

Imminent test of quantum gravity with gravitational waves

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with L. Modesto

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01/21– Quantum gravity

- Many proposals: string theory, loop quantum gravity, asymptotic safety, **nonlocal quantum gravity (v1.0-1.3)**, fractional gravity, . . .
- Some of them make contact with observations.
- Very few of them make falsifiable predictions.

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Nonlocal quantum gravity generates **falsifiable predictions (v2.0)**.

02/21– Stelle gravity

Stelle 1977, 1978:

$$\mathcal{L} = R + \gamma_0 R^2 + \gamma_2 R_{\mu\nu} R^{\mu\nu}, \quad \gamma_{0,2} = \text{const}$$

Renormalizable but **non-unitary** (spin-2 ghost). (Can be made unitary introducing fakeons [Anselmi & Piva 2017ab,2018].)

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Ostrogradski instability (ghost) from higher-order derivatives.

Example:

$$(\square + \alpha \square^2)\phi = 0 \quad \Rightarrow \quad \tilde{G}(k) = \frac{1}{k^2 - \alpha k^4} = -\frac{1}{k^2} + \frac{1}{k^2 - \alpha^{-1}}$$

03/21– Nonlocal quantum gravity v1.0-1.3

Krasnikov 1987; Kuz'min 1989; Tomboulis 1997; Modesto 2011; Gerwick et al. 2011

$$\begin{aligned}\mathcal{L} &= R + R\gamma_0(\square)R + R_{\mu\nu}\gamma_2(\square)R^{\mu\nu} + R_{\mu\nu\sigma\tau}\gamma_4(\square)R^{\mu\nu\sigma\tau} + \mathcal{V}(\mathcal{R}) + \mathcal{L}_m \\ &= R + G_{\mu\nu}\gamma(\square)R^{\mu\nu} + R_{\mu\nu\sigma\tau}\gamma_4(\square)R^{\mu\nu\sigma\tau} + \mathcal{V}(\mathcal{R}) + \mathcal{L}_m\end{aligned}$$

Minimal coupling to matter. Asymp. polynomial form factor:

$$\gamma(\square) = \frac{e^{\mathbf{H}(\square)} - 1}{\square}, \quad e^{\mathbf{H}(\square)} = e^{\gamma_E + \Gamma[0,p(\square)]} p(\square) \stackrel{\text{UV}}{\sim} e^{\gamma_E} p(\square)$$

- **v1.0:** $\gamma_4 \neq 0$, $\mathcal{V}(\mathcal{R}) = 0$
- **v1.1:** $\gamma_4 = 0$
- **v1.2:** $\gamma_4 = 0$, $\mathcal{V}(\mathcal{R}) \neq 0$, conformal symmetry
- **v1.3:** add a mass scale in $\gamma_{0,2}(\square)$ to get Starobinsky

04/21– Properties of nonlocal quantum gravity v1.0-1.3

- Well-defined Cauchy problem [GC, Modesto & Nardelli 2019]
- Finite number of physical degrees of freedom* [GC et al. 2019]
- Super-renormalizable or finite [Modesto & Rachwał 2014,2015]
- Perturbatively unitary [Briscese & Modesto 2019]
- Big-bang singularity resolved classically [Biswas, Mazumdar & Siegel, 2005; GC, Modesto & Nicolini 2014]
- Black-hole singularities resolved classically by nonlocality (**v1.0**, $\gamma_4 \neq 0$) [Modesto et al. 2015; Frolov & Zelnikov 2016; Edholm et al. 2016; Buoninfante et al. 2018; Giacchini & de Paula Netto 2019; Boos 2020] OR by conformal symmetry (**v1.2**, $\gamma_4 = 0$, finite) [Modesto & Rachwał 2016; Bambi et al. 2017ab; Zhou et al. 2019]
- Starobinsky inflation naturally embedded (**v1.3**) [Koshelev et al. 2016,2018,2020; Kumar & Modesto 2018; GC & Kuroyanagi 2021]
- **Forces not unified and no soon-falsifiable prediction**

