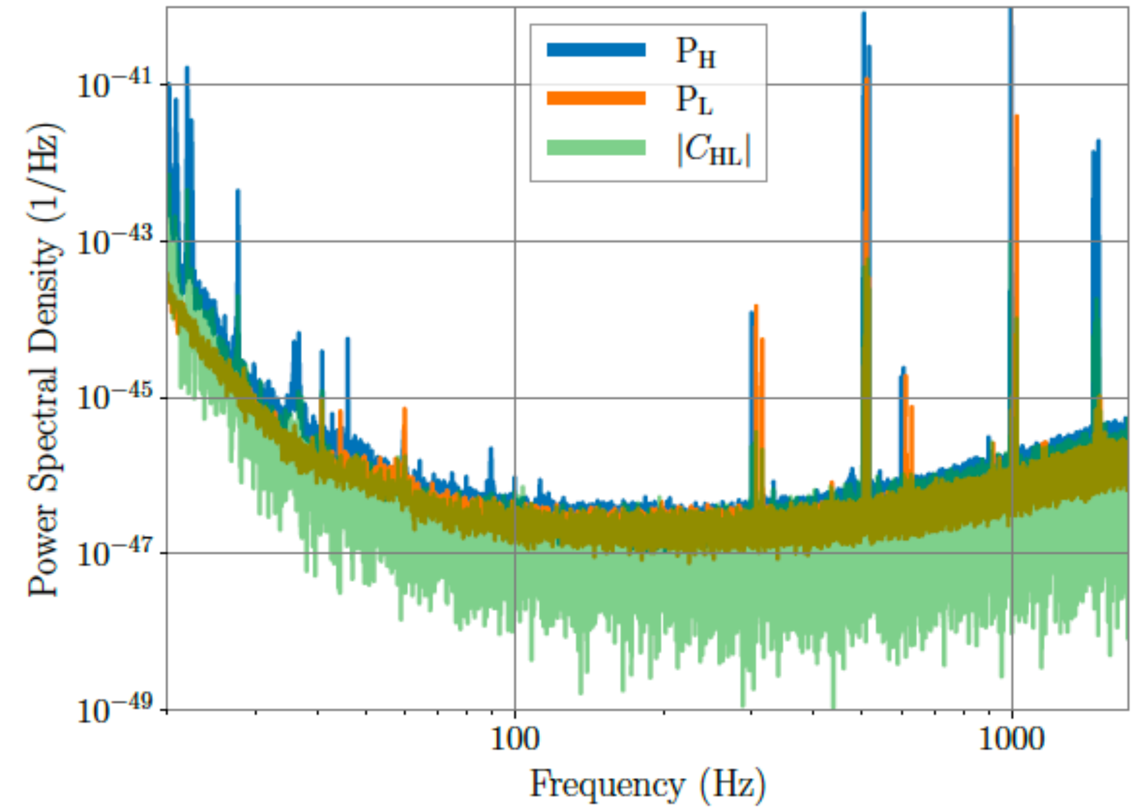
Variance: level of noise

strain data in discrete Fourier space

signal ↓ noise ↓

$$\tilde{s}_{I,f} = F_I \tilde{h}_f + \tilde{n}_{I,f}$$

↑ antenna pattern for + and x



cross-correlation

$$C_{IJ,f} = \frac{2}{T}\tilde{s}^*_{I,f}\tilde{s}_{J,f}$$

→ dominated by $\langle \tilde{h}_f^2 \rangle$

estimator

$$\hat{\Omega}_{\mathrm{GW},f} = \frac{\mathrm{Re}[C_{IJ,f}]}{\gamma_{IJ}(f) S_0(f)}$$

auto-correlation

$$P_{I,f} = \frac{2}{T}|\tilde{s}_{I,f}|^2$$

→ dominated by $\langle \tilde{n}_{I,f}^2 \rangle$

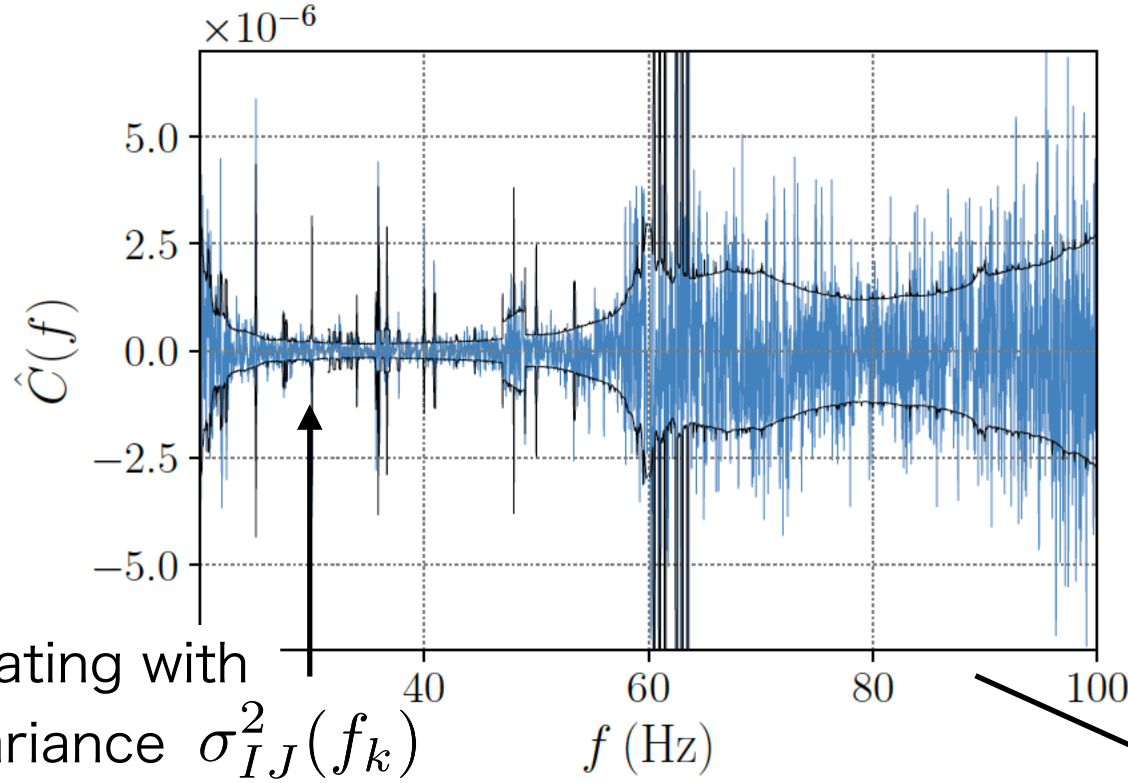
variance

$$\sigma^2_{IJ,k} = \frac{1}{2T\Delta f}\frac{P_{I,f}P_{J,f}}{\gamma^2_{IJ}(f)S^2_0(f)}$$

