APPLICATIONS OF THE TUNNELING POTENTIAL FORMALISM

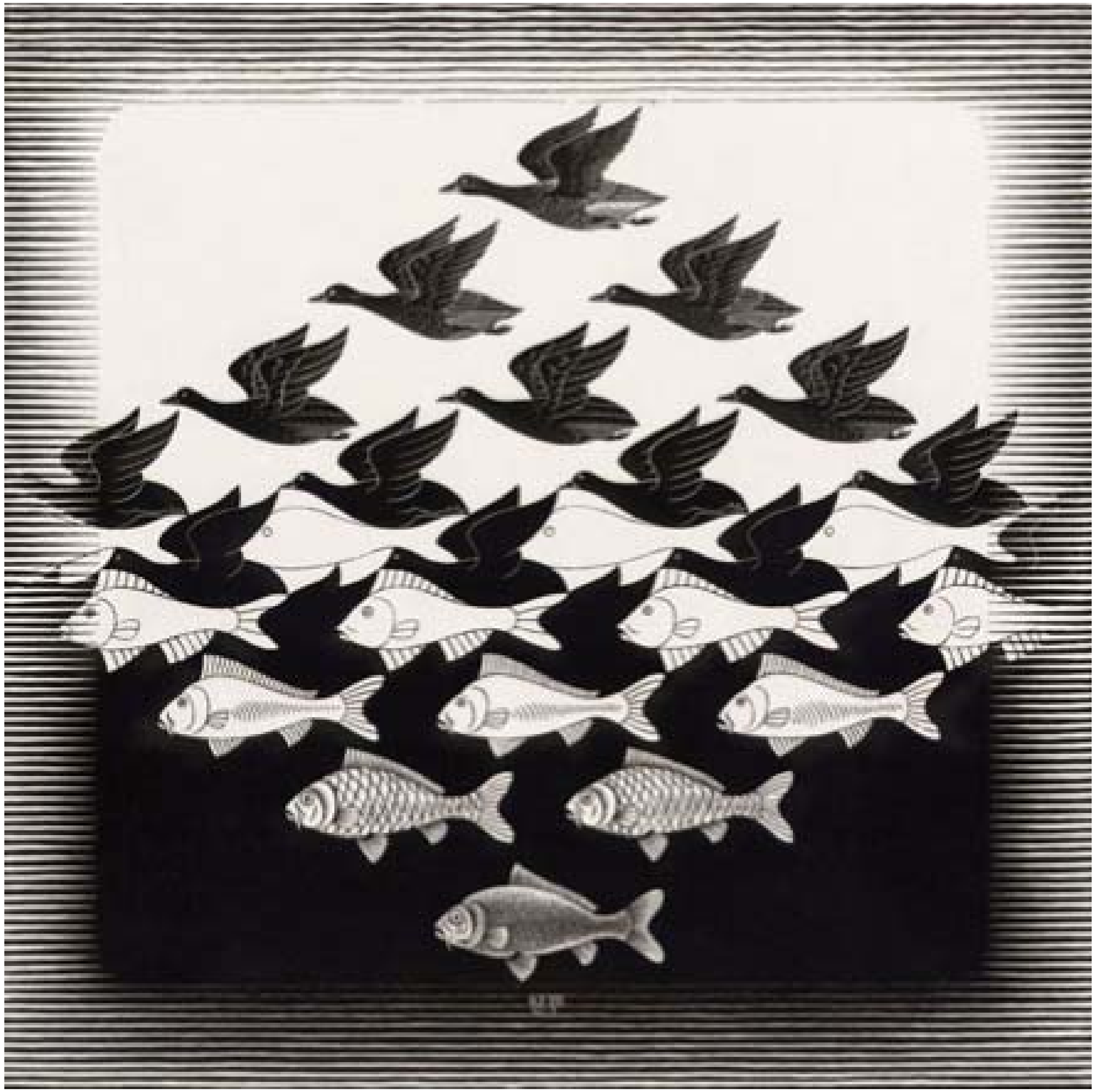
2/5/2024
CATCH 22+2
DIAS, DUBLIN

J.R.Espinosa

Instituto de Física Teórica
UAM-CSIC

1. TUNNELING POTENTIAL FORMALISM

JRE'1805

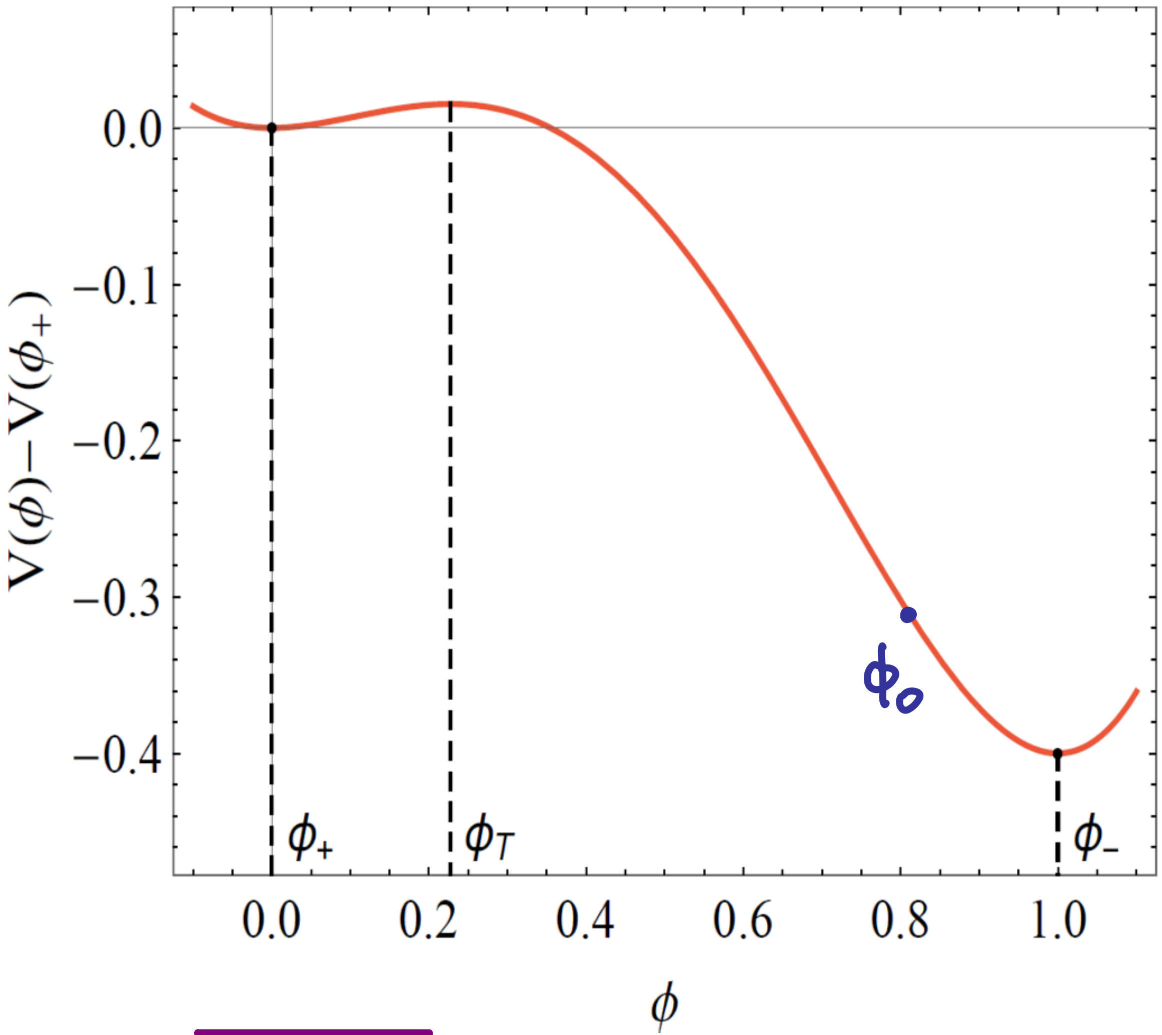


WHY VACUUM DECAY ?

- BSM/SM in false vacuum
- Early universe phase transitions
- Vacuum decay triggered by inflation
- String vacuum landscape

⋮

THE PROBLEM



$$\Gamma/V = A\, e^{-S}$$

calculate this

EUCLIDEAN FORMALISM

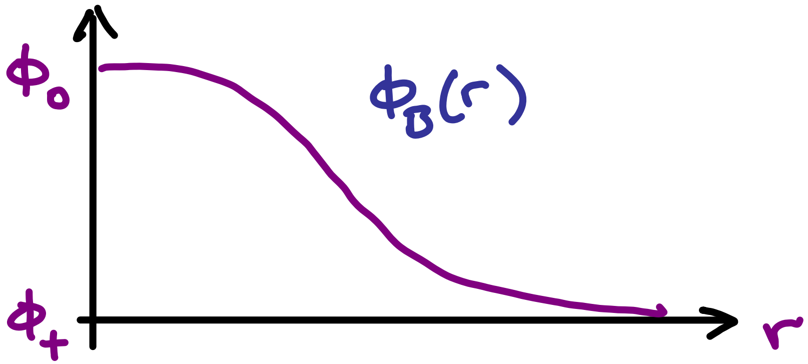
Coleman '77

Euclidean bounce $\phi_B(r)$, $O(4)$-sym, extremizes

$$S_E = 2\pi^2 \int_0^\infty dr\, r^3 \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$$

$\Rightarrow$ $$\ddot{\phi} + \frac{3}{r} \dot{\phi} = \frac{\partial V}{\partial \phi}$$

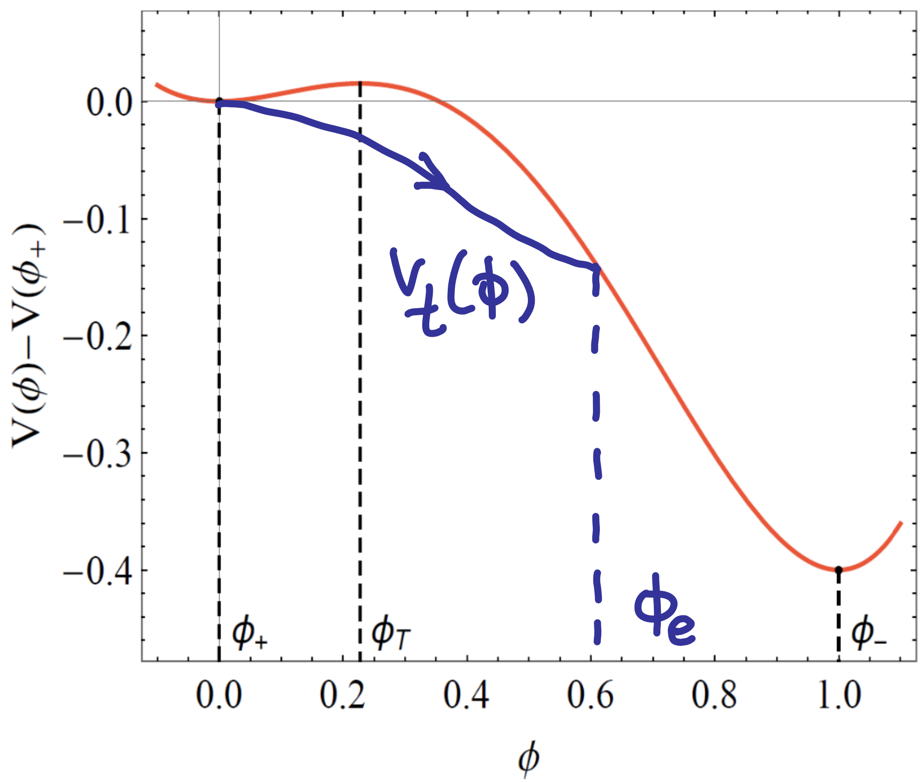
with $\phi(0) = \phi_0$ $\dot{\phi}(0) = 0$ $\phi(\infty) = \phi_+$



$$S = \Delta S_E \equiv S_E[\phi_B] - S_E[\phi_+]$$

TUNNELING POTENTIAL FORMALISM

JRE'1805



$$S[V_t] = 54\pi^2 \int_{\phi_+}^{\phi_e} \frac{(V - V_t)^2}{(-V_t')^3}\, d\phi$$

$$S = \underset{V_t}{\mathrm{Min}}\, S[V_t]$$

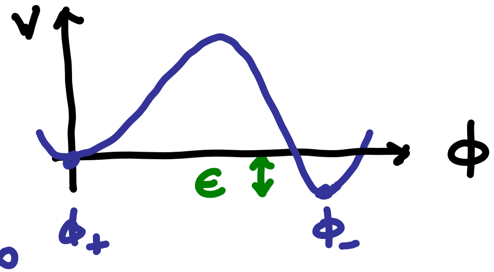
$\phi_e = \phi_0 = \phi(0)$, Euclidean

How to get $S[V_t]$

JRE '2305

LINK: $V_t = V - \frac{1}{2}\dot{\phi}^2 = -$[Euclidean Energy]