05/21– A nonlocal theory of all fields

$$S[\Phi_i] = \int d^D x \sqrt{|g|} [\mathcal{L}_{\text{loc}} + E_i F^{ij}(\Delta) E_j + \mathcal{V}(E_i)]$$

$$S_{\text{loc}} = \int d^D x \sqrt{|g|} \mathcal{L}_{\text{loc}}, \quad \mathcal{L}_{\text{loc}} = \frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_{\text{m}}(\Phi_i)$$

$$E_i := \frac{\delta S_{\text{loc}}}{\delta \Phi_i(x)}, \quad \Delta_{ki} := \frac{\delta E_i}{\delta \Phi_k} = \frac{\delta^2 S_{\text{loc}}}{\delta \Phi_k \delta \Phi_i}$$

$$\Phi_i \in \{g_{\mu\nu}, \Phi, \psi, A_\mu, \dots\}$$

$$F(\Delta) := \frac{e^{H(\Delta)-H(0)} - 1}{2\Delta}, \quad H(z) = \int_0^{p(z)} dw \frac{1 - e^{-w}}{w}$$

$$p(z) = b_0 - bz + z^4$$

06/21– Nonlocal quantum gravity v2.0: equations of motion

Extremals:

$$E_{\mu\nu} = \frac{1}{2} (M_{\text{Pl}}^2 G_{\mu\nu} - T_{\mu\nu})$$

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Nonlocal gravitational EOMs:

$$[\mathbf{e}^{\bar{\mathbf{H}}(\Delta)}]_{\mu\nu}^{\sigma\tau} E_{\sigma\tau} + O(E_{\mu\nu}^2) = 0$$

07/21– Conformal invariance

Field redefinitions:

$$g_{\mu\nu} =: \phi^2 \hat{g}_{\mu\nu}, \quad \Phi = \frac{\hat{\Phi}}{\phi}, \quad \psi = \frac{\hat{\psi}}{\phi^{\frac{3}{2}}}, \quad A_\mu = \hat{A}_\mu$$

Action invariant under Weyl transformations

$$\hat{g}'_{\mu\nu} = \Omega^2(x) \hat{g}_{\mu\nu}, \quad \phi' = \Omega^{-1}(x) \phi, \quad \dots$$

08/21– Properties of nonlocal quantum gravity v2.0

- Same basic characteristics as v1.0-1.2 [Modesto 2021].
- Super-renormalizable or finite [GC, Giacchini, Modesto, de Paula Netto & Rachwał 2023].
- Tree-level scattering amplitudes are the same as those of the underlying local theory [Modesto & GC 2021].
- Testable top-down cosmology [Modesto & GC 2022; GC & Modesto 2022] ←←

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Finiteness implies **conformal invariance** at the quantum level.

09/21– Two phases

$$F(\Delta) := \frac{e^{H(\Delta)-H(0)} - 1}{2\Delta}, \quad H(z) = \text{Ein}(z) = \int_0^{p(z)} dw \frac{1 - e^{-w}}{w}$$

$$p(z) = b_0 - bz + z^4$$

- 1 **Trans-Planckian conformal phase** $\Lambda_* \lesssim E \lesssim M_{\text{Pl}}$.
Manifest Weyl invariance.
- 2 **Post-Planckian Higgs phase** $\Lambda_*/\sqrt{b} =: \Lambda_{\text{hd}} \lesssim E \lesssim \Lambda_*$.
Intermediate UV regime $p(z) \simeq -bz$, broken Weyl symmetry.

10/21– Post-Planckian Higgs phase $\Lambda_{\text{hd}} \lesssim E \lesssim \Lambda_*$

Intermediate UV regime $p(z) \simeq -bz$:

$$\sqrt{\frac{\max(1, b_0)}{b}} \Lambda_* =: \Lambda_{\text{hd}} \lesssim E \lesssim \Lambda_*$$

Choice of vacuum: $\phi \rightarrow \langle \phi \rangle = \frac{M_{\text{Pl}}}{\sqrt{2}} + \langle \varphi \rangle \simeq \frac{M_{\text{Pl}}}{\sqrt{2}}$

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- Adding Λ_{cc} in $\mathcal{L}_{\text{loc}} = \mathcal{L}_{\text{loc}}[\phi^2 \hat{g}_{\mu\nu}, \phi^{-\Delta_i} \hat{\Phi}_i^{\text{SM}}]$ generates a $\sqrt{|g|} \Lambda_{\text{cc}} = \sqrt{|\hat{g}|} \Lambda_{\text{cc}} \phi^4$ term.