→ an indicator of the level of noise

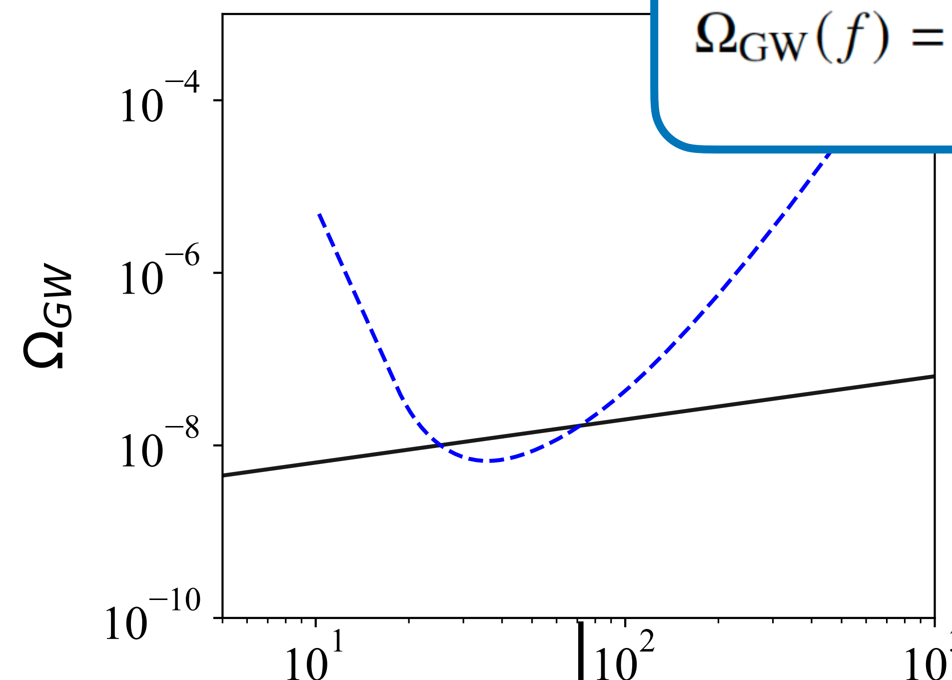
Likelihood analysis

data (cross-correlated spectrum)



fluctuating with the variance $\sigma^2_{IJ}(f_k)$

Your model



LVK analysis

Model = power-law

$$\Omega_{\rm GW}(f) = \Omega_\alpha \left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

Likelihood

$$p(\hat{C}^{IJ}_k|\mathbf{\Theta}) \propto \exp\left[-\frac{1}{2}\sum_{IJ}\sum_k\left(\frac{\hat{C}^{IJ}_k - \Omega_{\rm M}(f_k|\mathbf{\Theta})}{\sigma^2_{IJ}(f_k)}\right)^2\right]$$

variance

IJ: detector combinations

k: frequencies

Prior

Posterior distribution

$$p(\mathbf{\Theta}|C^{IJ}_k) \propto p(C^{IJ}_k|\mathbf{\Theta})p(\mathbf{\Theta})$$

Likelihood analysis

Posterior distribution $\propto$

$$p(\hat{C}_k^{IJ}|\mathbf{\Theta}) \propto \exp\left[-\frac{1}{2}\sum_{IJ}\sum_{k}\left(\frac{\hat{C}_k^{IJ}-\Omega_{\rm M}(f_k|\mathbf{\Theta})}{\sigma_{IJ}^2(f_k)}\right)^2\right]$$

