Natural Alignment and CP Violation Beyond the Standard Model

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Based on:

- P.S.B. Dev, AP, JHEP1412 (2014) 024
- AP, PRD93 (2016) 075012
- N. Darvishi, AP, PRD99 (2019) 115014; PRD101 (2020) 095008 . . .
- N. Darvishi, AP, J.-H. Yu, arXiv:2312.00882

Outline:

- **The Standard Theory of Electroweak Symmetry Breaking**

- **Brief history of symmetries for natural SM alignment**

- **SM Alignment in the 2HDM and Beyond**

- **CP Violation in 2HDM: Explicit, Spontaneous and Mixed**

- **Phenomenology at the LHC**

- **Summary**

Higgs Mechanism in the SM: $SU(3)_{colour} \otimes SU(2)_L \otimes U(1)_Y$

[P. W. Higgs '64; F. Englert, R. Brout '64.]
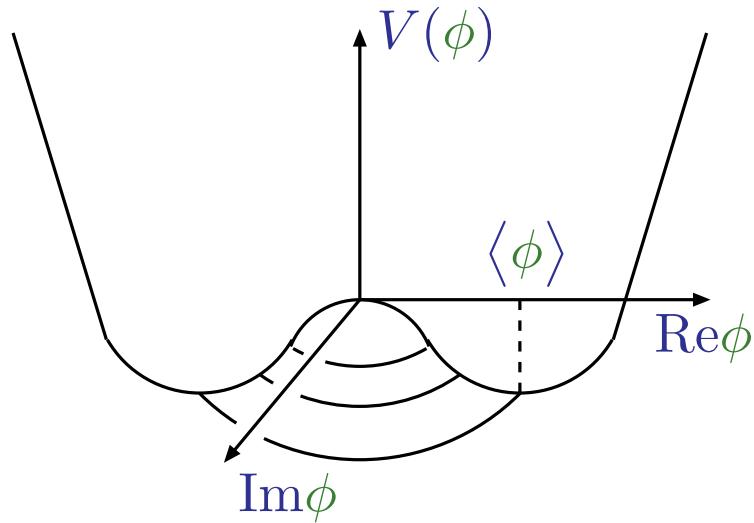
Higgs potential $V(\phi)$

$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \, .$$

Ground state:

$$\langle \phi \rangle = \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

carries weak charge, but no electric charge and colour.

Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

$$\mathbf{\Phi} \equiv \left(\phi \, , \, i\sigma^2 \phi^* \right) \quad \mapsto \quad \mathbf{\Phi}' \equiv U_L \, \mathbf{\Phi} \, U_C \, ,$$

with $U_L \in SU(2)_L$ and $U_C \in SU(2)_C$, and $SU(2)_L \otimes SU(2)_C / \mathbb{Z}_2 \simeq S\mathbb{O}(4)$.

On the SM Higgs-Boson at the LHC

ATLAS & CMS Results 2019:
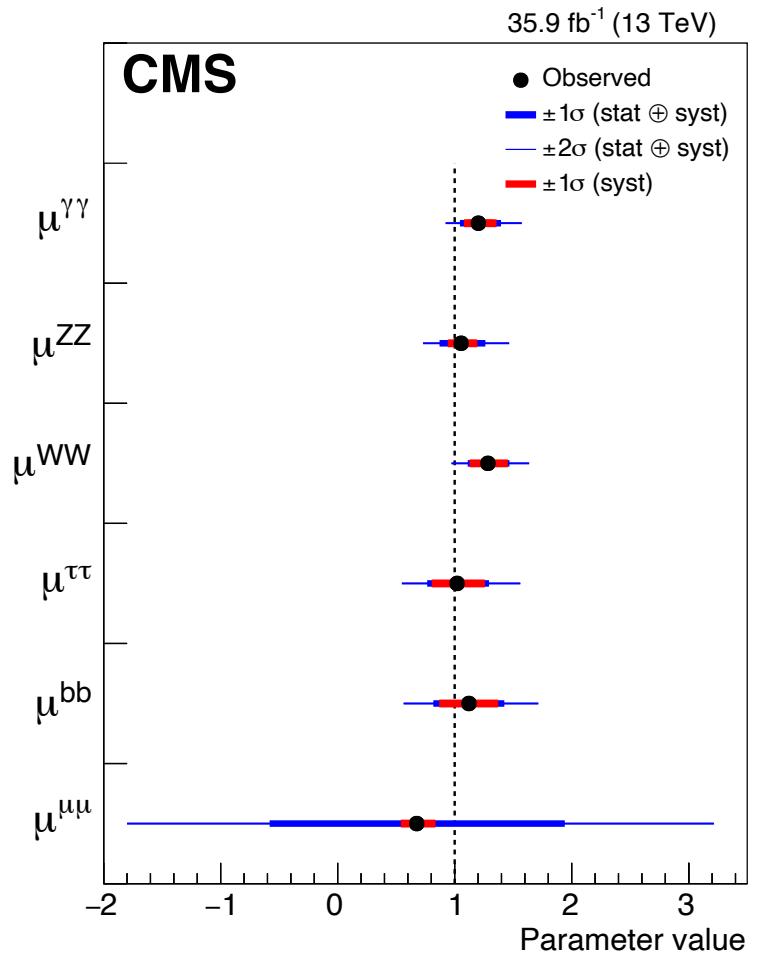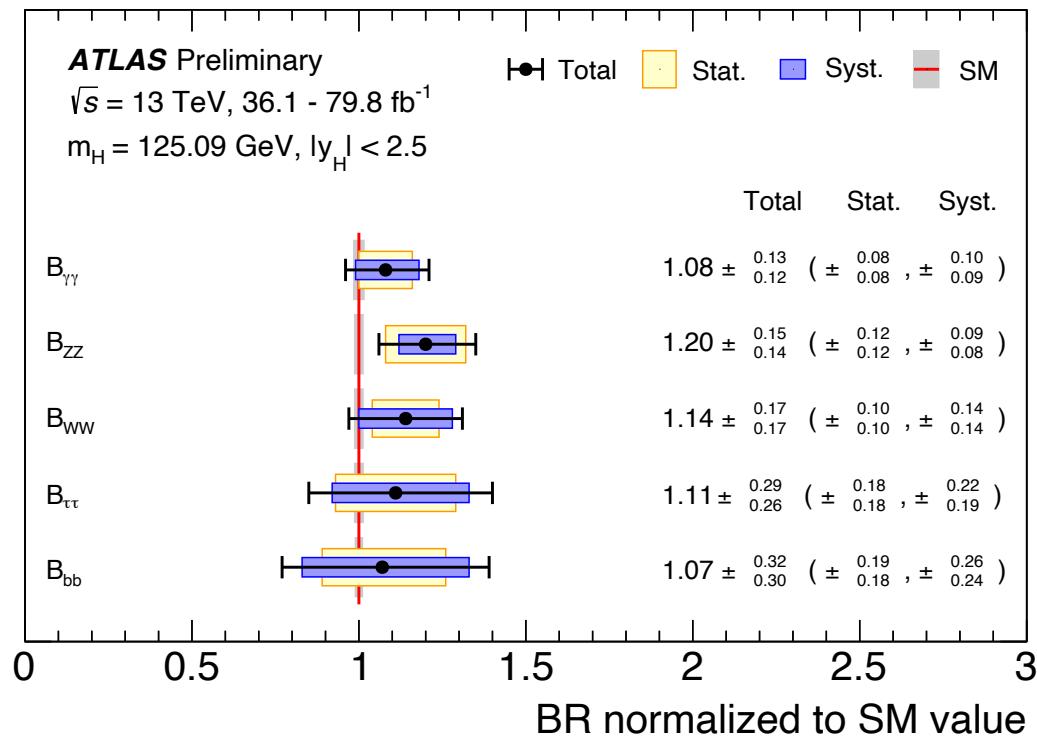
• Brief history of symmetries for natural SM alignment

– Flavour unitarity of the CKM mixing matrix
[Gell-Mann, Levy '60; Cabbibo '63; Kobayashi, Maskawa '73]

– GIM mechanism to explain the smallness of the strangeness-changing interaction at the quantum level (requires the existence of the c-quark)
[Glashow, Iliopoulos, Maiani '70]

– Conditions for diagonal neutral currents in Z-boson interactions to quarks
[Paschos '77]

– Natural diagonal neutral currents in Z- **&** multi-Higgs-boson interactions to quarks
[Glashow, Weinberg '77]

– Renormalizable models with partial flavour non-conservation at tree level (GIM suppressed).
[Branco, Grimus, Lavoura '96]

– Yukawa alignment in the 2HDM broken by RG effects
(no global symmetry protected)
[Pich, Tuzon '09]

– Natural Alignment Beyond the Standard Model
[Dev, Pilaftsis '14, AP '16 & this talk]

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;
Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
V = & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) \ .
\end{aligned}
$$

• Physical (CP-conserving) spectrum:

CP-even Higgs bosons H and h;   CP-odd scalar a;   charged scalars $h^\pm$.

• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} = \cos(\beta - \alpha) \ , \qquad g_{hVV} = \sin(\beta - \alpha) \ ,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

- **Global fit to SM mis-alignment**
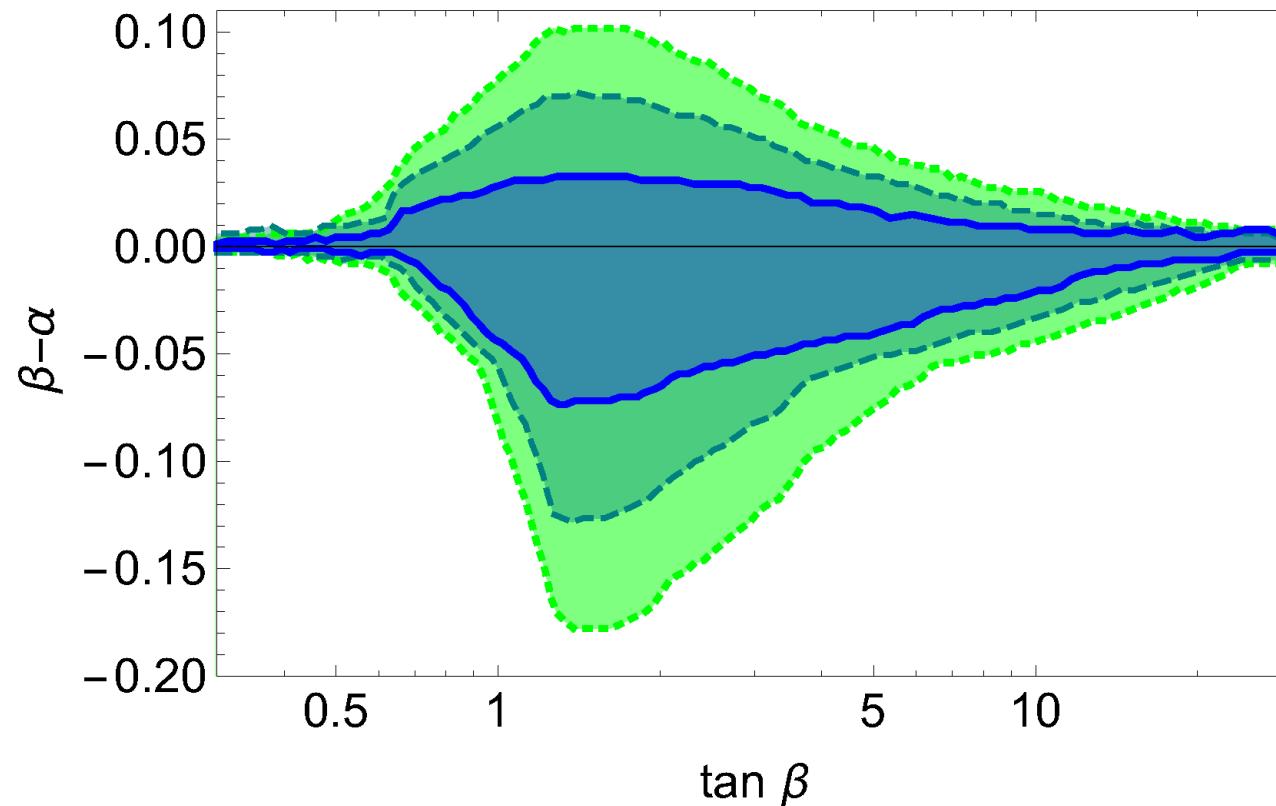
Pheno favoured limit $\beta \to \alpha$: $g_{HVV} = \cos(\beta - \alpha) \to g_{H_{\text{SM}}VV} = 1$.

- **SM Alignment** $\beta \to \alpha$:

(i) Decoupling:

[Georgi, Nanopoulos '79; Donoghue, Li '79;

Gunion, Haber '03; Ginzburg, Krawczyk '05.]

$$M_h^2 \;\simeq\; M_a^2 + \lambda_5 v^2 \;\simeq\; M_{h^\pm}^2 \;\gg\; v_{\mathsf{SM}}^2$$

$$M_H^2 \;\simeq\; 2\lambda_{\mathsf{SM}} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2} \left[s_\beta^2 \left(2\lambda_2 - \lambda_{345} \right) - c_\beta^2 \left(2\lambda_1 - \lambda_{345} \right) \;+\; \ldots \right]^2$$

(ii) Fine-tuning: [Krawczyk et al. '99; Carena, Low, Shah, Wagner '14; Dev, AP '14.]

$$\lambda_7 t_\beta^4 \;-\; (2\lambda_2 - \lambda_{345}) t_\beta^3 \;+\; 3(\lambda_6 - \lambda_7)\, t_\beta^2 \;+\; (2\lambda_1 - \lambda_{345}) t_\beta - \lambda_6 \;=\; 0$$

(iii) Natural SM Alignment (independent of $M_{h^\pm}$ and t_β):

[PSB Dev, AP '14

N Darvishi, AP, J-H Yu, arXiv:2312.00882]

$$\lambda_1 = \lambda_2 = \frac{\lambda_{345}}{2} \quad \text{(with } \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5\text{)}, \qquad \lambda_6 = \lambda_7 = 0$$

Symmetries:

1. Sp(4): $\lambda_4 = \lambda_5 = 0$

2. $\text{SU(2)}_{\text{HF}} \times \text{U(1)}_Y$: $\lambda_5 = 0$

3. $\mathcal{CP} \times \text{SO(2)}_{\text{HF}} \times \text{U(1)}_Y$: $\lambda_{3,4,5} \neq 0$

4*. $\mathcal{CP} \times \text{Sp(2)}_{\phi_1 \widetilde{\phi}_2} \times \text{Sp(2)}_{\phi_2 \widetilde{\phi}_1}$: $\lambda_4 = -\lambda_5$

5. $\text{SO(2)}_{\text{HF}} \times \text{Sp(2)}_{\phi_1 + \phi_2}$: $\lambda_4 = \lambda_5$

*Noted by Aiko, Kanemura '21

Sp(4) field multiplet:

[Battye, Brawn, AP, '11; AP, '12]

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2 \phi_1^* \\ i\sigma^2 \phi_2^* \end{pmatrix}, \qquad \mathbf{\Phi} = C\mathbf{\Phi}^*$$

References (*an incomplete list on SM Alignment in the 2HDM*)

- **On the SM Higgs basis (also Decoupling of FCNC Effects):**
  H. Georgi and D. V. Nanopoulos, Phys. Lett. B82 (1979) 95.

- **Alignment via Decoupling:**
  - J. F. Gunion, H. E. Haber, Phys. Rev. D67 (2003) 075019.
  - I. F. Ginzburg, M. Krawczyk, Phys. Rev. D72 (2005) 115013.

- **Alignment via Fine-tuning:**
  - P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, M. Krawczyk, Phys. Lett. B496 (2000) 195.
  - A. Delgado, G. Nardini, M. Quiros, JHEP1307 (2013) 054.
  - M. Carena, I. Low, N. R. Shah, C. E. M. Wagner, JHEP1404 (2014) 015.

- **Natural Alignment without Decoupling and without Fine-tuning:**
  - P.S.B. Dev, AP, JHEP1412 (2014) 024.
  - B. Grzadkowski, O. M. Ogreid, P. Osland, Phys. Rev. D94 (2016) 115002.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226;
  G.C. Branco, Phys. Rev. D22 (1980) 2901;
  G.C. Branco, M.N. Rebelo, Phys. Lett. B160 (1985) 117.

- **Z_2 symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.

- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.

- **Custodial $SU(2)_L$-preserving symmetry:**
  P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.

- **Bilinear formalism:**
  M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
  C. C. Nishi, Phys. Rev. D74 (2006) 036003.

- **$SU(2)_L \otimes U(1)_Y$-preserving symmetries:** $\underline{6}$
  I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
  P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.

- **Custodial $SU(2)_L$-preserving symmetries:** $\underline{7}$
  R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.

- **On completeness and uniqueness of classification:** $\underline{6} + \underline{7} = \underline{13}$
  AP, Phys. Lett. B706 (2012) 465.

- **Maximally Symmetric Two Higgs Doublet Model** [P.S.B. Dev, AP '14]

$$G_{\mathbf{\Phi}} = \mathsf{SU}(2)_L \otimes \mathsf{Sp}(4)/\mathsf{Z}_2 \simeq \mathsf{SU}(2)_L \otimes \mathsf{SO}(5).$$

$$\mathrm{V} = -\mu^2\left(|\Phi_1|^2 + |\Phi_2|^2\right) + \lambda\left(|\Phi_1|^2 + |\Phi_2|^2\right)^2 = -\frac{\mu^2}{2}\,\mathbf{\Phi}^\dagger\mathbf{\Phi} + \frac{\lambda}{4}\left(\mathbf{\Phi}^\dagger\mathbf{\Phi}\right)^2,$$

where [R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in \mathsf{SU}(2)_L : \ \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U_L\,\mathbf{\Phi},$$

such that under **global field transformations**, [AP, Phys. Lett. B706 (2012) 465.]

$$\mathsf{Sp}(4): \ \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U\,\mathbf{\Phi}, \quad \text{with } U \in \mathsf{U}(4) \ \& \ U\mathsf{C}U^\mathsf{T} = \mathsf{C} \equiv i\sigma^2 \otimes \sigma^0$$

$\mathsf{SU}(2)_L$ gauge kinetic terms remain invariant.

Breaking Effects: $-m_{12}^2\,\phi_1^\dagger\phi_2$, $\mathsf{U}(1)_Y$ coupling g', Yukawa couplings $\mathbf{Y}^{u,d}$.

For nHDM with $m < n$ inert scalar doublets, there are still **$\underline{3}$** continuous alignment symmetries in the **field space of the *non*-inert sector**:

$$\text{(i) } \mathrm{Sp}(2N_H) \times \mathcal{D}; \qquad \text{(ii) } \mathrm{SU}(N_H) \times \mathcal{D}; \qquad \text{(iii) } \mathrm{SO}(N_H) \times \mathcal{CP} \times \mathcal{D},$$

where $N_H = n - m$, $\mathcal{D}$ acts on the inert sector *only*, and $\mathcal{CP}$ is the canonical CP: $\Phi_i(t, \mathbf{x}) \to \Phi_i^*(t, -\mathbf{x})$ (with $i = 1, 2, \ldots, N_H$).

Symmetry invariants:

(i) $\quad S = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \ldots = \frac{1}{2} \mathbf{\Phi}^\dagger \mathbf{\Phi}$

(ii) $\quad D^a = \Phi_1^\dagger \sigma^a \Phi_1 + \Phi_2^\dagger \sigma^a \Phi_2 + \ldots$

(iii) $\quad T = \Phi_1 \Phi_1^\mathsf{T} + \Phi_2 \Phi_2^\mathsf{T} + \ldots$

Symmetric part of the scalar potential:

$$V_{\text{sym}} = -\mu^2 S + \lambda_S S^2 + \lambda_D D^a D^a + \lambda_T \, \mathsf{Tr}\left(T T^* \right).$$

Minimal Symmetry of Alignment: $\quad \mathsf{Z}_2^{\mathsf{EW}} \times \mathsf{Z}_2^{\mathsf{I}}$.

[N. Darvishi, AP, J.-H. Yu '2023]

$$R_\xi = \frac{\mathcal{V}_\xi(v_1, v_2 e^{i\xi})}{|\mathcal{V}_{\xi=0}|}$$

Weak basis:

$\text{Im}\lambda_5 = 0, \ \lambda_6 = \lambda_7$

$|r_{\text{CP}}| > 1$:
Explicit CPV, CPC

$1/2 < |r_{\text{CP}}| < 1$:
Explicit, Spontaneous, & Mixed CPV.

$0 < |r_{\text{CP}}| < 1/2$:
Spontaneous & Mixed CPV.

