Cutting rules and unitarity constraints for CP asymmetric processes

Peter Maták

In collaboration with Tomáš Blažek and Viktor Zaujec

COMENIUS UNIVERSITY BRATISLAVA

CATCH22+2

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CP asymetries and unitarity constraints

$$S^\dagger S = SS^\dagger \quad \rightarrow \quad \sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \text{for} \quad \mathrm{i}\,T = S - 1 \tag{1}$$

$$CPT \text{ symmetry} \quad \rightarrow \quad \Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{\overline{fi}}|^2 = |T_{fi}|^2 - |T_{if}|^2 \tag{2}$$

CP asymetries and unitarity constraints

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$$\sum_f \Delta|T_{fi}|^2 = \sum_i \Delta|T_{fi}|^2 = 0 \tag{3}$$

[Dolgov ’79; Kolb, Wolfram ’80; See also Hook ’11; Baldes, Bell, Petraki, Volkas ’14]

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[Dolgov '79; Kolb, Wolfram '80; See also Hook '11; Baldes, Bell, Petraki, Volkas '14]

$$\left.\begin{aligned} T_{fi} &= C_{\text{tree}} K_{\text{tree}} + C_{\text{loop}} K_{\text{loop}} \\ T_{if} &= C^*_{\text{tree}} K_{\text{tree}} + C^*_{\text{loop}} K_{\text{loop}} \end{aligned}\right\} \quad \Delta|T_{fi}|^2 = -4\,\mathrm{Im}[C_{\text{tree}} C^*_{\text{loop}}]\,\mathrm{Im}[K_{\text{tree}} K^*_{\text{loop}}] \tag{4}$$

CP asymetries and unitarity constraints

$$S^\dagger S \quad \rightarrow \quad T = T^\dagger + \mathrm{i}\, T^\dagger\, T \tag{5}$$

$$\Delta\big|\,T_{fi}\big|^2 = \Big|\,T^*_{if} + \mathrm{i}\sum_n T^\dagger_{fn}\,T_{ni}\Big|^2 - \big|\,T_{if}\big|^2 = -2\,\mathrm{Im}\left[\sum_n T_{if}\,T^\dagger_{fn}\,T_{ni}\right] + \Big|\sum_n T^\dagger_{fn}\,T_{ni}\Big|^2 \tag{6}$$

[Kolb, Wolfram ’80]

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[Kolb, Wolfram '80]

$$\text{No further on-shell cuts means } T^*_{if} = T_{fi} \quad \to \quad \Delta \big| T_{fi} \big|^2 = -2\, \mathrm{Im} \Big[\sum_n T_{if}\, T_{fn}\, T_{ni} \Big] \tag{7}$$

CP asymetries and unitarity constraints

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[Kolb, Wolfram '80]

No further on-shell cuts means $T^*_{if} = T_{fi} \quad \rightarrow \quad \Delta \left| T_{fi} \right|^2 = -2\, \mathrm{Im} \left[\sum_n T_{if}\, T_{fn}\, T_{ni} \right]$ (7)

$$\Delta \left| T_{fi} \right|^2 = \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \sum_n \mathrm{i}\, T_{if} \mathrm{i}\, T_{fn} \mathrm{i}\, T_{ni} \tag{8}$$

[Covi, Roulet, Vissani '98]

CP asymetries and unitarity constraints

2 April 1998

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Physics Letters B 424 (1998) 101–105

On the CP asymmetries in Majorana neutrino decays

Esteban Roulet [a], Laura Covi [b], Francesco Vissani [c]

[a] *Depto. de Física Teórica, Universidad de Valencia, E-46100 Burjasot, Valencia, Spain*
[b] *School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YB, United Kingdom*
[c] *Deutsches Elektronen Synchroton, DESY, 22603 Hamburg, Germany*

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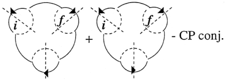

Fig. 3. Pictorial representation of the self-energy contribution to the cross section asymmetry ϵ_σ.

