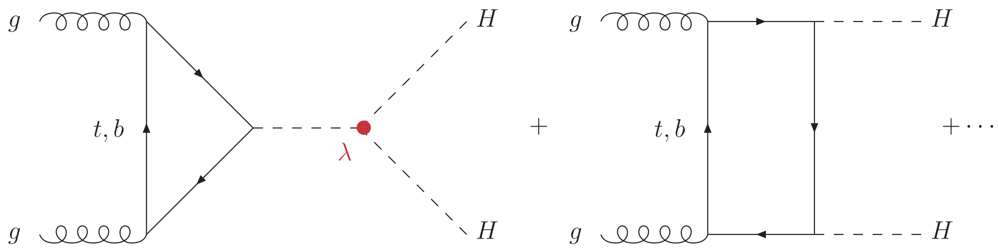
PAUL SCHERRER INSTITUT

PAIR PRODUCTION OF HIGGS BOSONS AT NLO

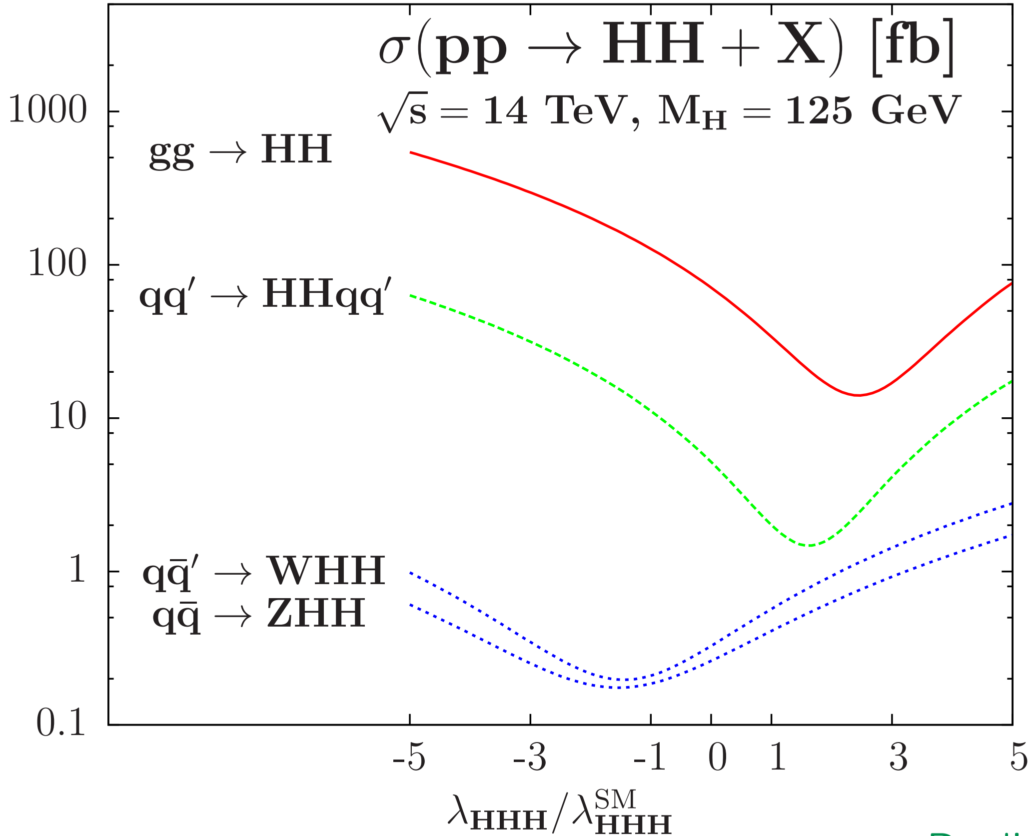
Michael Spira (PSI)

in collaboration with J. Baglio, A. Bhattacharya, F. Campanario, S. Carlotti, J. Chang, S. Glaus, J. Mazzitelli, J. Ronca, M. Mühlleitner and J. Schlenk

I INTRODUCTION



- third generation dominant: $t\,(b)$



$gg \to HH$:

$$\frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

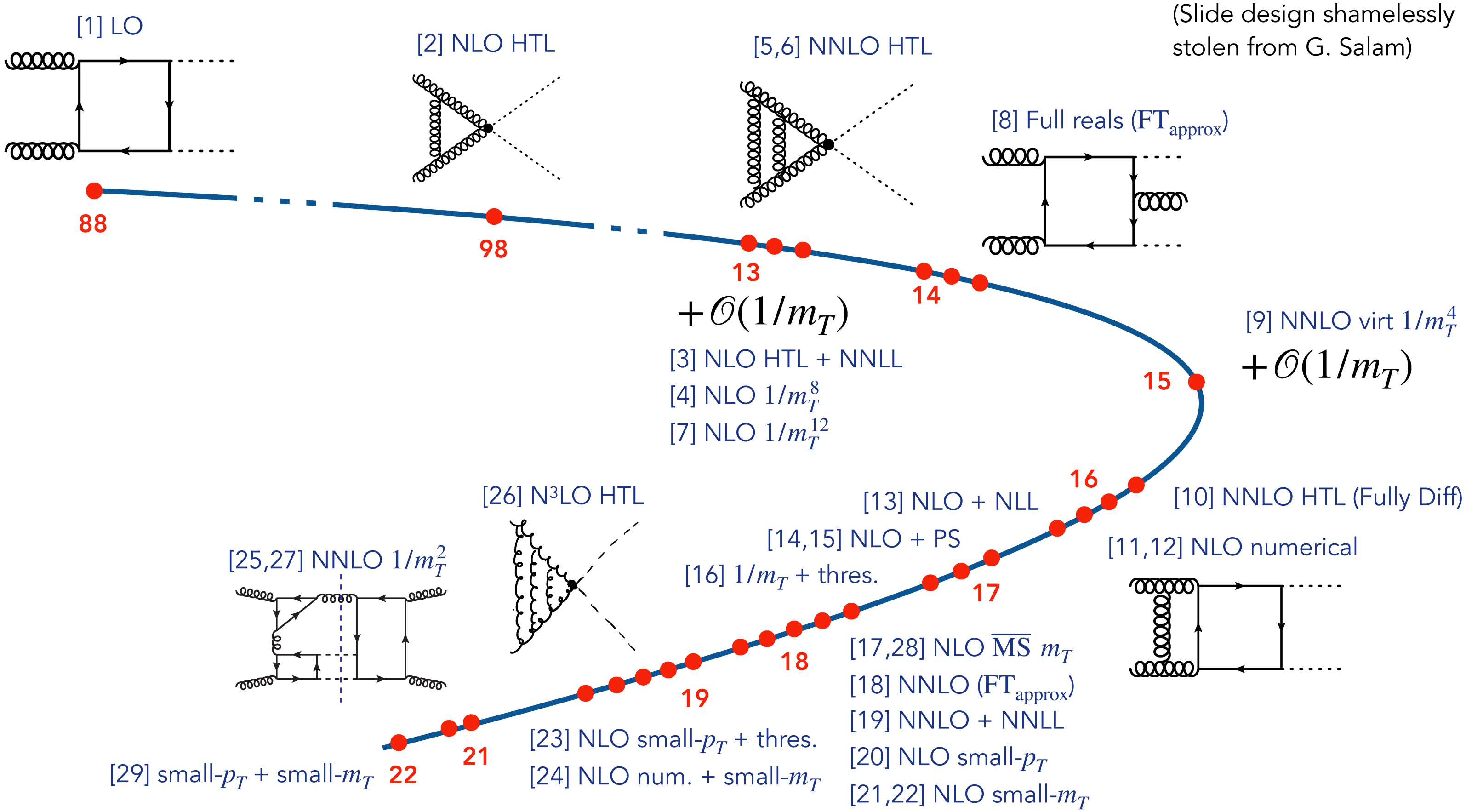
Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.