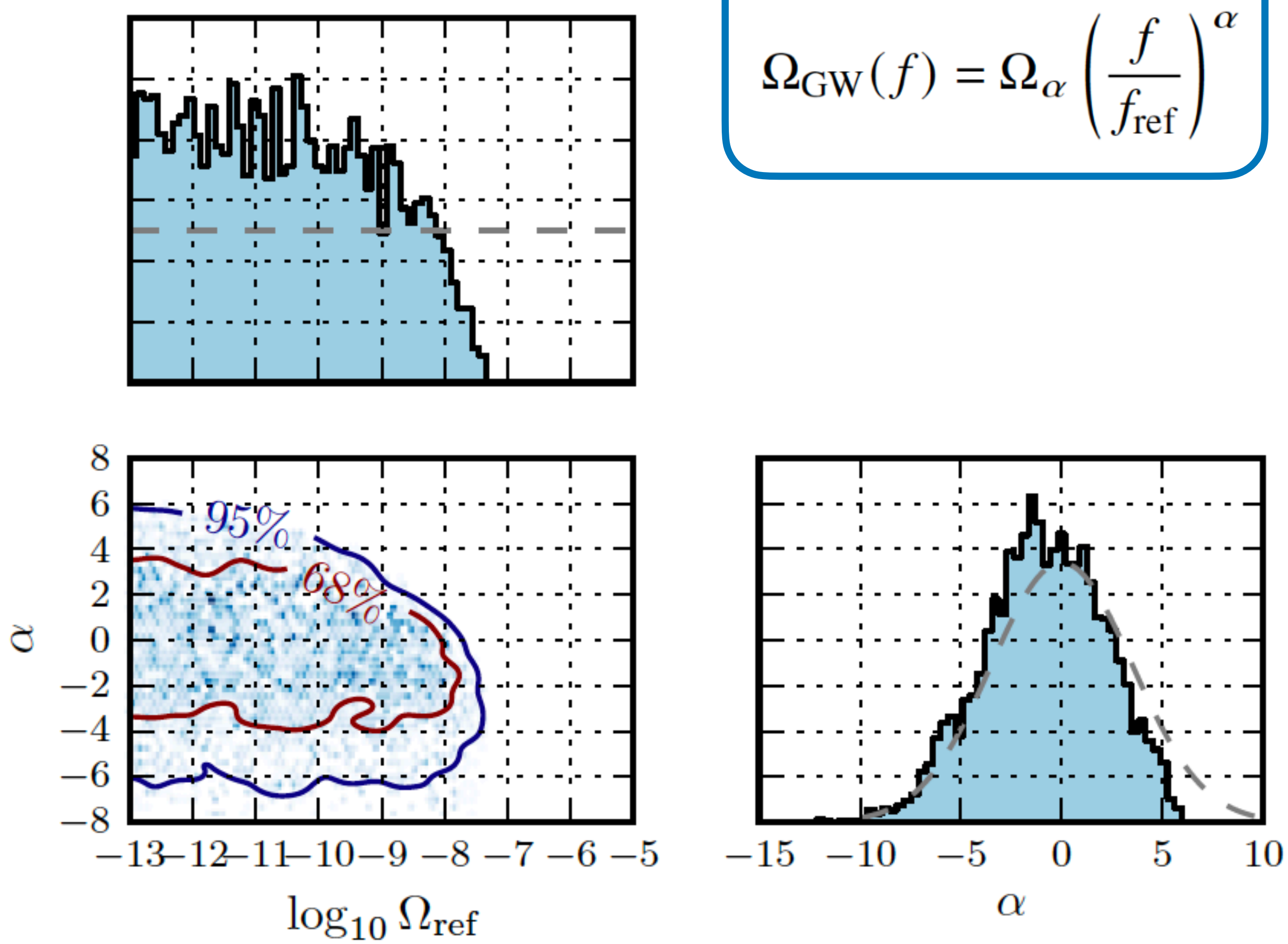
IJ: detector combinations
k: frequencies

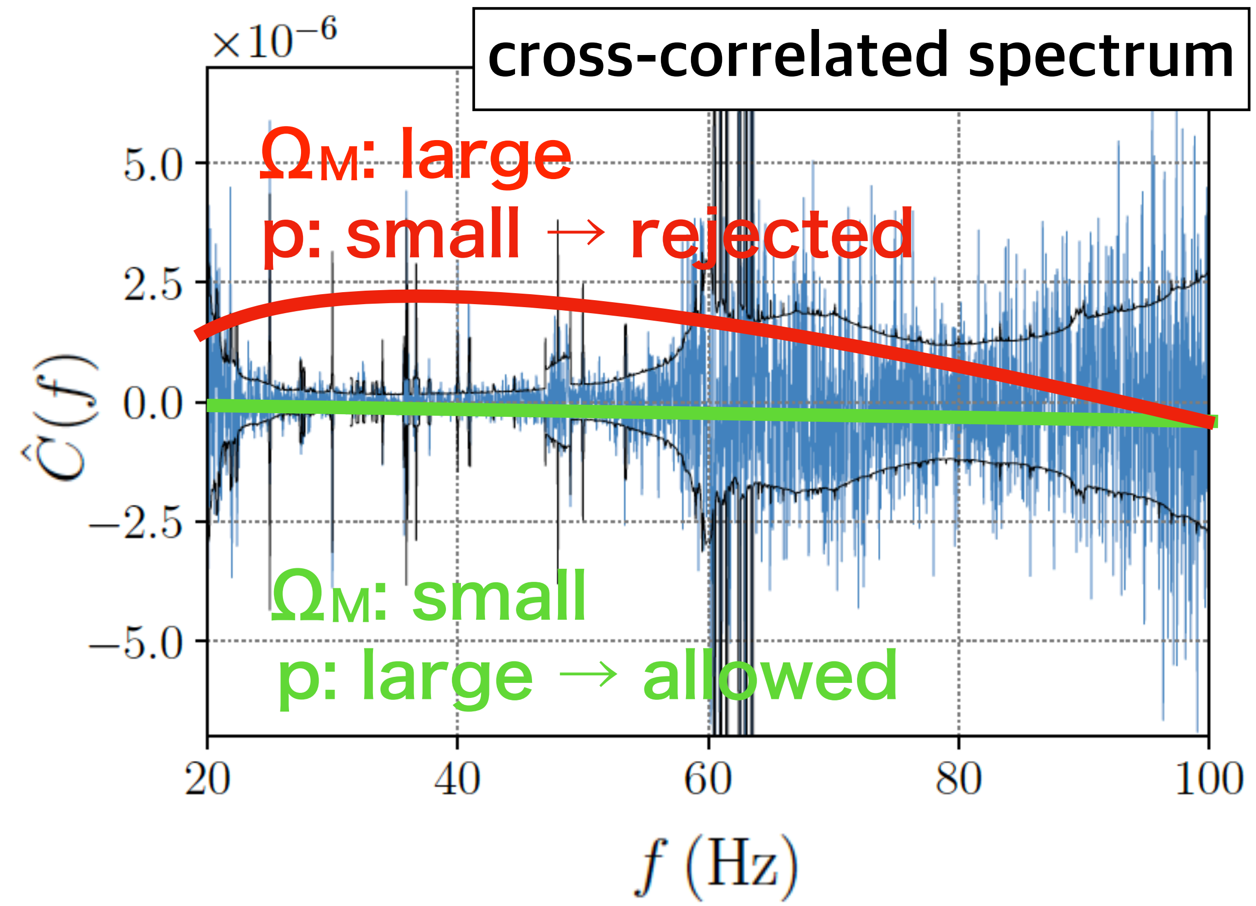
(for a flat & infinite-range prior)