CP-odd minimization condition:

$$\boxed{\sin 2\xi = 2\,|r_{\text{CP}}|\sin(\xi + \phi_{\text{CP}})}, \ r_{\text{CP}} = |r_{\text{CP}}|\,e^{i\phi_{\text{CP}}} = \frac{m_{12}^2 - \lambda_6 v^2}{\lambda_5 s_{2\beta} v^2}.$$
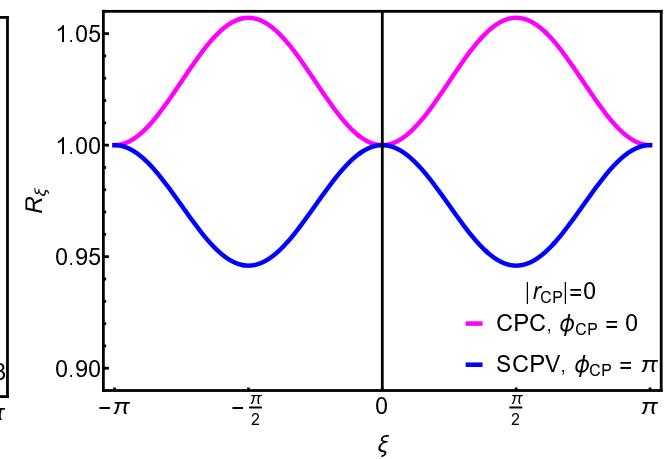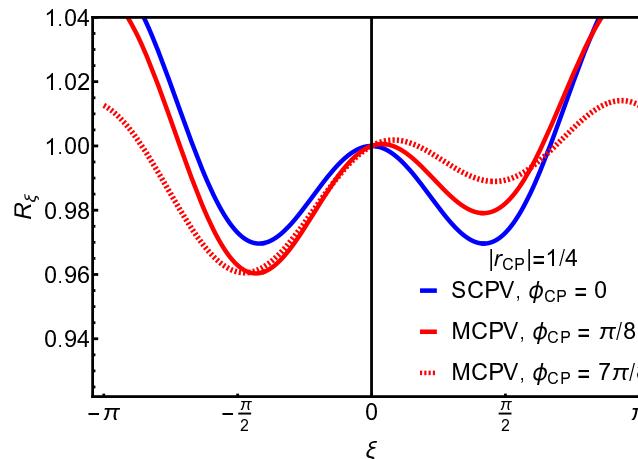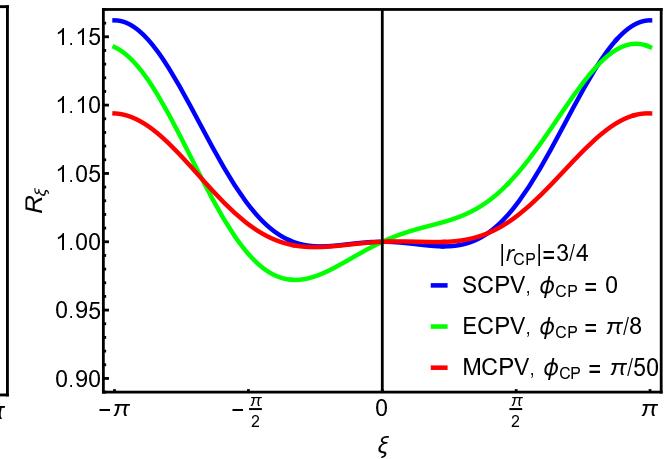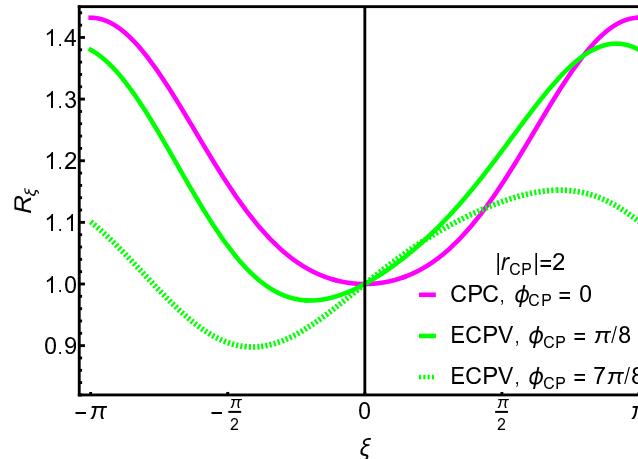
• CP Violation in Naturally Aligned (NA) 2HDM

No.	Generators	Continuous Syms	Parameters
8	T^{0-9}	SO(5)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$.
7	$T^{0,2,5,7,8,9}$	CP1×O(4)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = -\text{Re}(\lambda_5)$.
7′	$T^{0,1,4,6,8,9}$	CP2×O(4)′	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = \text{Re}(\lambda_5)$.
6	$T^{0,2,5,7,8,9}$	SO(4)	$m_{11}^2 = m_{22}^2$, $\text{Im}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = -\text{Re}(\lambda_5)$, $\text{Im}(\lambda_6) = \text{Im}(\lambda_7)$.
6′	$T^{0,1,4,6,8,9}$	SO(4)′	$m_{11}^2 = m_{22}^2$, $\text{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_3/2$, $\lambda_4 = \text{Re}(\lambda_5)$, $\text{Re}(\lambda_6) = \text{Re}(\lambda_7)$.
5	$T^{0,2,4,6}$	O(2)× O(3)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$, $\lambda_4 = \text{Re}(\lambda_5)$.
5′	$T^{0,1,5,7}$	O(2)′ × O(3)	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$, $\lambda_4 = -\text{Re}(\lambda_5)$.
4	T^{0-3}	O(3)× O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{34}/2$.
3	$T^{0,5,7}$	SO(3)	m_{11}^2, m_{22}^2, $\text{Im}(m_{12}^2)$, λ_1, λ_2, λ_3, $\lambda_4 = -\text{Re}(\lambda_5)$, $\text{Im}(\lambda_6)$, $\text{Im}(\lambda_7)$.
3′	$T^{0,4,6}$	SO(3)′	m_{11}^2, m_{22}^2, $\text{Re}(m_{12}^2)$, λ_1, λ_2, λ_3, $\lambda_4 = \text{Re}(\lambda_5)$, $\text{Re}(\lambda_6)$, $\text{Re}(\lambda_7)$.
2	$T^{0,2}$	CP1×O(2)×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$.
2′	$T^{0,1}$	CP2×O(2)′×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$.
1	$T^{0,2}$	O(2)×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\text{Im}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \lambda_{345}/2$, $\text{Im}(\lambda_6) = \text{Im}(\lambda_7)$.
1′	$T^{0,1}$	O(2)′×O(2)$_Y$	$m_{11}^2 = m_{22}^2$, $\text{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2 = \bar{\lambda}_{345}/2$, $\text{Re}(\lambda_6) = \text{Re}(\lambda_7)$.

Abbreviations: $\lambda_{34} \equiv \lambda_3 + \lambda_4$, $\quad \lambda_{345}\,(\bar{\lambda}_{345}) \equiv \lambda_3 + \lambda_4 + (-)\,\text{Re}(\lambda_5)$

– CP-violating breaking patterns in NA-2HDM

No.	Syms with NHAL	No.	Syms breaking of NHAL	Types of CPV after SB
8	SO(5)		—	
7	CP1×O(4)	6	SO(4)	ECPV/MCPV
7'	CP2×O(4)'	6'	SO(4)'	SCPV
5	O(2)×O(3)	6'	SO(4)'	SCPV
5'	O(2)'×O(3)	6	SO(4)	ECPV/MCPV
4	O(3)×O(2)$_Y$		—	
2	CP1×O(2)×O(2)$_Y$	1	O(2)×O(2)$_Y$	ECPV/MCPV
2'	CP2×O(2)'×O(2)$_Y$	1'	O(2)'×O(2)$_Y$	SCPV

SB: **S**oft symmetry-breaking **B**ilinear mass terms.

• EDM compatible scenarios with approximate natural alignment

Symmetries	CPV types	$\tan\beta$	ξ	ϕ_{12}	M_{H_1}	M_{H_2}	M_{H_3}	$M_{H\pm}$	g_{H_1VV}
O(2)×O(2)$_Y$	ECPV	1.608	0.002	-0.05	125.10	682.99	787.67	640.18	0.998
	MCPV	0.804	-1.93	0.27	125.21	225.11	270.50	380.54	0.989
O(2)′×O(2)$_Y$	SCPV	0.71	0.67	0	125.52	275.23	448.01	551.38	0.957
SO(4)	ECPV	0.827	0.002	-0.02	125.28	506.93	570.58	570.59	0.995
	MCPV	0.740	-1.25	0.44	125.40	408.49	433.98	433.99	0.984
SO(4)′	SCPV	0.742	-2.42	0	125.68	216.63	265	442.21	0.948

(A)   (C)   (F)   +   (G)
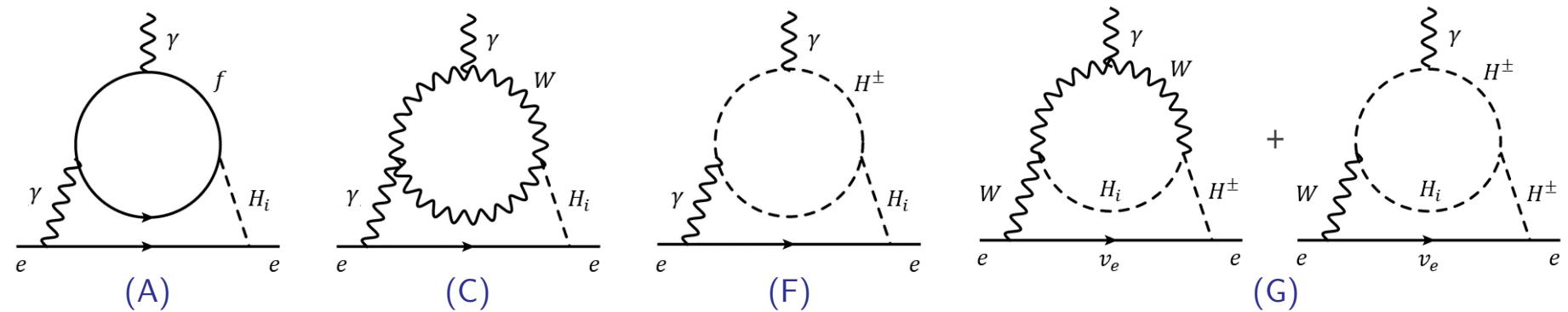
Symmetries	CPV types	The ratio of EDM contributions over the total EDM							EDM $[e \cdot cm]$
		(A)	(B)	(C)	(D)	(E)	(F)	(G)	
O(2)×O(2)$_Y$	ECPV	5.581	-0.072	-4.711	-0.308	0.0185	0.312	0.180	9.51×10^{-30}
	MCPV	2.008	-0.026	-0.942	-0.065	0.0015	0.028	-0.005	7.67×10^{-30}
O(2)′×O(2)$_Y$	SCPV	2.003	-0.026	-0.866	-0.079	0.0002	0.004	-0.037	3.86×10^{-30}
SO(4)	ECPV	1.851	-0.021	-0.832	-0.055	0.0005	0.009	0.048	2.87×10^{-30}
	MCPV	1.995	-0.021	-0.887	-0.077	-0.0013	-0.023	0.016	1.07×10^{-30}
SO(4)′	SCPV	-1.211	0.014	1.875	0.172	0.0002	0.004	0.146	3.55×10^{-30}
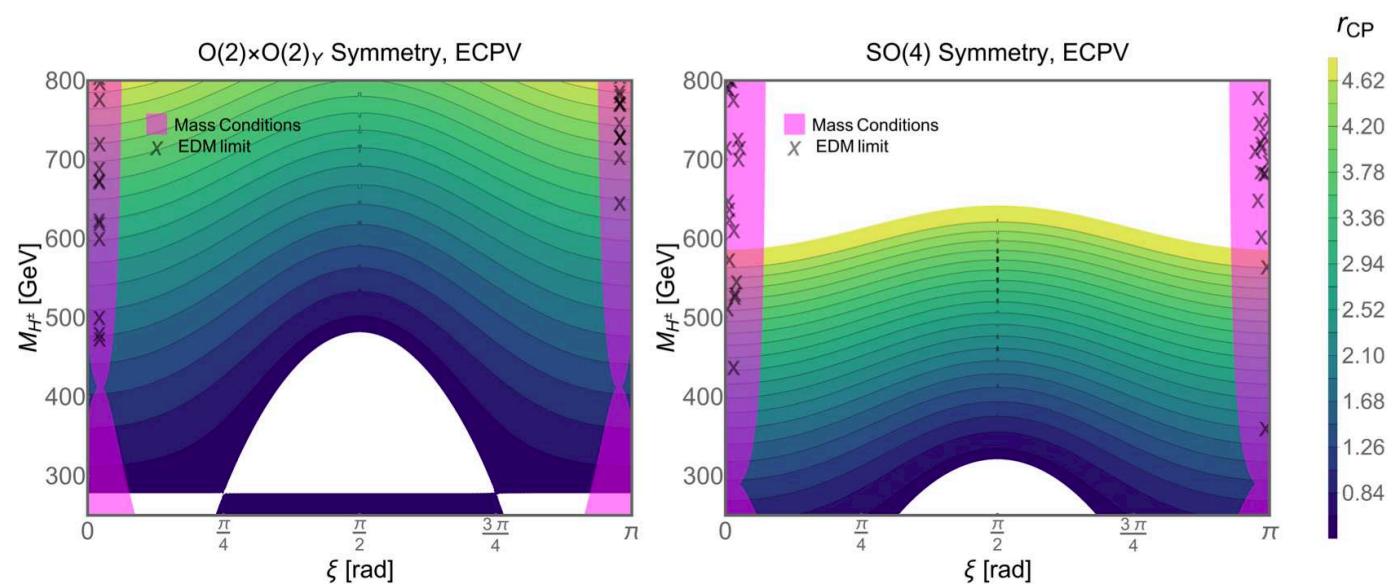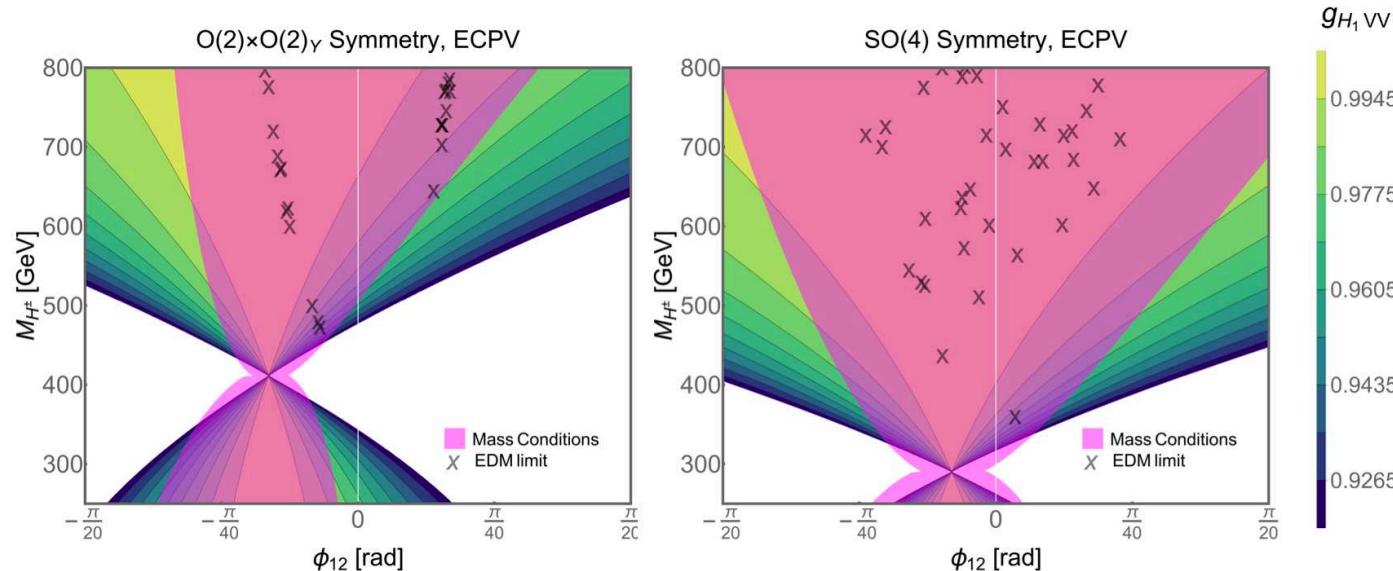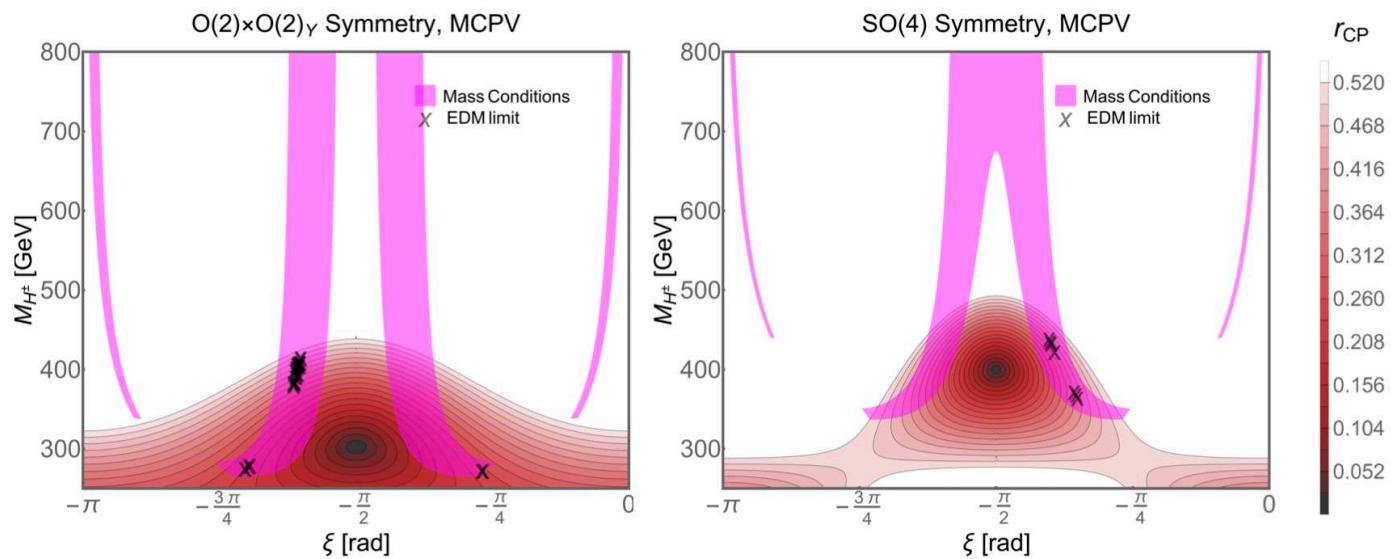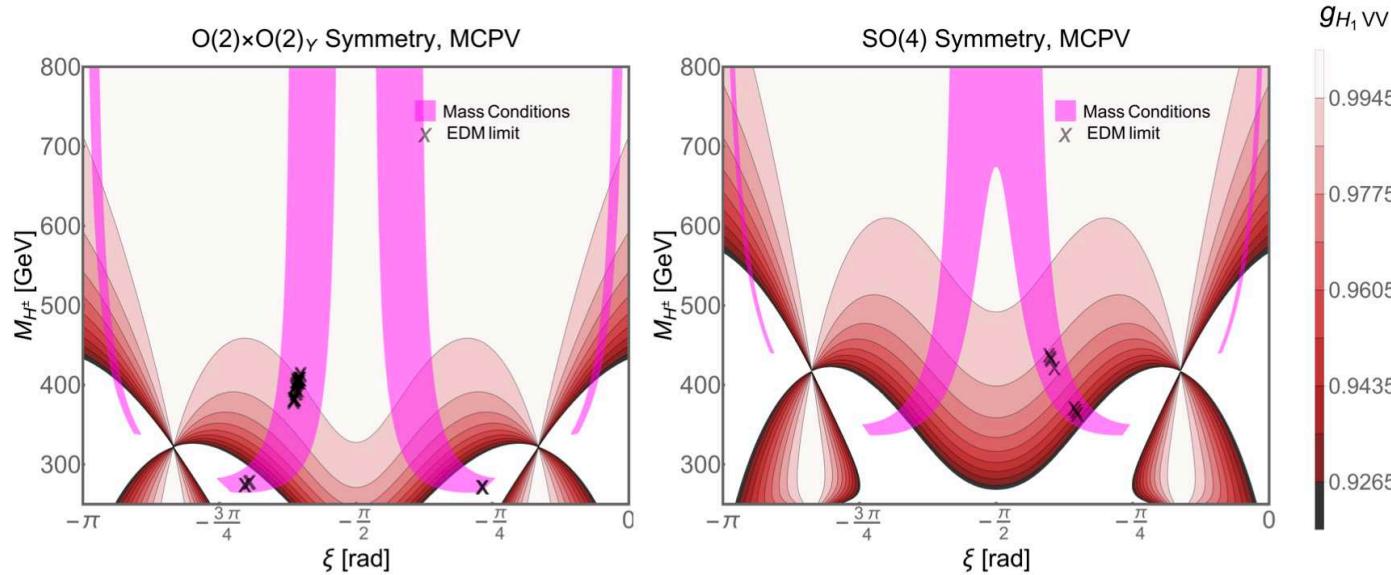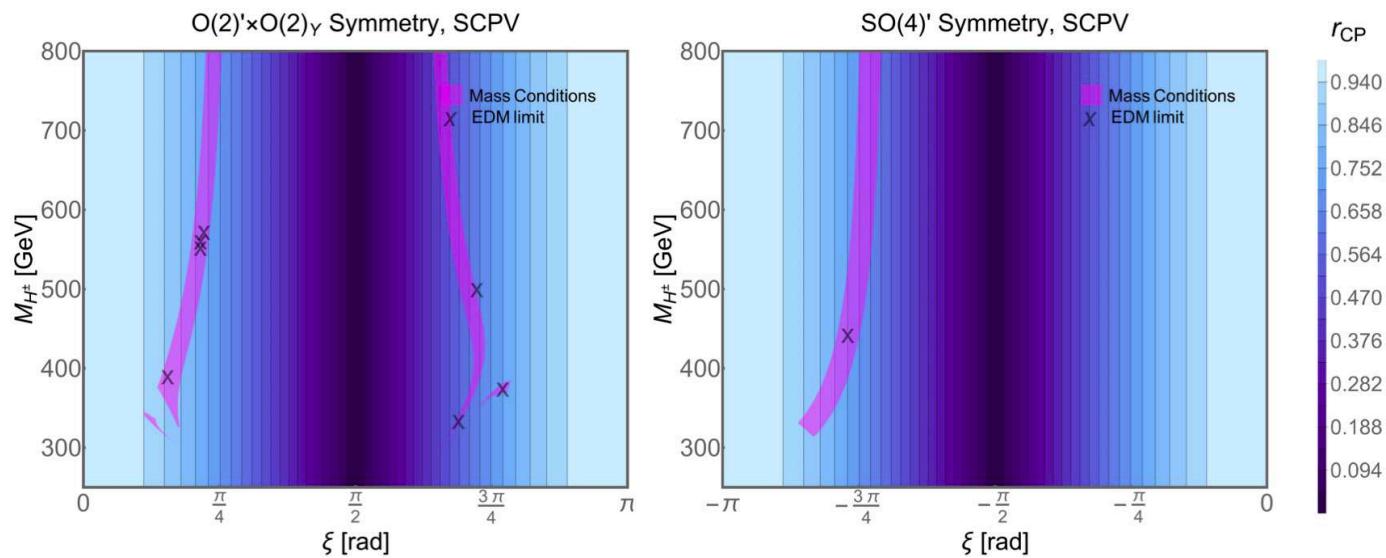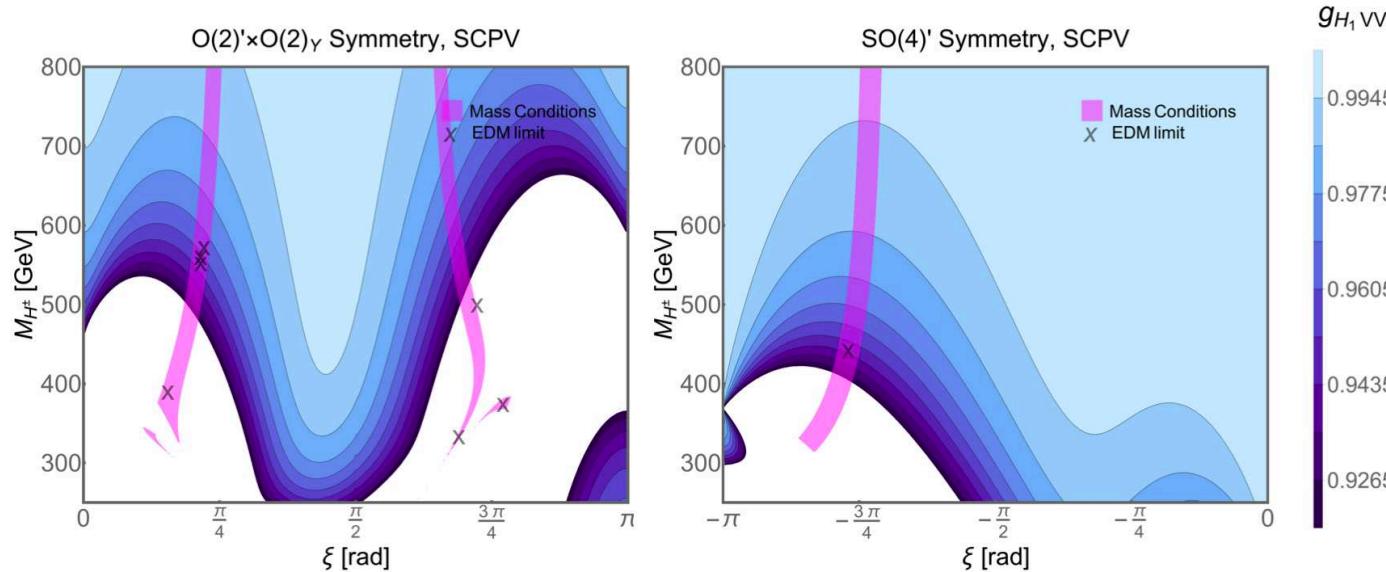
• **Branching ratios in the MS-2HDM**   [Dev, AP '14]
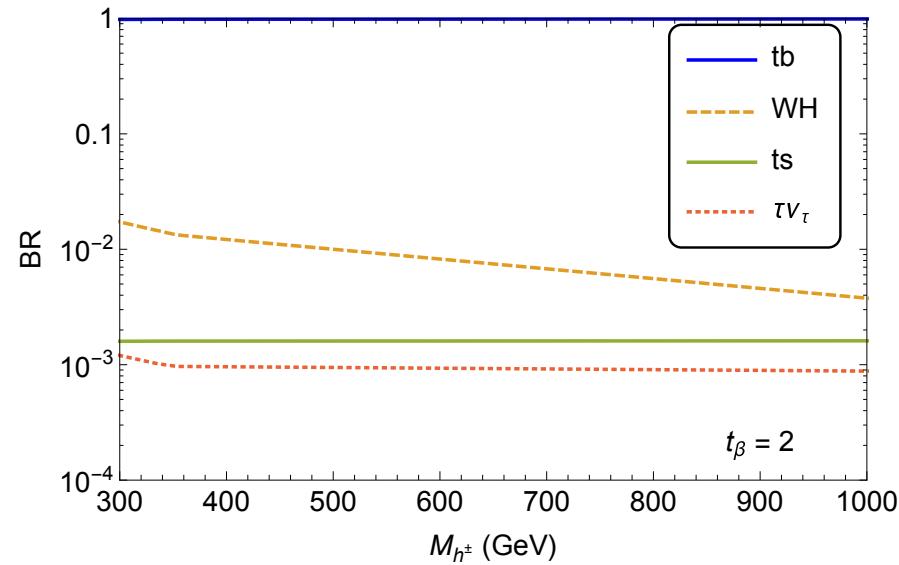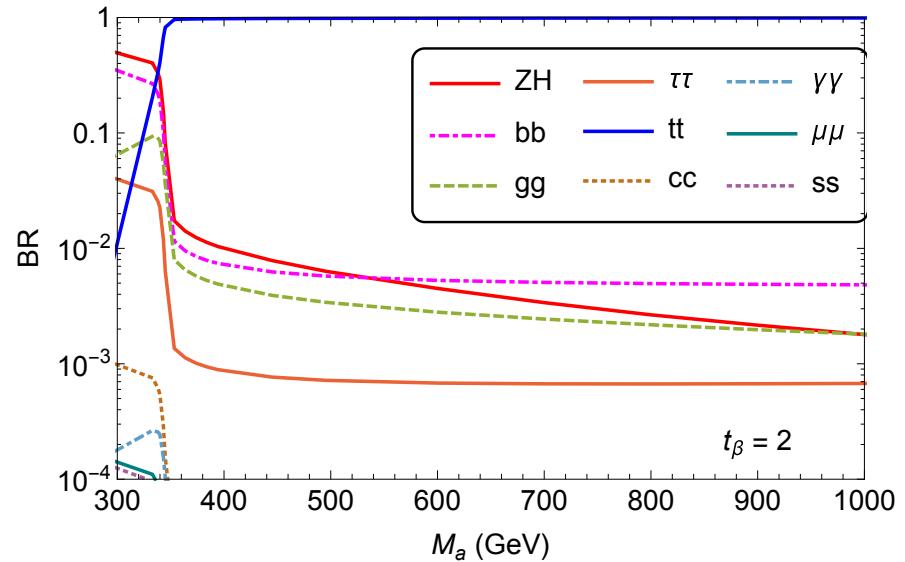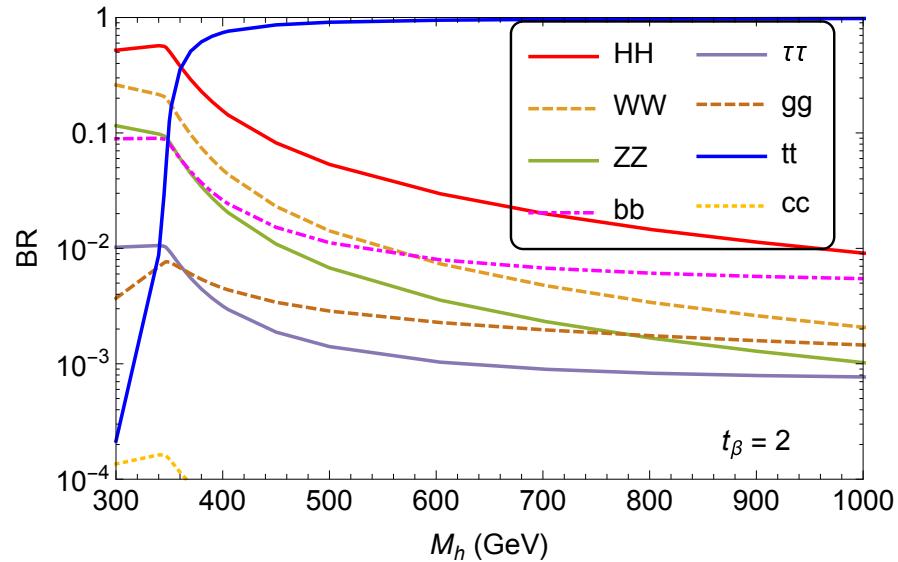
• Discovery channels for aligned Higgs doublets:

$-\ gg \to t\bar{b}h^- \to t\bar{b}\bar{t}b$

[Dev, AP '14;

Yvonne Peters, AP et al, PRD100 (2019) 035026]
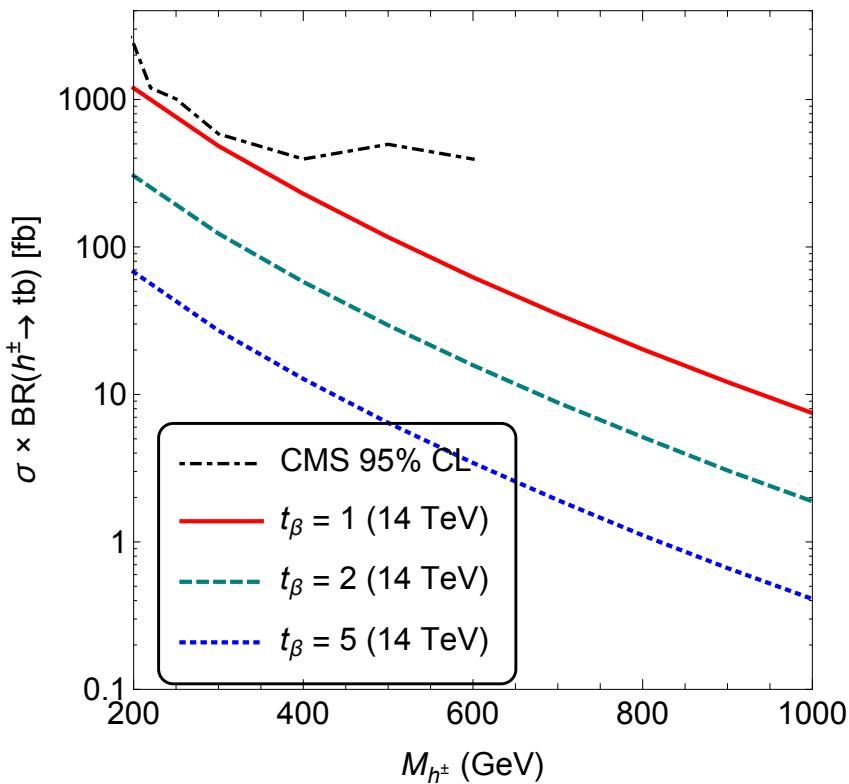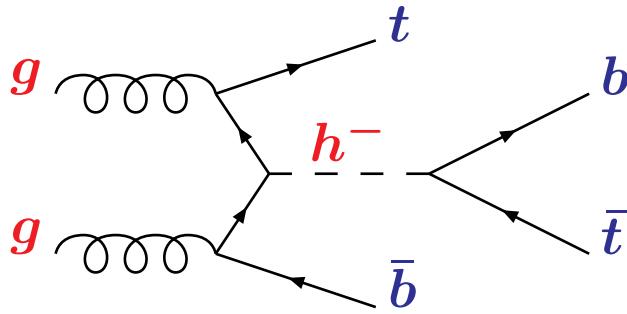
$p_T^\ell > 20$ GeV,

$|\eta^\ell| < 2.5$,

$\Delta R^{\ell\ell} > 0.4$,

$M_{\ell\ell} > 12$ GeV,

$|M_{\ell\ell} - M_Z| > 10$ GeV,

$p_T^j > 30$ GeV,

$|\eta^j| < 2.4$,

$\not{E}_T > 40$ GeV.

– $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$
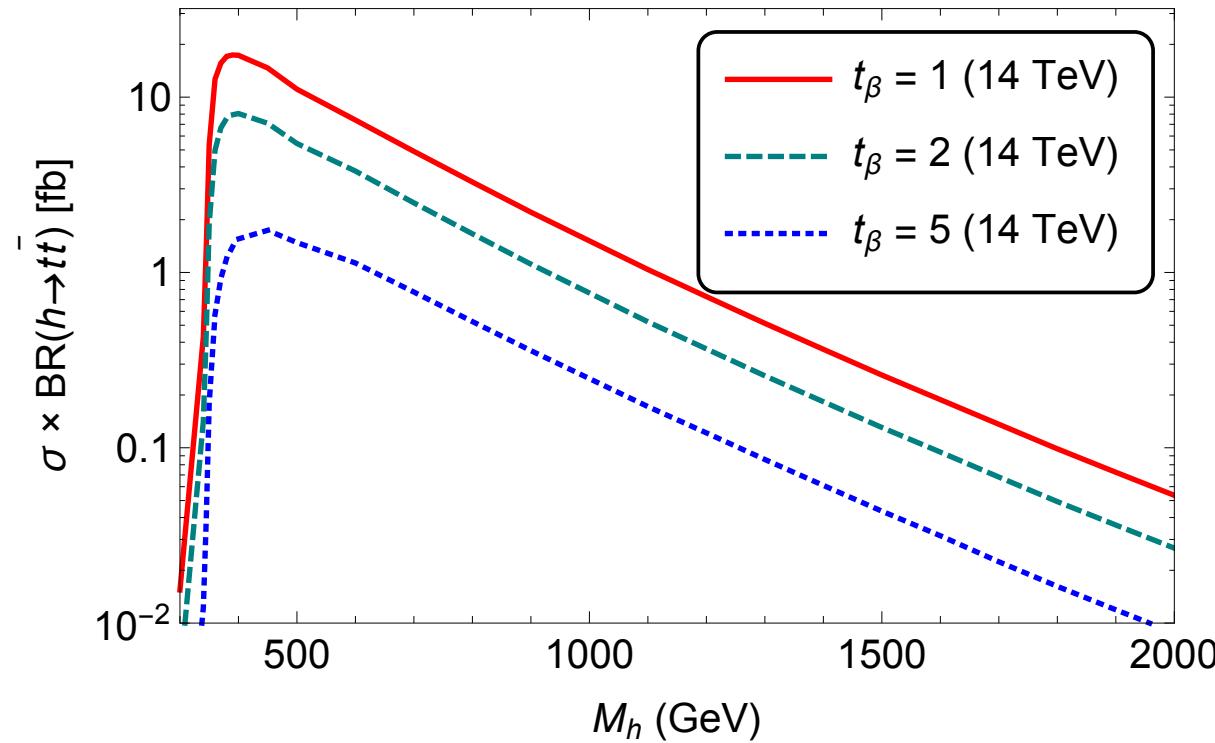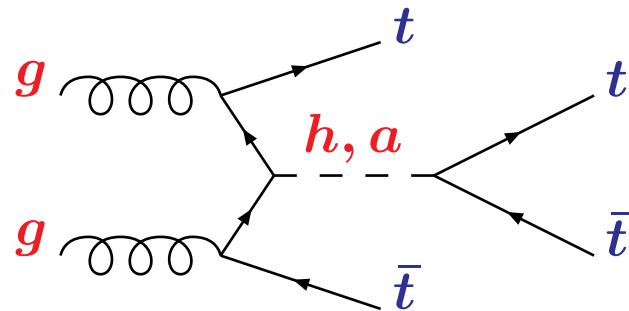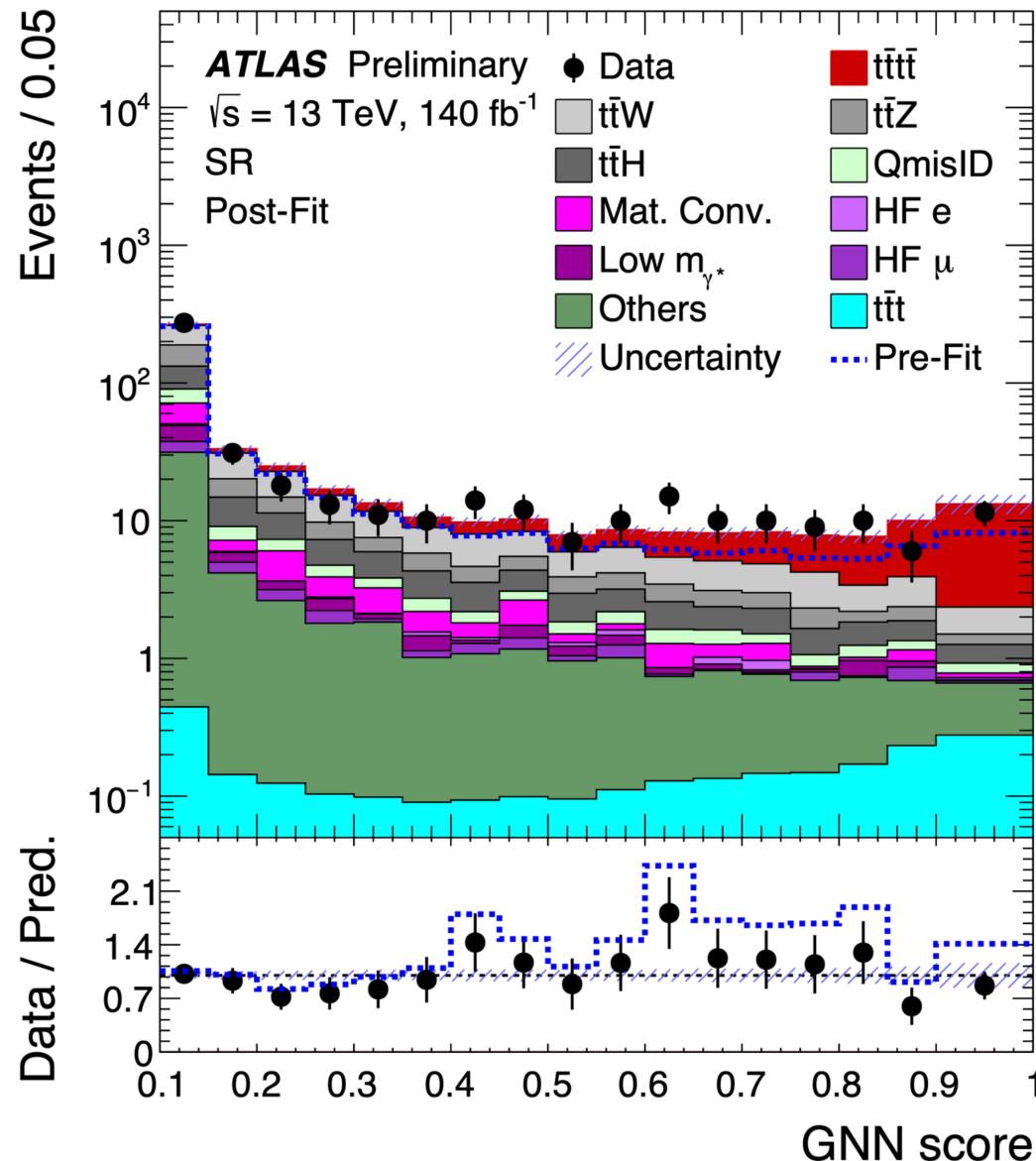
- **Other production channels for CP-violating NA Higgs bosons**

  - $gg \to (Z^*, H_2, H_3) \to ZH_1(+b\bar{b})$ [N. Darvishi, AP, J.-H. Yu '2023]
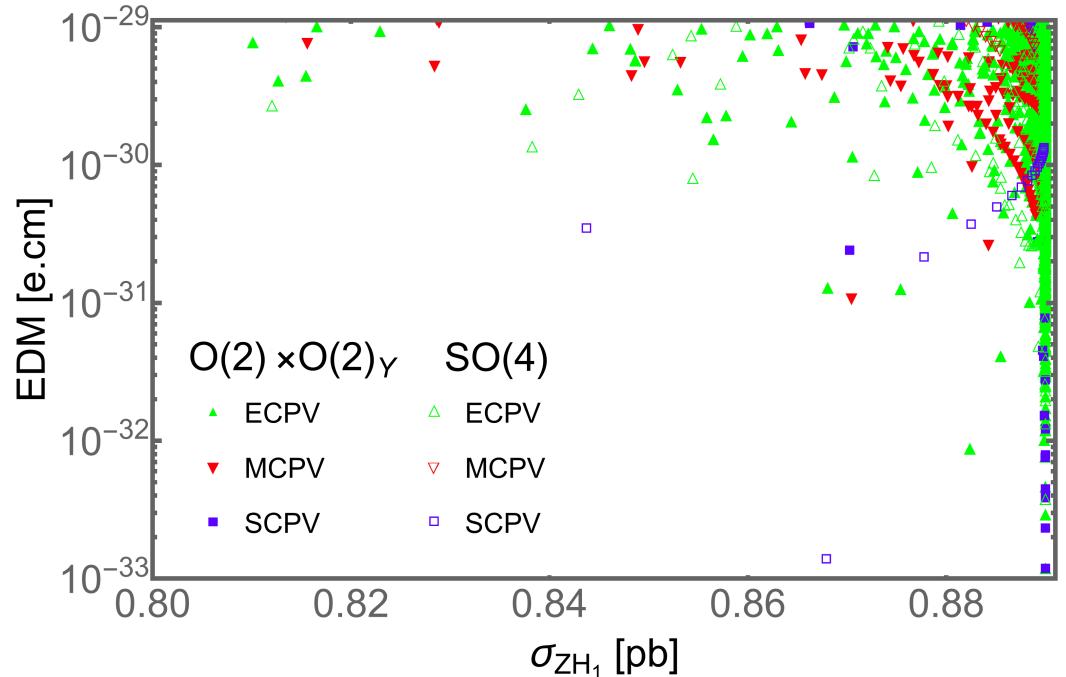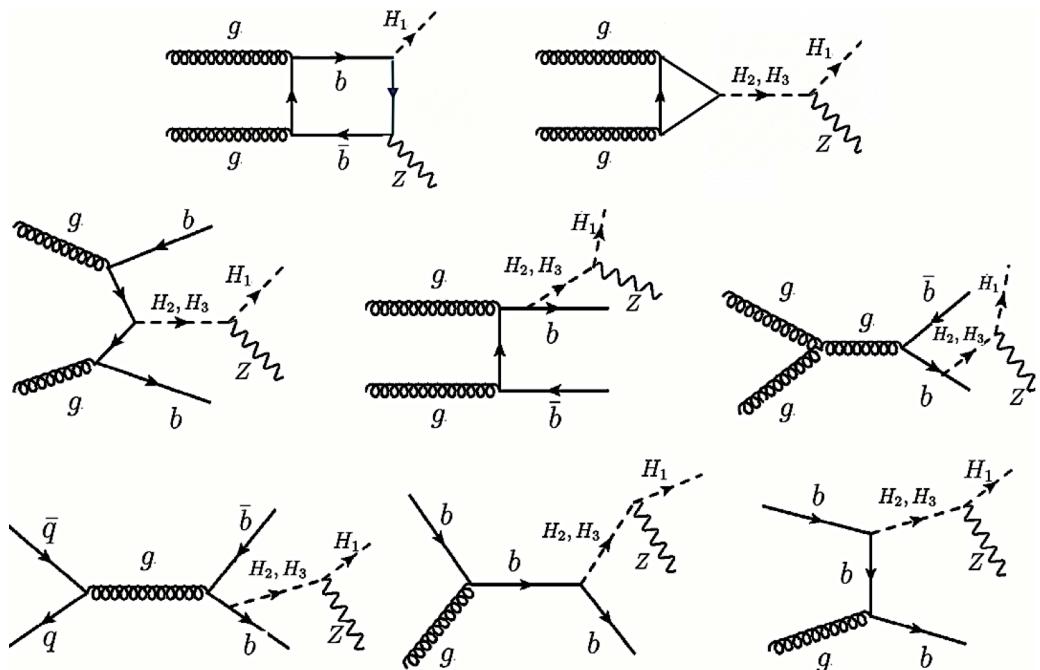
Omitted are graphs with $H_{2,3} \to Z$ and $H_{2,3}$ radiation from $\bar{b}$ quarks.