II $gg \to HH$

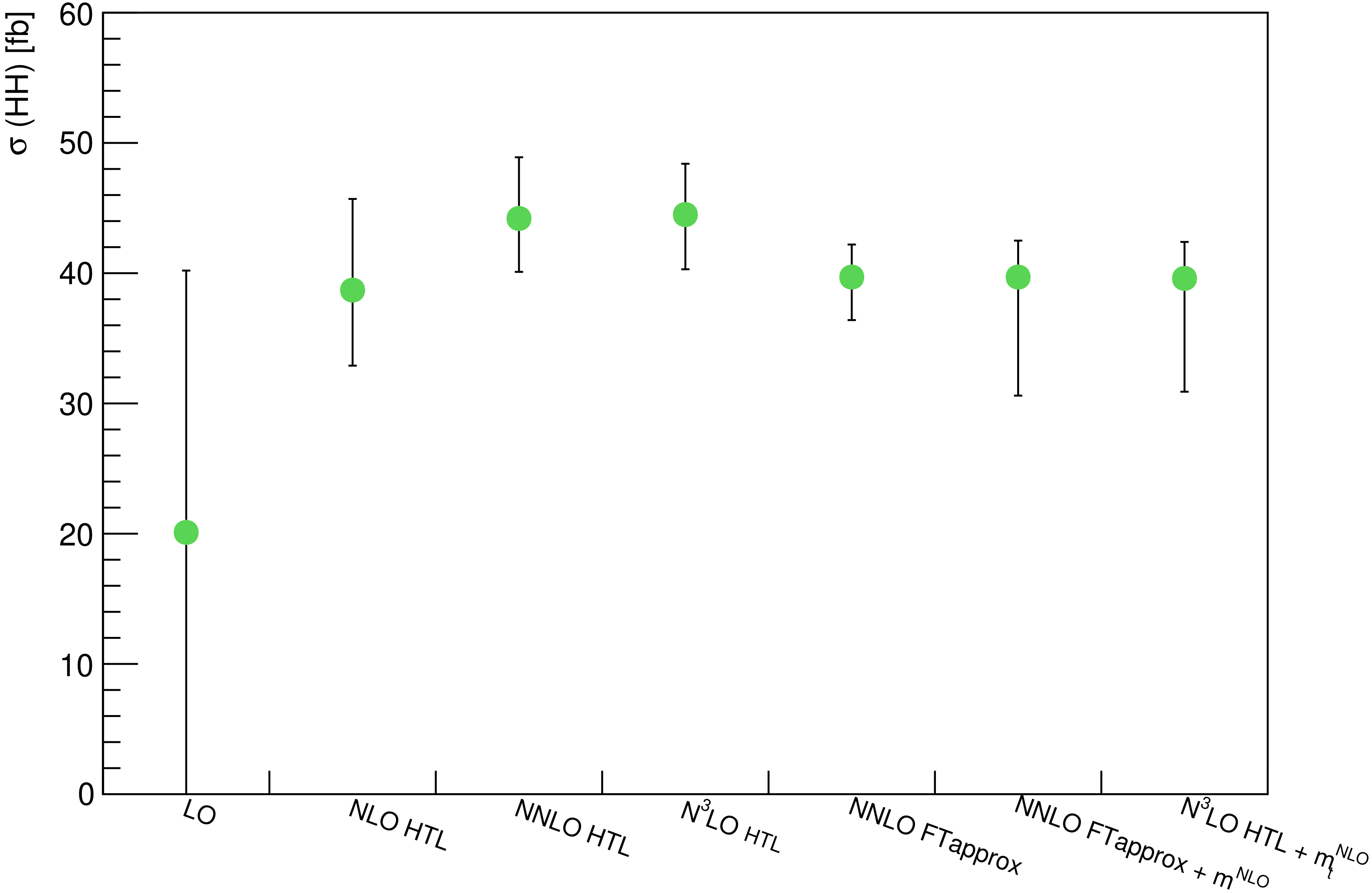
S. Jones

An approximate history (30 years in 30 seconds)

(Slide design shamelessly stolen from G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;

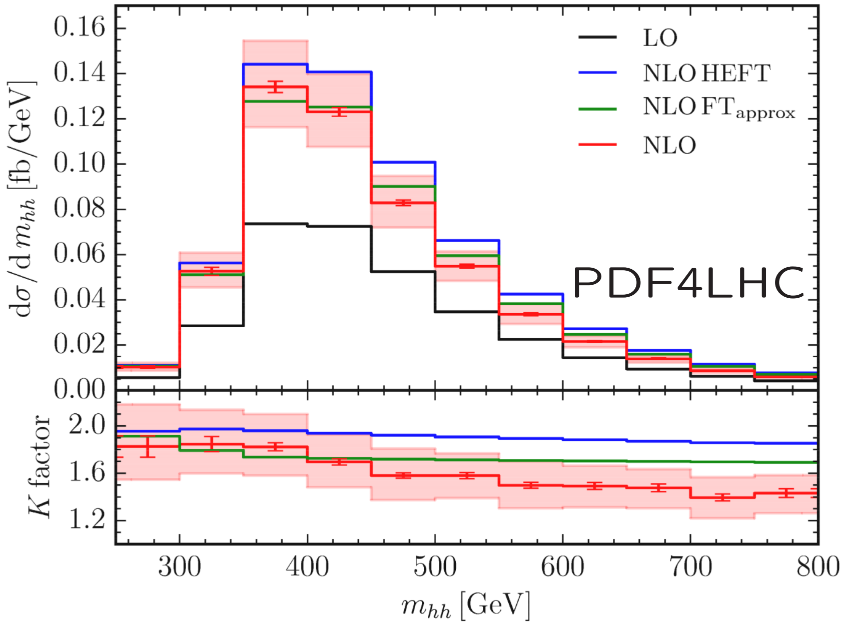


σ (HH) [fb]
60
50
40
30
20
10
0
LO
NLO HTL
NNLO HTL
N3LO HTL
NNLO FTapprox
NNLO FTapprox + m NLO
N3LO HTL + m t NLO

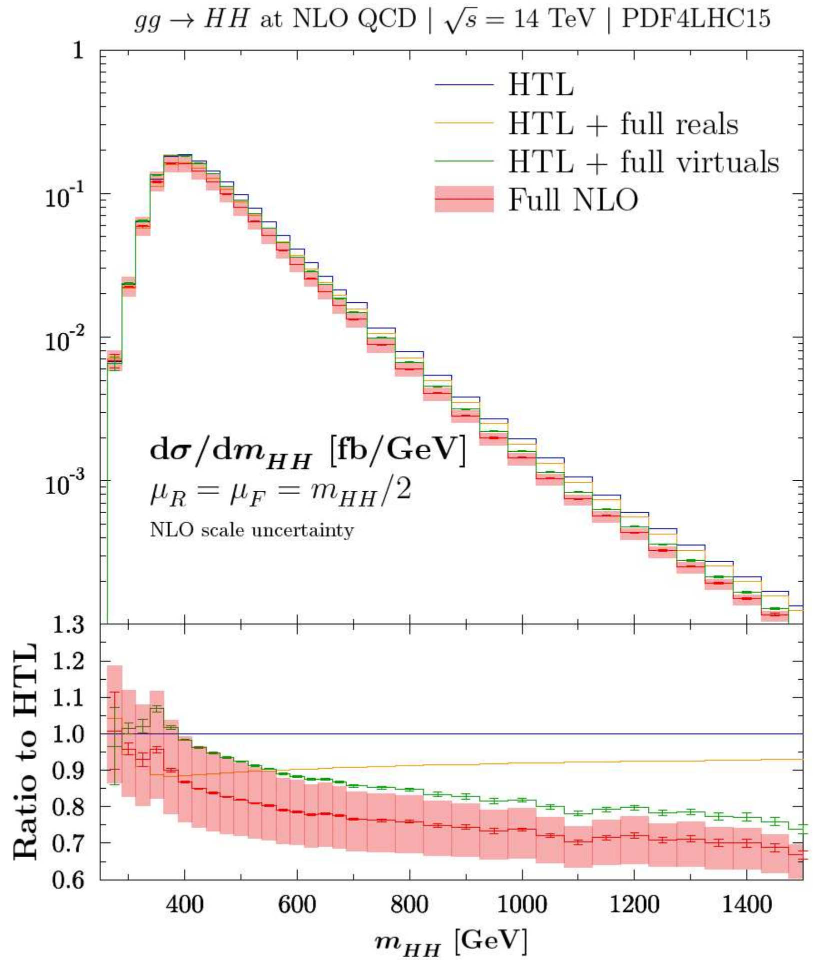
Full NLO calculation: top only, numerical integration