Thin-wall action

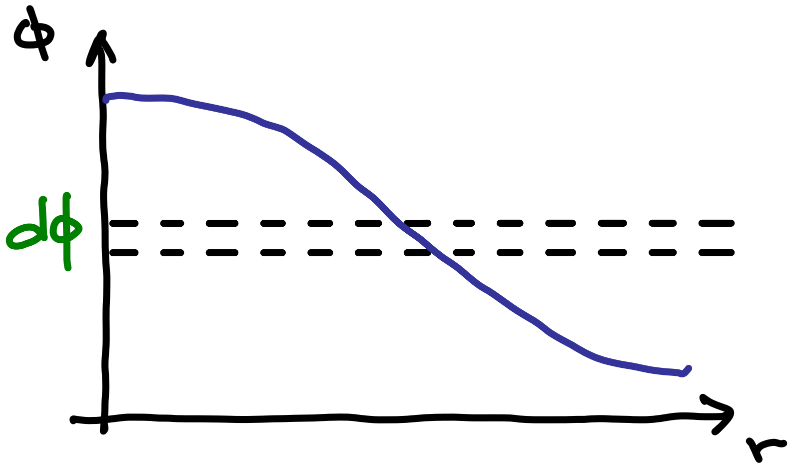


$$S_{tw} = \frac{27\pi^2\sigma^4}{2\epsilon^3}$$

$$\sigma = \int_{\phi_+}^{\phi_-} \sqrt{2(V-V_t)}\, d\phi = \int_0^\infty \dot{\phi}^2\, dr$$

From thin to thick

$$\sigma = \int_+^0 \sqrt{2(V-V_t)}\, d\phi$$



$$d\sigma = \sqrt{2(V-V_t)}\, d\phi \qquad d\epsilon = -V_t'\, d\phi$$

$$S[V_t] = \int_{\phi_+}^{\phi_0} dS_{tw} = 54\pi^2 \int_{\phi_+}^{\phi_0} \frac{(V-V_t)^2}{(-V_t')^3}\, d\phi$$

PROPERTIES & APPS OF V_t-FORM.

JRE'1805

★ V_t on same footing as V

Field can be a distraction.

★ V_t monotonic ⇨ Easy to approximate

★ $S[V_t]$ minimized ($\phi_B(r)$ saddle-point)

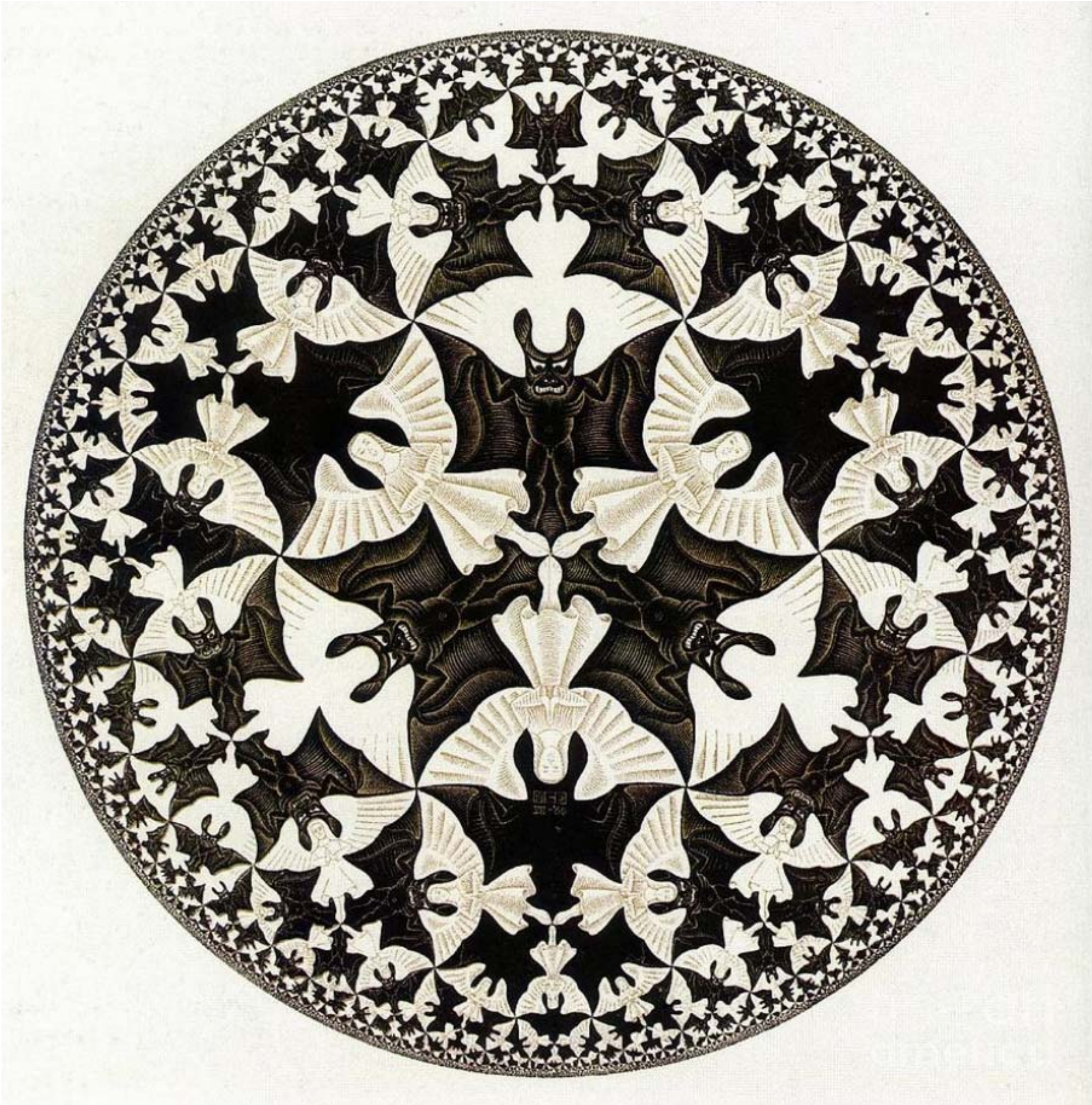
⇨ Good for numerics

⇨ Useful for multi-field case JRE, Konstandin'1811

★ Generalizes to any dimension

$d=3$ applicable to finite T phase transitions

2. GRAVITY CORRECTIONS



JRE '1808
JRE, Huertas, Fortin '2106
JRE, Fortin '2211

WHY ?

Gravity relevant if tunneling involves

$\Delta\phi \lesssim m_p$ or $\Delta V \lesssim m_p^4$ or $|V_+| \lesssim m_p^4$

Note $\Delta\phi \lesssim m_p$ already in SM...

When relevant :

Need to include reaction of the metric.

Gravity can cause qualitative changes

EUCLIDEAN FORMALISM W/GRAVITY

Coleman, De Luccia '80

Assuming $O(4)$

Euclidean bounce $\phi_B(r)$ and metric function $\rho(r)$

$$ds^2 = dr^2 + \rho^2(r)\, d\Omega_3^2$$

line-element S^3

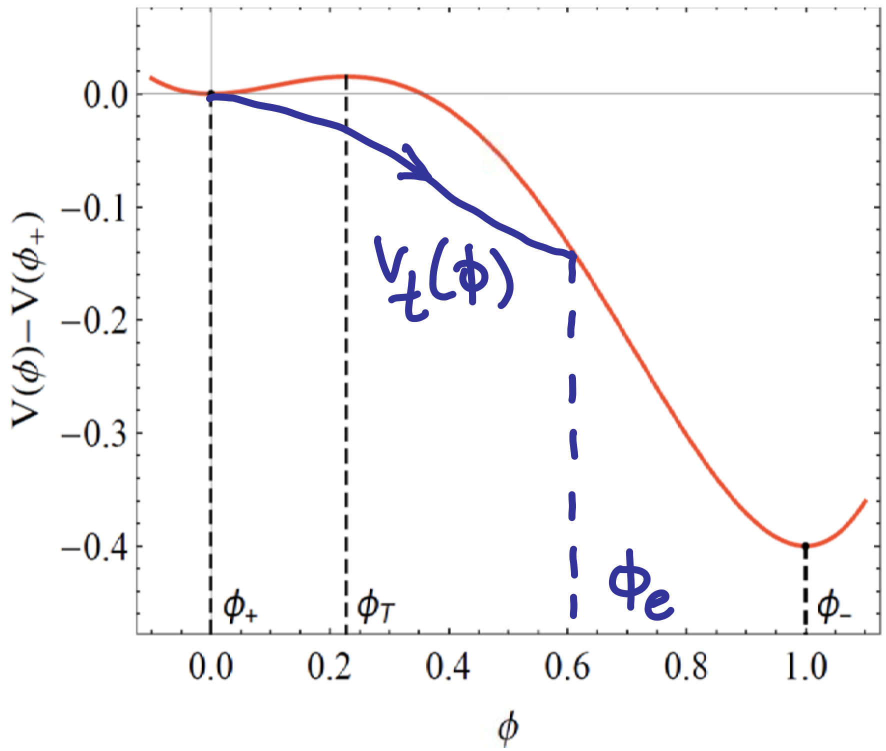
$$\Rightarrow \quad \ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{\partial V}{\partial \phi} \qquad \qquad \dot{\rho}^2 = 1 + \frac{\kappa \rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V\right)$$