Therefore, one can pictorially represent the self-energy contributions to $\sigma(\ell^c H^* \to \ell H)$ as in Fig. 2, where the cut blobs are the initial (i) and final (f) states, while the remaining blob stands for the one-loop self-energy. This last is actually the sum of two contributions, one with a lepton and one with an antilepton.

Now, in the computation of ϵ_σ, only the absorptive part of the loop will contribute, and the Cutkoski

Turning now to the vertex contributions, to see the cancellations we need to add the three contributions shown in Fig. 4 (including the tree level u-channel interfering with the one-loop self-energy diagram). In terms of cut diagrams, this can be expressed as in Fig. 5, where CP-conjugate stands for the same four diagrams with all the arrows reversed. Hence, the CP-conjugate contribution will exactly cancel the four diagrams, since changing the directions of the arrows just exchanges among themselves the first and fourth diagrams, as well as the second and third ones. We then see explicitly that the absorptive part of the self-energies are also playing here a crucial role, enforcing the cancellation of the CP violation produced by the vertex diagrams.

The only remaining diagrams to be considered are the interference of the one-loop vertex diagrams with the tree level u-channel. Pictorially, they are represented in Fig. 6, and they again cancel since the two

$$\Delta \left| T_{fi} \right|^2 = \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \sum_n \mathrm{i}\, T_{if} \mathrm{i}\, T_{fn} \mathrm{i}\, T_{ni} \qquad (8)$$

[Covi, Roulet, Vissani ’98]

Asymmetries and holomorphic cutting rules

$$S^\dagger S = 1 \quad \rightarrow \quad \mathrm{i}\,T^\dagger = \mathrm{i}\,T - \mathrm{i}\,T\mathrm{i}\,T^\dagger \tag{9}$$

$$|\,T_{fi}|^2 = -\mathrm{i}\,T^\dagger_{if}\mathrm{i}\,T_{fi} = -\mathrm{i}\,T_{if}\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} + \ldots \tag{10}$$

[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

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[Coster, Stapp '70; Bourjaily, Hannesdottir, *et al.* '21]

$$\begin{aligned}\Delta|\,T_{fi}|^2 = |\,T_{fi}|^2 - |\,T_{if}|^2 = &\sum_n \Big(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\Big) \\ &- \sum_{n,k} \Big(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni}\Big) \\ &+ \ldots\end{aligned} \tag{11}$$

[Blažek, Maták '21a]

Example I.

$$\mathcal{L} = \frac{1}{2}\bar{L}^c F_i L \bar{X}_i + \bar{e}^c G_i \nu \bar{X}_i + \text{H.c.} \tag{12}$$

[Heeck, Heisig, Thapa '23]

- Dirac leptogenesis, $\Delta Y_\nu = -\Delta Y_L - \Delta Y_e$. [Dick, Lindner, Ratz, Wright '00]
- Right-handed neutrino freezes-in out of thermal equilibrium.
- $M_i > T_{\text{reh}}$, X_i is absent in the universe and can be integrated out.

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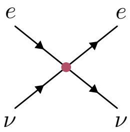
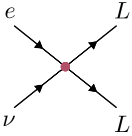
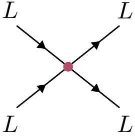


$= \mathrm{i}\sum_i \frac{F_i^\dagger F_i}{M_i^2}$,



$= \mathrm{i}\sum_i \frac{F_i^\dagger G_i}{M_i^2}$,



$= \mathrm{i}\sum_i \frac{G_i^\dagger G_i}{M_i^2}$

Example I.