Borowka *et al.*	Baglio *et al.*
tensor reduction	no tensor reduction
sector decomposition	IR, end-point subtraction
contour deformation	IBP, Richardson extrapolation
$m_t = 173$ GeV	$m_t = 172.5$ GeV

Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke
Baglio, Campanario, Glaus, Mühlleitner, Ronca, S., Streicher



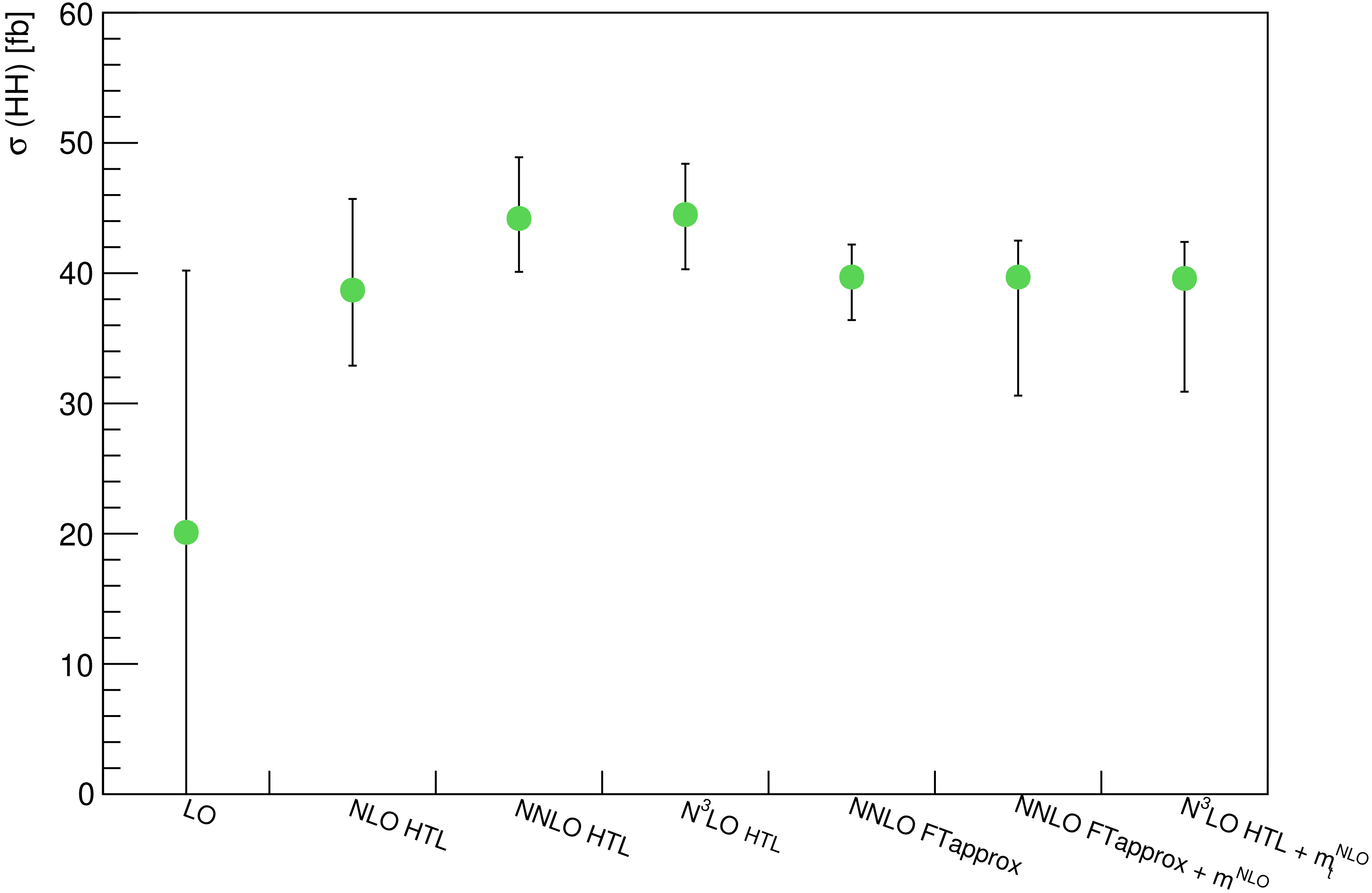
Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke



Baglio, Campanario, Glaus,
Mühlleitner, Ronca, S., Streicher

$\sigma_{NLO} =$	$32.91(10)^{+13.8\%}_{-12.8\%}\ fb$	$32.81(7)^{+13.5\%}_{-12.5\%}\ fb$
$\sigma^{HTL}_{NLO} =$	$38.75^{+18\%}_{-15\%}\ fb$	$38.66^{+18\%}_{-15\%}\ fb$
$m_t =$	173 GeV	172.5 GeV

⇒ -15% mass effects on top of LO



σ (HH) [fb]
60
50
40
30
20
10
0
LO
NLO HTL
NNLO HTL
N3LO HTL
NNLO FTapprox
NNLO FTapprox + m_t^NLO
N3LO HTL + m_t^NLO

uncertainties due to m_t

- use m_t, $\overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q$ $\rightarrow$ uncertainty = envelope:

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=300\ \text{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=400\ \text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=600\ \text{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \text{fb/GeV},$$

$$\frac{d\sigma(gg \rightarrow HH)}{dQ}\Big|_{Q=1200\ \text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

- bin-by-bin interpolation:

$$\sigma(gg \rightarrow HH) = 32.81^{+4\%}_{-18\%}\ \text{fb}$$

final combined ren./fac. scale and m_t scale/scheme unc. @ NNLO$_{FTapprox}$:

$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%} \text{ fb}$$

$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 36.69^{+6\%}_{-23\%} \text{ fb}$$