Constraint by O3 data

Model = power-law

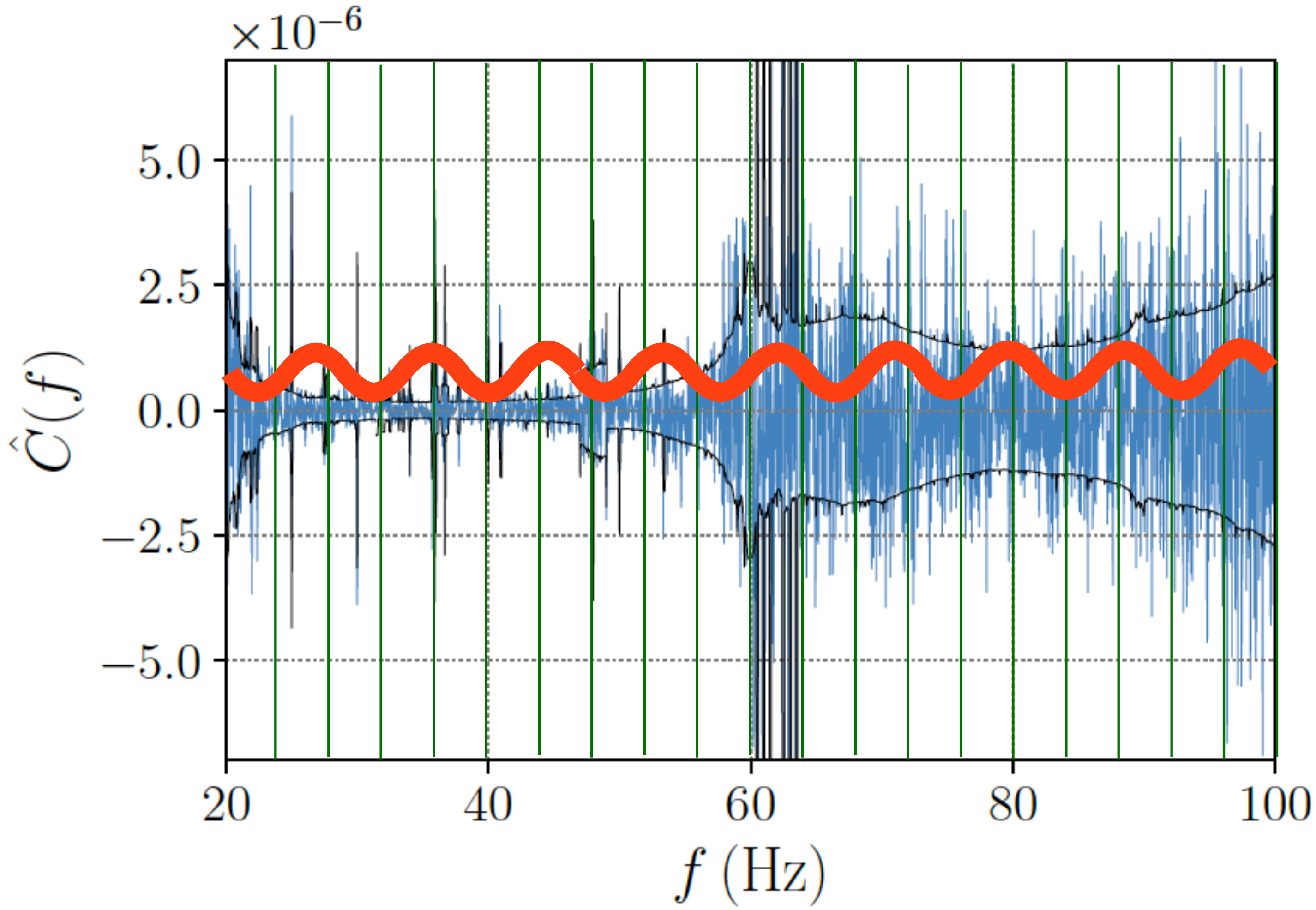
$$\Omega_{\rm GW}(f) = \Omega_\alpha\left(\frac{f}{f_{\rm ref}}\right)^\alpha$$





LVK collaboration, PRD 104, 022004 (2021)

Probing GWs with features



For LISA,
see the talk by G. Nardini on Friday and
C. Caprini et al. JCAP, 11 (2019) 017

LVK analysis

Frequency resolution 1/32Hz

Likelihood

$$p(\hat{C}_k^{IJ}|\mathbf{\Theta}) \propto \exp\left[-\frac{1}{2}\sum_{IJ}\sum_k \left(\frac{\hat{C}_k^{IJ} - \Omega_{\rm M}(f_k|\mathbf{\Theta})}{\sigma_{IJ}^2(f_k)}\right)^2\right]$$