$\kappa \equiv 1/m_p^2$

$$S = \Delta S_E = S_E[\phi_B, \rho_B] - S_E[\phi_+, \rho_+]$$

V_t FORMALISM w/ GRAVITY

JRE'1808



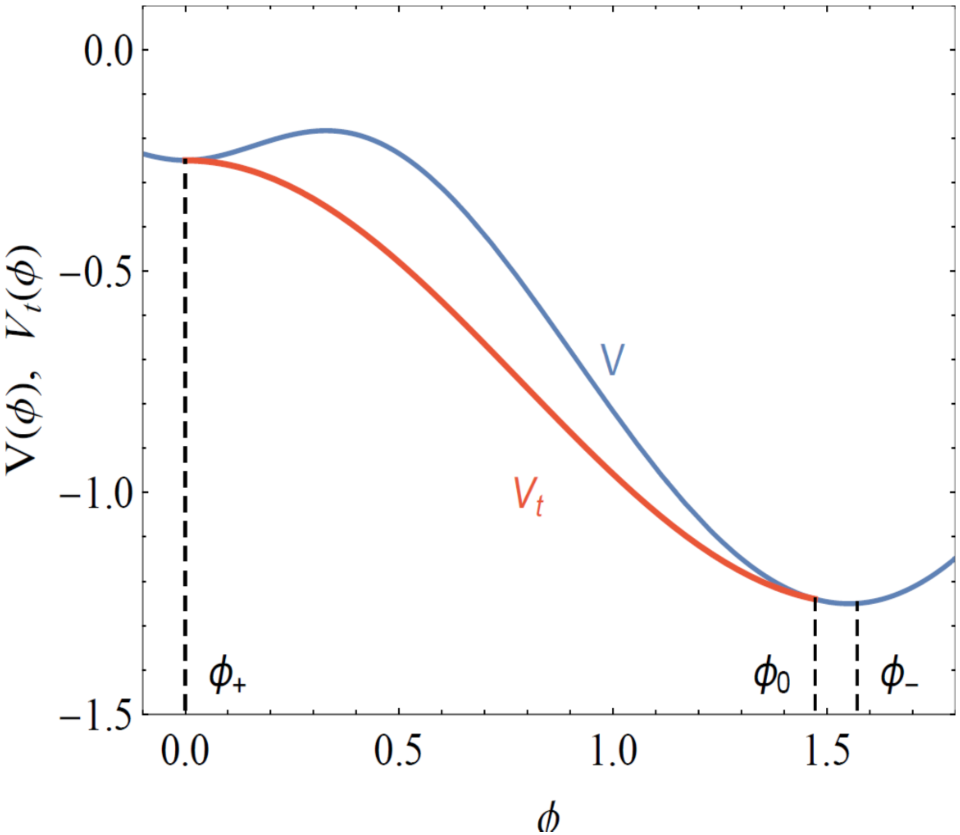
$$S[V_t] = \frac{6\pi^2}{\kappa^2}\int_{\phi_+}^{\phi_e} \frac{(D+V_t')^2}{D V_t^2}\, d\phi \qquad = \Delta S_E$$

with

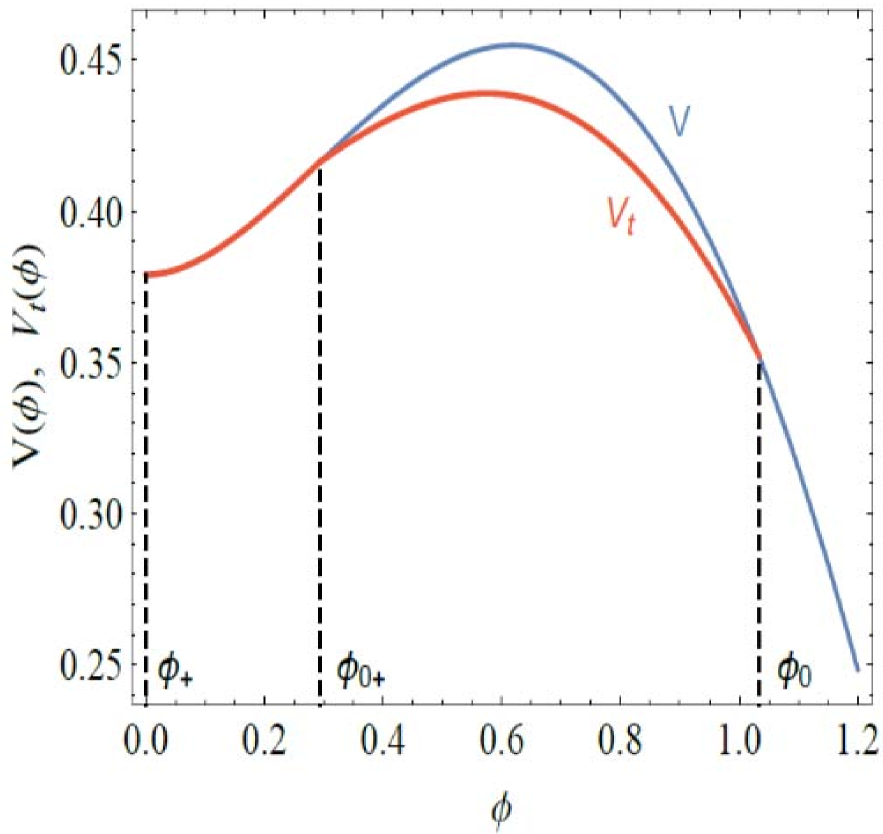
$$D = \sqrt{(V_t')^2 + 6\kappa (V - V_t) V_t} \qquad \kappa \equiv 1/m_P^2$$

PROPERTIES & APPS OF V_t-FORM.

Universal formula, valid for AdS, Minkowski or dS



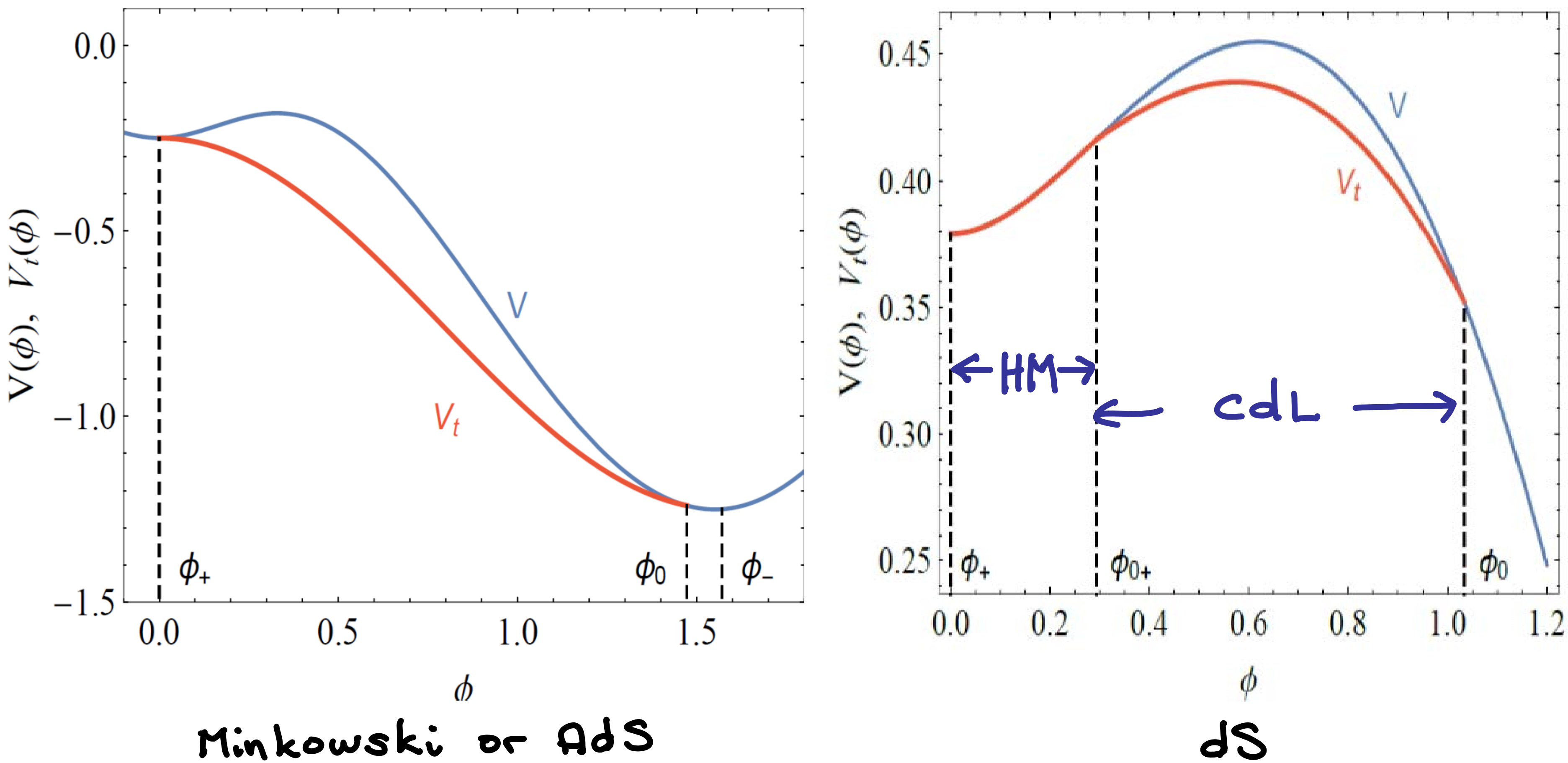
Minkowski or AdS



dS

PROPERTIES & APPS OF V_t-FORM.

Universal formula, valid for AdS, Minkowski or dS



(Th. int: Brown, Weinberg, 0706)

HAWKING-MOSS RATE

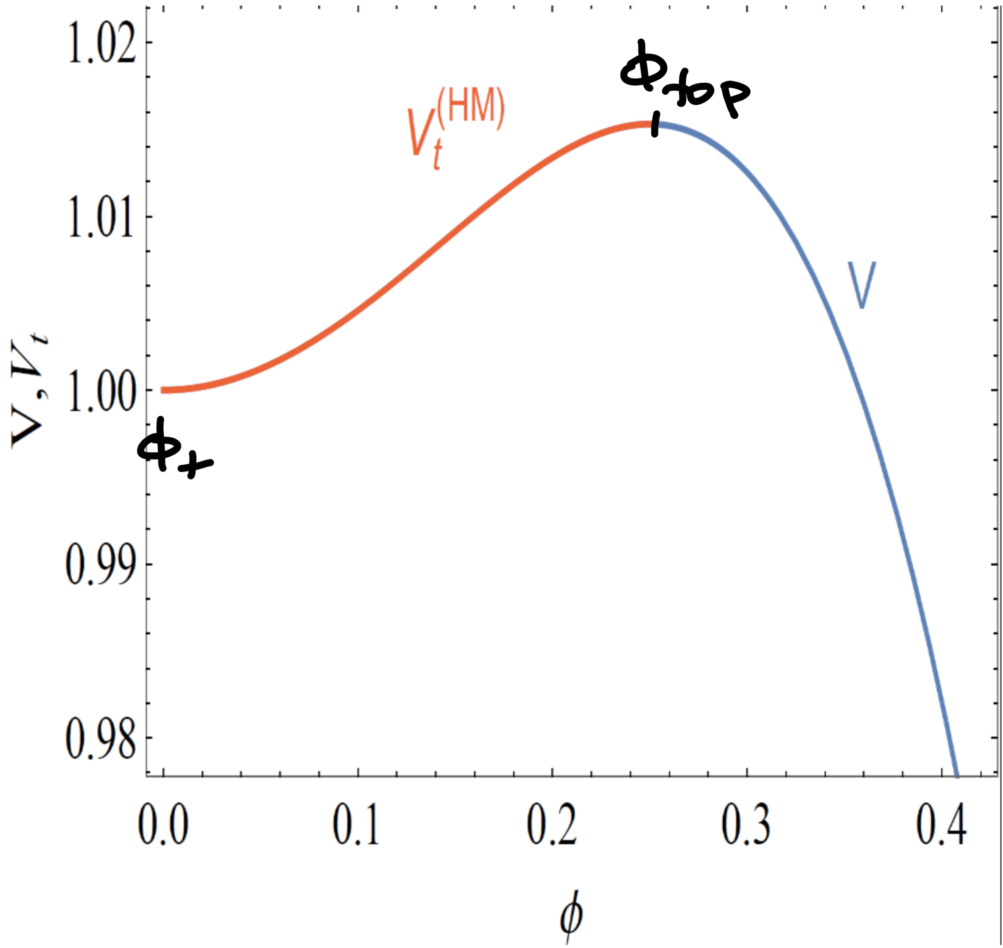
For dS, $V_+ \uparrow$ $\Rightarrow$ no CdL bounce

Hawking-Moss decay '82

with tunneling action

$$S_{HM} = \frac{24\pi^2}{\kappa^2}\left(\frac{1}{V_+} - \frac{1}{V_{top}}\right)$$

$S[V_t]$ reproduces this ✓



GLOBAL PICTURE (dS)