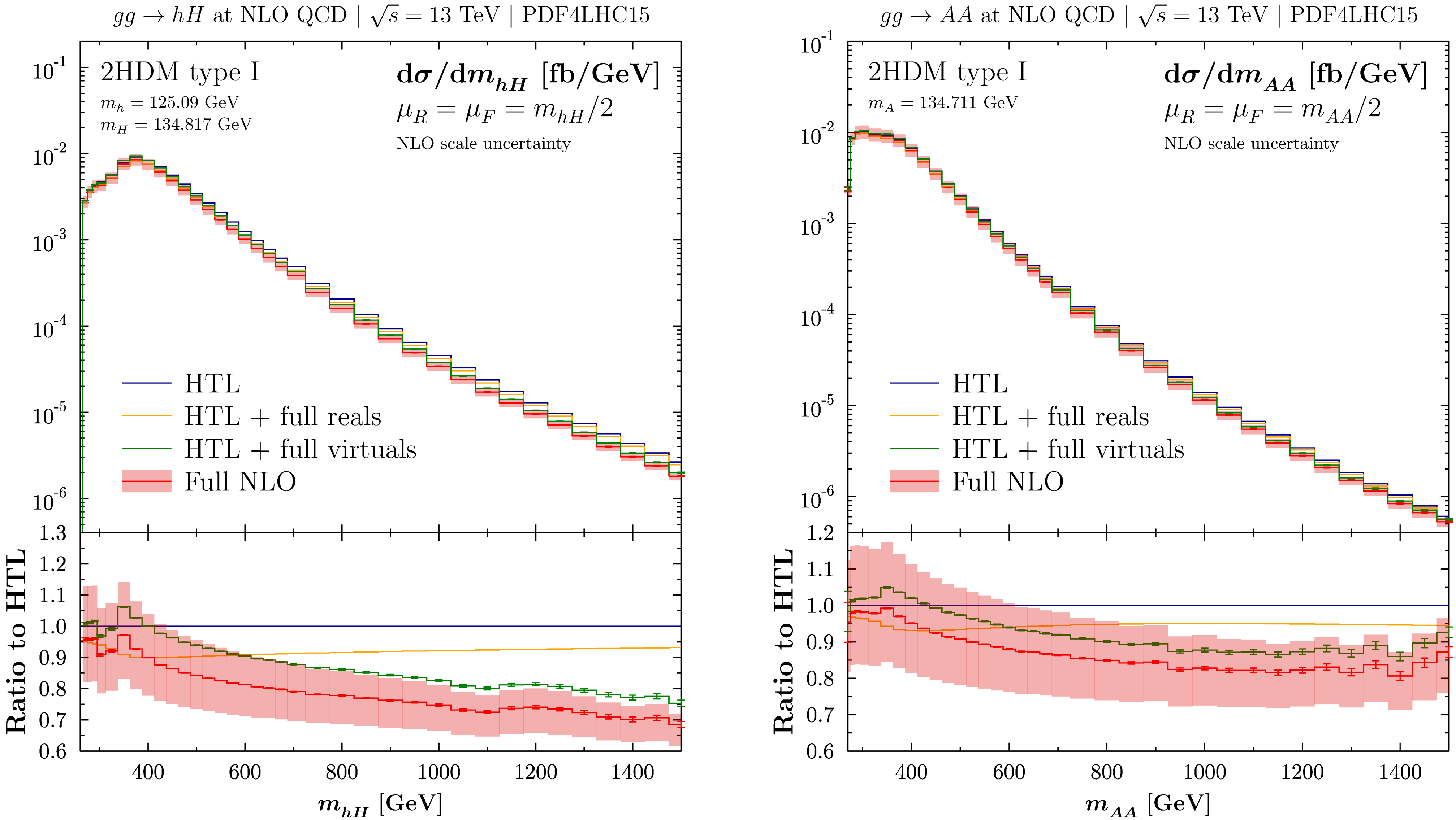
$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 139.9^{+5\%}_{-22\%} \text{ fb}$$

$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1224^{+4\%}_{-21\%} \text{ fb}$$

2HDM [type I]: $gg \to hh, hH, HH, AA$ [no $hA, HA \to$ DY-like]

$M_h = 125.09$ GeV $\quad M_H = 134.817$ GeV $\quad M_A = 134.711$ GeV

$\text{tg}\beta = 3.759 \quad \alpha = -0.102 \quad m_{12}^2 = 4305$ GeV^2 $\quad \Rightarrow \cos(\beta-\alpha) = 0.157$



Baglio, Campanario, Glaus, Mühlleitner, Ronca, S.

combined uncertainties

$$13\ \text{TeV}: \quad \sigma(gg \to hH) = 1.592(1)^{+21\%}_{-24\%}\ \text{fb}$$
$$14\ \text{TeV}: \quad \sigma(gg \to hH) = 1.876(1)^{+21\%}_{-24\%}\ \text{fb}$$
$$27\ \text{TeV}: \quad \sigma(gg \to hH) = 7.036(4)^{+18\%}_{-23\%}\ \text{fb}$$
$$100\ \text{TeV}: \quad \sigma(gg \to hH) = 60.49(4)^{+16\%}_{-25\%}\ \text{fb}$$

$$13\ \text{TeV}: \quad \sigma(gg \to AA) = 1.643(1)^{+26\%}_{-21\%}\ \text{fb}$$
$$14\ \text{TeV}: \quad \sigma(gg \to AA) = 1.927(1)^{+26\%}_{-22\%}\ \text{fb}$$
$$27\ \text{TeV}: \quad \sigma(gg \to AA) = 7.012(4)^{+23\%}_{-21\%}\ \text{fb}$$
$$100\ \text{TeV}: \quad \sigma(gg \to AA) = 58.12(3)^{+22\%}_{-22\%}\ \text{fb}$$

Is this everything?

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No. . .

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No. . .

electroweak corrections. . .

(i) y_t: HTL for $ggH(H)$ coupling + full corrections to HHH vertex

Mühlleitner, Schlenk, S.

(ii) y_t: analytical results for $ggHH$ coupling in the HEL

Davies, Mishima, Schönwald, Steinhauser, Zhang

and close to the production threshold

Davies, Schönwald, Steinhauser, Zhang

(iii) λ: elw. corrections due to the Higgs self-interactions

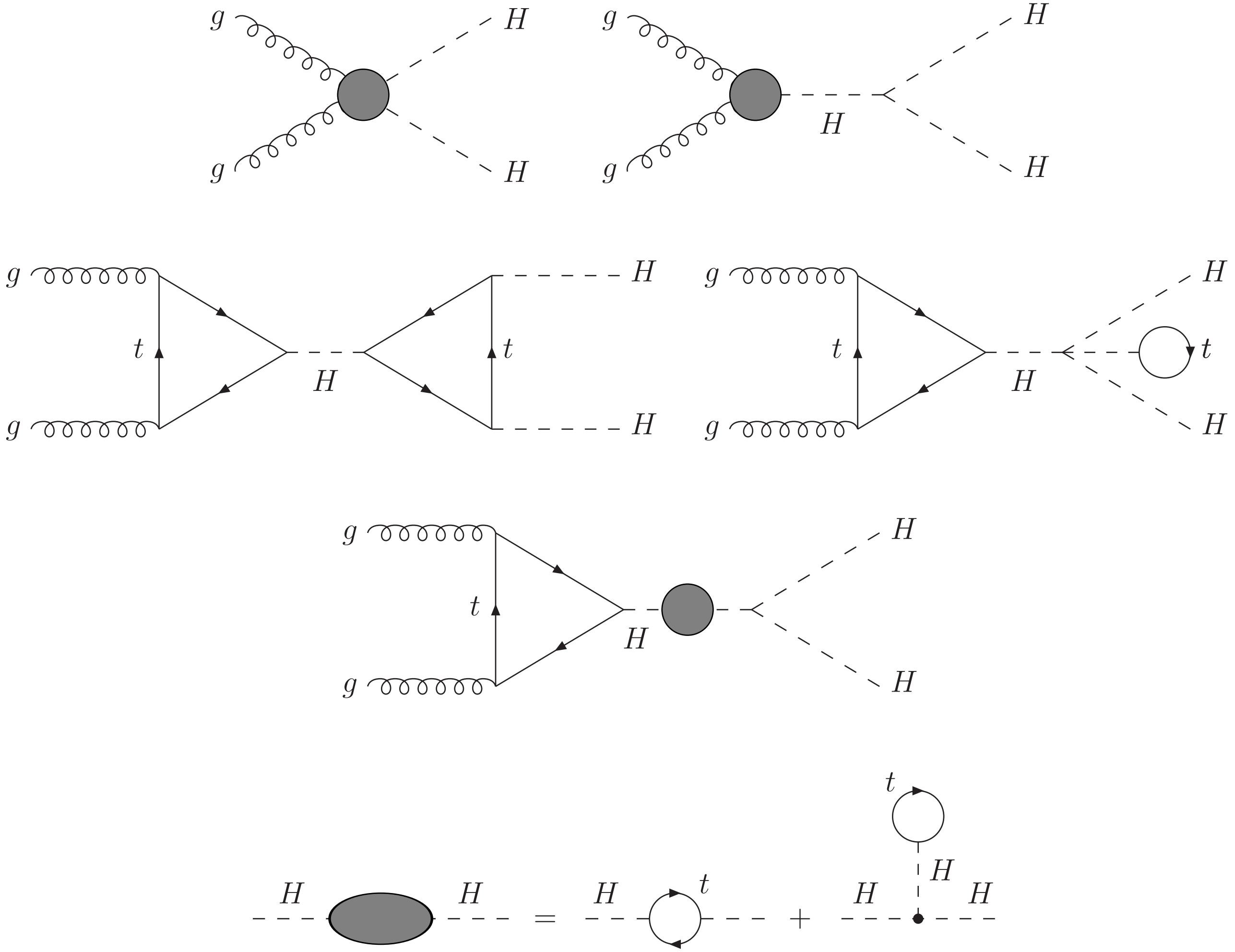
Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao

(iv) full elw. corrections ($\leftarrow$ to be checked)

Bi, Huang, Huang, Ma, Yu

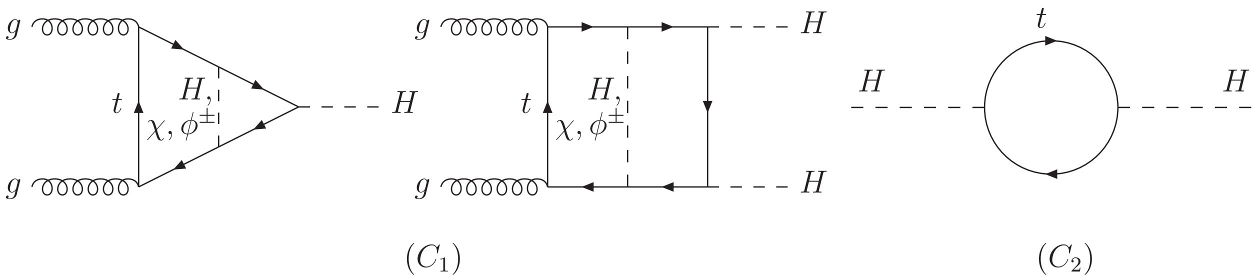
Top-induced elw. corrections

Mühlleitner, Schlenk, S.



(i) effective $ggH(H)$ couplings:

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \log\left(1 + C_2 \frac{H}{v}\right)$$



- $C_1 = 1 - 3x_t$: genuine vertex corrections $[x_t = G_F m_t^2/(8\sqrt{2}\pi^2)]$

Djoaudi, Gambino
Chetyrkin, Kniehl, Steinhauser

- $C_2 = 1 + 7x_t/2$ $[= 1 + \delta Z_H/2 - \delta v/v]$: universal corrections Kniehl, Spira

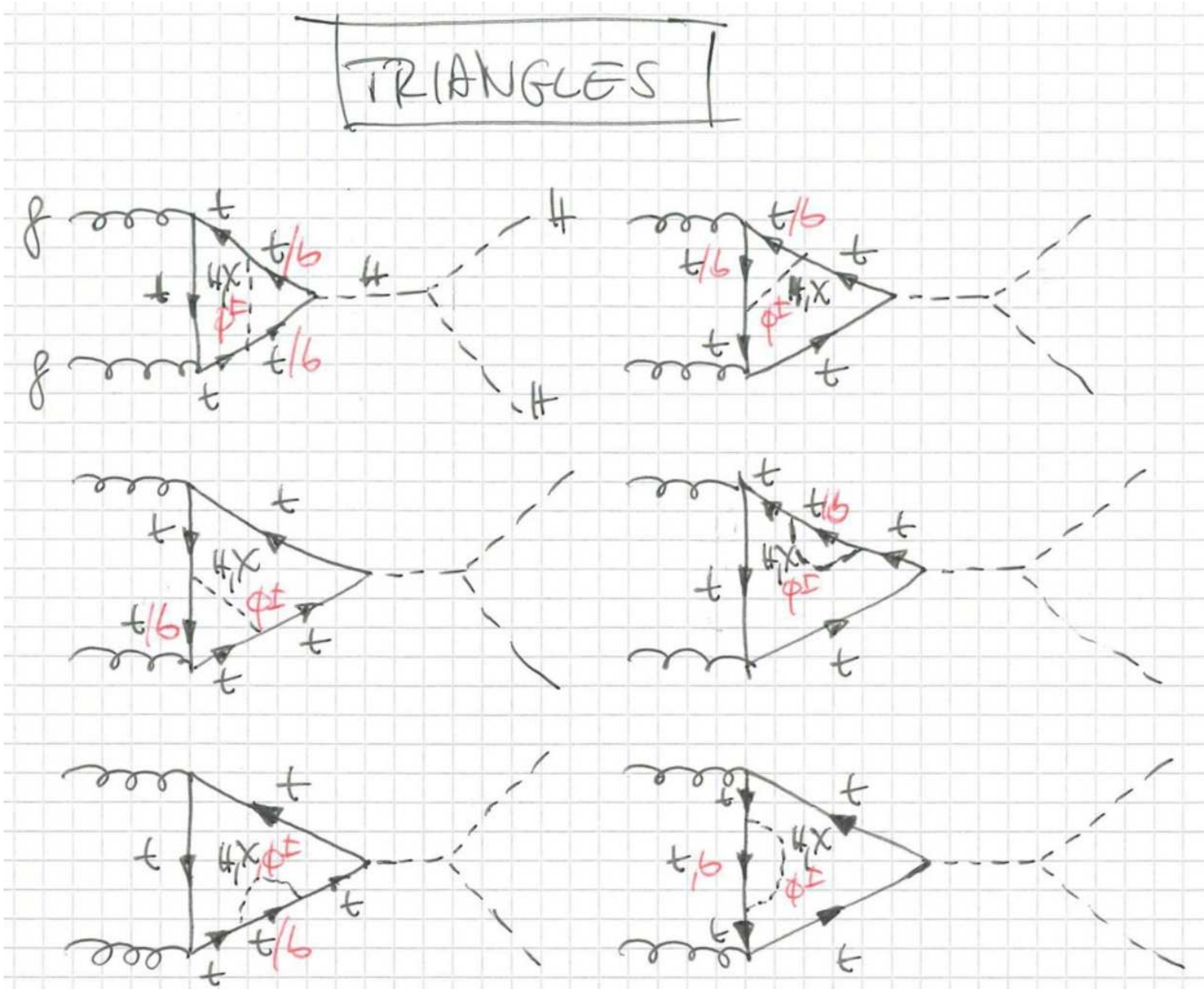
Kwiatkowski, Steinhauser

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} \left\{ (1+\delta_1)\frac{H}{v} + (1+\eta_1)\frac{H^2}{2v^2} + \mathcal{O}(H^3) \right\}$$