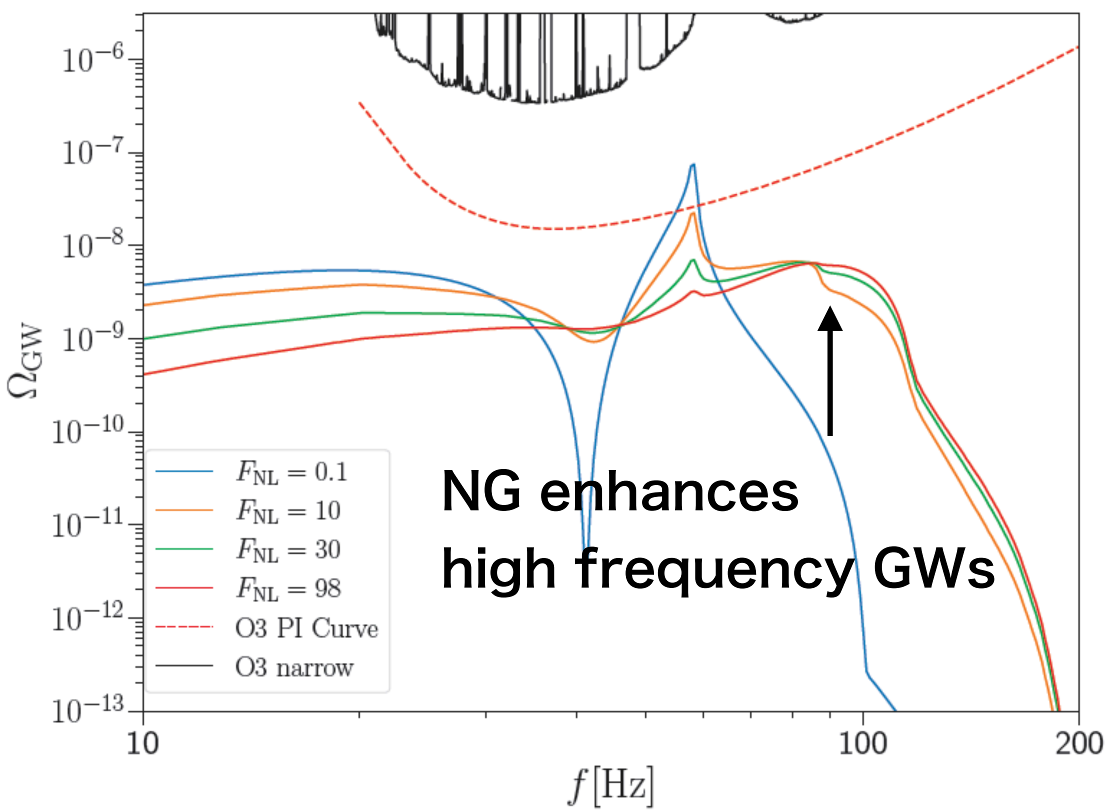
k: frequencies

→ Features can be detected if the signal has high enough SNR (at least 1) in the corresponding frequency bin.
(Information can be summed up for features wider than the binning)

① LVK O3 constraint on scalar induced GWs

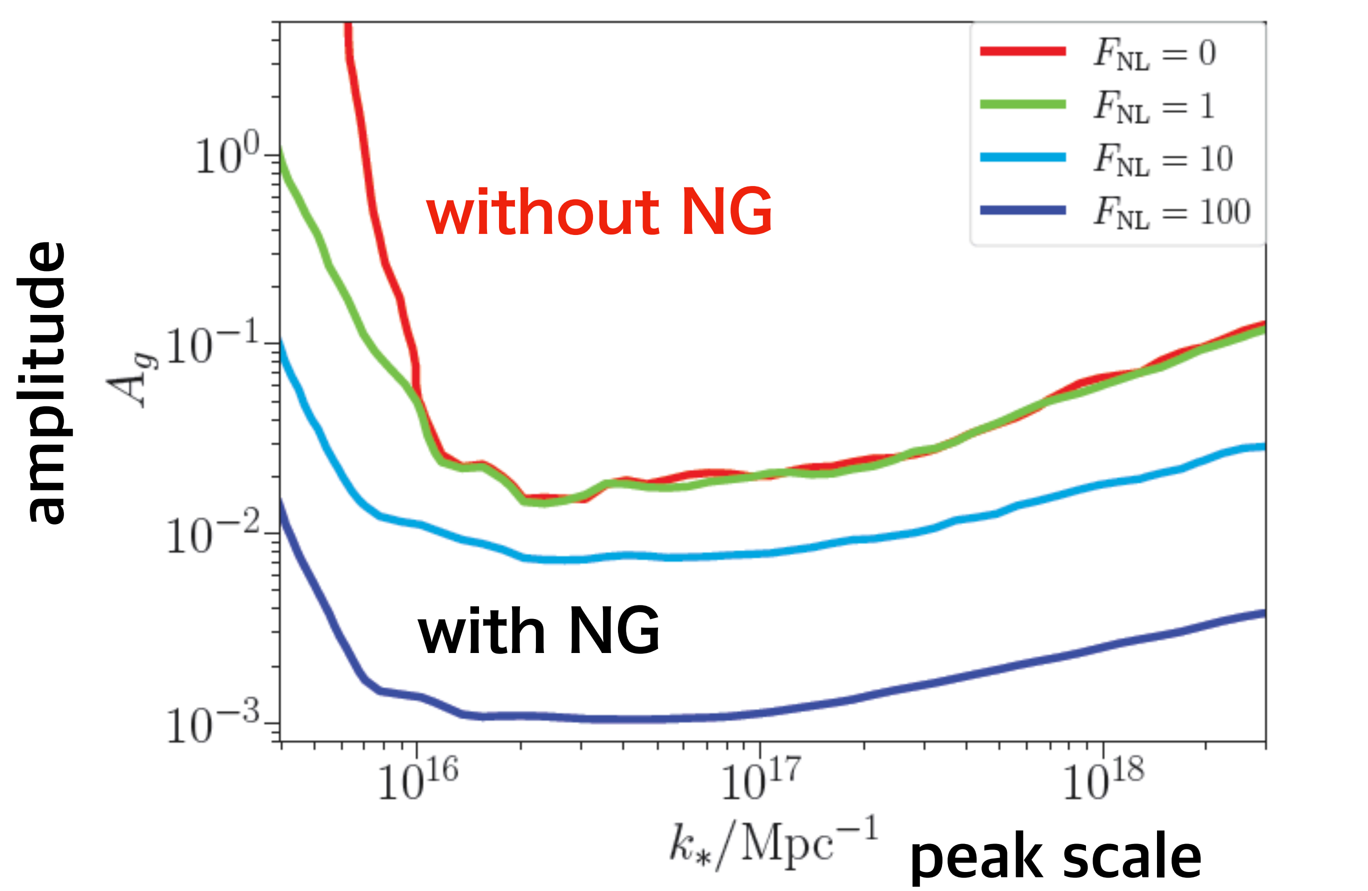
→ constraint on primordial black holes (PBHs)

GW spectrum



Many inflationary models predicting large curvature perturbations (and producing PBHs) exhibit Non-Gaussianity (NG)

- ultra slow roll inflation
- multi field inflation
- couplings leading to particle production, etc.