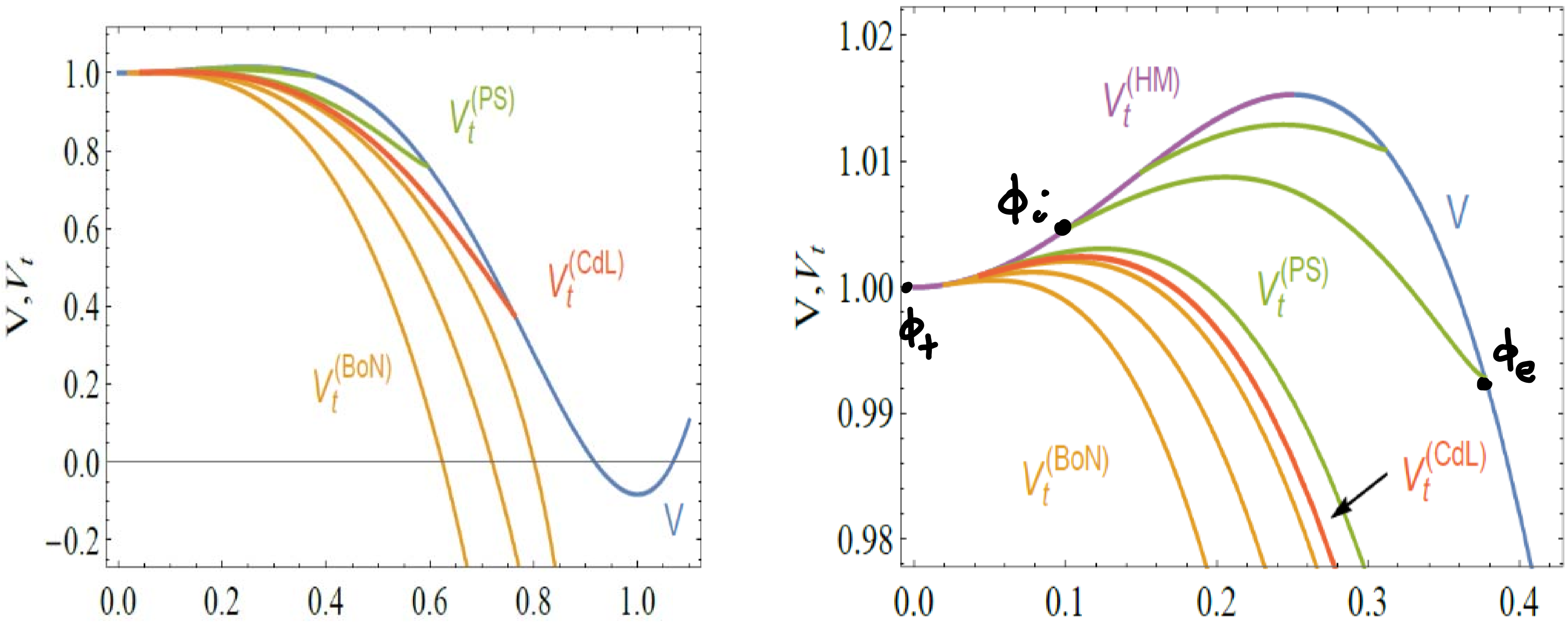
Family of solutions $V_t(\phi_i;\phi)$ of EoM for V_t

$\delta S/\delta V_t = 0 \Rightarrow$

$$0 = (4V_t' - 3V')\,V_t' + 6(V - V_t)\left[V_t'' + \kappa\,(3V - 2V_t)\right]$$

BCs $\quad V_t(\phi_i) = V(\phi_i) \qquad V_t'(\phi_i) = \frac{3}{4}V'(\phi_i)$

GLOBAL PiCTURE (dS)



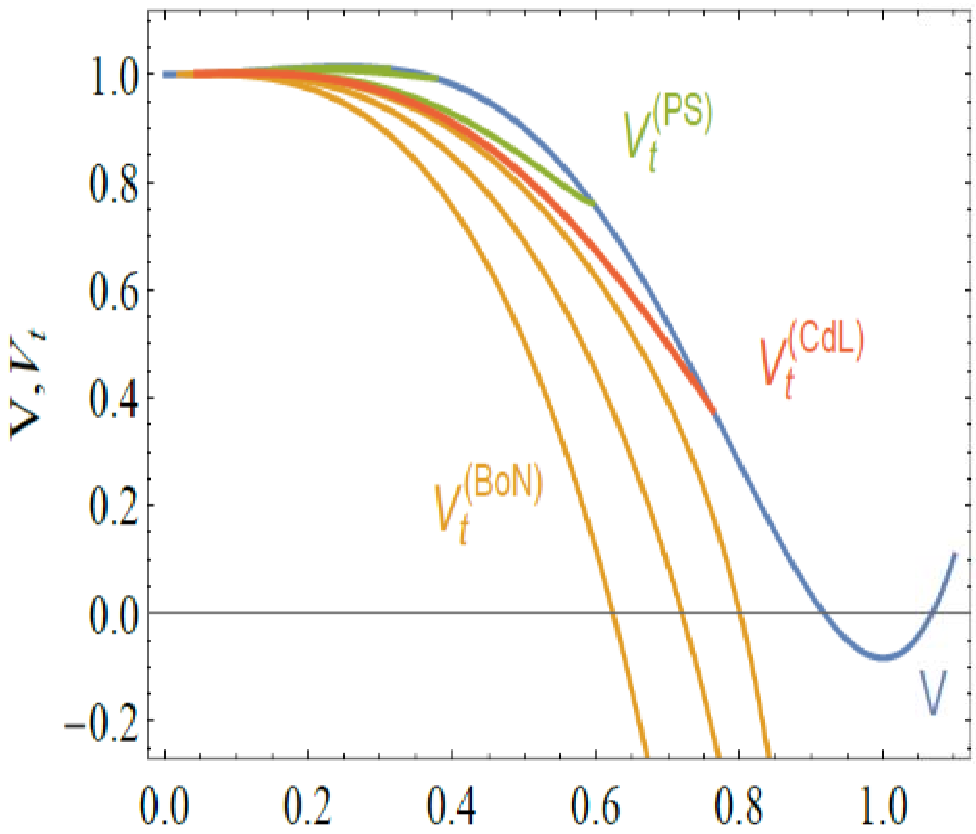
PS = "pseudo-bounces"

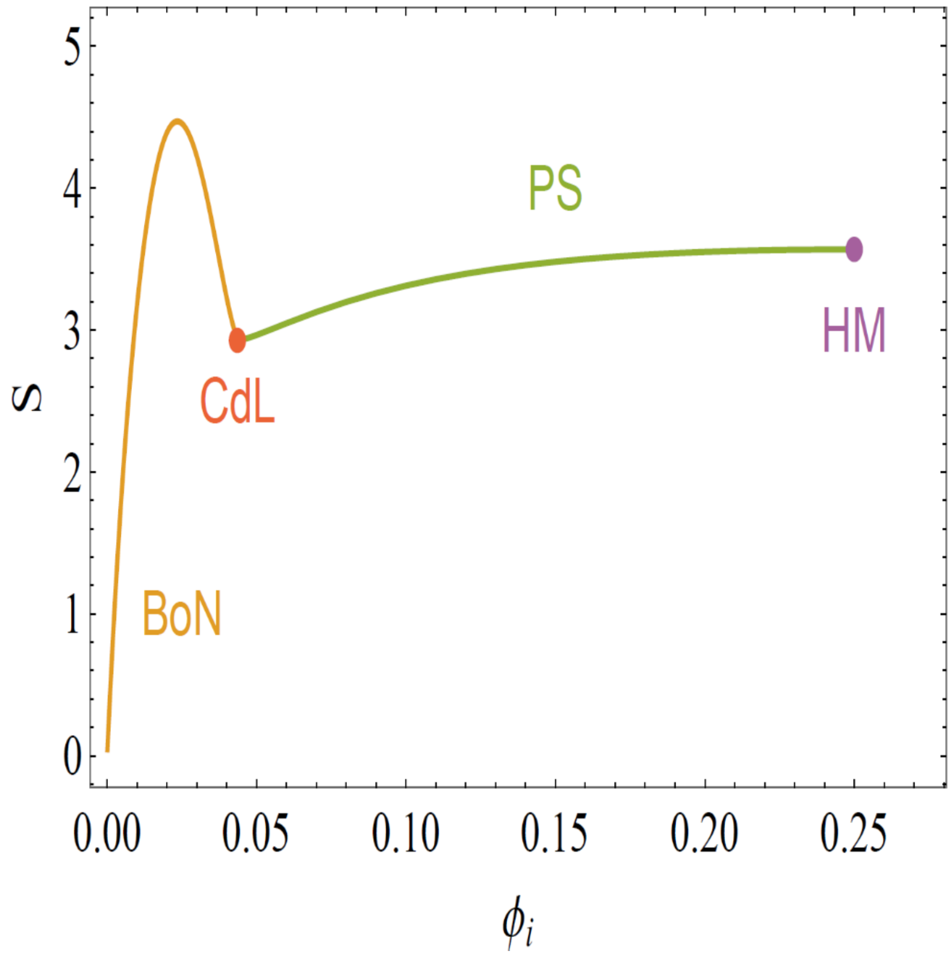
BoNs = Bubble of nothing decays

Relevant if ϕ is a size modulus.

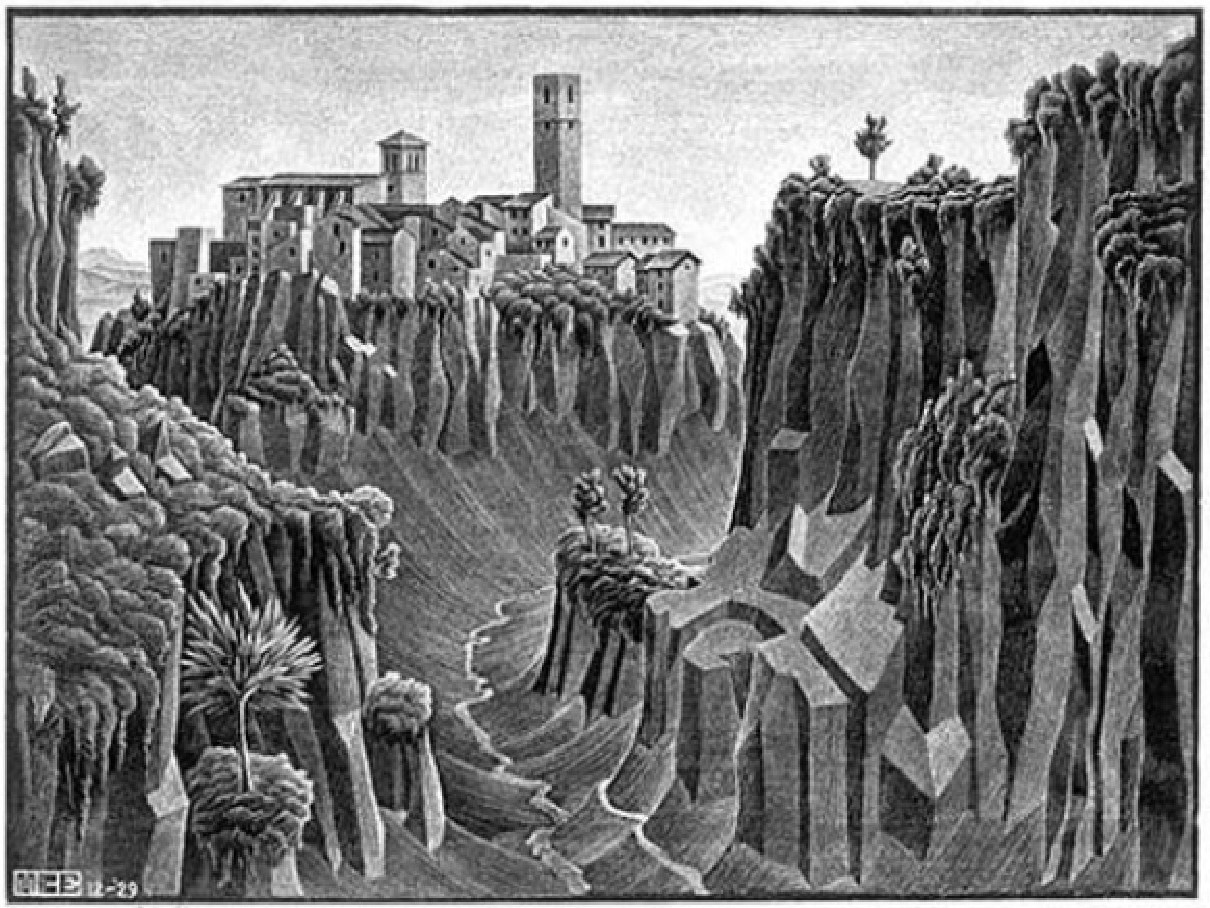
GLOBAL PICTURE (dS)