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- Modify $\mathcal{L}_{\text{loc}} = \mathcal{L}_1^{\text{loc}}[\phi^2 \hat{g}_{\mu\nu}, \phi^{-\Delta_i} \hat{\Phi}_i^{\text{SM}}] + \mathcal{L}_2^{\text{loc}}[\phi, \hat{g}_{\mu\nu}, \hat{\Phi}_i^{\text{SM}}]$ and add Weyl-invariant dilaton-Higgs potential [Bars et al. 2006,2014]:

$$\mathcal{L}_2^{\text{loc}}[\phi, \mathfrak{h}] = \lambda(\mathfrak{h}^\dagger \mathfrak{h} - \alpha \phi^2)^2 + \lambda' \phi^4$$

11/21– Post-Planckian Higgs phase $\Lambda_{\text{hd}} \lesssim E \lesssim \Lambda_*$

Local quadratic action + one-loop log quantum corrections:

$$\mathcal{L}_{\Lambda_{\text{hd}}} = \frac{M_{\text{Pl}}^2}{2} R + \mathcal{L}_{\text{m}} + E_{\mu\nu} E^{\mu\nu} + \mathcal{V}(E_{\mu\nu}) + \mathcal{L}_Q$$

$$\mathcal{L}_Q = \beta_R R \ln \left(-\frac{\square}{\Lambda_*^2 \delta_0^2} \right) R + \beta_{\text{Ric}} R_{\mu\nu} \ln \left(-\frac{\square}{\Lambda_*^2} \right) R^{\mu\nu}$$

$\delta_0, \beta_R, \beta_{\text{Ric}}$ numerical constants (not beta functions!).

Overconstrained **Stelle limit**:

$$E_{\mu\nu} E^{\mu\nu} = -\frac{M_{\text{Pl}}^4}{4} \left(R_{\mu\nu} R^{\mu\nu} + \frac{D-4}{4} R^2 \right)$$

$$\mathcal{L}_{\Lambda_{\text{hd}}}^{D=4} = -\frac{e^{\tilde{\gamma}_E} M_{\text{Pl}}^2 b}{2\Lambda_*^2} R_{\mu\nu} R^{\mu\nu} + \mathcal{L}_{\text{m}} + \mathcal{L}_Q$$

12/21– Early universe

I. Solutions of hot-big-bang problems

All problems of the hot big-bang model are solved **without inflation** in the **conformal phase**

II. Primordial perturbations

Quasi-scale-invariant primordial spectra generated by **quantum and thermal fluctuations** in the **Higgs phase**

III. Testable prediction

Large tensor-to-scalar ratio, **observable by BICEP Array and LiteBIRD**

13/21– Conformal invariance

- Conformal invariance as an alternative to inflation is not new [Antoniadis et al. 1997,2012; Amelino-Camelia et al. 2013,2015; Agrawal et al. 2020] . . .
- . . . but until now it was not known how to make it work in a fundamental theory.
- We provide a concrete setting thanks to UV finiteness. . .
- . . . and extract rigid, falsifiable predictions on the tensor sector.
- Such rigidity comes from the fact that our cosmological model is derived *directly* from the full theory.

14/21– Flat spacetime approximation

Two assumptions

- 1 Quadratic-gravity limit, $\Lambda_{\text{hd}} \lesssim E \lesssim \Lambda_*$.
- 2 One-loop quantum corrections in the propagator subdominant with respect to classical part, $E \gtrsim \Lambda_*/10$.

$$\Rightarrow E \simeq \frac{\Lambda_*}{10}$$

15/21– Tensor spectrum

From sub-horizon quantum fluctuations. Graviton two-point correlation function:

$$\langle h_{ij}(x)h_{kl}(x') \rangle = -\mathbf{C}\Lambda_*^{2\epsilon_2} P_{ijkl}^{(2)}(\partial_x) \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot (x-x')}}{k^{4+2\epsilon_2}} = -P_{ijkl}^{(2)}(\partial_x) \int_0^{+\infty} \frac{dk}{k} \Delta_h^2(k) \frac{\sin(kr)}{kr}$$

$$\mathbf{C} := \frac{4\Lambda_*^2}{b e^{\tilde{\gamma}_E} M_{\text{Pl}}^2}, \quad \epsilon_2 = -\frac{2\beta_{\text{Ric}}\Lambda_*^2}{b e^{\tilde{\gamma}_E} M_{\text{Pl}}^2}$$

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Tensor spectrum:

$$\mathcal{P}_t(k) := 2\Delta_h^2(k) = \frac{n_t}{(2\pi)^2 \beta_{\text{Ric}}} (\ell_* k)^{n_t}$$

Tensor spectral index:

$$n_t := \frac{d \ln \mathcal{P}_t}{d \ln k} = -2\epsilon_2 = \frac{4\beta_{\text{Ric}}}{b e^{\tilde{\gamma}_E}} \frac{\Lambda_*^2}{M_{\text{Pl}}^2} > 0$$

16/21– Scalar spectrum

From thermal fluctuations. Radiation-dominated universe.