Assumption:
local type non-Gaussianity

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + F_{\rm NL}\zeta_g^2(\mathbf{x})$$

Note: Parametrization with $F_{\rm NL}$ covers limited cases

R. Inui, S. Jaraba, SK, S. Yokoyama, arXiv: 2311.05423 to be published in JCAP

② Log oscillation features

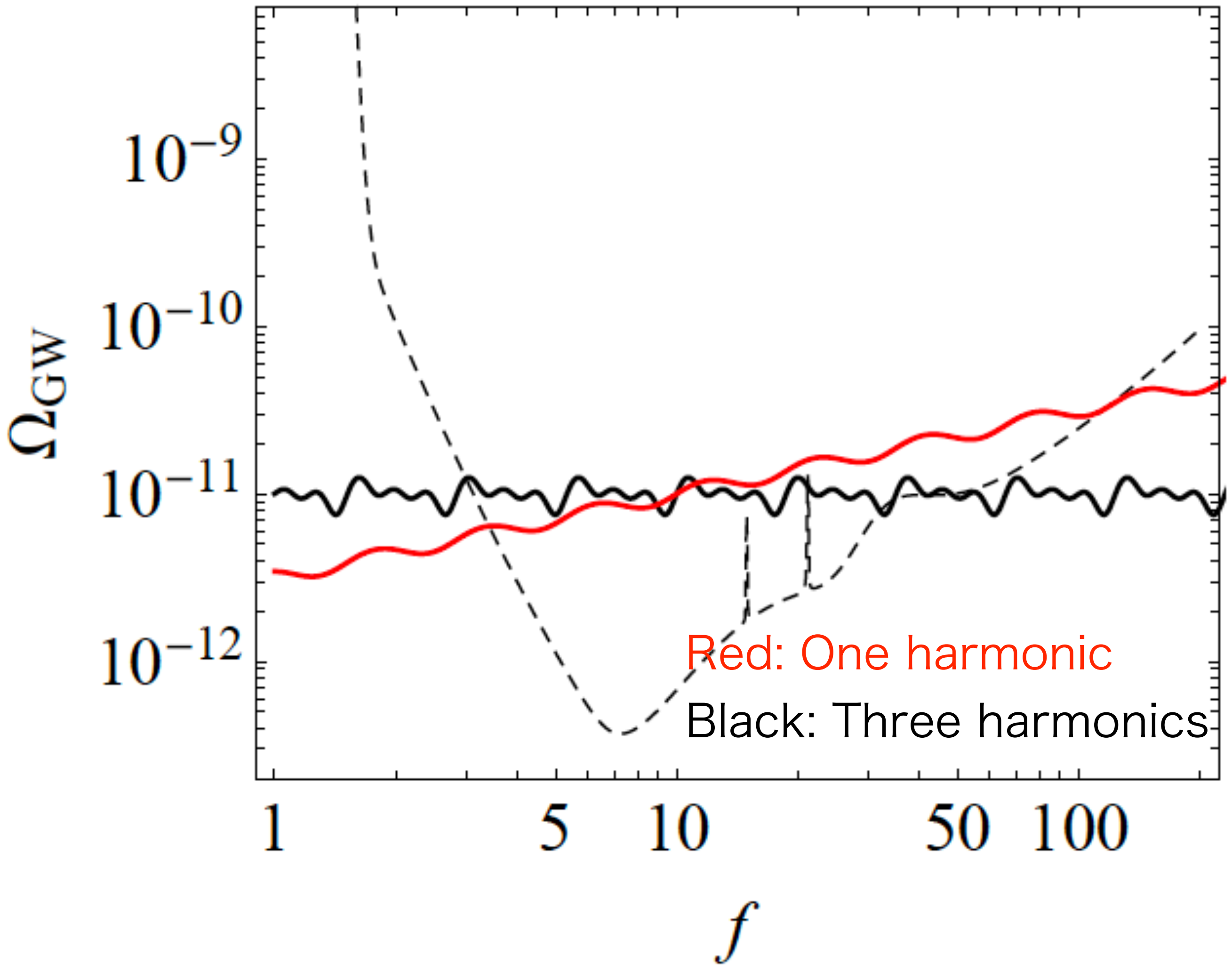
Some early universe models can exhibit log oscillation features

