

Exploring loop-induced 1st order EW phase transition in the Higgs EFT

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EW phase transition

EWSB has been confirmed

⇒ EWPT exists in thermal history of Universe

Next target!

Aspect of the PT is crucial for EW Baryogenesis

Required extended Higgs models

Example: 2HDM with softly broken Z2

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)} \\ + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + (\text{h.c.}) \right]$$

$$\Phi_1 \text{ and } \Phi_2 \Rightarrow \underbrace{h, H, A^0, H^\pm}_{\substack{\uparrow \text{ CP even} \\ \uparrow \text{ CP odd} \\ \uparrow \text{ charged}}} \oplus \text{Goldstone bosons}$$

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + ia_i) \end{bmatrix} \quad (i = 1, 2)$$

Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix} \\ \begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

masses

$$m_h^2 = v^2 \left(\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda}{2} \sin^2 2\beta \right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

$$m_{A, H, H^\pm}^2 \simeq M^2 + \lambda_i v^2$$

$$\frac{v_2}{v_1} \equiv \tan \beta$$

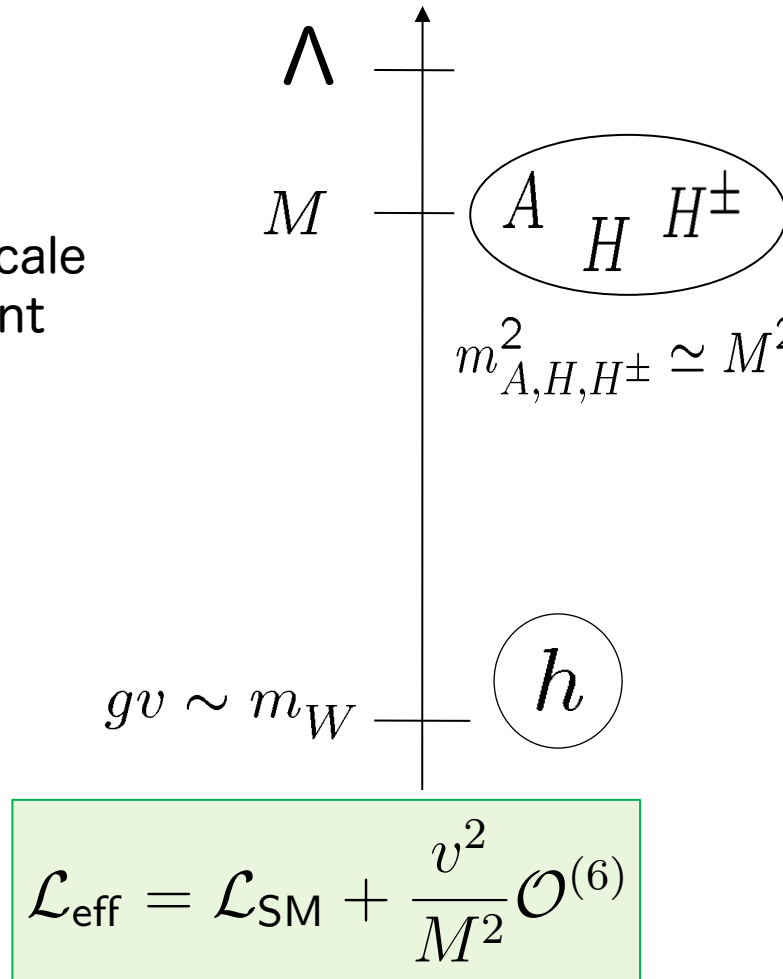
$$M_{\text{soft}} \left(= \frac{m_3}{\sqrt{\cos \beta \sin \beta}} \right):$$

soft-breaking scale
of the discrete symm.