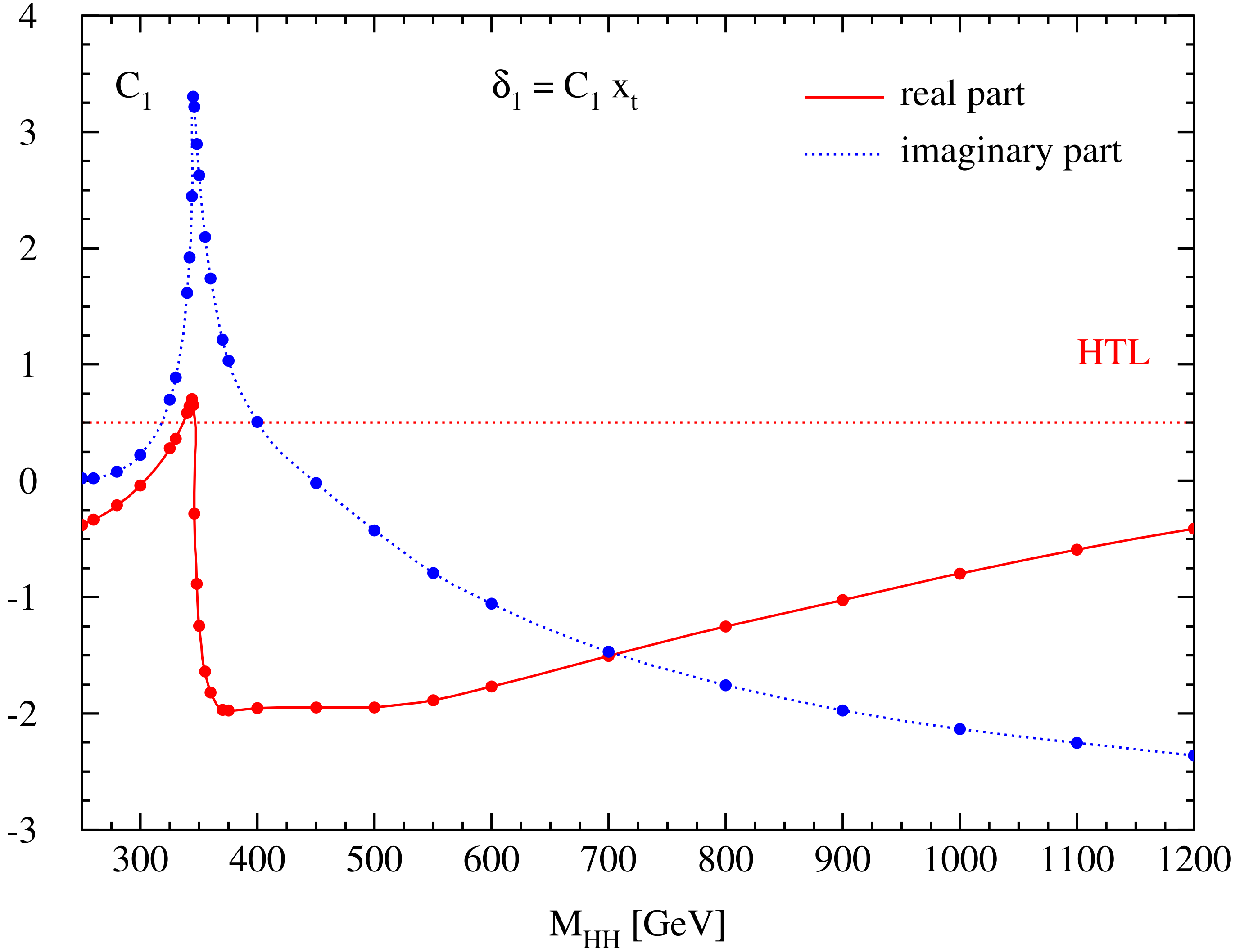
$$\delta_1 = \frac{x_t}{2} + \mathcal{O}(x_t^2) \qquad \eta_1 = 4x_t + \mathcal{O}(x_t^2)$$

[HEL: Davies, Mishima, Schönwald, Steinhauser, Zhang]

Full top-mass dependence (wave-function ren. adjusted appropriately)



PRELIMINARY



Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, S.

(ii) effective $HHH(H)$ couplings:

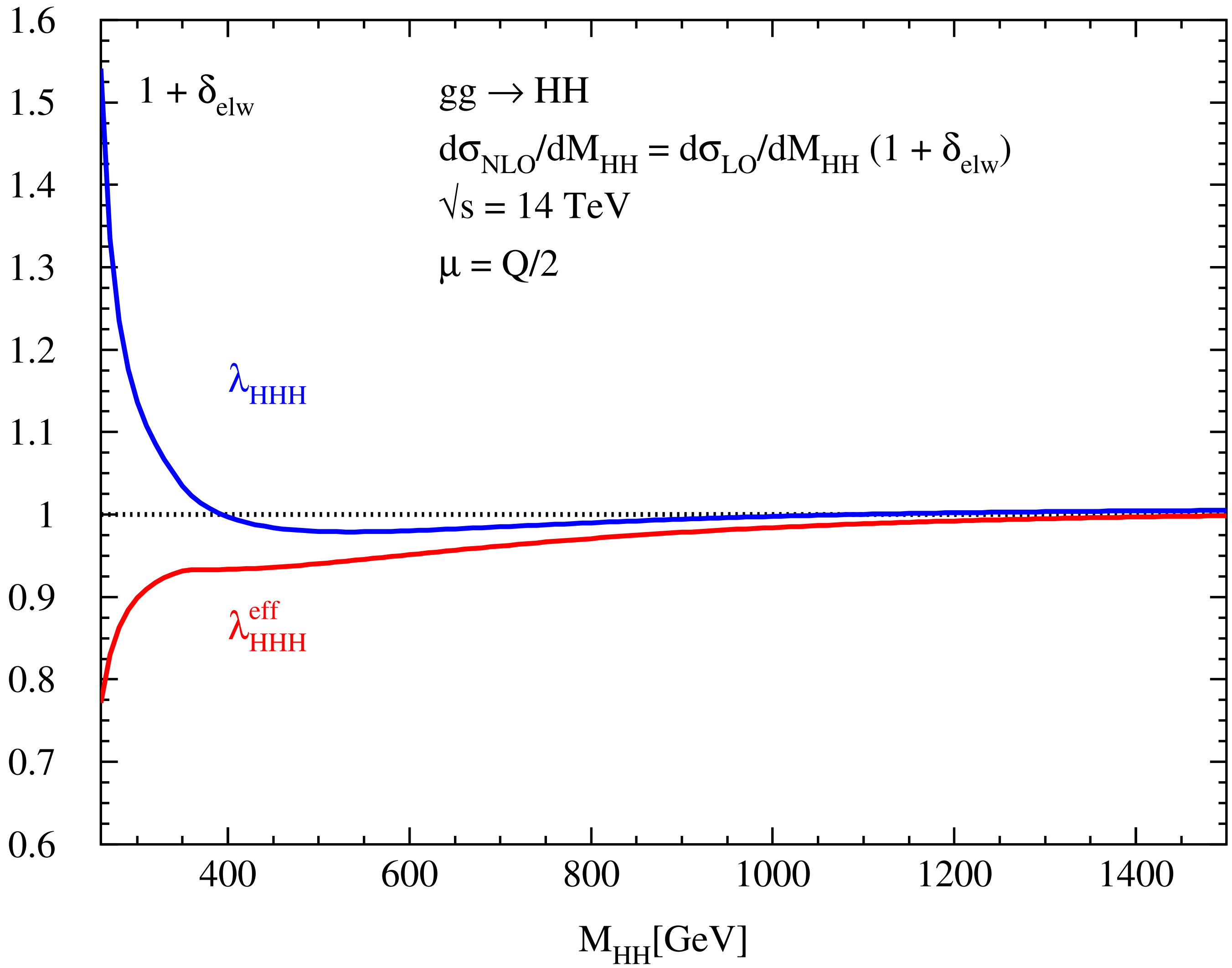
- effective Higgs potential: Coleman, Weinberg

$$
\begin{aligned}
V_{eff} &= V_0 + V_1 \\
V_0 &= \mu_0^2|\phi|^2 + \frac{\lambda_0}{2}|\phi|^4 \\
V_1 &= \frac{3\overline{m}_t^4}{16\pi^2}\Gamma(1+\epsilon)(4\pi^2)^\epsilon\left(\frac{1}{\epsilon} + \log\frac{\bar{\mu}^2}{\overline{m}_t^2} + \frac{3}{2}\right) \\
\phi &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix} \qquad \overline{m}_t = m_t\left(1+\frac{H}{v}\right)
\end{aligned}
$$