Ex. 1) Multi-fractional spacetimes

G. Calcagni, PRL 104, 251301 (2010)

G. Calcagni & SK, JCAP 03, 019 (2021)

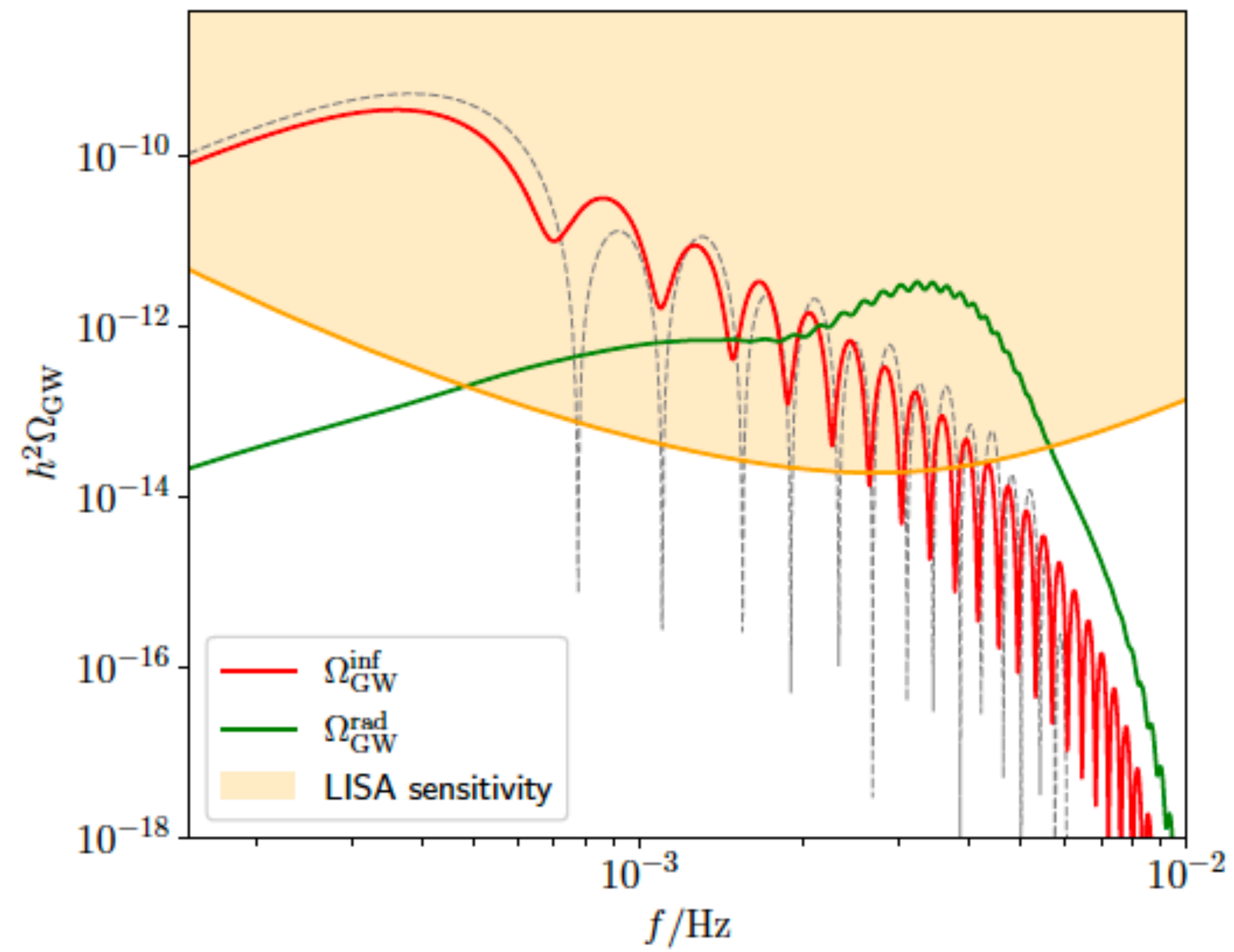
Manifestation of discrete scale invariance (geometry)



Ex. 2) Scalar induced GWs

Fumagalli et al., JHEP 03 (2022) 196

The excited state triggered by a sharp feature during inflation



G. Calcagni & SK, CQG 41, 015031 (2024)

Fisher prediction for one harmonic case

$$\Omega_{\rm GW}(f) = \Omega_0 \left(\frac{f}{f_*}\right)^{n_{\rm t}} \left[1 + A_1 \sin\left(\omega \ln \frac{f}{f_*} + \Phi_1\right)\right]$$