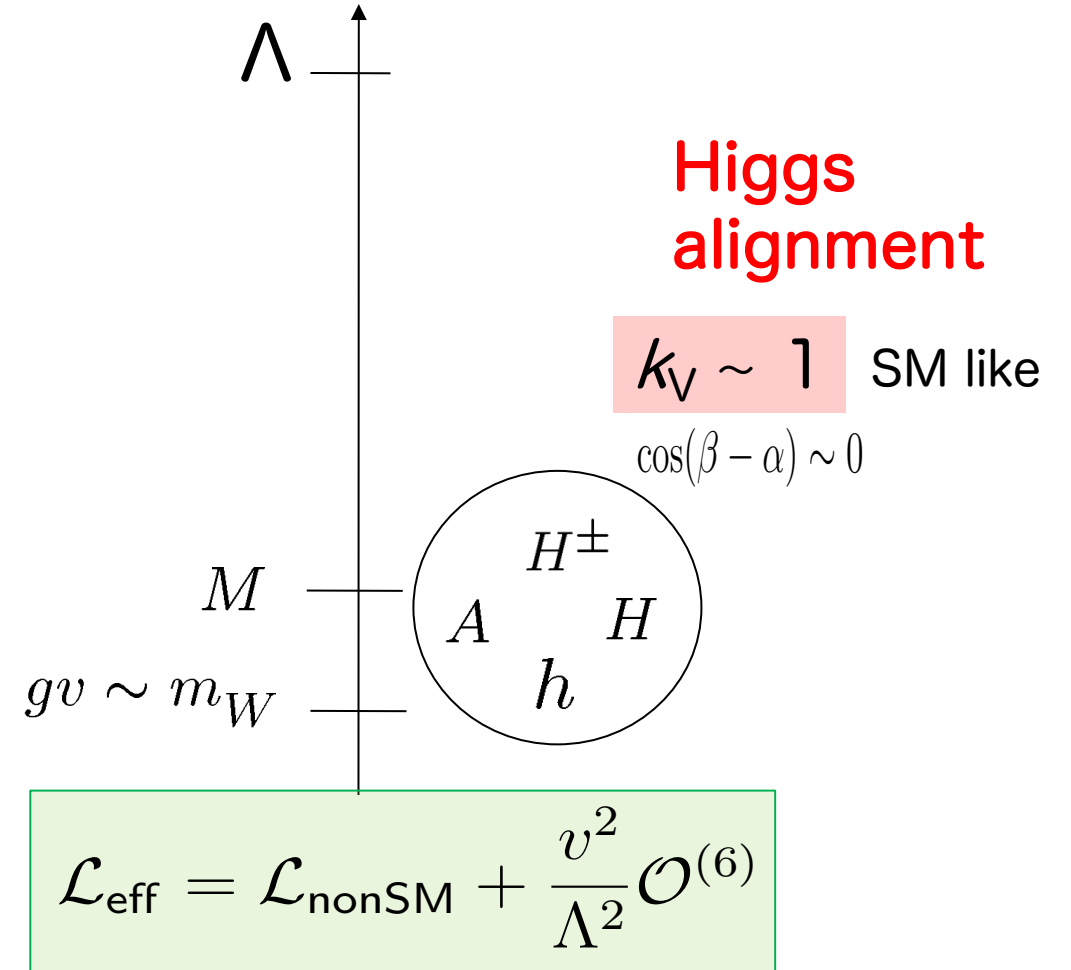
How SM-like is realized?

Λ : Cutoff

M : Mass scale irrelevant to VEV



Effective Theory is the SM
Decoupling ($M \gg v$)



Effective Theory is an extended Higgs sector
Alignment and Non-Decoupling ($M \sim v$)

EW Baryogenesis

Sakharov Conditions

Kuzmin, Ruvakov, Shaposhnikov (1985)

- 1) B non-conservation \Rightarrow Sphaleron transition at high T
- 2) C and CP violation \Rightarrow C violation (SM is a chiral theory)
CP in BSM sectors
- 3) Departure from thermal equilibrium \Rightarrow **EWPT is strongly 1st OPT**