● SUMMARY

- **5 symmetries for natural alignment *without* decoupling in 2HDM:**

  (i) Sp(4)   (ii) SU(2)$_{HF}$ (iii) SO(2)$_{HF}$ × $\mathcal{CP}$

  (iv) Sp(2)$_{\phi_1 \widetilde{\phi}_2}$ × Sp(2)$_{\phi_2 \widetilde{\phi}_1}$ × $\mathcal{CP}$ (v) SO(2)$_{HF}$ × Sp(2)$_{\phi_1 + \phi_2}$

- **Soft breaking $\longrightarrow$ minimal alignment symmetry: Z_2^{EW} × Z_2^{I}**

  $\rightarrow$ **Naturally aligned (NA) heavy Higgs sector is Z_2^{EW} odd.**

- **Three forms of CPV in 2HDM: Explicit, Spontaneous and Mixed.**
  Maximising CPV along softly-broken symmetry directions:

  (i) SO(2)$_{HF}$ (ii) Sp(2)$_{\phi_1 \widetilde{\phi}_2}$ × Sp(2)$_{\phi_2 \widetilde{\phi}_1}$ $\simeq$ SO(4),

  compatible with current EDM limits.

- $t\bar{t}t\bar{t}$ **channel is a powerful probe for NA-2HDMs**

- ZH_1 **channel can test CP-violating NA Higgs bosons at the LHC.**

Back-Up Slides

[Dev, AP '14; N. Darvishi, AP '19]

Symmetry-breaking of Sp(4)/$Z_2 \sim$ SO(5):

● Soft breaking (e.g. through m_{12}^2):

$$M_H^2 \;=\; 2\lambda_2 v^2, \qquad M_h^2 \;=\; M_a^2 \;=\; M_{h^\pm}^2 \;=\; \frac{\mathsf{Re}(m_{12}^2)}{s_\beta c_\beta}$$

Heavy Higgs spectrum is degenerate at tree level.

● Explicit breaking through RG running (two loops):

$$\mathsf{Sp(4)}/\mathsf{Z}_2 \otimes \mathsf{SU(2)}_L \quad \xrightarrow{\;g' \neq 0\;} \quad \mathsf{SU(2)}_{\mathsf{HF}} \otimes \mathsf{U(1)}_Y \otimes \mathsf{SU(2)}_L$$

$$\xrightarrow{\;\mathbf{Y}^{u,d}\;} \quad \mathsf{U(1)}_{\mathsf{PQ}} \otimes \mathsf{U(1)}_Y \otimes \mathsf{SU(2)}_L$$

$$\xrightarrow[\langle \Phi_{1,2} \rangle]{\;m_{12}^2\;} \quad \mathsf{U(1)}_{\mathsf{em}}$$

- **Quartic Coupling Unification (two loops)**
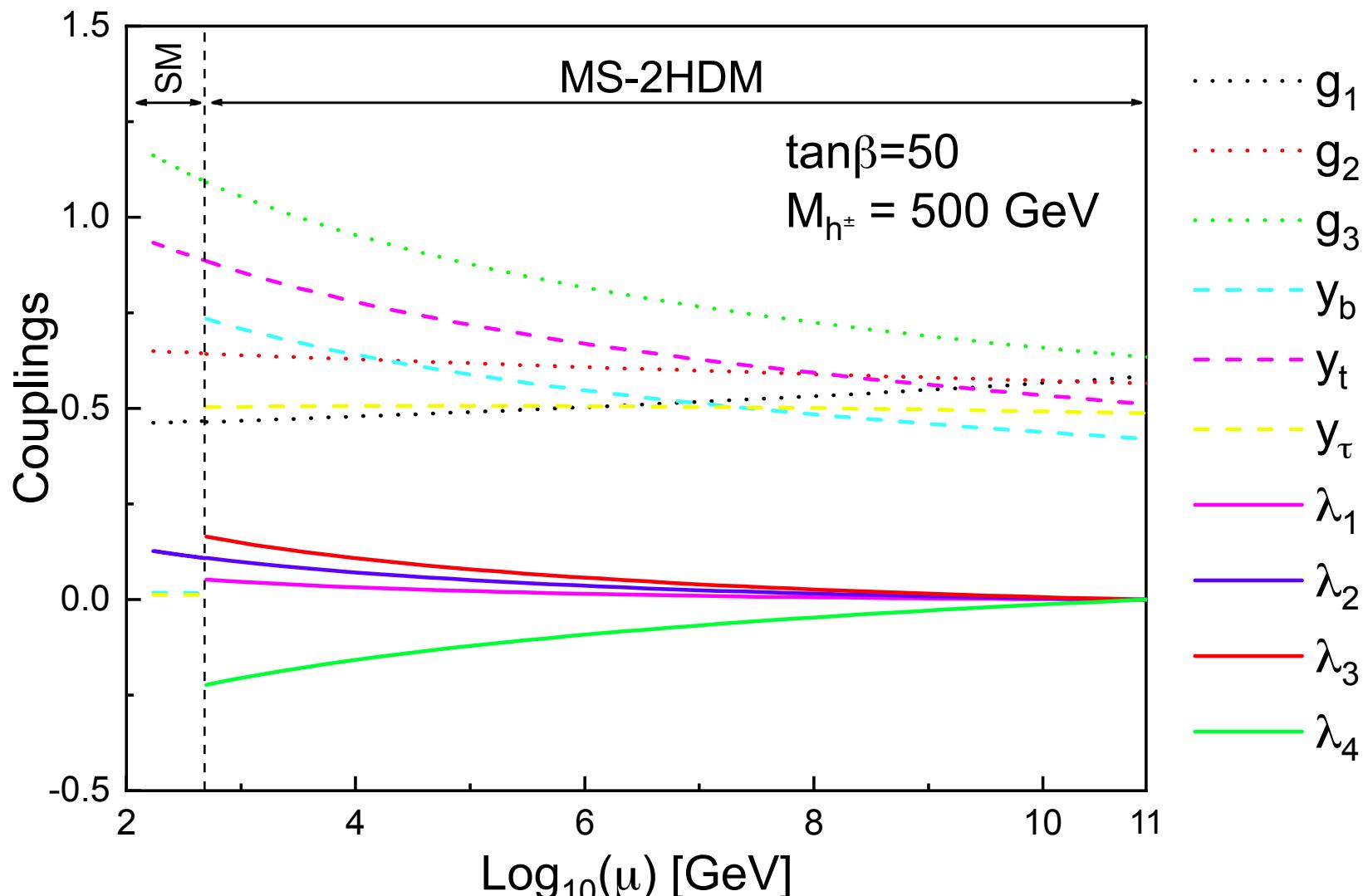
First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order m_{Pl})

[N. Darvishi, AP '19]

SM

MS-2HDM

tanβ=50
$M_{h^\pm}$ = 500 GeV

Couplings

$\text{Log}_{10}(\mu)$ [GeV]

g_1
g_2
g_3
y_b
y_t
y_τ
λ_1
λ_2
λ_3
λ_4

[N. Darvishi, AP '19]

– Misalignment in the MS-2HDM

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_{\mathcal{S}}^2 = \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} \quad \xrightarrow[\textbf{approx.}]{\textbf{seesaw}} \quad M_H^2 \simeq \widehat{A} - \frac{\widehat{C}^2}{\widehat{B}} \quad \& \quad M_h^2 \simeq \widehat{B} \gg \widehat{A}, \widehat{C}$$

Light-to-heavy scalar mixing:

$$\theta_{\mathcal{S}} \equiv \frac{\widehat{C}}{\widehat{B}} = \frac{v^2 s_\beta c_\beta \left[s_\beta^2 \left(2\lambda_2 - \lambda_{34}\right) - c_\beta^2 \left(2\lambda_1 - \lambda_{34}\right) \right]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2 \left(\lambda_1 + \lambda_2 - \lambda_{34}\right)} \ll 1$$

Higgs couplings to $V = W, Z$:

$$g_{HVV} \simeq 1 - \frac{1}{2}\theta_{\mathcal{S}}^2\,, \qquad g_{hVV} \simeq -\theta_{\mathcal{S}}$$

Higgs couplings to quarks:

$$g_{Huu} \simeq 1 + t_\beta^{-1}\theta_{\mathcal{S}}\,, \qquad\qquad g_{Hdd} \simeq 1 - t_\beta\,\theta_{\mathcal{S}}\,,$$
$$g_{huu} \simeq -\theta_{\mathcal{S}} + t_\beta^{-1}\,, \qquad\qquad g_{hdd} \simeq -\theta_{\mathcal{S}} - t_\beta\,.$$

Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h\pm} = 500\,$GeV.

[N. Darvishi, AP '19]

Couplings	ATLAS	CMS	$\tan\beta = 2$	$\tan\beta = 20$	$\tan\beta = 50$
$\|g_{HZZ}^{\text{low-scale}}\|$	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$\|g_{HZZ}^{\text{high-scale}}\|$			0.9981	0.9999	0.9999
$\|g_{Htt}^{\text{low-scale}}\|$	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$\|g_{Htt}^{\text{high-scale}}\|$			1.0987	1.0003	1.0001
$\|g_{Hbb}^{\text{low-scale}}\|$	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$\|g_{Hbb}^{\text{high-scale}}\|$			0.8810	0.9449	0.9427

$\rightarrow$ Misalignment predictions consistent with experiment

- **Maximally Symmetric Three Higgs Doublet Model (MS-3HDM)**

Breaking pattern:

$$\text{Sp}(6)/\text{Z}_2 \otimes \text{SU}(2)_L \quad \xrightarrow{g' \neq 0} \quad \text{SU}(3)_{\text{HF}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L$$

$$\xrightarrow[\text{Type V}]{\mathbf{Y}^{u,d,e}} \quad \text{U}(1)_{\text{PQ}} \otimes \text{U}(1)'_{\text{PQ}} \otimes \text{U}(1)_Y \otimes \text{SU}(2)_L$$

$$\xrightarrow[\text{soft } m_{ij}^2]{\langle \Phi_{1,2,3} \rangle} \quad \text{U}(1)_{\text{em}}$$

– Quartic Coupling Unification in the MS-3HDM

Input parameters: $\tan \beta_1 = v_2/v_1$, $\tan \beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,

$M_{h_{1,2}^{\pm}}$ and $h_1^{\pm} h_2^{\mp}$-mixing angle: σ

Low-Scale Quartic Coupling Unification

$M_{h_1^{\pm}}$=500 GeV

$\tan\beta_1$=50

$\tan\beta_2$=0.018

λ_{11} λ_{1122}
λ_{22} λ_{1221}
λ_{33} λ_{1133}
λ_{1331}
λ_{2233}
λ_{2332}

Quartic Couplings

$\text{Log}_{10}(\mu)$ [GeV]
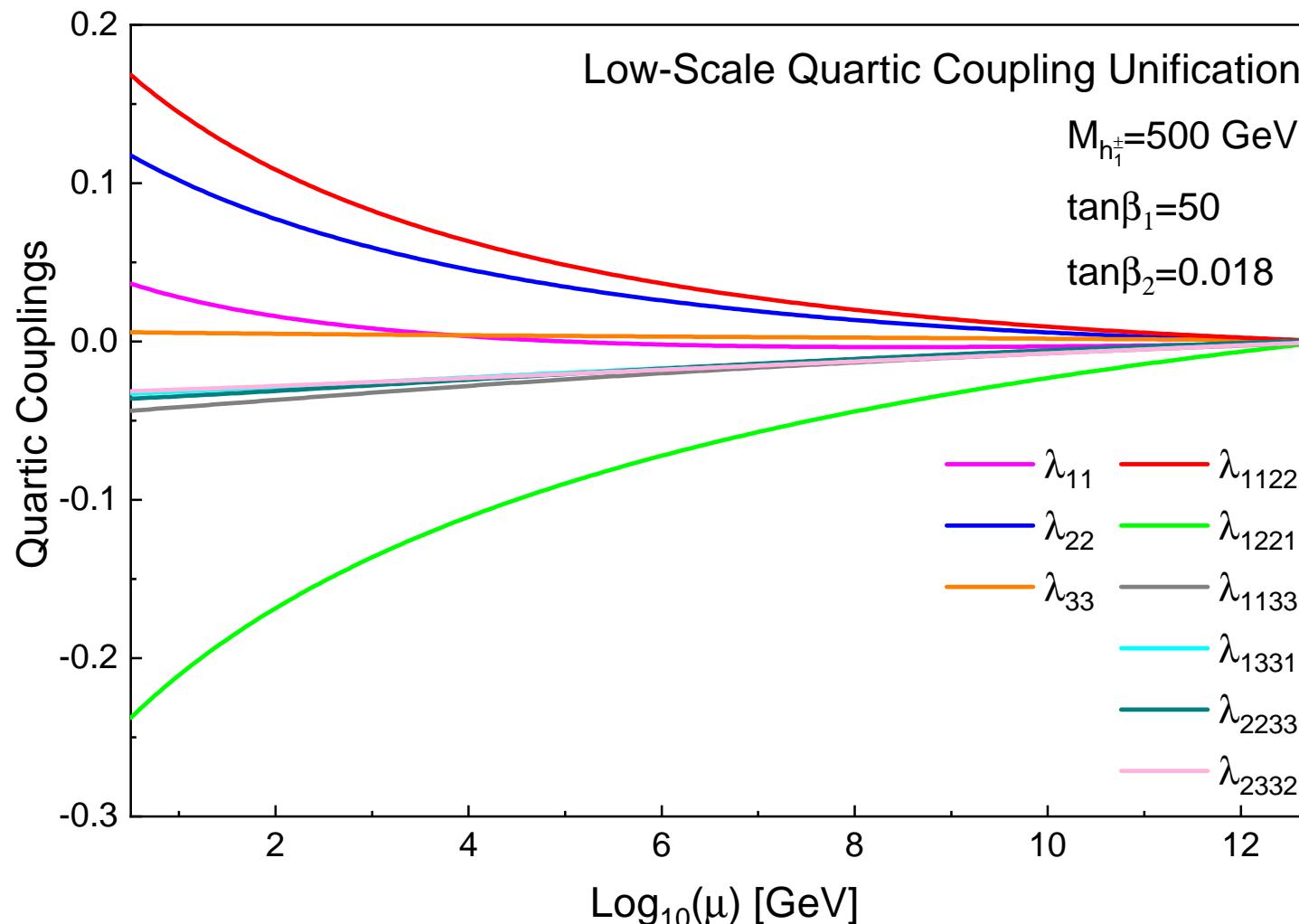
– Misalignment predictions in the MS-3HDM

High-Scale

Low-Scale

$1 - g_{HVV}^2$

$1 - g_{Ht\bar{t}}^2$

$1 - g_{Hb\bar{b}}^2$

$1 - g_{H\tau\bar{\tau}}^2$

$M_{h_1^\pm} = 500$ GeV

$\tan\beta_1 = 50$

$\tan\beta_2 = 0.018$

Misalignment

$\mathrm{Log}_{10}(\mu)$ [GeV]
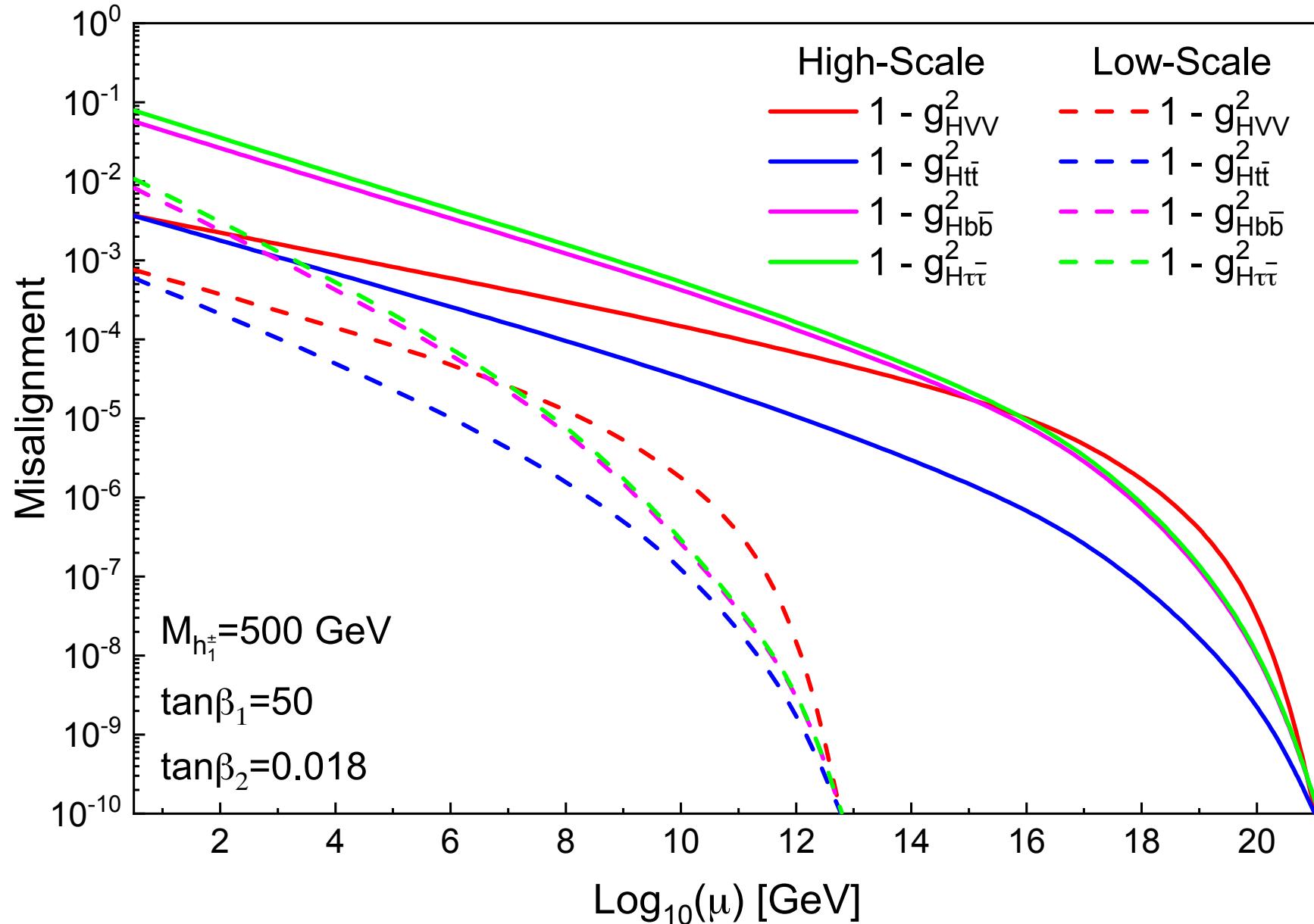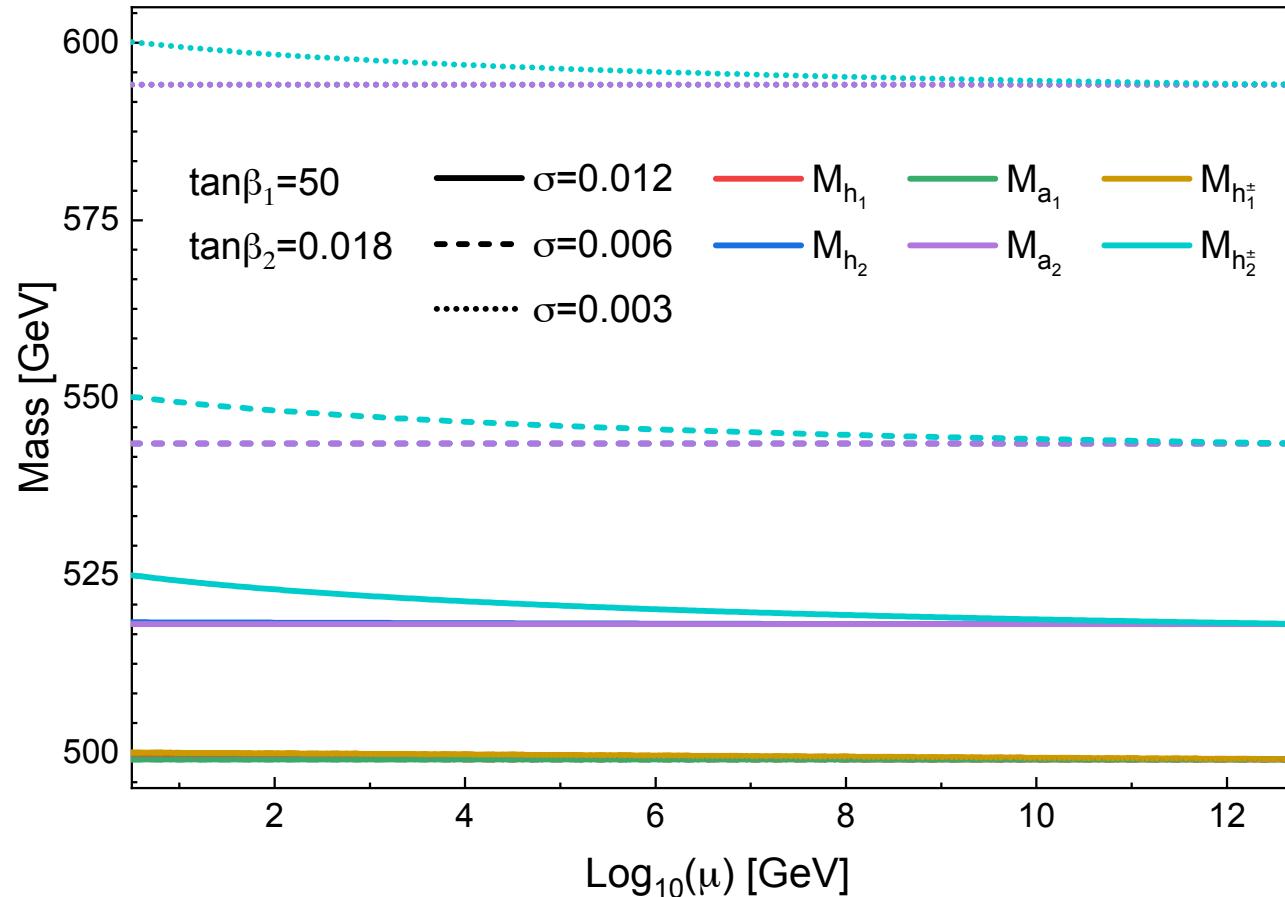
Predictions:

Alignment of masses: $M_{h_1} \sim M_{a_1} \sim M_{h_1^\pm}$ $M_{h_2} \sim M_{a_2} \sim M_{h_2^\pm}$

Alignment of all heavy-sector mixing angles in the Higgs basis: $\alpha \simeq \rho \simeq \sigma$

• 2HDM potential

$$V = -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1)$$

$$+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

$$+ \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1)$$

$$+ \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1) \,.$$

• Physical spectrum (CP-conserving limit):

CP-even Higgs bosons H and h;   CP-odd scalar a;   charged scalars $h^\pm$.

• Higgs coupling to gauge bosons $V = W, Z$:

$$g_{HVV} = \cos(\beta - \alpha)\,, \qquad g_{hVV} = \sin(\beta - \alpha)\,,$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

• Symmetries of the 2HDM Potential

Introduce the $SU(2)_L$-covariant 8D complex field multiplet

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in SU(2)_L : \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U_L \mathbf{\Phi}.$$

$\mathbf{\Phi}$ satisfies the **Majorana constraint**

$$\mathbf{\Phi} = \mathsf{C}\,\mathbf{\Phi}^*,$$

where C is the **charge conjugation** 8D matrix

$$\mathsf{C} = \sigma^2 \otimes \sigma^0 \otimes \sigma^2 = \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2).$$

• The SO(1,5) Bilinear Formalism

Introduce the *null* 6-Vector

$$
\mathsf{R}^A = \boldsymbol{\Phi}^\dagger \, \Sigma^A \, \boldsymbol{\Phi} = \begin{pmatrix} \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\ \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\ -i \left[\phi_1^\dagger \phi_2 - \phi_2^\dagger \phi_1 \right] \\ \phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \\ \phi_1^\mathsf{T} i\sigma^2 \phi_2 - \phi_2^\dagger i\sigma^2 \phi_1^* \\ -i \left[\phi_1^\mathsf{T} i\sigma^2 \phi_2 + \phi_2^\dagger i\sigma^2 \phi_1^* \right] \end{pmatrix} ,
$$

with $A = \mu, \ 4, \ 5,$ and

$$
\Sigma^\mu = \frac{1}{2} \begin{pmatrix} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^\mathsf{T} \end{pmatrix} \otimes \sigma^0 ,
$$

$$
\Sigma^4 = \frac{1}{2} \begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0 , \qquad \Sigma^5 = \frac{1}{2} \begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix} \otimes \sigma^0 .
$$

- **The 2HDM Potential in the SO(1,5) Formalism**

$$V_{\text{2HDM}} = -\frac{1}{2} M_A \, \mathsf{R}^A + \frac{1}{4} L_{AB} \, \mathsf{R}^A \, \mathsf{R}^B \,,$$

with

$$M_A = \left(\mu_1^2 + \mu_2^2, \quad 2\text{Re}(m_{12}^2), \quad -2\text{Im}(m_{12}^2), \quad \mu_1^2 - \mu_2^2, \quad 0, \quad 0 \right) \,,$$

$$L_{AB} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \text{Re}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 & 0 & 0 \\ \text{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \text{Re}(\lambda_5) & -\text{Im}(\lambda_5) & \text{Re}(\lambda_6 - \lambda_7) & 0 & 0 \\ -\text{Im}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_5) & \lambda_4 - \text{Re}(\lambda_5) & -\text{Im}(\lambda_6 - \lambda_7) & 0 & 0 \\ \lambda_1 - \lambda_2 & \text{Re}(\lambda_6 - \lambda_7) & -\text{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} .$$

- **Unitary Field Transformations:**   [AP, Phys. Lett. B706 (2012) 465.]

$$\text{Sp}(4): \quad \boldsymbol{\Phi} \mapsto \boldsymbol{\Phi}' = U \boldsymbol{\Phi} \,, \quad \text{with} \quad U \in \mathsf{U}(4) \quad \underline{\text{and}} \quad UCU^{\mathsf{T}} = \mathsf{C}$$

$$\text{SO}(5): \quad \mathsf{R}^{\mathsf{I}} \mapsto \mathsf{R}'^{\mathsf{I}} = \mathsf{O}^{\mathsf{I}}{}_{\mathsf{J}} \mathsf{R}^{\mathsf{J}} \,, \quad \text{with} \quad \mathsf{O} \in \mathsf{SO}(5) \subset \mathsf{SO}(1,5)$$

$$\Longrightarrow \quad \text{SO}(5) \sim \text{Sp}(4)/\mathbf{Z}_2$$

- **Symmetry Breaking Scenarios and pseudo-Goldstone Bosons**

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo-Goldstone Bosons	
1	$Z_2 \times O(2)$	T^0	D_{CP1}	$-$	0	
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	$-$	0	
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	$-$	0	
4	$O(2) \times O(2)$	$T^3,\ T^0$	$-$	T^3	1	(a)
✓ 5	$Z_2 \times [O(2)]^2$	$T^2,\ T^0$	D_{CP1}	T^2	1	(h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3},\ T^0$	$-$	$T^{1,2}$	2	$(h,\ a)$
7	$SO(3)$	$T^{0,4,6}$	$-$	$T^{4,6}$	2	$(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2	$(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2	$(h^\pm)$
10	$O(2) \times O(3)$	$T^3,\ T^{0,8,9}$	$-$	T^3	1	(a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	$-$	$T^{3,5,7}$	3	$(a,\ h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3	$(a,\ h^\pm)$
✓ 13	$SO(5)$	$T^{0,1,2,\ldots,9}$	$-$	$T^{1,2,8,9}$	4	$(h,\ a,\ h^\pm)$

✓ : Natural SM Alignment $\longmapsto$

• **Symmetries** in **2HDMEFTs**

$$V_{\text{2HDMEFT}} = -\frac{1}{2} M_A \, \mathsf{R}^A + \frac{1}{4} L_{AB} \, \mathsf{R}^A \mathsf{R}^B$$

$$+ \frac{1}{\Lambda^2} K_{ABC} \, \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C + \frac{1}{\Lambda^4} Z_{ABCD} \, \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C \mathsf{R}^D + \cdots$$

No. of couplings: $\quad N^{(\text{dim}=2n)} = \frac{1}{6}(n+1)(n+2)(n+3)$

$$N^{(\text{dim} \leq 4)} = 14, \quad N^{(\text{dim} \leq 6)} = 34, \quad N^{(\text{dim} \leq 8)} = 69, \ \ldots, \ N^{(\text{dim} \leq 20)} = 1000$$

Symmetry restrictions:

$$M_A \big[T^a \big]^{A'}_{\ A} = 0, \quad L_{A'B} \big[T^a \big]^{A'}_{\ A} + L_{AB'} \big[T^a \big]^{B'}_{\ B} = 0,$$

$$K_{A'BC} \big[T^a \big]^{A'}_{\ A} + K_{AB'C} \big[T^a \big]^{B'}_{\ B} + K_{ABC'} \big[T^a \big]^{C'}_{\ C} = 0,$$

$$Z_{A'BCD} \big[T^a \big]^{A'}_{\ A} + Z_{AB'CD} \big[T^a \big]^{B'}_{\ B} + Z_{ABC'D} \big[T^a \big]^{C'}_{\ C} + Z_{ABCD'} \big[T^a \big]^{D'}_{\ D} = 0,$$

where $T^a \in \mathfrak{g}$ are the generators of the symmetry subgroup $G \subseteq \text{SO}(5)$.

No.	Symmetry	Non-zero parameters of Symmetric 2HDMEFT Potential	Dim
1	CP1	μ_1^2, μ_2^2, $\mathrm{Re}(m_{12}^2)$, λ_1, λ_2, λ_3, λ_4, $\mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6)$, $\mathrm{Re}(\lambda_7)$ $\kappa_1, \ldots, \kappa_6$, $\mathrm{Re}(\kappa_7, \ldots, \kappa_{13})$ $\zeta_1, \ldots, \zeta_9$, $\mathrm{Re}(\zeta_{10}, \ldots, \zeta_{22})$	$D \geq 4$
2	Z_2	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4, λ_5 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6, κ_8, κ_9 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5 ζ_6, ζ_7, ζ_8, ζ_9, ζ_{10}, ζ_{13}, ζ_{14}, ζ_{15}, ζ_{16}	$D \geq 4$
3	Z_3	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6, κ_7 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9, ζ_{11}, ζ_{12}	$D \geq 6$
4	Z_4	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9, ζ_{10}	$D \geq 8$
5	CP2	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, λ_5, $\lambda_6 = -\lambda_7$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, $\kappa_8 = \kappa_9$, $\kappa_{11} = -\kappa_{12}$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, ζ_{10}, $\zeta_{11} = -\zeta_{12}$, ζ_{13}, $\zeta_{14} = \zeta_{15}$, ζ_{16}, $\zeta_{17} = -\zeta_{18}$, $\zeta_{19} = -\zeta_{20}$, $\zeta_{21} = -\zeta_{22}$	$D \geq 4$
6	CP3	$\mu_1 = \mu_2$, $\lambda_1 = \lambda_2$, λ_3, λ_4 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, κ_7 $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, $\zeta_{11} = \zeta_{12}$	$D \geq 6$
7	CP4	$\mu_1 = \mu_2$, $\lambda_1 = \lambda_2$, λ_3, λ_4 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, $\kappa_8 = -\kappa_9$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, ζ_{10}, $\zeta_{14} = -\zeta_{15}$	$D \geq 6$

8	$\mathrm{U(1)_{PQ}}$	μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4 κ_1, κ_2, κ_3, κ_4, κ_5, κ_6 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, ζ_6, ζ_7, ζ_8, ζ_9	$D \geq 4$
9	$\mathrm{CP1 \otimes SO(2)_{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, λ_4, $\mathrm{Re}(\lambda_5) = 2\lambda_1 - \lambda_{34}$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6$, $\mathrm{Re}(\kappa_8) = \mathrm{Re}(\kappa_9) = \frac{1}{2}(3\kappa_1 - \kappa_3 - \kappa_5)$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, ζ_6, $\zeta_7 = \zeta_8$, ζ_9, $\mathrm{Re}(\zeta_{10}) = -\frac{1}{4}\mathrm{Re}(\zeta_{13}) + \frac{1}{2}\mathrm{Re}(\zeta_{14}) - \frac{1}{4}\mathrm{Re}(\zeta_{16})$, $\mathrm{Re}(\zeta_{13}) = \frac{1}{6}(4\zeta_1 + 2\zeta_3 - 4\zeta_4 - 4\zeta_6 + 2\zeta_7 - \zeta_9)$, $\mathrm{Re}(\zeta_{14}) = \mathrm{Re}(\zeta_{15}) = \frac{1}{2}(4\zeta_1 - \zeta_4 - \zeta_7)$, $\mathrm{Re}(\zeta_{16}) = \frac{1}{2}(4\zeta_1 - 2\zeta_3 + 2\zeta_4 - \zeta_9)$	$D \geq 4$
10	$\mathrm{SU(2)_{HF}}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = 2\lambda_1 - \lambda_3$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 3\kappa_1 - \kappa_3$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 2\zeta_1 + \zeta_3 - 2\zeta_4$, $\zeta_7 = \zeta_8 = 4\zeta_1 - \zeta_4$, $\zeta_9 = 4\zeta_1 - 2\zeta_3 + 2\zeta_4$	$D \geq 4$
11	$\mathrm{Sp(2)_{\phi_1 + \phi_2}}$	μ_1^2, μ_2^2, $\mathrm{Re}(m_{12}^2)$, λ_1, λ_2, λ_3, $\lambda_4 = \mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6)$, $\mathrm{Re}(\lambda_7)$ κ_1, κ_2, κ_3, κ_4, $\kappa_5 = 2\mathrm{Re}(\kappa_8)$, $\kappa_6 = 2\mathrm{Re}(\kappa_9)$, $\mathrm{Re}(\kappa_7) = \frac{1}{3}\mathrm{Re}(\kappa_{10})$, $\mathrm{Re}(\kappa_{11})$, $\mathrm{Re}(\kappa_{12})$, $\mathrm{Re}(\kappa_{13})$ ζ_1, ζ_2, ζ_3, ζ_4, ζ_5, $\zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13})$, $\zeta_7 = 2\mathrm{Re}(\zeta_{14})$, $\zeta_8 = 2\mathrm{Re}(\zeta_{15})$, $\zeta_9 = 2\mathrm{Re}(\zeta_{16})$, $\mathrm{Re}(\zeta_{11}) = \frac{1}{3}\mathrm{Re}(\zeta_{17})$, $\mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{18})$, $\mathrm{Re}(\zeta_{19})$, $\mathrm{Re}(\zeta_{20})$, $\mathrm{Re}(\zeta_{21})$, $\mathrm{Re}(\zeta_{22})$	$D \geq 4$

12	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, $\mathrm{Re}(m_{12}^2)$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = \mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6)=\mathrm{Re}(\lambda_7)$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2\mathrm{Re}(\kappa_8) = 2\mathrm{Re}(\kappa_9)$, $\mathrm{Re}(\kappa_7) = \frac{1}{3}\mathrm{Re}(\kappa_{10})$, $\mathrm{Re}(\kappa_{11}) = \mathrm{Re}(\kappa_{12})$, $\mathrm{Re}(\kappa_{13})$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2\mathrm{Re}(\zeta_{14}) = 2\mathrm{Re}(\zeta_{15})$, $\zeta_9 = 2\mathrm{Re}(\zeta_{16})$ $\mathrm{Re}(\zeta_{11}) = \mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}) = \frac{1}{3}\mathrm{Re}(\zeta_{18})$, $\mathrm{Re}(\zeta_{19}) = \mathrm{Re}(\zeta_{20})$, $\mathrm{Re}(\zeta_{21}) = \mathrm{Re}(\zeta_{22})$	$D \geq 4$
13	$\mathrm{CP2} \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 = \mathrm{Re}(\lambda_5)$, $\mathrm{Re}(\lambda_6) = -\mathrm{Re}(\lambda_7)$ $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2\mathrm{Re}(\kappa_8) = 2\mathrm{Re}(\kappa_9)$, $\mathrm{Re}(\kappa_{11}) = -\mathrm{Re}(\kappa_{12})$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2\mathrm{Re}(\zeta_{14}) = 2\mathrm{Re}(\zeta_{15})$, $\zeta_9 = 2\mathrm{Re}(\zeta_{16})$ $\mathrm{Re}(\zeta_{11}) = -\mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}) = -\frac{1}{3}\mathrm{Re}(\zeta_{18})$, $\mathrm{Re}(\zeta_{19}) = -\mathrm{Re}(\zeta_{20})$, $\mathrm{Re}(\zeta_{21}) = -\mathrm{Re}(\zeta_{22})$	$D \geq 4$
14	$\mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{Sp}(2)_{\phi_1\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, λ_4 $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$, $\kappa_5 = \kappa_6$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$, ζ_6, $\zeta_7 = \zeta_8 = \frac{1}{2}\zeta_9$	$D \geq 4$
15	$\mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	μ_1^2, μ_2^2, λ_1, λ_2, λ_3 κ_1, κ_2, κ_3, κ_4 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5	$D \geq 4$
16	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$	$D \geq 4$
17	$\mathrm{Sp}(4)$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$	$D \geq 4$

• Symmetries of multi-HDM Potentials

Prime bilinear invariants:

▶ Maximal block:
$$\begin{cases} \mathrm{Sp}(2n): & S_n = \Phi^+\Phi & \text{with } \Phi = \begin{pmatrix} \phi \\ i\sigma^2\phi^* \end{pmatrix} \\ \mathrm{SU}(n): & D_n^a = \phi^\dagger\sigma^a\phi & \text{and } \phi = (\phi_1, \phi_2, \cdots, \phi_n)^\mathsf{T} \\ \mathrm{SO}(n): & T_n = \phi\phi^\mathsf{T} \end{cases}$$

▶ Minimal block:
$$\begin{cases} \mathrm{Sp}(2): & \begin{cases} S_{ii} = \phi_i^\dagger\phi_i & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_i^* \end{pmatrix} \\ S_{ij} = \phi_i^\dagger\phi_j + \phi_j^\dagger\phi_i & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_j^* \end{pmatrix} \& \begin{pmatrix} \phi_j \\ i\sigma^2\phi_i^* \end{pmatrix} \end{cases} \\[4ex] \mathrm{SU}(2)\times\mathrm{U}(1): & \begin{cases} D_{ij}^a = \phi_i^\dagger\sigma^a\phi_i + \phi_j^\dagger\sigma^a\phi_j = D_{ji}^a & \text{for } \begin{pmatrix} \phi_i \\ \phi_j \end{pmatrix} \\ D_{ij}'^a = \phi_i^\dagger\sigma^a\phi_i + (i\sigma^2\phi_j^*)\sigma^a(i\sigma^2\phi_j^*) & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_j^* \end{pmatrix} \end{cases} \\[4ex] \mathrm{SO}(2): & \begin{cases} T_{ij} = \phi_i\phi_i^\mathsf{T} + \phi_j\phi_j^\mathsf{T} = T_{ji} & \text{for } \begin{pmatrix} \phi_i \\ \phi_j \end{pmatrix} \end{cases} \end{cases}$$

$$\boxed{\text{The symmetric potential} \rightarrow V_{\mathsf{sym}} = -\mu^2 S_n + \lambda_S S_n^2 + \lambda_D D_n^2 + \lambda_T T_n^2}$$

- **Discrete Symmetries**

[Earlier studies: Ivanov, Vdovin '12; V Keus et al '13; Ivanov, Varzielas '19, . . .]

$\longrightarrow$ **Generalized CP (GCP) transformations:**

$$\mathsf{GCP}[\phi_i] = G_{ij}\phi_j^* \qquad G_{ij} \in \mathsf{SU}(n)$$

$\longrightarrow$ **Abelian Discrete Symmetries:**

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2, \quad Z_3 \times Z_3, \quad \cdots, \quad Z_n, \quad \cdots,$$

where $Z_n = \{1, \omega, \cdots, \omega^{(n-1)}\}$ with $\omega^n = 1$.