→ 5 free parameters

$p_i = [\Omega_0, n_{\rm t}, \omega, A_1, \Phi_1]$

basic shape = power law form

oscillation

$f_* = 10$ Hz

fiducial values

$\Omega_0 = 10^{-11}$

$n_{\rm t} = 0$

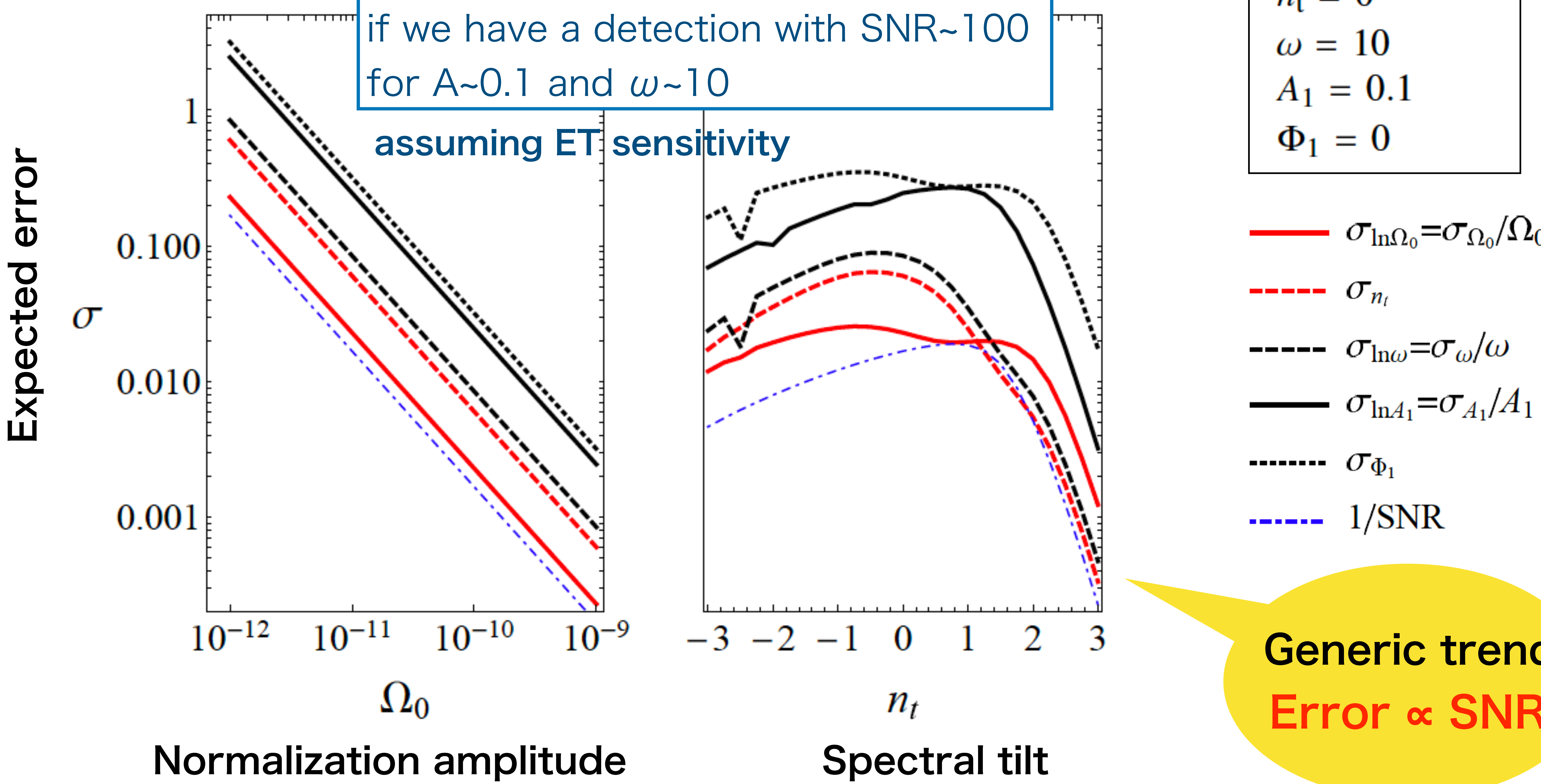
$\omega = 10$

$A_1 = 0.1$

$\Phi_1 = 0$

O(10%) precision
if we have a detection with SNR~100
for A~0.1 and ω~10

assuming ET sensitivity



Generic trend

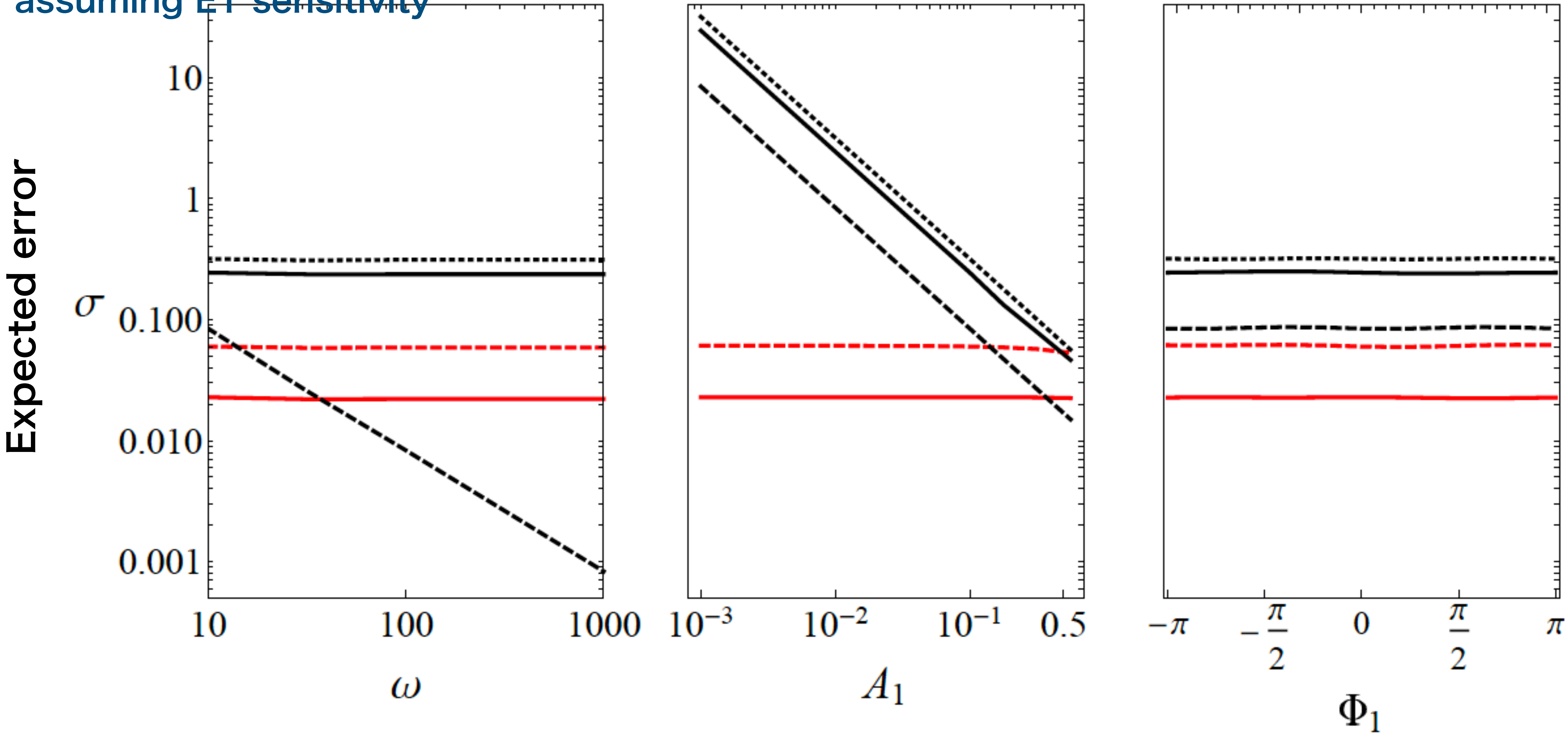
Error ∝ SNR^{-1}

Fisher prediction for one harmonic case

Dependence on oscillation parameters

- errors on Ω_0 and n_t are unchanged
- larger ω → smaller error on ω
- larger A_1 → smaller error on ω, A_1, Φ_1

assuming ET sensitivity



Oscillation frequency | Oscillation amplitude | Oscillation phase

Summary

Features in the stochastic GW background can be a unique probe of high-energy physics/beyond-GR theories

- We can detect the features if we have high enough SNR (at least 1) in the corresponding frequency bin.
  (Information can be summed up for features wider than the binning)

- **Inui et al. (+SK) arXiv: 2311.05423**
  We have provided constraint on scalar induced GWs using LVK O3 data, by taking into account the effect of non-Gaussianity in curvature perturbations.

- **G. Calcagni & SK, CQG 41, 015031 (2024)**
  We investigated the detectability of log-oscillation features in future GW experiments. The generic finding is that the errors generically decrease as 1/SNR for all parameters.