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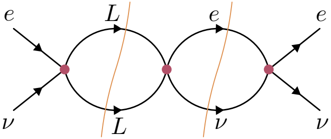
[Heeck, Heisig, Thapa '23]

$\Delta|T_{\nu e \to LL}|^2 =$



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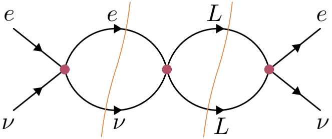

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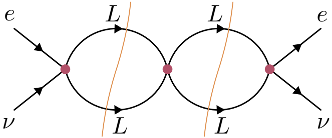

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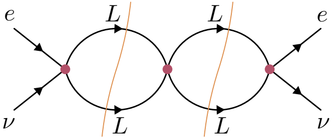

$= 0$

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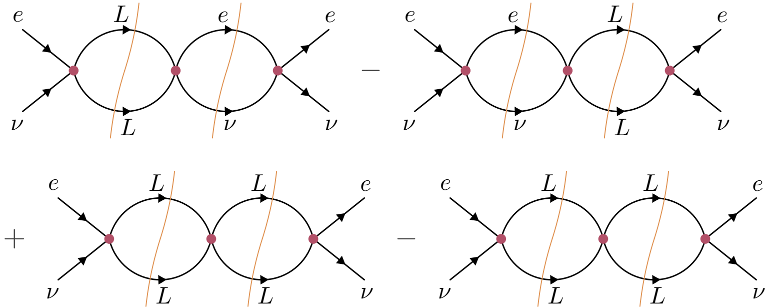


$$= 0 \tag{13}$$

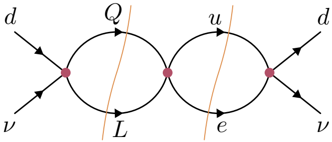
$$\Delta|T_{\nu e\to LL}|^2 + \Delta|T_{\nu e\to \nu e}|^2 = 0 \tag{14}$$

Example II.

$$\mathcal{L} = \bar{Q}^c F_i L \bar{X}_i + \bar{d}^c G_i \nu \bar{X}_i + \bar{u}^c K_i e \bar{X}_i + \text{H.c.} \tag{15}$$

[Heeck, Heisig, Thapa '23]

$\Delta |T_{\nu d \to LQ}|^2 =$



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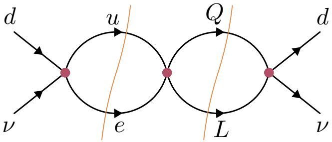

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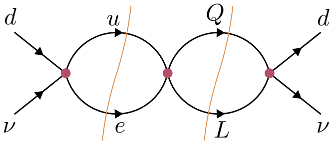
$\Delta |T_{\nu d \to eu}|^2 =$



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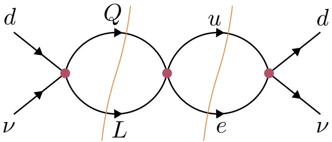

Example II.

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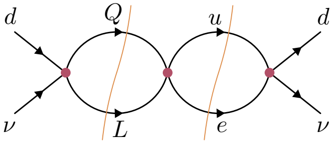
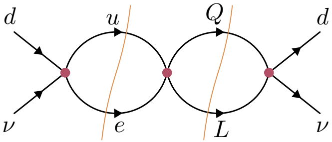
[Heeck, Heisig, Thapa '23]

$\Delta |T_{\nu d \to LQ}|^2 =$ 

$-$ 

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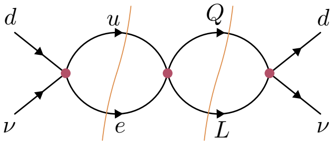
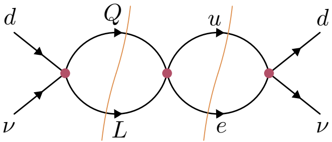
$\Delta |T_{\nu d \to eu}|^2 =$ 

$-$ 

$$\Delta |T_{\nu d \to LQ}|^2 = -\Delta |T_{\nu d \to eu}|^2 \neq 0 \tag{17}$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

Vacuum diagrams and complex phases

- Used to represent the weak-basis invariants. [Botella, Nebot, Vives '06]
- Reversing the arrows on charged-particle propagators must lead to a topologically inequivalent vacuum diagram.

$$\Delta|T_{fi}|^2 = \sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}\right) - \dots \tag{18}$$

Vacuum diagrams and the Boltzmann equation

Change in # of particles $\leftrightarrow$ average # of interactions the particles participate in