- after renormalization

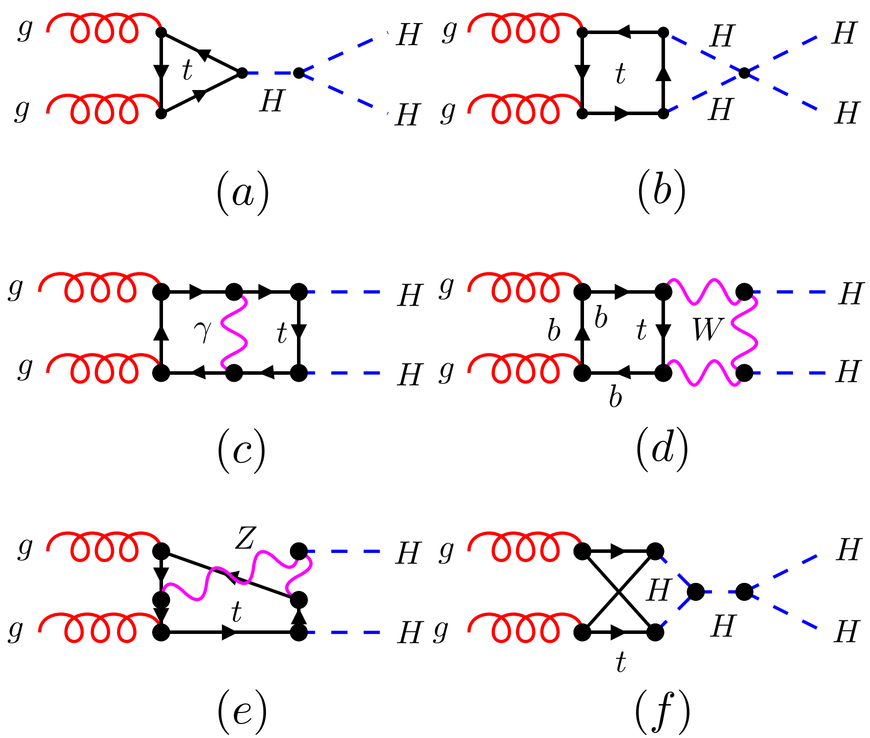
$$
\begin{aligned}
\lambda_{HHH}^{eff} &= 3\frac{M_H^2}{v} + \Delta\lambda_{HHH}\,, \qquad \lambda_{HHHH}^{eff} = 3\frac{M_H^2}{v^2} + \Delta\lambda_{HHHH} \\
\Delta\lambda_{HHH} &= -\frac{3m_t^4}{\pi^2 v^3}\,, \qquad \Delta\lambda_{HHHH} = -\frac{12m_t^4}{\pi^2 v^4}
\end{aligned}
$$

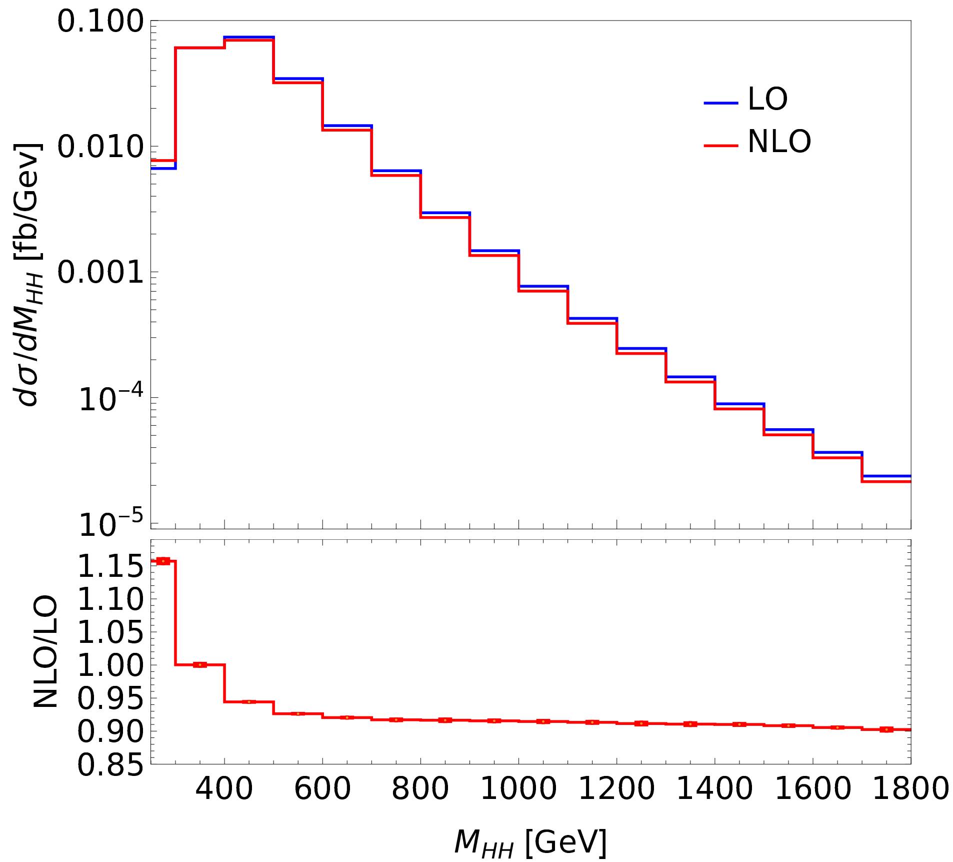
$$
\boxed{\lambda_{HHH}^{eff} = 3\frac{M_H^2}{v} - \frac{3m_t^4}{\pi^2 v^3} \approx 0.91 \times 3\frac{M_H^2}{v}}
$$



$$\sigma = 1.002 \times \sigma_{LO} \qquad (\lambda_{HHH})$$
$$\sigma = 0.938 \times \sigma_{LO} \qquad (\lambda^{eff}_{HHH})$$

Mühlleitner, Schlenk, S.





Bi, Huang, Huang, Ma, Yu

IV CONCLUSIONS

• scale and scheme uncertainties due to m_t relevant for large momenta

• Higgs pair production: m_t effects on top of LO $\sim -15\%$ for σ_{tot} [larger for distributions]

• uncertainties due to factorization/renormalization scale and m_t scale/scheme choice @NNLO$_{FT approx}$ $\lesssim 25\%$

• combined uncertainties available for λ dependence, too.