$\longrightarrow$ **Non-Abelian Discrete Symmetries**

• Typical **Non**-Abelian Discrete Symmetries

— *Permutation group* S_N $\xrightarrow{\text{with order}}$ $N!$

— *Alternating group* A_N $\xrightarrow{\text{with order}}$ $N!/2$

— *Dihedral group* D_N $\xrightarrow{\text{isomorphic to}}$ $Z_N \rtimes Z_2$

— *Binary Dihedral group* Q_{2N} $\xrightarrow{\text{with order}}$ $4N$

— *Tetrahedral group* $T_{N(\text{prime number})}$ $\xrightarrow{\text{isomorphic to}}$ $Z_N \rtimes Z_3$

— *Dihedral-like groups*:

$$\Sigma(2N^2) \cong (Z_N \times Z'_N) \rtimes Z_2 \qquad \Delta(3N^2) \cong (Z_N \times Z'_N) \rtimes Z_3$$

$$\Sigma(3N^3) \cong Z_N \times \Delta(3N^2) \qquad \Delta(6N^2) \cong (Z_N \times Z'_N) \rtimes S_3$$

— *Crystal-like groups* $\Sigma(M\phi)$, with $\phi = 1, 2, 3$:

$$\Sigma(60\phi), \quad \Sigma(168\phi), \quad \Sigma(36\phi), \quad \Sigma(72\phi), \quad \Sigma(216\phi), \quad \Sigma(360\phi)$$

No.	Symmetry	Non-zero parameters for 3HDM potentials
1	CP1	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, Re(λ_{1212}), Re(λ_{1313}), Re(λ_{2323}), Re(λ_{1213}), Re(λ_{2113}), Re(λ_{1223}), Re(λ_{2123}), Re(λ_{1323}), Re(λ_{1332}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
2	Z_2	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{13}^2, \lambda_{1212}, \lambda_{1313}, \lambda_{2323}, \lambda_{1232}, \lambda_{1113}, \lambda_{2213}, \lambda_{3313}$ and H.c.$\}$
2′	Z_2'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{23}^2, \lambda_{1212}, \lambda_{1313}, \lambda_{2323}, \lambda_{1213}, \lambda_{1123}, \lambda_{2223}, \lambda_{3323}$ and H.c.$\}$
3	$Z_2 \otimes Z_2'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}, \lambda_{1313}, \lambda_{2323}$ and H.c.$\}$
4	Z_3	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1213}, \lambda_{1323}, \lambda_{2123}$ and H.c.$\}$
5	Z_4	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}, \lambda_{1323}$ and H.c.$\}$
5′	Z_4'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1313}, \lambda_{3212}$ and H.c.$\}$
6	^{a}U(1)	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1323}$ and H.c.$\}$
6′	^{b}U(1)′	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{12}^2, \lambda_{1212}, \lambda_{1112}, \lambda_{2212}, \lambda_{3312}, \lambda_{1332}$ and H.c.$\}$
7	U(1) $\otimes$ U(1)′	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}
8	$Z_2 \otimes$ U(1)′	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$ and H.c.$\}$
9	CP1 $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
10	CP1 $\otimes$ Z_2 $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212})
11	U(1) $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}
12	CP2	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1313}$)=Re($\lambda_{2323}$), Re($\lambda_{1212}$), $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$

13	$CP2 \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1221}, \lambda_{1133} = \lambda_{2233}, Re(\lambda_{1212}),$ $\{\lambda_{1112} = -\lambda_{2212} \text{ and H.c.}\}$
14	$SO(2)_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1221}, \lambda_{1133} = \lambda_{2233}, \lambda_{1331} = \lambda_{2332},$ $Re(\lambda_{1313}) = Re(\lambda_{2323}), Re(\lambda_{1212}) = 2\lambda_{11} - (\lambda_{1122} + \lambda_{1221}),$
15	D_3	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133} = \lambda_{2233}, \lambda_{1221}, \lambda_{1331} = \lambda_{2332},$ $\{\lambda_{2131} = -\lambda_{1232}, \lambda_{1323} \text{ and H.c.}\}$
16	D_4	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133} = \lambda_{2233}, \lambda_{1221}, \{\lambda_{1212} \text{ and H.c.}\},$ $\lambda_{1331} = \lambda_{2332} = Re(\lambda_{3231})$
17	$D_3 \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133} = \lambda_{2233}, \lambda_{1221}$
18	$D_4 \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133} = \lambda_{2233}, \lambda_{1221}, Re(\lambda_{1212})$
19	$SO(2)_{\phi_1,\phi_2} \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = Re(\lambda_{1212}) = \lambda_{11} - \frac{1}{2}\lambda_{1122}$
20	$SU(2)_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122} = 2\lambda_{11} - \lambda_{1221}, \lambda_{1221},$ $\lambda_{1133} = \lambda_{2233}, \lambda_{1331} = \lambda_{2332}$
21	$SU(2)_{\phi_1,\phi_2} \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2, \mu_3^2, \lambda_{11} = \lambda_{22}, \lambda_{33}, \lambda_{1122} = 2\lambda_{11} - \lambda_{1221}, \lambda_{1133} = \lambda_{2233},$ λ_{1221}
22	$Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2, \mu_2^2, \mu_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), \lambda_{11}, \lambda_{22}, \lambda_{33},$ $\lambda_{1122}, \lambda_{1133}, \lambda_{2233}, \lambda_{1221} = Re(\lambda_{1212}), \lambda_{1331} = Re(\lambda_{1313}),$ $\lambda_{2332} = Re(\lambda_{2323}), Re(\lambda_{1213}) = Re(\lambda_{2113}), Re(\lambda_{1223}) = Re(\lambda_{2123}),$ $Re(\lambda_{1323}) = Re(\lambda_{1332}), Re(\lambda_{1112}), Re(\lambda_{2212}), Re(\lambda_{3312}),$ $Re(\lambda_{1113}), Re(\lambda_{2213}), Re(\lambda_{3313}), Re(\lambda_{1123}), Re(\lambda_{2223}), Re(\lambda_{3323})$
23	$Z_2 \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2, \mu_2^2, \mu_3^2, Re(m_{13}^2), \lambda_{11}, \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133}, \lambda_{2233},$ $\lambda_{1221} = Re(\lambda_{1212}), \lambda_{1331} = Re(\lambda_{1313}), \lambda_{2332} = Re(\lambda_{2323}),$ $Re(\lambda_{1113}), Re(\lambda_{2213}), Re(\lambda_{3313})$
23'	$Z_2' \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2, \mu_2^2, \mu_3^2, Re(m_{23}^2), \lambda_{11}, \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133}, \lambda_{2233},$ $\lambda_{1221} = Re(\lambda_{1212}), \lambda_{1331} = Re(\lambda_{1313}), \lambda_{2332} = Re(\lambda_{2323}),$ $Re(\lambda_{1123}), Re(\lambda_{2223}), Re(\lambda_{3323})$
24	$Z_2 \otimes Z_2' \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2, \mu_2^2, \mu_3^2, \lambda_{11}, \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133}, \lambda_{2233},$ $\lambda_{1221} = Re(\lambda_{1212}), \lambda_{1331} = Re(\lambda_{1313}), \lambda_{2332} = Re(\lambda_{2323})$
25	$Z_4 \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2, \mu_2^2, \mu_3^2, \lambda_{11}, \lambda_{22}, \lambda_{33}, \lambda_{1122}, \lambda_{1133}, \lambda_{2233}, \lambda_{1221} = Re(\lambda_{1212})$

26	$(CP1 \rtimes S_2) \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \mathrm{Re}(m_{12}^2),\ \mathrm{Re}(m_{13}^2)=\mathrm{Re}(m_{23}^2),\ \lambda_{11} = \lambda_{22},\ \lambda_{33},$ $\lambda_{1122},\ \lambda_{1133} = \lambda_{2233},\ \lambda_{1221}= \mathrm{Re}(\lambda_{1212}),$ $\lambda_{1331}=\lambda_{2332}=\mathrm{Re}(\lambda_{1313})=\mathrm{Re}(\lambda_{2323}),\ \mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1332}),$ $\mathrm{Re}(\lambda_{1223}) = \mathrm{Re}(\lambda_{2123})=\mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{2113}),$ $\mathrm{Re}(\lambda_{3313}) = \mathrm{Re}(\lambda_{3323}),\ \mathrm{Re}(\lambda_{1112}) = \mathrm{Re}(\lambda_{2212}),$ $\mathrm{Re}(\lambda_{3312}),\ \mathrm{Re}(\lambda_{1113}) = \mathrm{Re}(\lambda_{1123}) = \mathrm{Re}(\lambda_{2213}) = \mathrm{Re}(\lambda_{2223})$
27	$D_4 \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} =\mathrm{Re}(\lambda_{1212})$
28	$Sp(2)_{\phi_1+\phi_2} \otimes Sp(2)_{\phi_3}$	$\mu_1^2,\ \mu_2^2,\ \mu_3^2,\ \mathrm{Re}(m_{12}^2),\ \lambda_{11},\ \lambda_{22},\ \lambda_{33},\ \lambda_{1122},\ \lambda_{1133},\ \lambda_{2233},$ $\lambda_{1221}= \mathrm{Re}(\lambda_{1212}),\ \mathrm{Re}(\lambda_{1112}),\ \mathrm{Re}(\lambda_{2212}),\ \mathrm{Re}(\lambda_{3312})$
29	$Sp(2)_{\phi_1\phi_2}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233},\ \lambda_{1221},$ $\lambda_{1331} = \lambda_{2332}$
30	$Sp(2)_{\phi_1\phi_2} \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233},\ \lambda_{1221}$
31	A_4	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \{\lambda_{1212} = \lambda_{1313} = \lambda_{2323},\ \text{and H.c.}\}$
32	S_4	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \mathrm{Re}(\lambda_{1212}) = \mathrm{Re}(\lambda_{1313}) = \mathrm{Re}(\lambda_{2323})$
33	$SO(3)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},$ $\mathrm{Re}(\lambda_{1212}) = \mathrm{Re}(\lambda_{1313}) = \mathrm{Re}(\lambda_{2323}) = 2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$
34	$S_4 \otimes Sp(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33} = \frac{1}{2}\lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332}=\mathrm{Re}(\lambda_{1212}) = \mathrm{Re}(\lambda_{1313}) = \mathrm{Re}(\lambda_{2323})$
35	$\Delta(54)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \{\lambda_{1213} = \lambda_{2123} = \lambda_{3231}\ \text{and H.c.}\}$
36	$\Sigma(36)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332},\ \mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1232})=$ $\frac{3}{4}(2\lambda_{11} - \lambda_{1122} - \lambda_{1221})$
37	$Sp(2)_{\phi_1} \otimes Sp(2)_{\phi_2} \otimes Sp(2)_{\phi_3}$	$\mu_1^2,\ \mu_2^2,\ \mu_3^2,\ \lambda_{11},\ \lambda_{22},\ \lambda_{33},\ \lambda_{1122},\ \lambda_{1133},\ \lambda_{2233}$
38	$Sp(4) \otimes Sp(2)_{\phi_3}$	$\mu_1^2 = \mu_2^2,\ \mu_3^2,\ \lambda_{11} = \lambda_{22} = \frac{1}{2}\lambda_{1122},\ \lambda_{33},\ \lambda_{1133} = \lambda_{2233}$
39	$SU(3) \otimes U(1)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33},\ \lambda_{1122} = \lambda_{1133} = \lambda_{2233},$ $\lambda_{1221} = \lambda_{1331} = \lambda_{2332} = 2\lambda_{11} - \lambda_{1122}$
40	$Sp(6)$	$\mu_1^2 = \mu_2^2 = \mu_3^2,\ \lambda_{11} = \lambda_{22} = \lambda_{33} = \frac{1}{2}\lambda_{1122} = \frac{1}{2}\lambda_{1133} = \frac{1}{2}\lambda_{2233}$

• Vacuum Topology of the 2HDM

$\mathrm{G}_{\mathrm{HF/CP}}$	$\mathrm{H}_{\mathrm{HF/CP}}$	$\mathcal{M}_{\Phi}^{\mathrm{HF/CP}}$	Topological Defect
Z_2	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{U(1)}_{\mathrm{PQ}} \simeq S^1$	$\mathbf{I}$	S^1	Vortex
$\mathrm{SO(3)}_{\mathrm{HF}}$	$\mathrm{SO(2)}_{\mathrm{HF}}$	S^2	Global Monopole
$\mathrm{CP1} \simeq \mathrm{Z}_2$	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{CP2} = \mathrm{Z}_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
$\mathrm{CP1} \otimes \mathrm{SO(2)}$	$\mathrm{CP1}$	S^1	Vortex

• **Energy density of the topological defect** $\phi_{1,2}(\mathbf{r})$:

$$\mathcal{E}(\phi_1, \phi_2) \;=\; (\nabla \phi_1^\dagger) \cdot (\nabla \phi_1) + (\nabla \phi_2^\dagger) \cdot (\nabla \phi_2) + \mathrm{V}(\phi_1, \phi_2) \;+\; \mathrm{V}_0 \,.$$

• **Gradient flow approach to numerically find** $\phi_{1,2}(\mathbf{r})$

$$-\frac{\delta E[\phi_{1,2}]}{\delta \phi_{1,2}(\mathbf{r}, \tau)} \;=\; \frac{\partial \phi_{1,2}(\mathbf{r}, \tau)}{\partial \tau} \;\to\; 0 \,, \quad \text{for } \tau \gg 1 \,.$$

• $\mathbf{Z_2}$ Domain Walls

$$\begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} \quad \xleftrightarrow{\text{U(1)}_Y} \quad \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix}$$

$$\text{Z}_2 \updownarrow \qquad\qquad\qquad \updownarrow \text{Z}_2$$

$$\begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} \quad \xleftrightarrow{\text{U(1)}_Y} \quad \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}$$
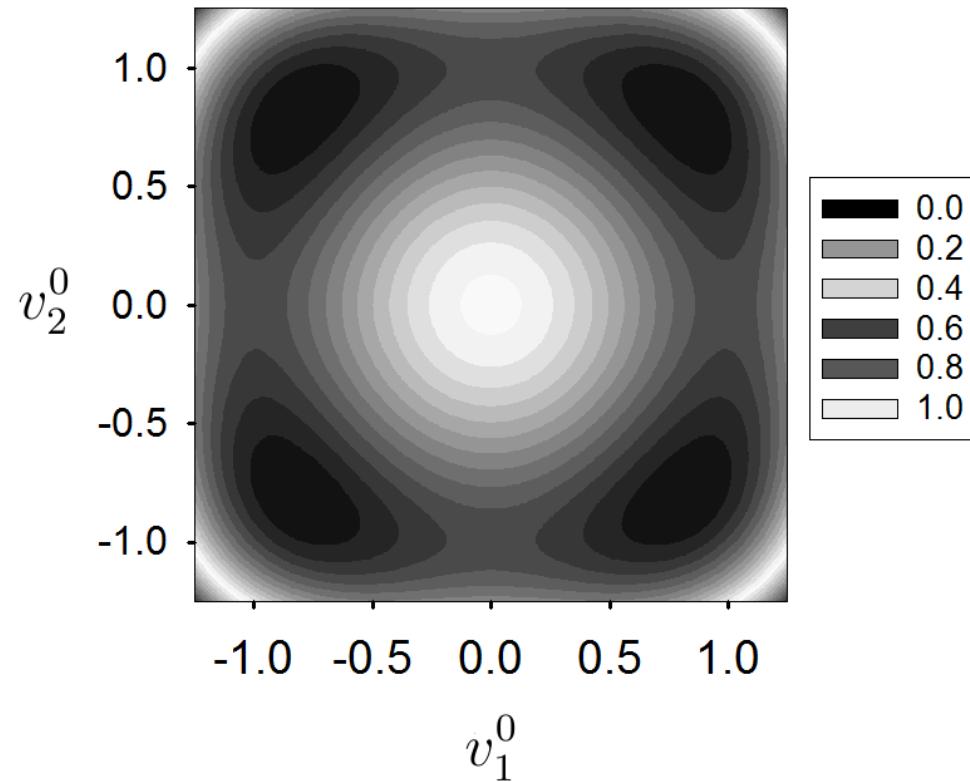
– Spatial profile of the $\mathbf{Z}_2$ domain wall

[R Battye, G Brown, AP, JHEP08 (2011) 020.]

Introduce dimensionless quantities:

$$\hat{x} = \mu_2 x , \quad \hat{v}^0_{1,2}(\hat{x}) = \frac{v^0_{1,2}(\hat{x})}{\eta} , \quad \widehat{E} = \frac{\lambda_2 E}{\mu_2^3} , \quad \text{with } \eta = \frac{\mu_2}{\sqrt{\lambda_2}} .$$

Legend:
- $\hat{v}^0_1(\hat{x})$
- $\hat{v}^0_2(\hat{x})$
- $\hat{\mathcal{E}}(\hat{x})$

$\hat{x} = \mu_2 x$

$\hat{x} = \mu_2 x$
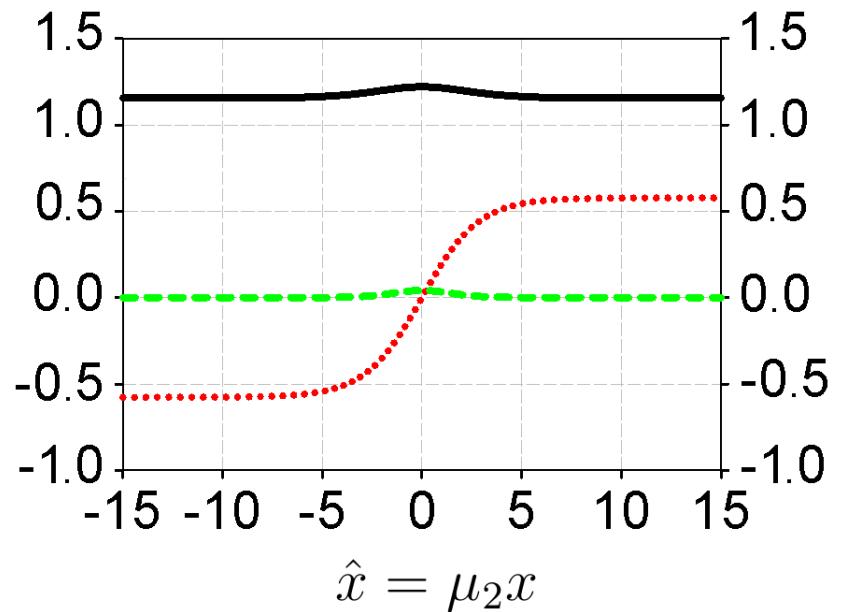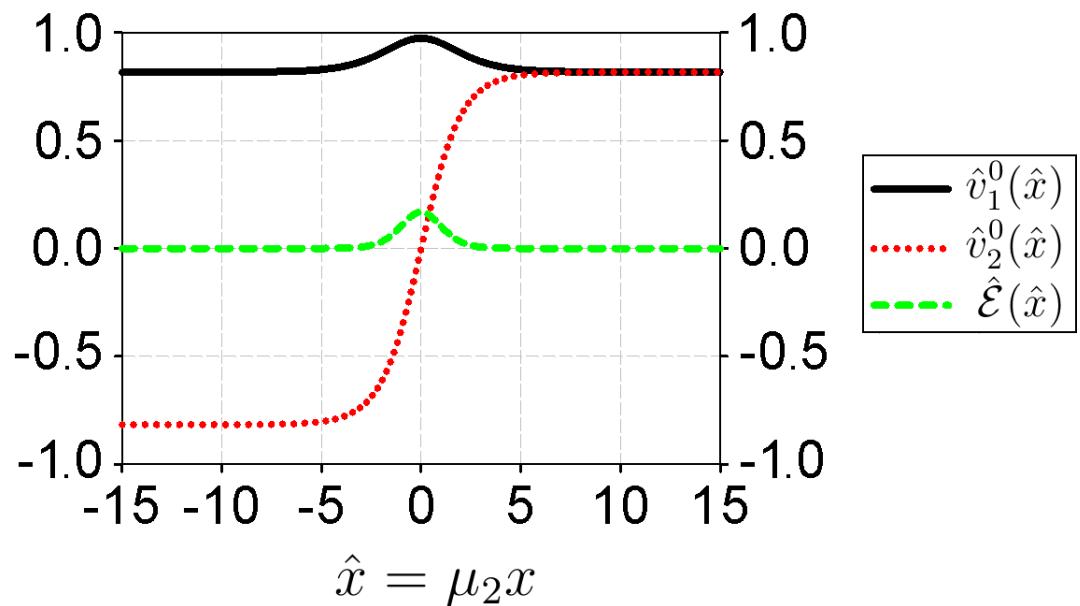
• Charge-**Violating** Domain Walls in the 2HDM

– Relatively **gauge-rotated** vacua at the boundaries:

$$\Phi_1(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(-\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} v_+ \to 0 \\ -v_2\, e^{-i\xi} \end{pmatrix},$$

$$\Phi_1(+\infty) = U(+\infty) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(+\infty) = U(+\infty) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ +v_2\, e^{+i\xi} \end{pmatrix},$$

$$U(x) = e^{i\theta(x)} \exp\left(\frac{i\, G^i(x)\, \sigma^i}{v_{\mathsf{SM}}\, 2} \right), \qquad \text{with} \quad U(-\infty) = \mathbf{1}_2.$$