Finite action :





3. Pseudo-Bounces



PSEUDOBOUNCES

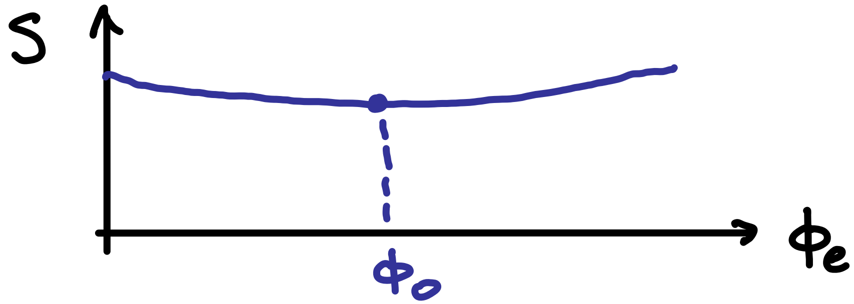
JRE '1908

★ Minimize $S[V_t]$ if ϕ_e held fixed

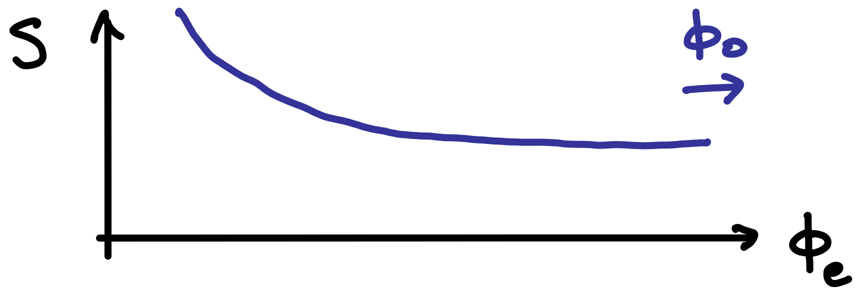
⇨ Not true extremals $\quad dS/d\phi_e \neq 0$

Relevant if

- $dS/d\phi_e$ small



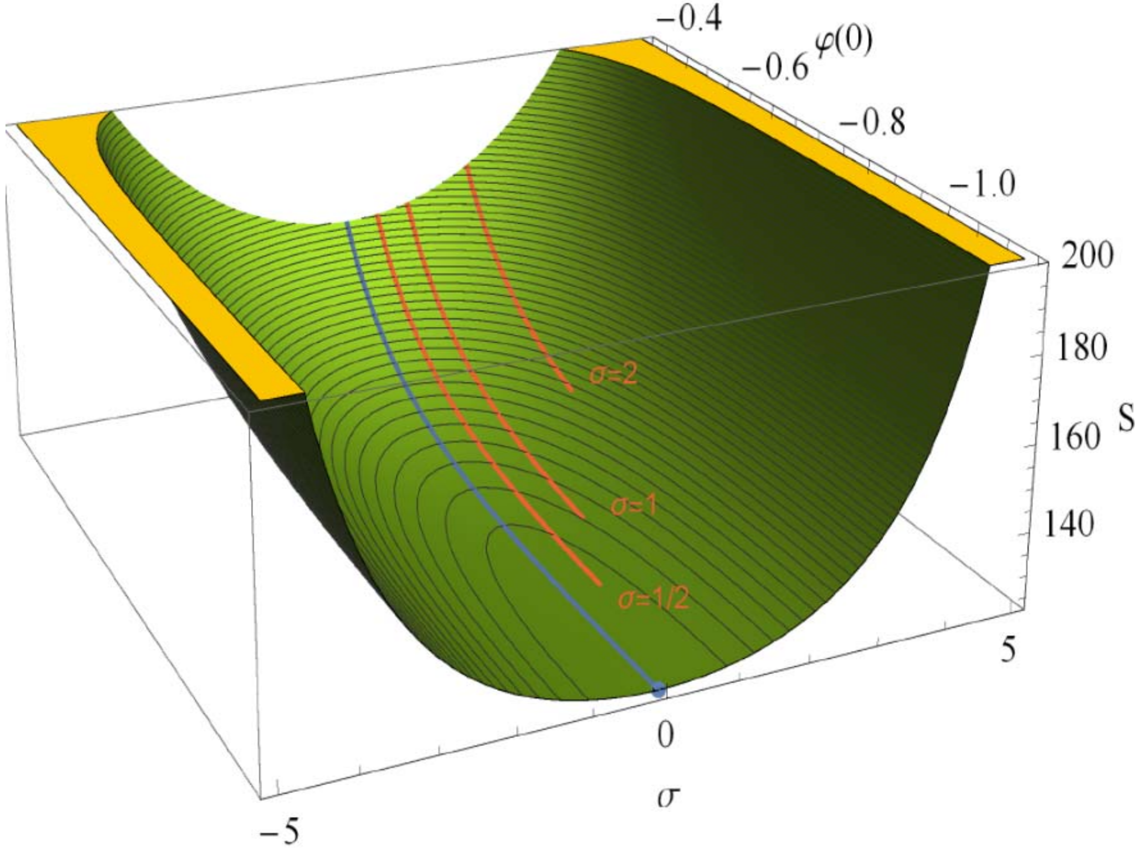
- $\phi_0 \to \infty$ (no bounce)



PSEUDOBOUNCES

Track bottom of action valley in config. space (better than constrained instantons)

JRE, Huertas '2106



CONCLUSIONS

Tunneling potential formalism

simple and useful

- Vacuum decay / Finite T transitions
- Gravity effects on vacuum decays
- Pseudo-bounces

CONCLUSIONS

Further applications

- Bubble-of-nothing decays w/ Blanco-Pillado, Huertas, Sousa '2312
- Q-balls w/ Heeck, Sokhashvili '2307
- Positive Energy theorems in AdS maxima w/ Jinno
- Flat / curved Domain Walls w/ C. Bachas, Z. Chen

More? Euclidean solutions $\Rightarrow$ V_t solutions

BACK UP SLIDES

V_t FOR THERMAL TRANSITIONS

General d

$$S_d[V_t] = \frac{(d-1)^{d-1}(2\pi)^{d/2}}{\Gamma(1+d/2)} \int_{\phi_+}^{\phi_0} \frac{(V-V_t)^{d/2}}{|V_t'|^{d-1}}\, d\phi$$

Thermal tr : d=3

$$S_3[V_t] = \frac{16\pi}{3} \int_{\phi_+}^{\phi_0} \frac{[2(V-V_t)]^{3/2}}{(V_t')^2}\, d\phi$$

V : thermally corrected potential

DE-EUCLIDEANIZE

JRE '1805

⇨ Get rid of Euclidean quantities in terms of V & V_t ones

LINK : $V_t = V - \frac{1}{2}\dot{\phi}^2$

⇨ $\dot{\phi} = -\sqrt{2(V-V_t)}$ $\quad \ddot{\phi} = \frac{d}{d\phi}\left(-\sqrt{2(V-V_t)}\right)\dot{\phi} = V' - V_t'$

EoM : $\ddot{\phi} + \frac{3}{r}\dot{\phi} = V'$ ⇨ $r = \frac{3\sqrt{2(V-V_t)}}{-V_t'}$

$\frac{d}{dr}(\ldots)$ ⇨ $(4V_t' - 3V')V_t' + 6(V-V_t)V_t'' = 0$

Action gives this EoM for V_t

DE-EUCLIDEANIZE

JRE '1808

⇨ Get rid of Eucl. quant. in terms of V & V_t

$$V_t = V - \frac{1}{2}\dot{\phi}^2$$

As before. Now

$$r = \frac{3\sqrt{2(V-V_t)}}{-V_t'} \rightarrow \rho = \frac{3\sqrt{2(V-V_t)}}{D}$$

$$\frac{d}{d\xi}(\ldots) \Rightarrow 0 = (4V_t' - 3V')V_t' + 6(V-V_t)\left[V_t'' + \kappa(3V - 2V_t)\right]$$

Action gives this EoM for V_t

GRAVITATIONAL QUENCHING

For AdS, Minkowski ($V_+ \leq 0$)

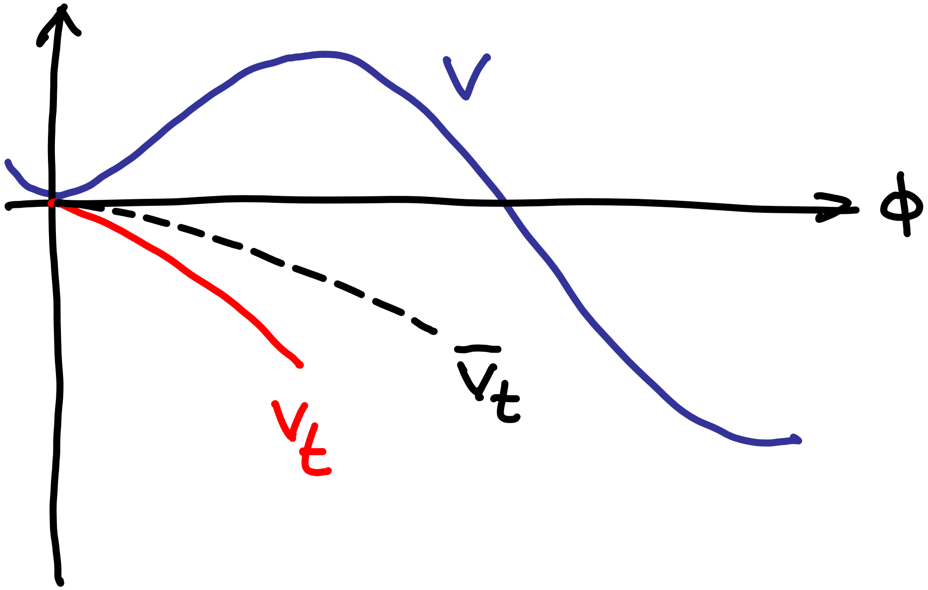
gravitational quenching of decay possible

Coleman, De Luccia '80

$$D = \sqrt{\underset{\vee \atop 0}{(V_t')^2} + \underbrace{6\kappa(V - V_t)V_t}_{\wedge \atop 0}} \quad \text{must be real} \quad \Rightarrow$$