• top-induced electroweak corrections: small for total cxn, larger for distributions

• full elw. corrections $\sim 5 - 10\%$ (in absolute terms)

BACKUP SLIDES

$$\sigma_{\mathrm{NLO}}(pp \to HH + X) = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\sigma_{\mathrm{LO}} = \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s)$$

$$\Delta\sigma_{\mathrm{virt}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s) \; C$$

$$\Delta\sigma_{gg} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log \frac{M^2}{\tau s} + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z} \right)_+ \right\}$$

$$\Delta\sigma_{gq} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1-z)^2} + d_{gq}(z) \right\}$$

$$\Delta\sigma_{q\bar{q}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z} \, \hat{\sigma}_{\mathrm{LO}}(Q^2 = z\tau s) \; d_{q\bar{q}}(z)$$

$$C \to \pi^2 + \frac{11}{2} + C_{\triangle\triangle}, \quad d_{gg} \to -\frac{11}{2}(1-z)^3, \quad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

- m_t scale/scheme uncertainties at LO:

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300\ \text{GeV}} = 0.01656^{+62\%}_{-2.4\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=400\ \text{GeV}} = 0.09391^{+0\%}_{-20\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=600\ \text{GeV}} = 0.02132^{+0\%}_{-48\%}\ \text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200\ \text{GeV}} = 0.0003223^{+0\%}_{-56\%}\ \text{fb/GeV}$$

$$\begin{aligned} F_i &= F_{i,LO} + \Delta F_i \\ \Delta F_i &= \Delta F_{i,HTL} + \Delta F_{i,mass} \end{aligned}$$

- pole mass:

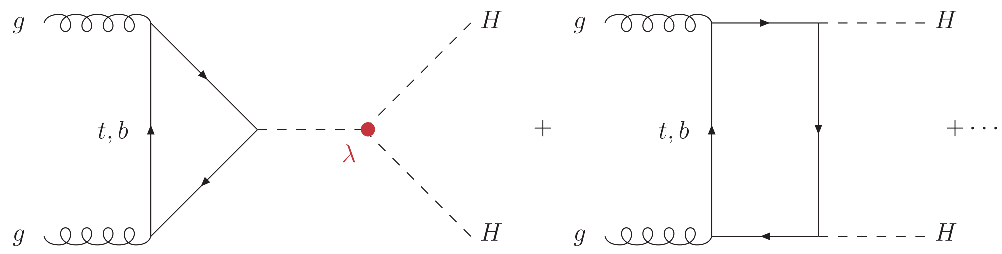
$$\begin{aligned} F_{1,LO} &\to 4\frac{m_t^2}{\hat{s}} \\ F_{2,LO} &\to -\frac{m_t^2}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\} \end{aligned}$$

- $\overline{\text{MS}}$ mass:

$$\begin{aligned} F_{1,LO} &\to 4\frac{\overline{m}_t^2(\mu_t)}{\hat{s}} \\ F_{2,LO} &\to -\frac{\overline{m}_t^2(\mu_t)}{\hat{s}\hat{t}(\hat{s}+\hat{t})}\{(\hat{s}+\hat{t})^2 L_{1ts}^2 + \hat{t}^2 L_{ts}^2 + \pi^2[(\hat{s}+\hat{t})^2 + \hat{t}^2]\} \end{aligned}$$

• different scales for y_t in triangle (Q) and box (M_H) diagrams?

$\rightarrow$ has to hold at all orders



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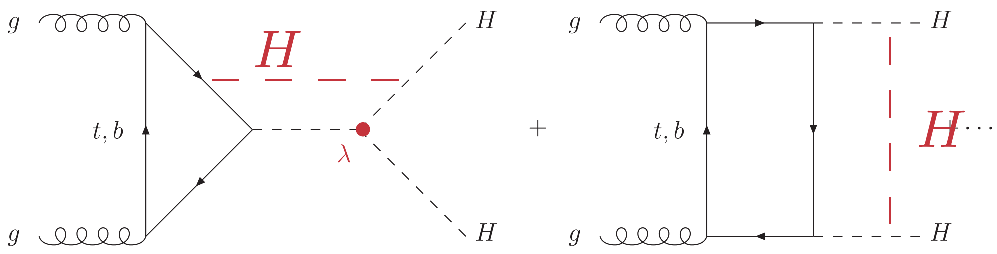

elw. corrections

$\Rightarrow$ same scales in all diagrams

$$\sigma_{\mathrm{LO}} = \int_{\tau_0}^{1} d\tau \; \frac{d\mathcal{L}^{gg}}{d\tau} \; \hat{\sigma}_{\mathrm{LO}}(Q^2 = \tau s)$$

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} g(x, \mu_F) g\left(\frac{\tau}{x}, \mu_F\right)$$

$$\hat{\sigma}_{LO} = \frac{G_F^2 \alpha_s^2(\mu_R)}{512(2\pi)^3} \int_{\hat{t}_-}^{\hat{t}_+} d\hat{t} [|C_\triangle F_\triangle + F_\Box|^2 + |G_\Box|^2]$$

$$\hat{t}_\pm = -\frac{1}{2}\left[Q^2 - 2M_H^2 \mp Q^2 \sqrt{1 - 4\frac{M_H^2}{Q^2}}\right]$$

$$\lambda_{HHH} = 3\frac{M_H^2}{v}$$

$$C_\triangle = \frac{\lambda_{HHH} v}{(Q^2 - M_H^2)}$$

$$\text{HTL: } F_\triangle \to 2/3, \qquad F_\Box \to -2/3, \qquad G_\Box \to 0$$

$$C_\triangle F_\triangle \to C_\triangle F_\triangle (1 + \Delta_\triangle)$$

$$F_\Box \to F_\Box (1 + \Delta_\Box)$$

$$\Delta_\triangle = \delta_1 + \Delta_{HHH}$$

$$\Delta_\Box = \eta_1$$

$$
\begin{aligned}
\Delta_{HHH} &= \Delta_{vertex} + \Delta_{self} + \Delta_{CT} \\
\Delta_{vertex} &= \frac{m_t^4}{v^2 M_H^2}\frac{8}{(4\pi)^2}\Big\{B_0(Q^2;m_t,m_t) + 2B_0(M_H^2;m_t,m_t) \\
&\quad + \left(4m_t^2 - \frac{Q^2+2M_H^2}{2}\right) C_0(Q^2,M_H^2,M_H^2;m_t,m_t,m_t)\Big\} + \frac{T_1}{vM_H^2} \\
\Delta_{self} &= \frac{\Sigma_H(Q^2)}{Q^2-M_H^2} + \frac{1}{2}\Sigma'_H(M_H^2) \\
\Delta_{CT} &= \frac{\delta M_H^2}{Q^2-M_H^2} + \frac{\delta\lambda_{HHH}}{\lambda_{HHH}} \\
\Sigma_H(Q^2) &= 3\,\frac{T_1}{v} + 6\,\frac{m_t^2}{(4\pi)^2 v^2}\Big\{2A_0(m_t) + (4m_t^2-Q^2)B_0(Q^2;m_t,m_t)\Big\} + \mathcal{O}(m_t^0) \\
\Sigma'_H(Q^2) &= 6\,\frac{m_t^2}{(4\pi)^2 v^2}\Big\{(4m_t^2-Q^2)B'_0(Q^2;m_t,m_t) - B_0(Q^2;m_t,m_t)\Big\} + \mathcal{O}(m_t^0) \\
\frac{T_1}{v} &= -12\,\frac{m_t^2}{(4\pi)^2 v^2}A_0(m_t) \\
\frac{\delta\lambda_{HHH}}{\lambda_{HHH}} &= \frac{\delta M_H^2}{M_H^2} + \frac{1}{2}\frac{\Sigma_W(0)}{M_W^2} \\
\frac{\Sigma_W(0)}{M_W^2} &= 2\,\frac{T_1}{vM_H^2} + \frac{2m_t^2}{(4\pi)^2 v^2}\Big\{B_0(0;m_t,0) + 2B_0(0;m_t,m_t) + m_t^2 B'_0(0;m_t,0)\Big\} + \mathcal{O}(m_t^0) \\
\delta M_H^2 &= -\Sigma_H(M_H^2)
\end{aligned}
$$