$$\delta h \xrightarrow{\delta\text{EOM}} \delta T_{00} = \delta\rho \Big|_{E \simeq \frac{\Lambda_*}{10}} \xrightarrow{\text{Poisson eq.}} \delta\Phi \Big|_{\text{lss}} \longrightarrow \frac{\delta T_{\text{CMB}}}{T_{\text{CMB}}} \Big|_{\text{lss}}$$

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Scalar spectrum:

$$\mathcal{P}_s(k) = \frac{9}{4} \Delta_{\Phi}^2(k) = \frac{15}{64} \frac{(n_s - 1)^2}{(2\pi)^2 \beta_{\text{Ric}}} (\ell_* k)^{n_s - 1}$$

Scalar spectral index:

$$n_s - 1 := \frac{d \ln \mathcal{P}_s}{d \ln k} = 2\epsilon_2 = -\frac{4\beta_{\text{Ric}}\Lambda_*^2}{b e^{\tilde{\gamma}_E} M_{\text{Pl}}^2} < 0$$

17/21– Tensor-to-scalar ratio and consistency relation

Tensor-to-scalar ratio:

$$r := \frac{\mathcal{P}_t(k_0)}{\mathcal{P}_s(k_0)} = \frac{64}{15(1 - n_s)} (\ell_* k_0)^{2(1 - n_s)}$$

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Consistency relation:

$$n_t = 1 - n_s > 0$$

Typical of non-inflationary scenarios such as string-gas cosmology [Brandenberger 2015; Bernardo et al. 2020] and new ekpyrotic model [Brandenberger & Wang 2020ab].

18/21– Primordial GWs

PLANCK Legacy with $dn_s/d \ln k = 0$ [Aghanim et al. 2020],
 $k_0 = 0.05 \text{ Mpc}^{-1}$, n_t and r uniquely specified:

$$n_t \approx 0.0351, \quad r_{0.05} = 0.009 - 0.011$$

For $\ell_* = \ell_{\text{PJ}}$:

$$r_{0.05} = 0.011$$

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within 4 years [Ade et al. 2021]. Detection if $r > 0.009$, implication if $r \sim 0.006 - 0.009$, exclusion if $r < 0.006$.

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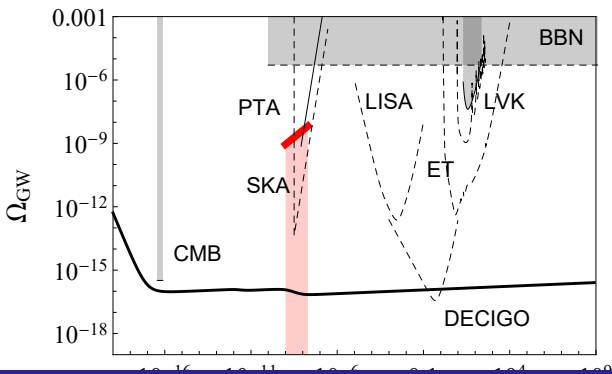
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- **LiteBIRD**: $\sigma(r) \sim 0.001$ (exclusion if $r < 0.002$) [Allys et al. 2022].

19/21– Stochastic GW background

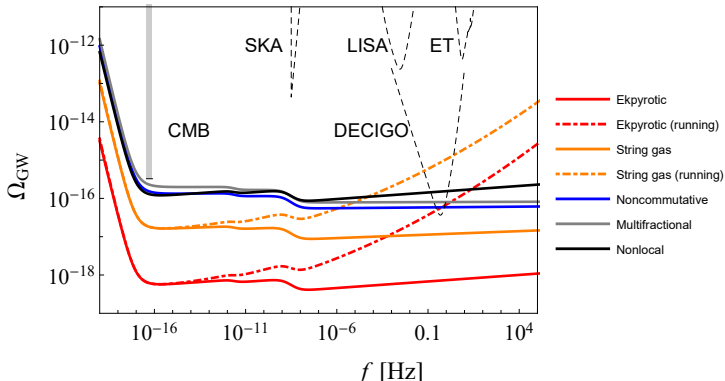
Blue-tilted tensor spectrum at CMB scales, stochastic GW background observable by DECIGO. Can serve as discriminator against other high- r models.



20/21– Quantum gravity and GWs

GC & Kuroyanagi, to appear (2024)

Only DECIGO will be able to see a stochastic background from blue-tilted quantum-gravity-motivated primordial spectra.



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- Investigate value of β_{Ric} .
- Dark energy? H_0 and σ_8 ?

TO BE CONTINUED . . .