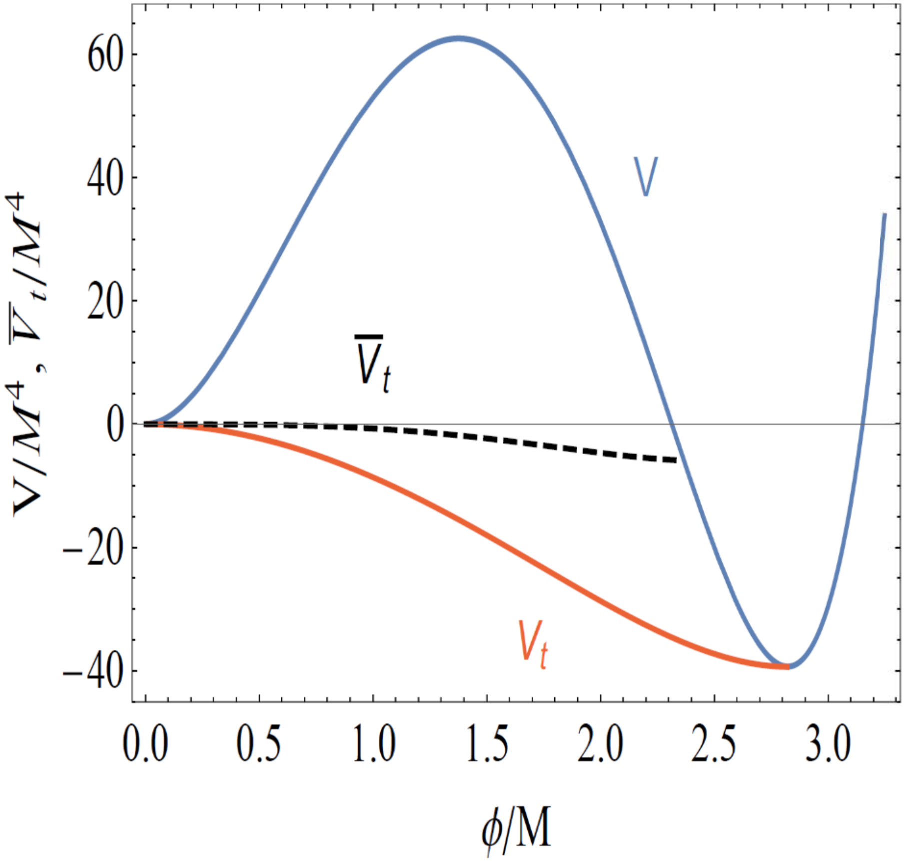
$$|V_t'| > |\bar{V}_t'| = \sqrt{6\kappa(V - \bar{V}_t)(-\bar{V}_t)}$$

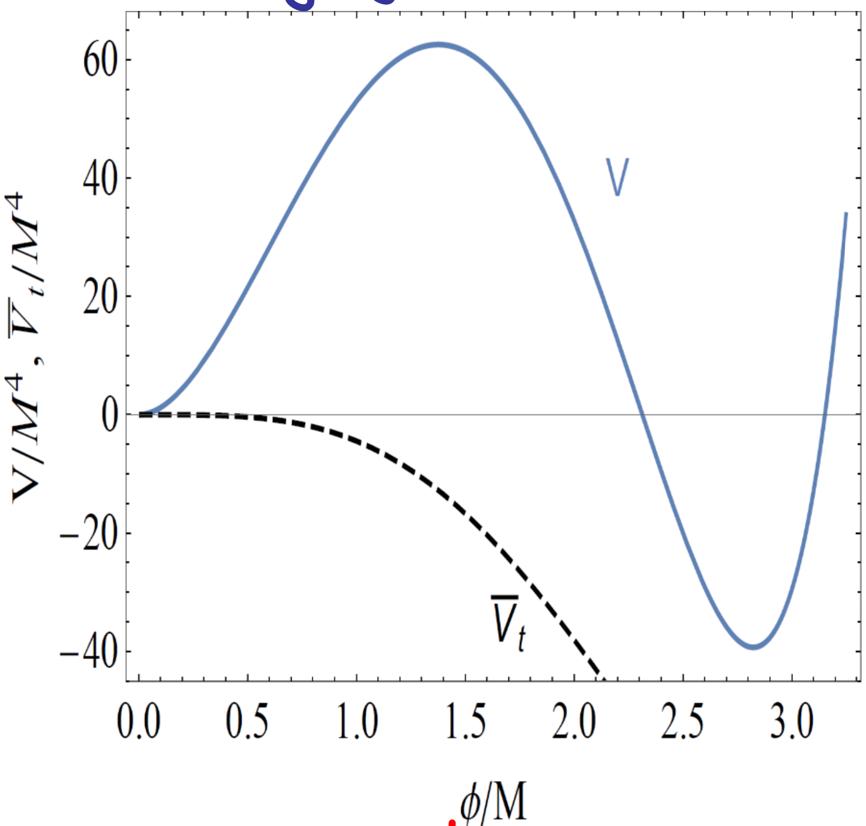
$V_t < \bar{V}_t$



GRAVITATIONAL QUENCHING

Weak grav. effects → Strong grav. effects





Decays

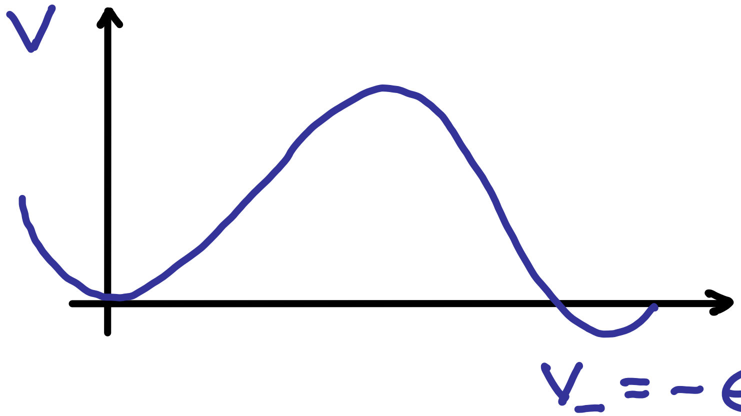
No decay

NO EXIT

Positive Energy theorem

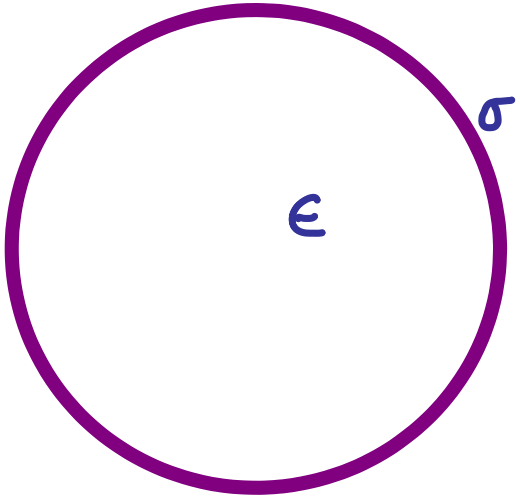
JRE'2005

GRAVITATIONAL QUENCHING



No gravity

$$E \sim -\epsilon R^3 + \sigma R^2 = 0$$



large $R \sim \sigma / \epsilon$

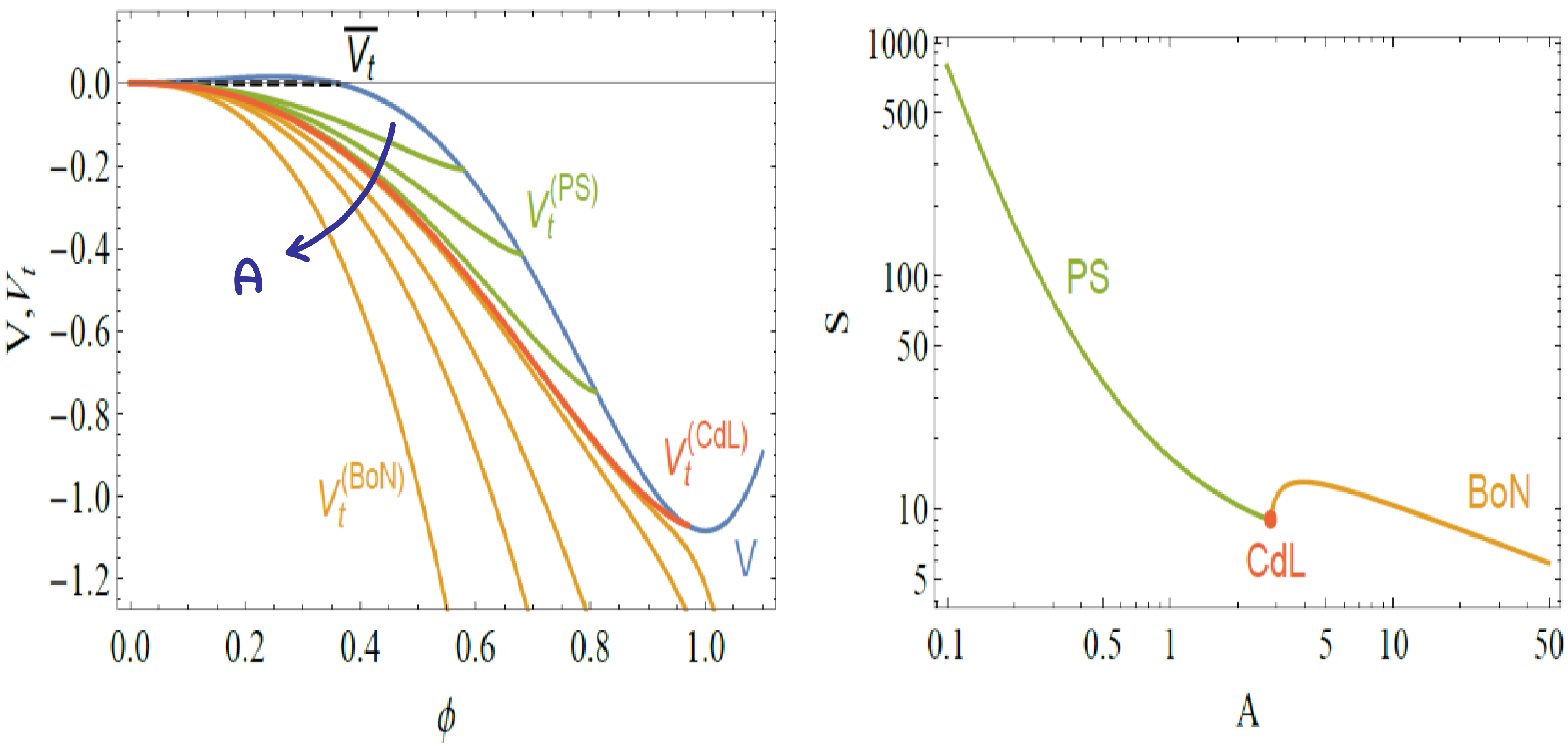
With gravity

$$E \sim -\epsilon R^2 + \sigma R^2$$

$E = 0$ not guaranteed

GLOBAL PICTURE (Mink.)