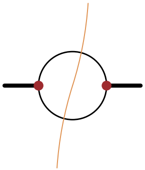
$$\dot{n}_{i_1} + 3Hn_{i_1} = \sum_{\text{all reactions}} -\gamma_{fi} + \gamma_{if} \tag{19}$$

$$\begin{aligned}
\gamma_{fi}^{\text{eq}} &= \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i^{\text{eq}}(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] (2\pi)^4 \delta^{(4)}(p_f - p_i) \big| M_{fi} \big|^2 \\
&= \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i^{\text{eq}}(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} \Big(- \mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \dots \Big)
\end{aligned} \tag{20}$$

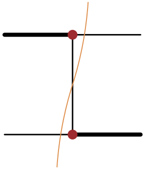
$$[\mathrm{d}\boldsymbol{p}] = \frac{\mathrm{d}^3 \boldsymbol{p}}{(2\pi)^3 2E_{\boldsymbol{p}}} \qquad\qquad |T_{fi}|^2 = V_4 (2\pi)^3 \delta^{(3)}(\boldsymbol{p}_f - \boldsymbol{p}_i) |M_{fi}|^2$$

Vacuum diagrams and the Boltzmann equation

$$\gamma^{\mathrm{eq}}_{fi} = \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f^{\mathrm{eq}}_i(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} \Big(-\mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \ldots \Big) \tag{21}$$



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2](2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$



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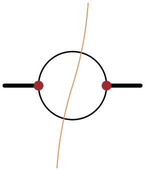
Vacuum diagrams and the Boltzmann equation

$$\gamma_{fi}^{\mathrm{eq}} = \int \prod_{\forall i} [\mathrm{d}\boldsymbol{p}_i]\, f_i^{\mathrm{eq}}(p_i) \int \prod_{\forall f} [\mathrm{d}\boldsymbol{p}_f] \frac{1}{V_4} \Big(-\mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \dots \Big) \tag{21}$$



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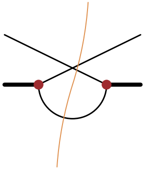
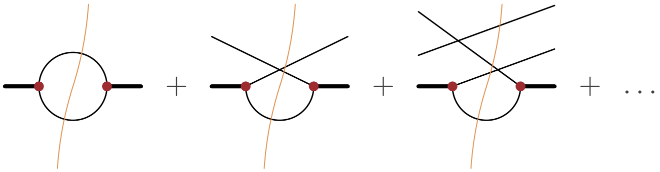


$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \mathrm{e}^{-E_{k_1}/T} (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2)$$

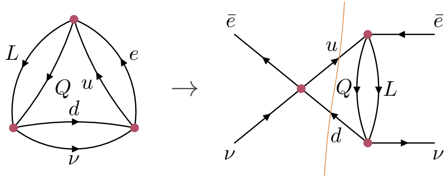
Vacuum diagrams and the Boltzmann equation



$$\int [\mathrm{d}\boldsymbol{p}_1] \mathrm{e}^{-E_{p_1}/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \left[1 + \frac{1}{\mathrm{e}^{E_{k_1}/T} - 1} \right] (2\pi)^4 \delta^{(4)}(p_1 - k_1 - k_2) \tag{22}$$

[Blažek, Maták '21b]

Vacuum diagrams and the Boltzmann equation



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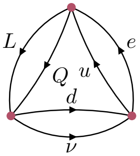
Vacuum diagrams and the Boltzmann equation



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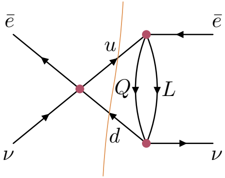

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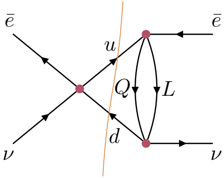

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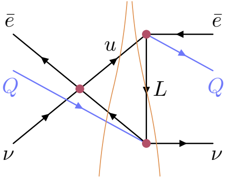