CATCH 2024   Natural Alignment and CP Violation Beyond the SM   A. PILAFTSIS
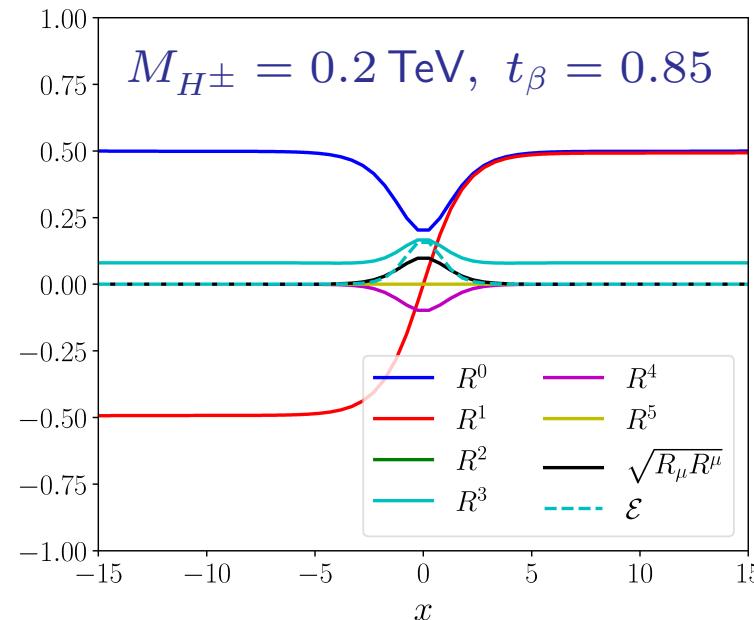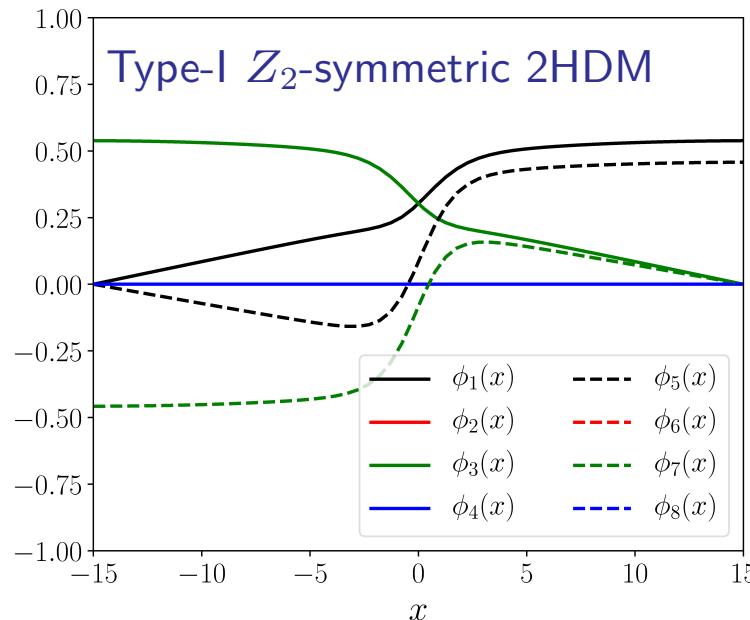
– 3D DW network in the Type-I Z_2-symmetric 2HDM
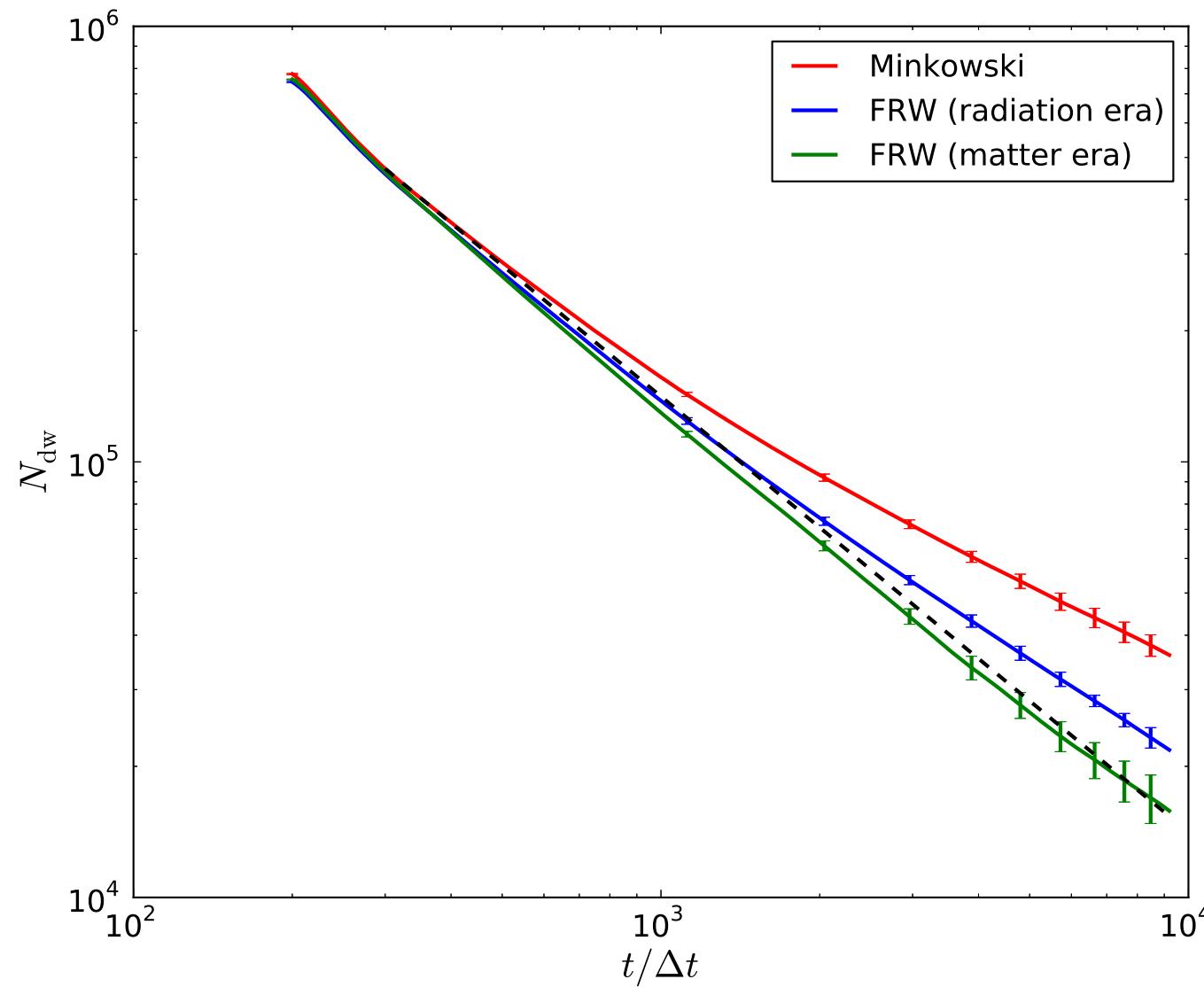
– **QCD** instantons in Type-II Z_2-symmetric **2HDM**

[R Battye, AP, D Viatic, PRD102 (2020) 123536;
RD Peccei, HR Quinn '77]

$$V_{\text{inst}} \sim \Lambda_{\text{QCD}}^4 \left[\left(\frac{\Phi_1^\dagger \Phi_2}{v_{\text{SM}}^2} \right)^{n_{\mathsf{G}}} - \left(\frac{\Phi_1^\dagger \Phi_2 \, e^{i\theta_{\text{QCD}}}}{v_{\text{SM}}^2} \right)^{n_{\mathsf{G}}} \right] + \text{ H.c.}$$

$$\lesssim \frac{\Lambda_{\text{QCD}}^4}{v_{\text{SM}}^2} \, s_\beta^2 c_\beta^2 \left(1 - \cos \left(n_{\mathsf{G}} \theta_{\text{QCD}} \right) \right) \Phi_1^\dagger \Phi_2 \; + \; \text{H.c.} \, ,$$

$$\implies \quad \theta_{\text{QCD}} \; \gtrsim \; \frac{10^{-11}}{\sin \beta \cos \beta}$$

From neutron **EDM** limit: $\theta_{\text{QCD}} \lesssim 10^{-11} - 10^{-10}$

Loose constraint: $\boxed{\mathbf{0.3 \lesssim \tan \beta \lesssim 3}}$

– **Biased** initial conditions in Z_2-symmetric 2HDMs

$$N_{\mathrm{dw}} = At^{-n}\exp(-\alpha t)$$

Legend (left plot):
- $\varepsilon = 0.03$
- $\alpha = 0.00016$, $n = 0.88512$
- $\varepsilon = 0.04$
- $\alpha = 0.00056$, $n = 0.76261$
- $\varepsilon = 0.05$
- $\alpha = 0.00106$, $n = 0.66388$
- $\varepsilon = 0.06$
- $\alpha = 0.00176$, $n = 0.50527$

Right plot legend: $y = 0.0533x - 0.0015$
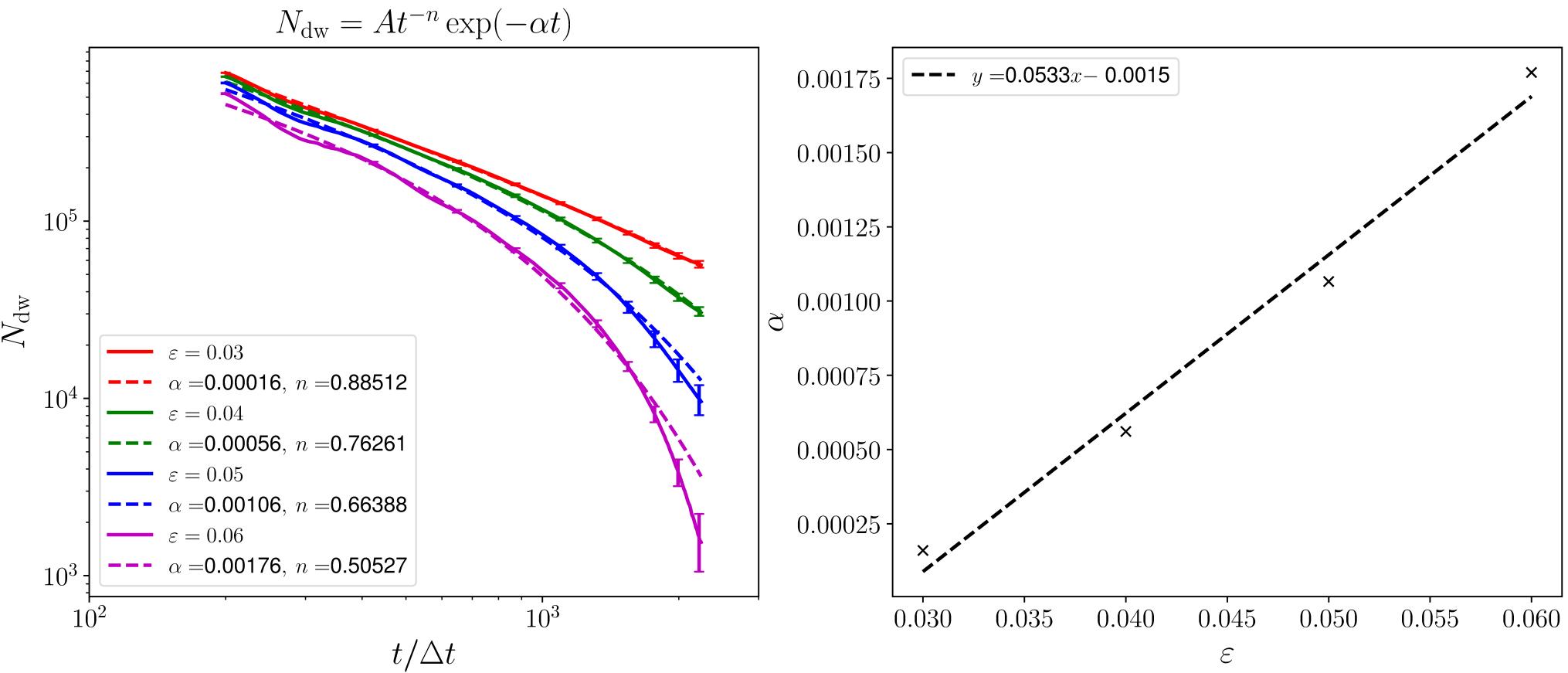
Avoidance of DW domination in the Universe:

$$\varepsilon \;>\; \frac{640\pi}{3}\frac{A\widehat{E}}{e}\left(\frac{v_{\mathsf{SM}}^{3/2}}{M_{\mathsf{Pl}}}\right)^2 \;\simeq\; 2.5\times 10^{-29}A\widehat{E}\ \text{GeV}, \quad \text{with } A, \widehat{E}\sim 1.$$

● **U(1)$_{\rm PQ}$ Vortices**   [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2^0(r)e^{in\chi} \end{pmatrix}.$$
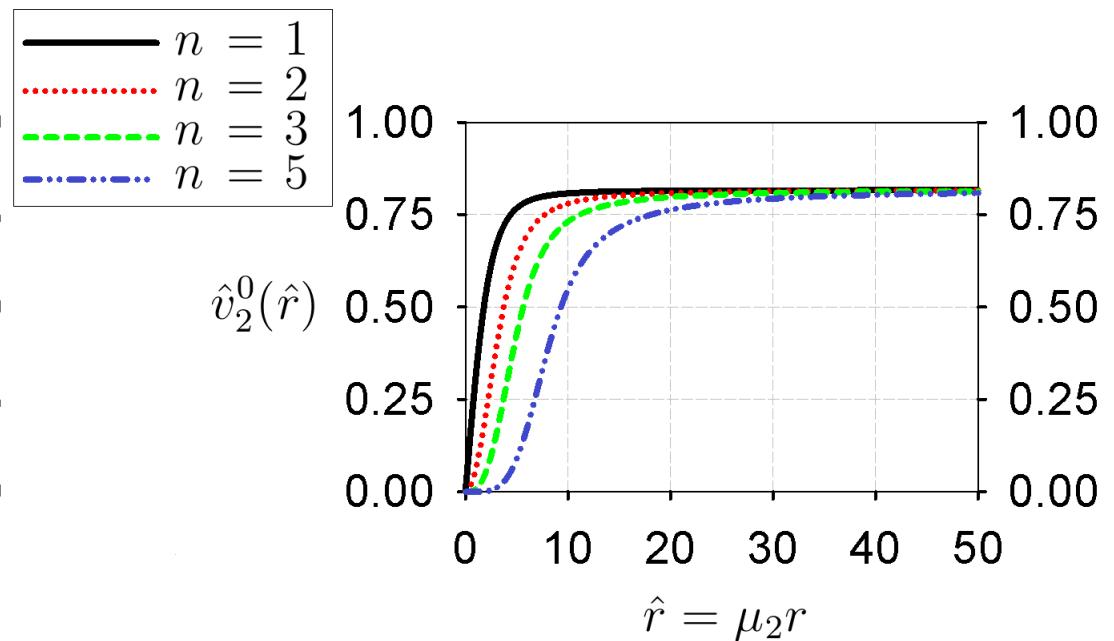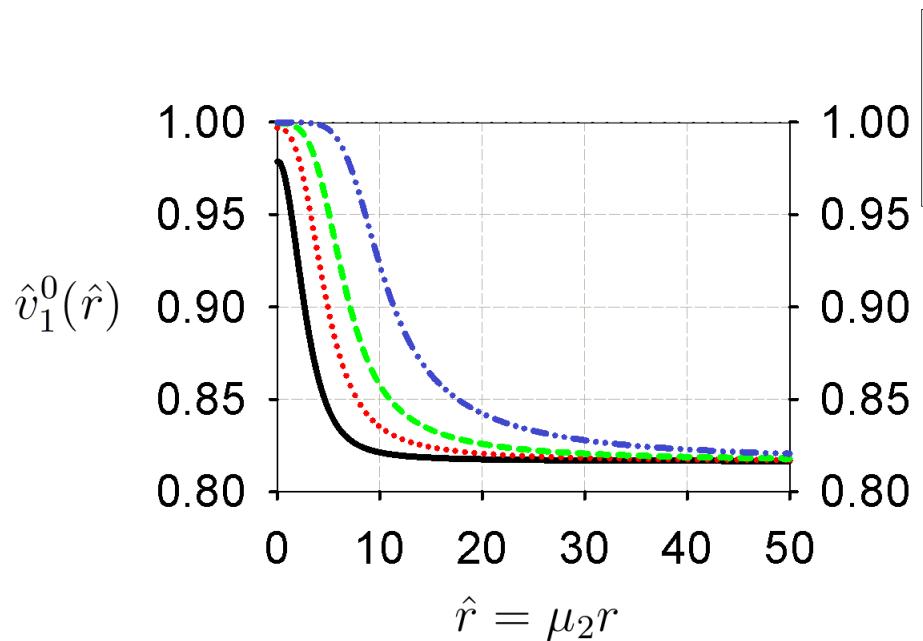
Energy dependence of the U(1)$_{\mathrm{PQ}}$ Vortex

Energy per unit length:

$$\mathrm{E} \;=\; 2\pi \int_0^\infty r\, dr \;\; \mathcal{E}(\phi_1, \phi_2) \;,$$

with

$$\mu^2 \;=\; \frac{\mu_1^2}{\mu_2^2} \;.$$

$$\phi_1(r,\chi) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r)\cos(n\chi) \end{pmatrix}, \qquad \phi_2(r,\chi) \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -v(r)\sin(n\chi) \end{pmatrix}.$$
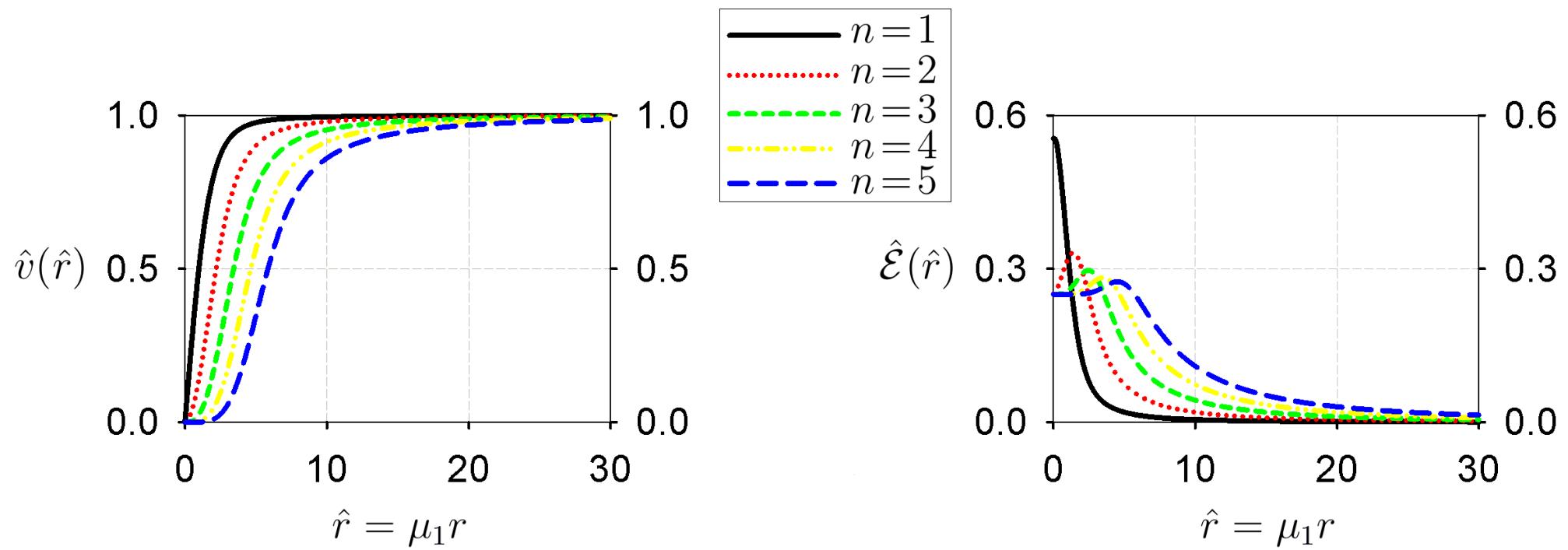
- **SO(3)$_{\mathrm{HF}}$ Global Monopole**   [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r,\chi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r)\sin\chi \end{pmatrix}, \qquad \phi_2(r,\chi,\psi) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(r)e^{i\psi}\cos\chi \end{pmatrix}.$$
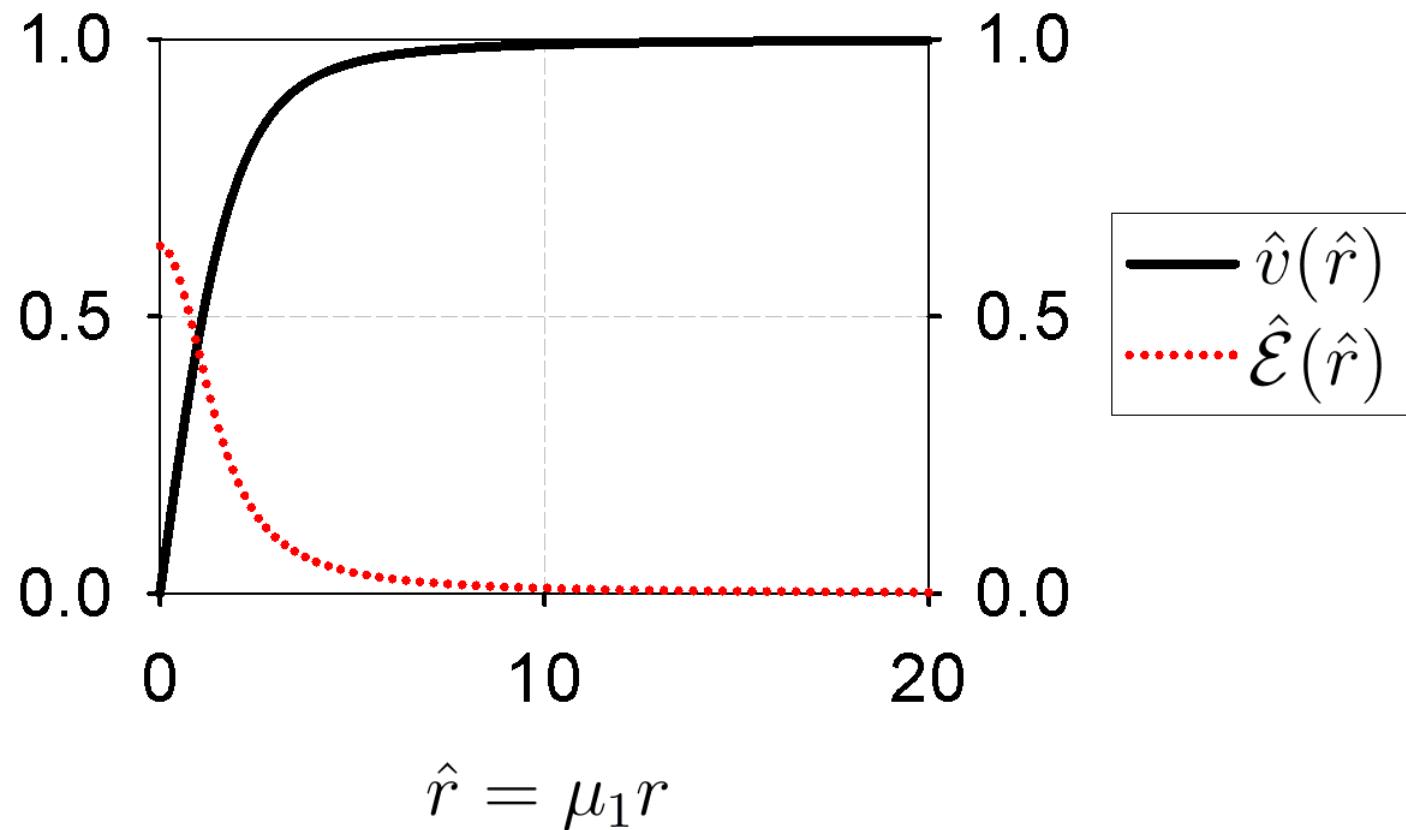
$$\hat{r} = \mu_1 r$$