Extension of the Higgs sector is required

Condition of Strongly First OPT

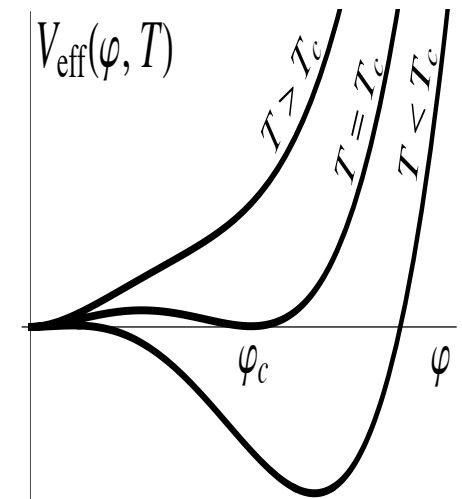
In the broken phase, sphaleron should quickly decouple to avoid wash out

$$\Gamma_{\text{sph}} < H$$



$$\frac{\varphi_c}{T_c} \gtrsim 1$$

Physics of Higgs potential



1st OPT by nondecoupling quantum effect

Effective Potential
at finite T (HTE)

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - \underline{ET}\varphi^3 + \frac{\lambda_T}{4}\varphi^4 + \dots$$

$$\frac{\varphi_c}{T_c} \gtrsim 1$$

SM: The condition cannot be satisfied

Non-minimal Higgs can satisfy it due to **non-decoupling quantum effects**

$$\frac{\phi_C}{T_C} \simeq \frac{1}{3\pi v m_h^2} \left\{ 6m_W^2 + 3m_Z^2 + \underbrace{\sum_{\Phi} n_{\Phi} m_{\Phi}^3 \left(1 - \frac{M^2}{m_{\Phi}^2}\right)^{3/2}}_{\text{Quantum effects of } \Phi (= H, A, H^+, \dots)} \right\} > 1 \quad (\text{when } M \ll m_{\Phi})$$

Prediction: Large deviation in **the hhh coupling**

$$\lambda_{hhh} \simeq \frac{3m_h^2}{v} \left\{ 1 - \frac{m_t^4}{\pi^2 v^2 m_h^2} + \underbrace{\sum_{\Phi} n_{\Phi} \frac{m_{\Phi}^4}{12\pi^2 v^2 m_h^2} \left(1 - \frac{M^2}{m_{\Phi}^2}\right)^3}_{\text{Quantum effects of } \Phi} \right\} > \lambda_{hhh}^{\text{SM}}$$