Family of solutions $V_t(A;\phi)$

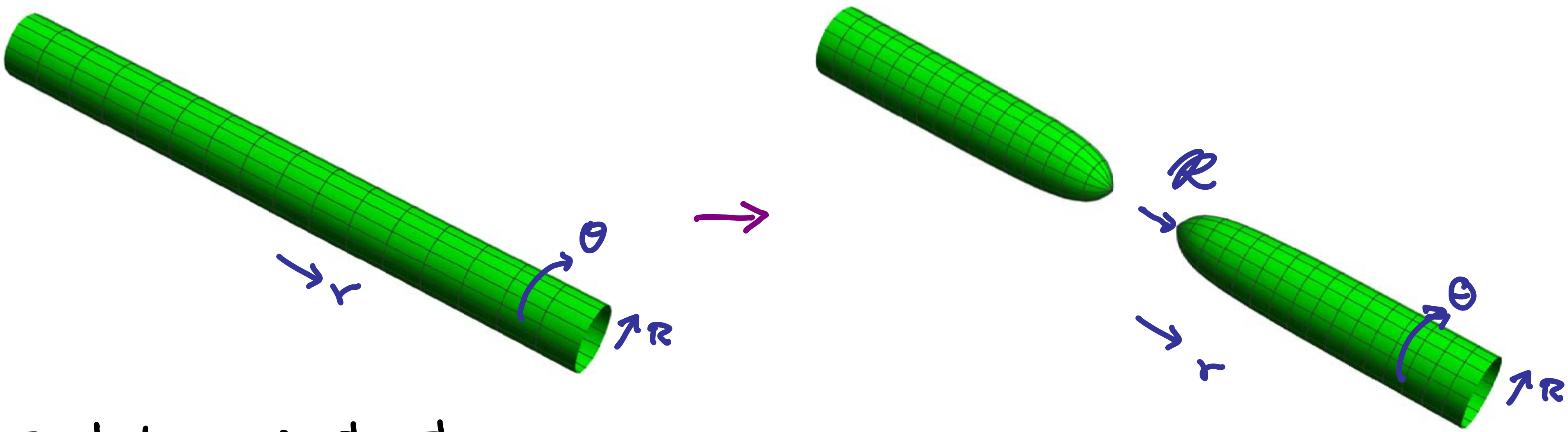


4. BUBBLE OF NOTHING DECAYS

BUBBLE OF NOTHING DECAYS

Decays of spacetimes with compactified dim. like

5d KK ($M^4 \times S^1$) Witten' 82



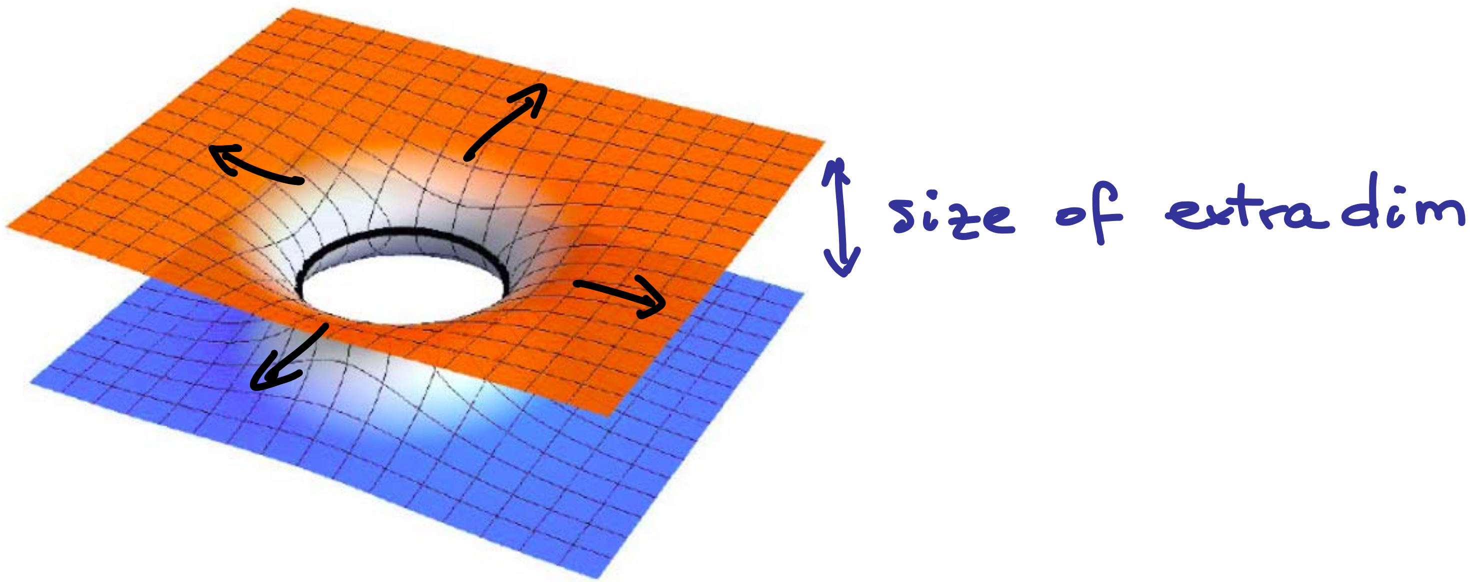
Euclidean instanton:

$$ds^2 = \frac{dr^2}{1-R^2/r^2} + r^2 d\Omega_3^2 + \underbrace{R^2\left(1-\frac{R^2}{r^2}\right)}_{R(r)^2 \to 0 \text{ at } r \to \mathcal{R} = R} d\theta^2$$

with $\mathcal{S} = (\pi R m_p)^2$

BUBBLE OF NOTHING DECAYS

End-product of tunneling process is a hole in space-time



It's smooth, w/o singularities, curvature not large

But it grows, eating the whole space

WHY BONS?

- Relevant for string vacua landscape

- BoNs thought to be ultimate cause of decay of non-SUSY vacua in string compactifications
  García-Etxebarría, Montero, Sousa, Valenzuela '2005

- Relevant for cobordism conjecture

4d VIEW

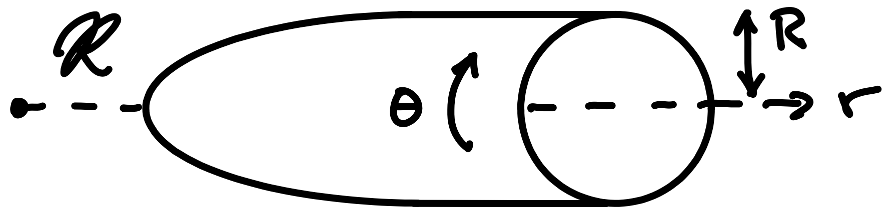
Dine, Fox, Gorbatov '0405

5d → 4d + Scalar ϕ (geometric modulus)

$$R_{S^1}^2 = R^2\left(1 - \frac{R^2}{r^2}\right)$$

$$\parallel$$

$$R^2 e^{-2\sqrt{2\kappa/3}\,\phi}$$



and $ds_4^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2$

BoN reduces to a singular CdL problem

Action reproduced (paying attention to boundary terms)

4d VIEW

Dine, Fox, Gorbatov '0405

5d → 4d + Scalar ϕ (geometric modulus)

$$R_{S^1}^2 = \underbrace{R^2\left(1-\frac{R^2}{r^2}\right)}_{5d} = \underbrace{R^2 e^{-2\sqrt{2\kappa/3}\,\phi}}_{4d}$$

$$\begin{cases} \text{BoN core}: & r \to R & \phi \to \infty \\ \text{False vac}: & r \to \infty & \phi \to 0 \end{cases}$$

and $ds_4^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2$

with $\dfrac{d\xi}{dr} = \dfrac{1}{(1-R^2/r^2)^{1/4}}$ (BoN core: $\xi \to 0$)

BoN reduces to a CdL problem

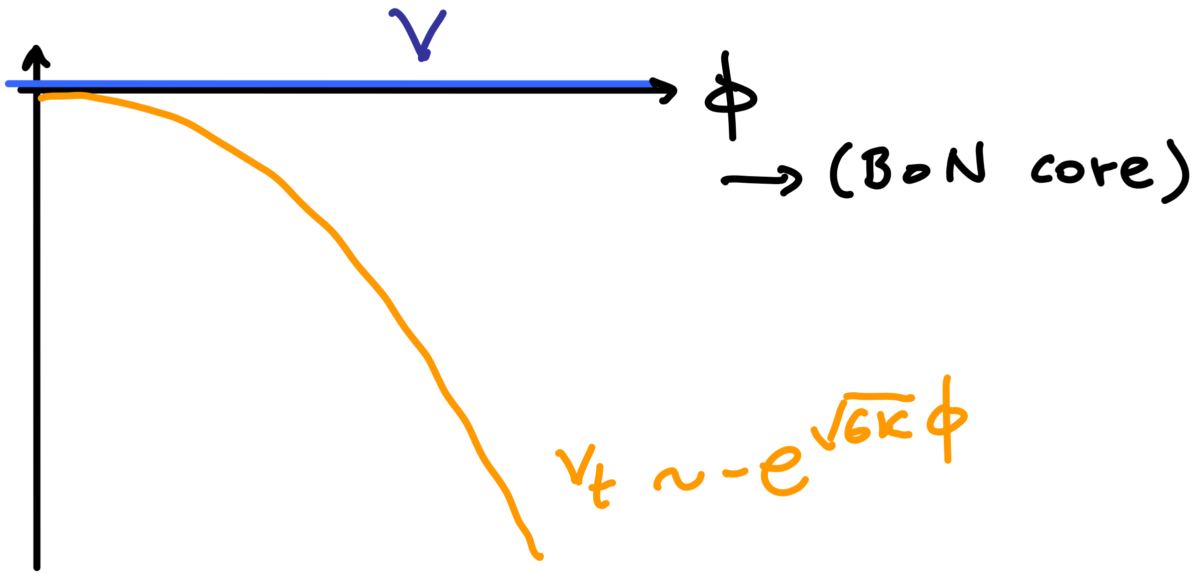
$\phi(0) = \infty$ $\quad\dot\phi(0) = -\infty$ $\quad\phi(\infty) = \phi_+ = 0$

Action reproduced (paying attention to boundary terms)

V_t FOR BONS

Blanco-Pillado, JRE, Huertas, Sousa '2312

Witten's BoN $\quad V_t = -6 m_p^2 R^2 \sinh^3(\sqrt{2\kappa/3}\,\phi)$, $\quad V=0$

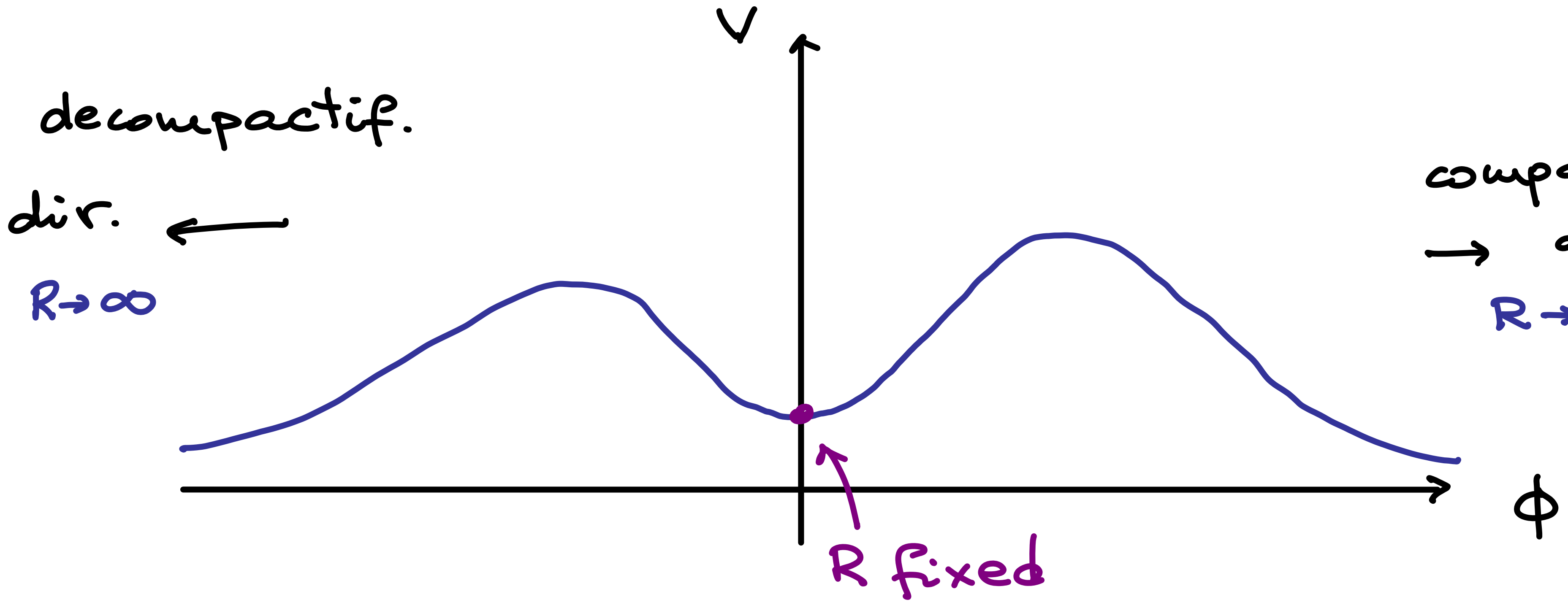


BoN tunneling to $-\infty$ AdS

★ $S[V_t]$ ✓ without additional boundary terms

Allows bottom-up approach to study which $V(\phi)$ admits BoN decays $\quad$ Draper, García-García, Lillard '2105

BONS WITH $V(\phi)$?