– **Branching ratios** in the **MS-2HDM**   [Dev, AP '14]
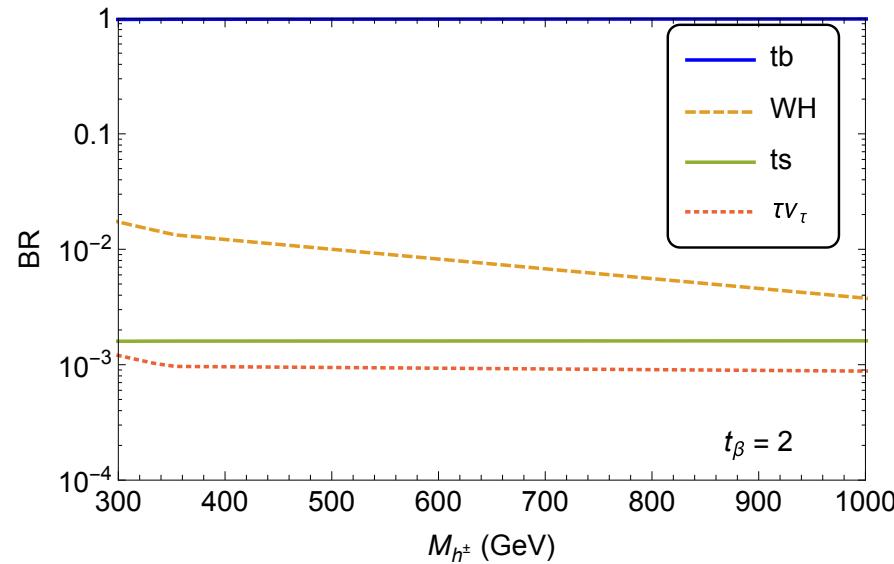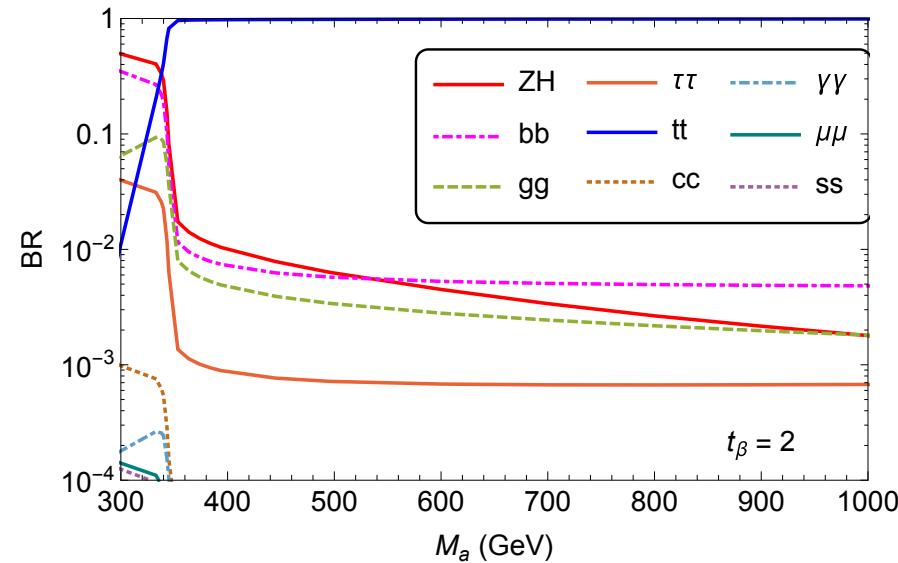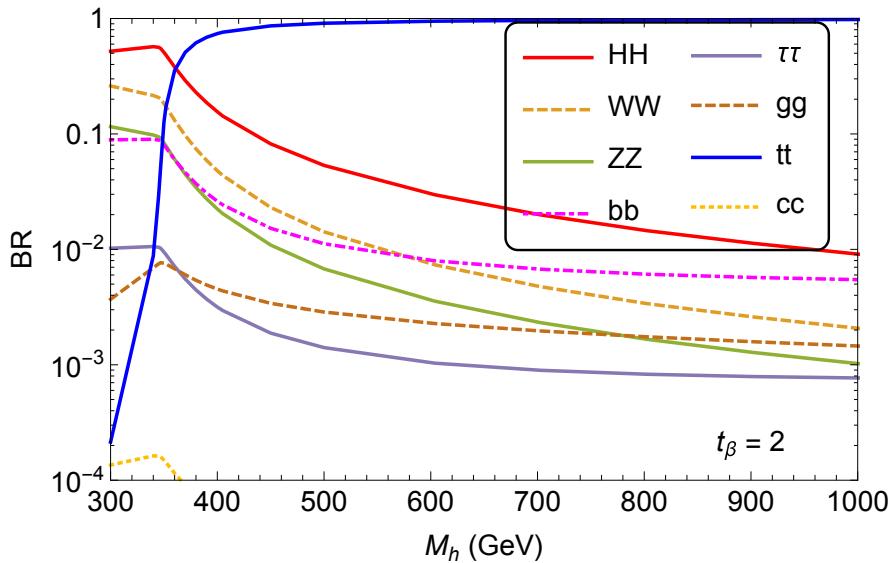
– Discovery channels for aligned Higgs doublets:

• $gg \rightarrow t\bar{b}h^- \rightarrow t\bar{b}\bar{t}b$

[Dev, AP '14]

$p_T^\ell > 20$ GeV,

$|\eta^\ell| < 2.5$,

$\Delta R^{\ell\ell} > 0.4$,

$M_{\ell\ell} > 12$ GeV,

$|M_{\ell\ell} - M_Z| > 10$ GeV,

$p_T^j > 30$ GeV,

$|\eta^j| < 2.4$,
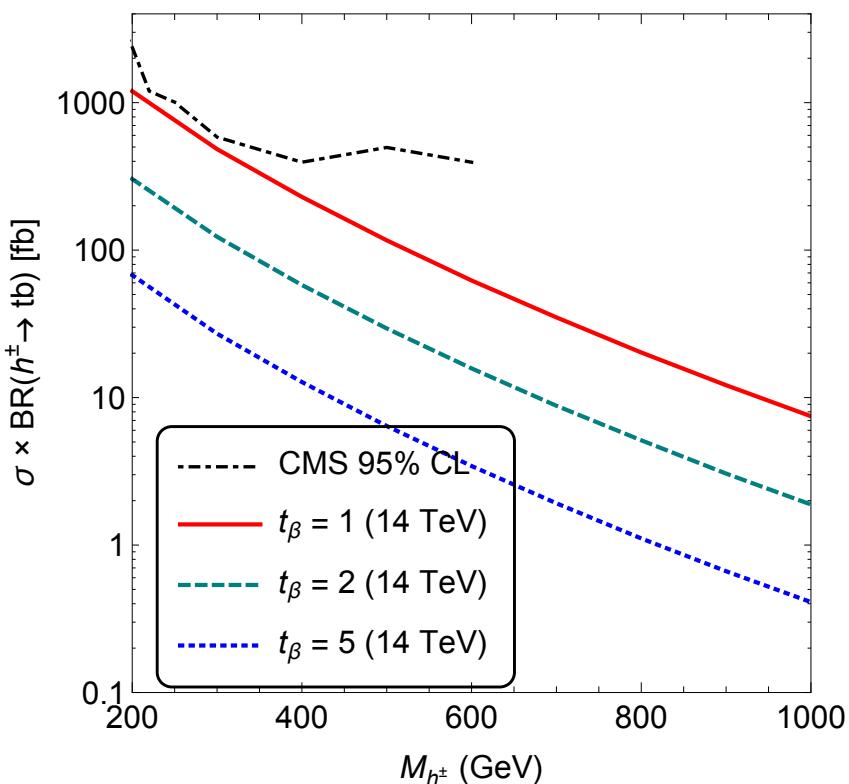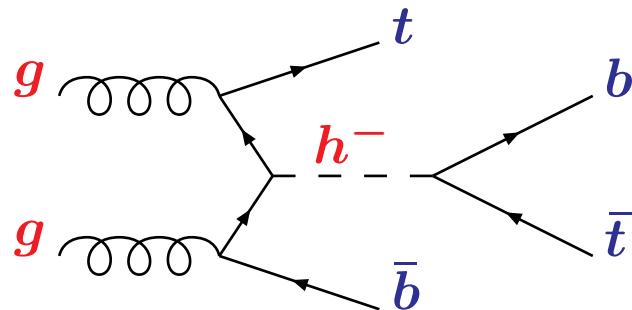
$\not{E}_T > 40$ GeV.

- $gg \to t\bar{t}(h,a) \to t\bar{t}t\bar{t}$
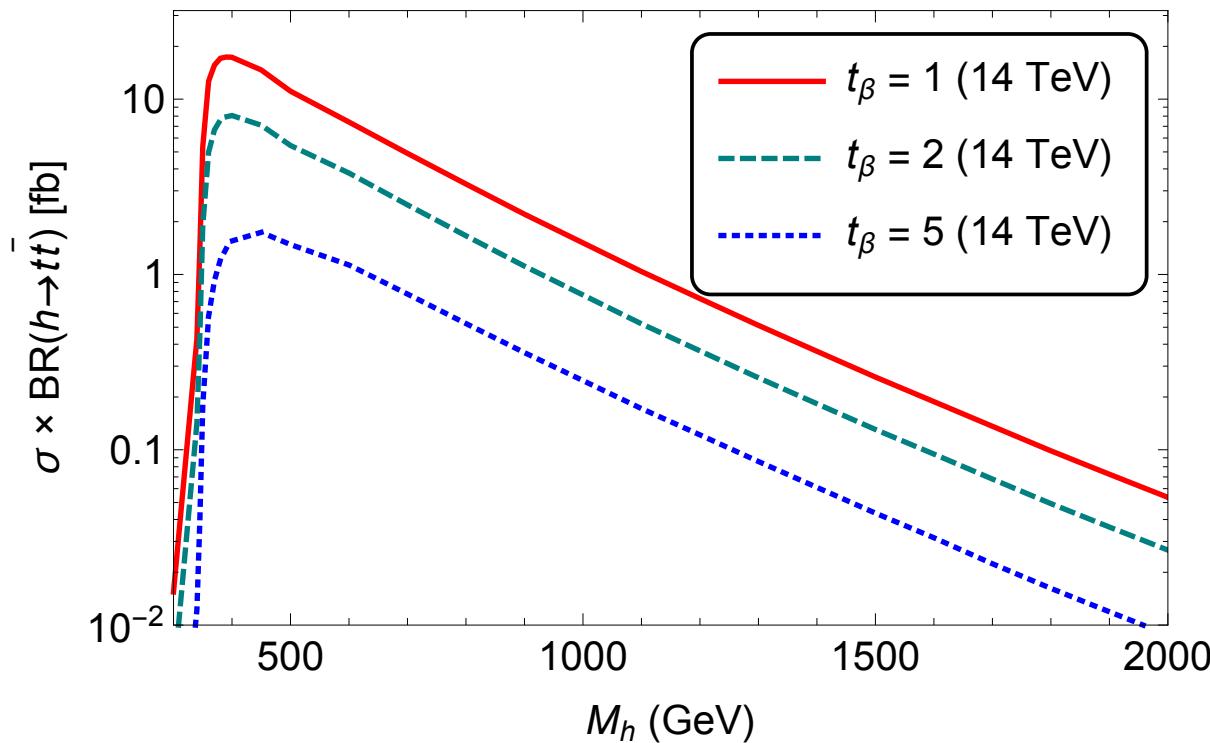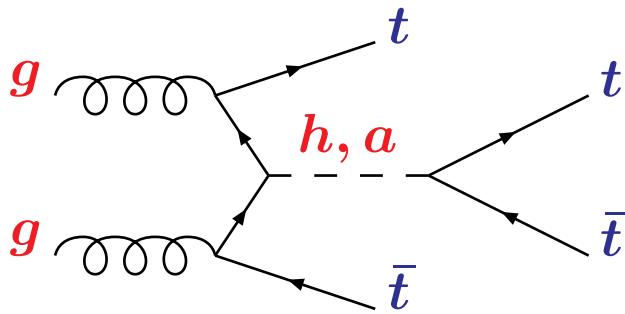
Observation of $t\bar{t}t\bar{t}$ with the ATLAS detector [ATLAS, arXiv:2303.15061]

Reconstruction BDT trained on 57 observables:

- $\Delta R(b_i, l^a)$, $\Delta\eta(b_i, l^a)$, $\Delta\phi(b_i, l^a)$, $p_T^{b_i+l^a}$, $m(b_i, l^a)$,
  where $i = tH, t$ and $a = +, -$

- $\left| m(l^+, b_{tH}) - m(l^-, b_t) \right|$ and $\left| m(l^-, b_{tH}) - m(l^+, b_t) \right|$

- $p_T^{b_j}$, where $j = tH, H, t$

- $\Delta R(b_{tH}, b_k)$, $\Delta\eta(b_{tH}, b_k)$, $\Delta\phi(b_{tH}, b_k)$, $p_T^{b_{tH}+b_k}$, $m(b_{tH}, b_k)$, where $k = H, t$

- $\Delta R(t_{H^a}, b_H)$, $\Delta\eta(t_{H^a}, b_H)$, $\Delta\phi(t_{H^a}, b_H)$, $p_T^{t_{H^a}, b_H}$, $m(t_{H^a}, b_H)$,
  where $a = +, -$

- $\Delta R(t_{H^a}, t_c)$, $\Delta\eta(t_{H^a}, t_c)$, $\Delta\phi(t_{H^a}, t_c)$, where $(H^a, t_c) = (H^+, \bar{t})$ or (H^-, t)

- $m(H^a) - m(b_H)$, where $a = +, -$

- $m(H^+) - m(\bar{t})$ and $m(H^-) - m(t)$

- $p_T^{H^\pm + t_{\text{other}}}$

- $m(H^\pm, t_{\text{other}})$

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]

L = 150 fb^{-1}

$\sqrt{s}$ = 13 TeV

Expected exclusion

H$^{\pm}$ mass [GeV]

tanβ
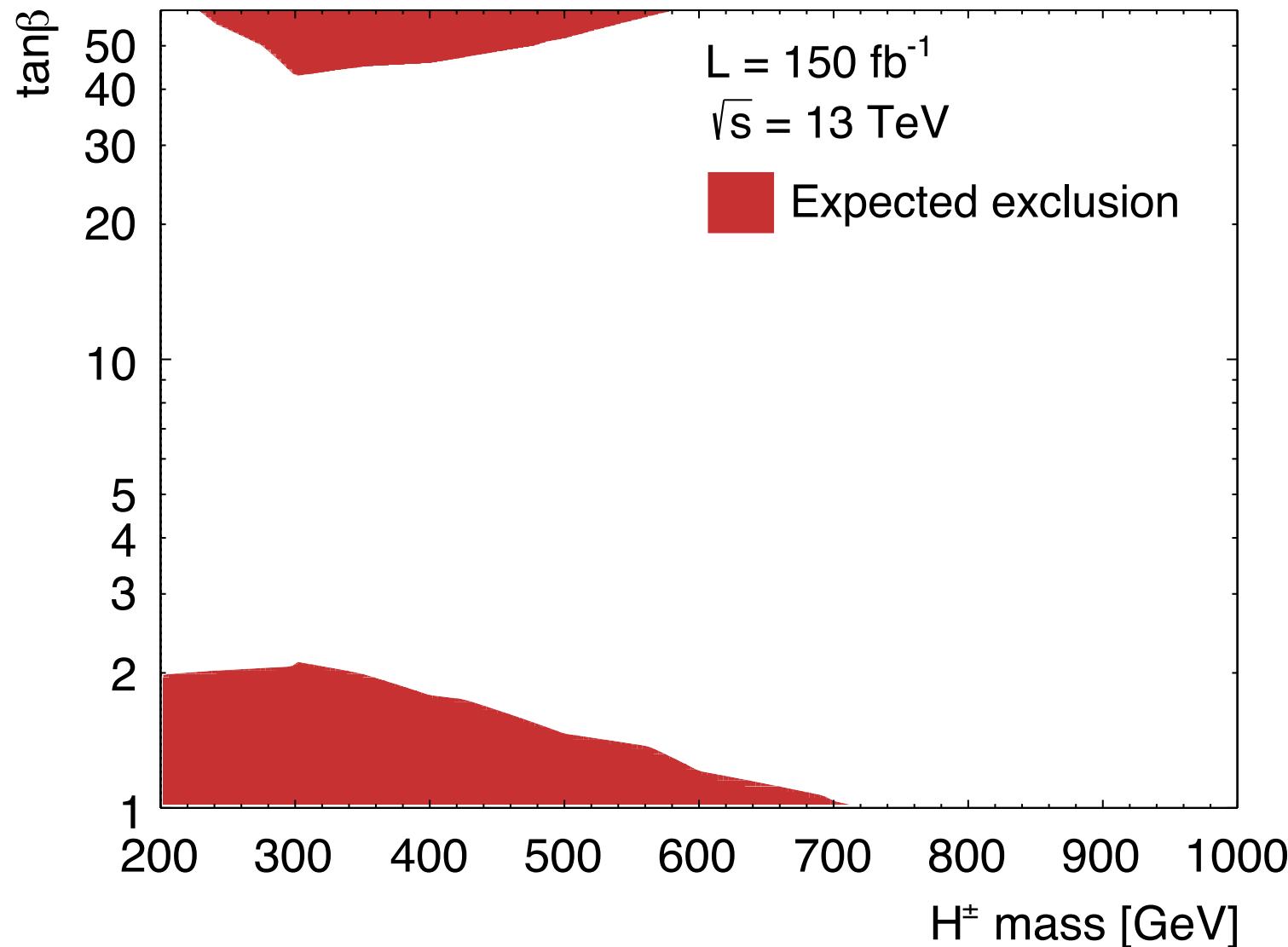
– n**HDM potential** with m **inert scalar doublets:**

$$V_{n\text{HDM}} = V_{\text{sym}} + V_{\text{inert}} + \Delta V_{\text{soft}} ,$$

– <u>**3**</u> **continuous alignment symmetries** in the **field space of the active EWSB sector** $(N_H = n - m)$:

  (i) $\text{Sp}(2N_H) \times \mathcal{D}$ (ii) $\text{SU}(N_H) \times \mathcal{D}$ (iii) $\text{SO}(N_H) \times \mathcal{CP} \times \mathcal{D}$,

  where $\mathcal{D}$ acts on the inert sector *only*.

– **Symmetry invariants:**

(i) $S = \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \ldots = \frac{1}{2} \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi}$

(ii) $D^a = \Phi_1^\dagger \sigma^a \Phi_1 + \Phi_2^\dagger \sigma^a \Phi_2 + \ldots$

(iii) $T = \Phi_1 \Phi_1^\mathsf{T} + \Phi_2 \Phi_2^\mathsf{T} + \ldots$

– **Symmetric part of the scalar potential:**

$$V_{\text{sym}} = -\mu^2 S + \lambda_S S^2 + \lambda_D D^a D^a + \lambda_T \text{Tr}\,(T\,T^*) .$$

– **Inert part of the scalar potential:**

$$V_{\text{inert}} \;=\; \widehat{m}^2_{\hat{a}\hat{b}}\, \widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}} \;+\; \lambda_{\hat{a}\hat{b}\hat{c}\hat{d}}\,(\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\widehat{\Phi}_{\hat{d}}) \;+\; \lambda_{\hat{a}\hat{b}cd}\,(\widehat{\Phi}^\dagger_{\hat{a}}\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\Phi_d)$$

$$+\; \lambda_{a\hat{b}\hat{c}d}\,(\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}}\Phi_d) \;+\; \left[\lambda_{ab\hat{c}\hat{d}}\,(\Phi^\dagger_a\widehat{\Phi}_{\hat{b}})(\Phi^\dagger_c\widehat{\Phi}_{\hat{d}}) \;+\; \text{H.c.} \right]$$

$$\mathsf{Z}^{\mathsf{I}}_2: \quad \Phi_a \;\to\; \Phi_a \quad (a = 1, 2, \ldots, N_H)\,, \qquad \widehat{\Phi}_{\hat{b}} \;\to\; -\,\widehat{\Phi}_{\hat{b}} \quad (\hat{b} = \hat{1}, \hat{2}, \ldots, \widehat{m})$$

– **Soft-symmetry Breaking:**

$$\Delta V_{\text{soft}} \;=\; m^2_{ab}\, \Phi^\dagger_a\Phi_b$$

– **Minimal Symmetry of Alignment in the Higgs basis:**

$$\mathsf{Z}^{\mathsf{EW}}_2: \quad \Phi'_1 \;\to\; \Phi'_1\,, \qquad \Phi'_{a'} \;\to\; -\,\Phi'_{a'} \qquad (a' = 2, 3, \ldots, N_H)$$

where m^2_{ab} becomes diagonal.

$$\Longrightarrow \qquad \boxed{\text{Minimal Alignment Symmetry:} \quad \mathsf{Z}^{\mathsf{EW}}_2 \times \mathsf{Z}^{\mathsf{I}}_2} \qquad \text{[AP '16]}$$