Grojean, Servant, Wells, 2005

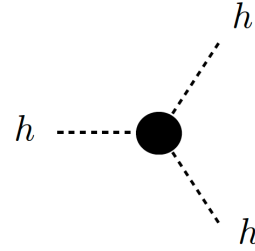
SK, Y. Okada, E. Senaha, 2005

Test of strongly 1st OPT

Strongly 1st OPT

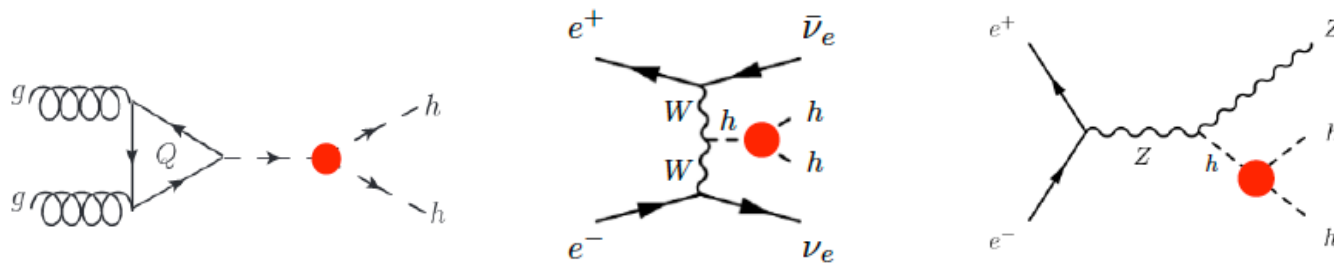
→ A large deviation in the hhh coupling

SK, Y. Okada, E. Senaha, 2005



Example →
Aligned 2HDM:
viable scenario
of EWBG

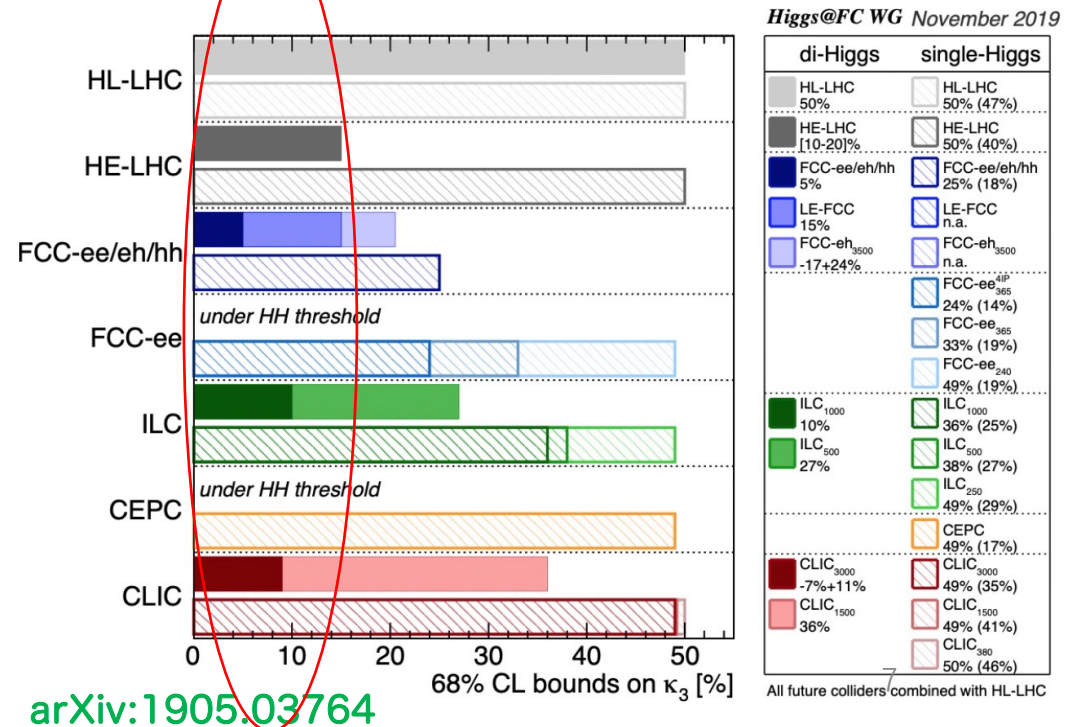
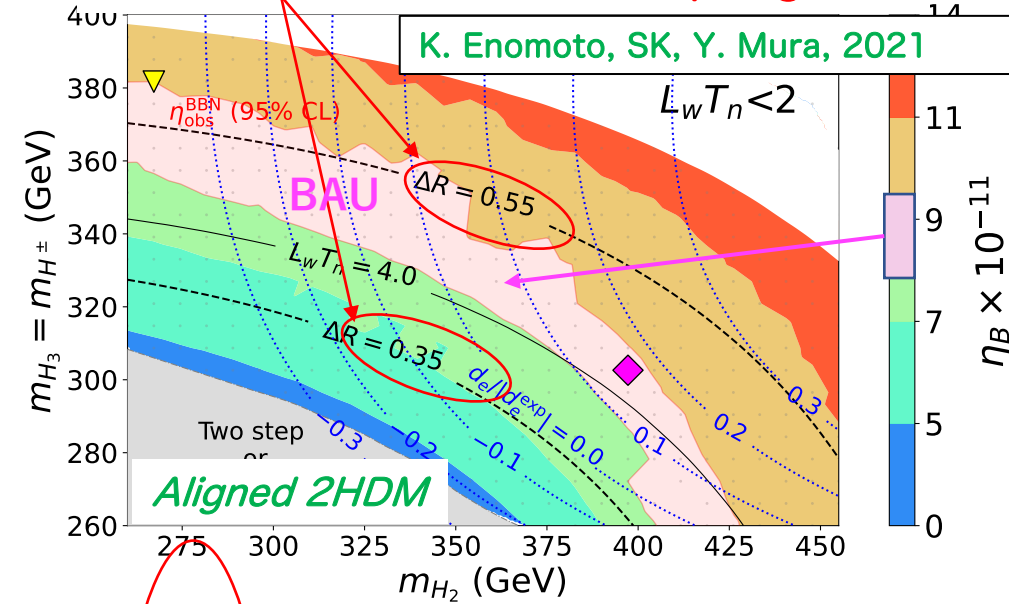
The hhh coupling can be measured at HL-LHC, or future e⁺e⁻ colliders