★ Universal asymptotic V, V_t behaviours at $\phi \to \infty$ via

$$0 = (4V_t' - 3V')V_t' + 6(V - V_t)\left[V_t'' + \kappa(3V - 2V_t)\right]$$

→ which V, V_t can be obtained from extra-d theories?

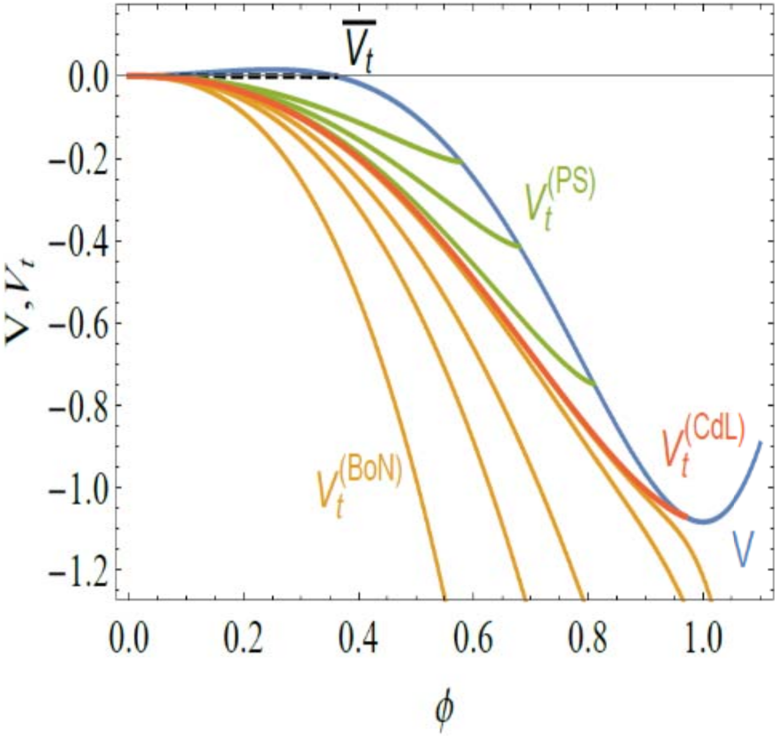
→ Is BoN always the dominant decay?

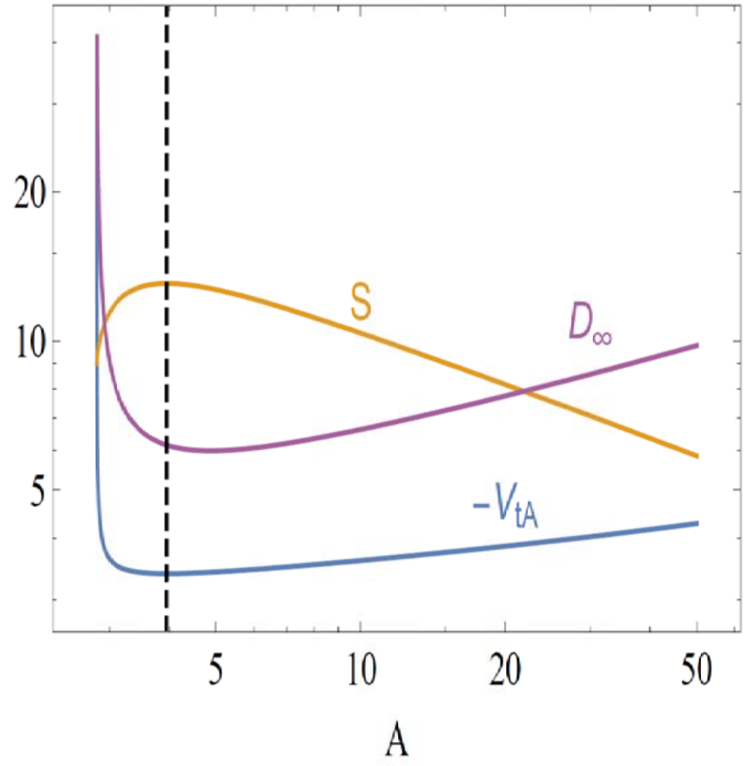
$\phi \to \infty$ (BoN CORE) ASYMPTOTICS

Example: $\mathbb{M}^4 \times S^1$, $V(\phi) \neq 0$

4d EFT :

$$V_t \simeq V_{tA}(A)\, e^{\sqrt{6\kappa}\,\phi} + \dots \qquad D \simeq D_\infty(A)\, e^{\sqrt{8\kappa/3}\,\phi} + \dots$$





$\phi \to \infty$ (BON CORE) ASYMPTOTICS

Example: $\mathbb{M}^4 \times S^1$, $V(\phi) \neq 0$

4d EFT:

$$V_t \simeq V_{tA}(A)\, e^{\sqrt{6\kappa}\,\phi} + \dots \qquad D \simeq D_\infty(A)\, e^{\sqrt{8\kappa/3}\,\phi} + \dots$$

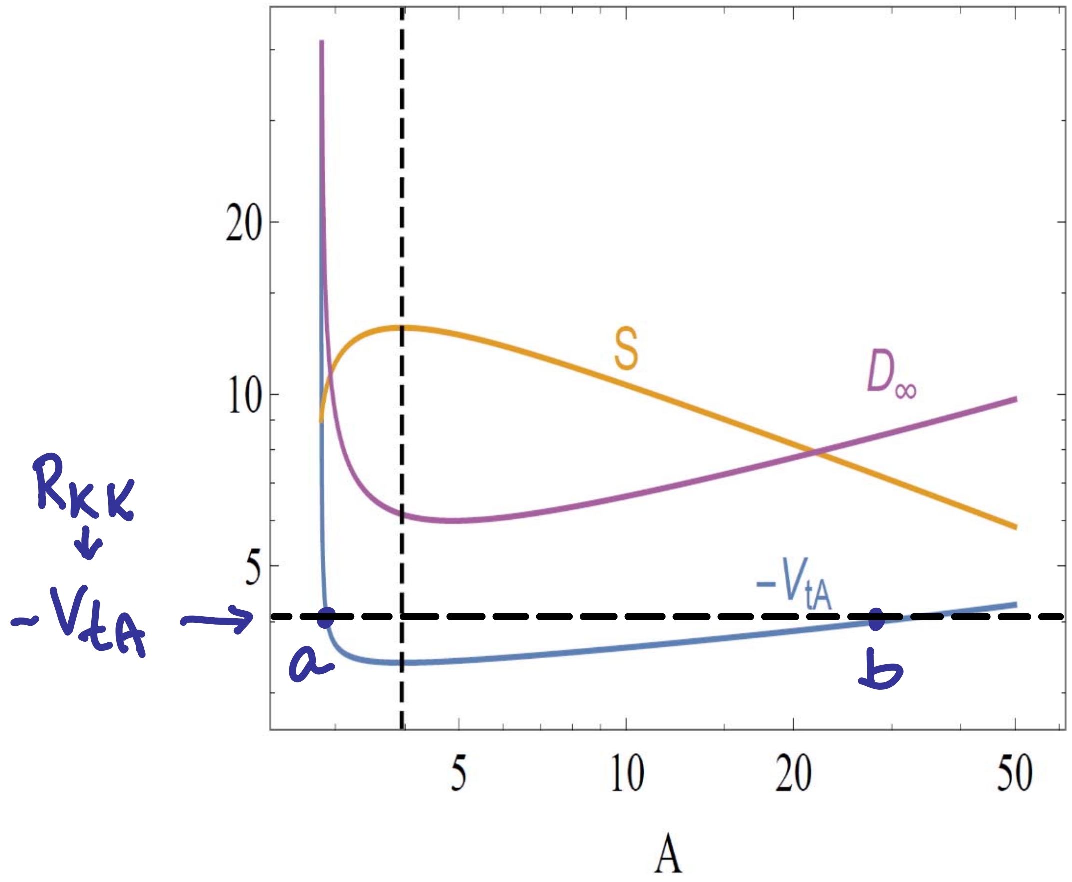
5d input:

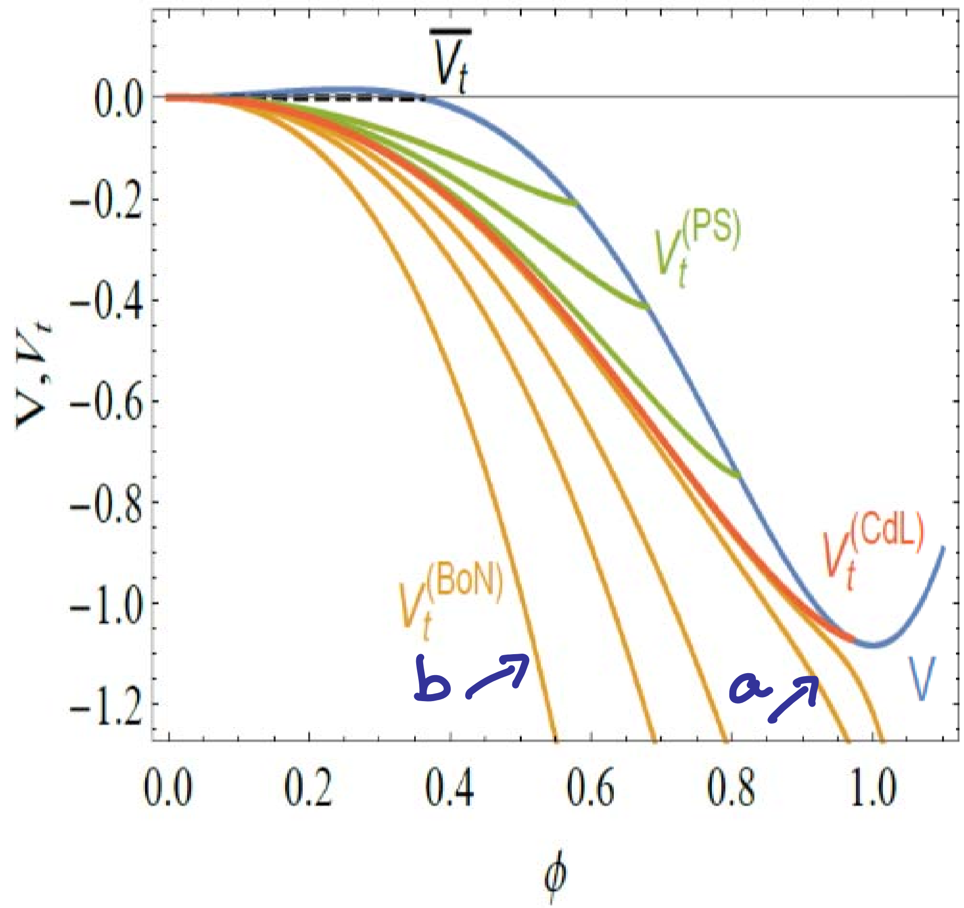
$$V_{tA} = \frac{c}{\kappa R_{KK}^2} \qquad D_\infty = \frac{c'}{\mathcal{R}\, R_{KK}\sqrt{\kappa}}$$

BoN radius ↲ ↖ KK radius

For fixed R_{KK} ⇨ fixed V_{tA}

WHICH V_t^{BoN} ?





Choose sol. with lowest S and D_∞ fixes $\mathcal{R}$

If R_{KK} gives $-V_{tA} < -V_{tA}^{min}$ ⇒ No BoN decay