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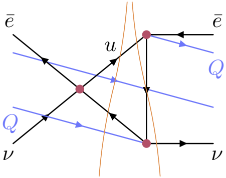
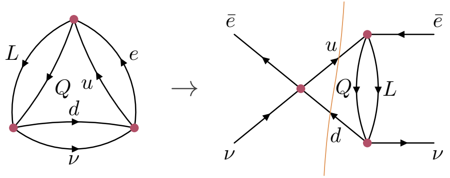


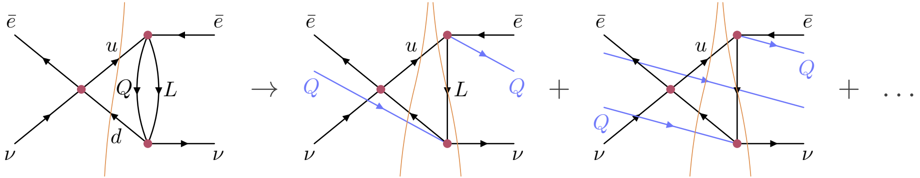
$+ \ \dots$ (24)

$$\frac{\mathrm{i}}{p^2 + \mathrm{i}\epsilon} \rightarrow 2\pi \sum_{w=1}^{\infty} \big(-1\big)^{w} \big[f^{\mathrm{eq}}(|p^0|)\big]^{w} \delta(p^2) = -2\pi f_{\mathrm{FD}}(|p^0|)\delta(p^2) \tag{25}$$

Vacuum diagrams and the Boltzmann equation



(23)



(24)

$$\Delta\gamma^{\text{eq}}_{\nu\bar{e}(Q)\to u\bar{d}(Q)} + \Delta\gamma^{\text{eq}}_{(\nu)\bar{e}Q\to(\nu)\bar{L}u} = 0 \qquad (26)$$

The assumeptions we made

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ -\sum_p F_p a_p^\dagger a_p \right\}, Z = \prod_p Z_p \tag{27}$$

$$\mathrm{e}^{-E_p/T} \to \mathrm{e}^{-F_p} = \frac{f_p}{1 \pm f_p} \qquad f_p = \mathrm{Tr}\left[a_p^\dagger a_p \rho\right] \tag{28}$$

$$\rho' = S\rho S^\dagger \quad \to \quad (1 + \mathrm{i}\,T)\rho(1 - \mathrm{i}\,T + \mathrm{i}\,T\mathrm{i}\,T - \dots) \tag{29}$$

The collision term for the Boltzmann equation is obtained as $\mathrm{Tr}\left[a_p^\dagger a_p\left(\rho - \rho'\right)\right] / V_4$.

[McKellar, Thomson '94; Blažek, Maták '21b]

What else can be done?

- Thermal masses from anomalous thresholds. [Blažek, Maták '22]

- CPT and unitarity constraints generalized to thermal-corrected asymmetries. In the seesaw type-I leptogenesis at $\mathcal{O}(Y^4 Y_t^2)$ they look like

$$\Delta\gamma^{\text{eq}}_{N_i Q\to lt} + \Delta\gamma^{\text{eq}}_{N_i(Q)\to lH(Q)} + \Delta\gamma^{\text{eq}}_{N_i(Q)\to \bar{l}\bar{H}(Q)} + \Delta\gamma^{\text{eq}}_{N_i Q\to \bar{l}QQ\bar{t}} = 0, \tag{30}$$

$$m^2_{H,Y_t}(T)\frac{\partial}{\partial m_H^2}\bigg|_0 \left(\Delta\gamma^{\text{eq}}_{N_i\to lH} + \Delta\gamma^{\text{eq}}_{N_i\to \bar{l}\bar{H}}\right) = 0. \tag{31}$$

[Blažek, Maták, Zaujec '22]

- No double-counting from intermediate states. Resonant dark matter annihilation at fixed order. [Maták '24; see also Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

Conclusions

- CPT and unitarity constraints can be formulated diagrammatically at any perturbative order.

- Vacuum diagrams must not be invariant under the arrow reversal to come with a complex phase.

- Completing the Boltzmann equations by all possible unitary cuts represents thermal corrections (even when you do not notice that).

Thank you for your attention!