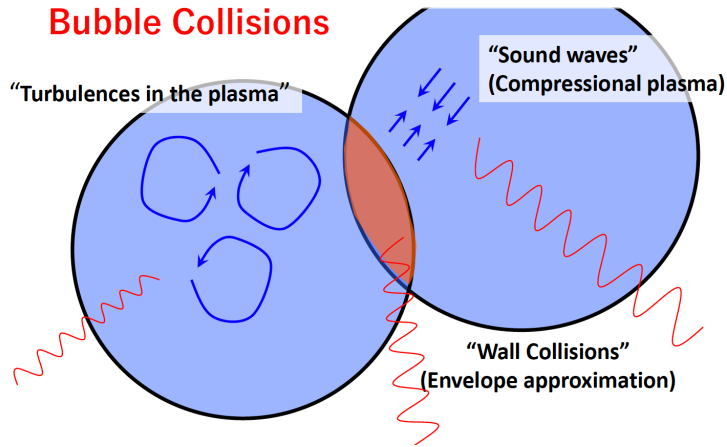
EW Baryogenesis can be directly tested by the hhh measurement

H_{γγ} can also be sensitive to non-decoupling effects

Deviation in the hhh coupling (%)

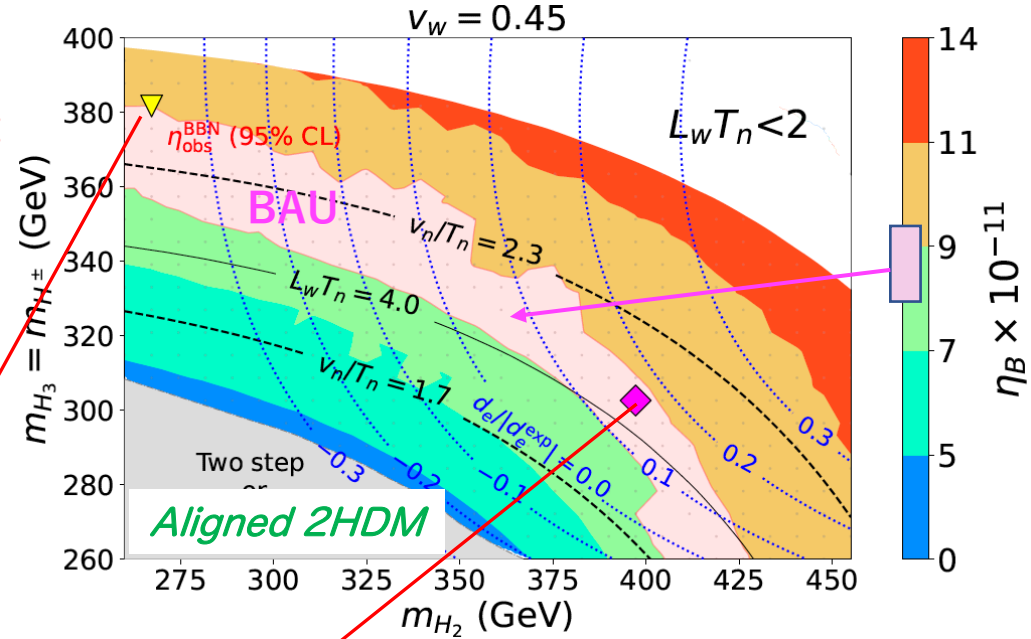


GW from 1stOPT

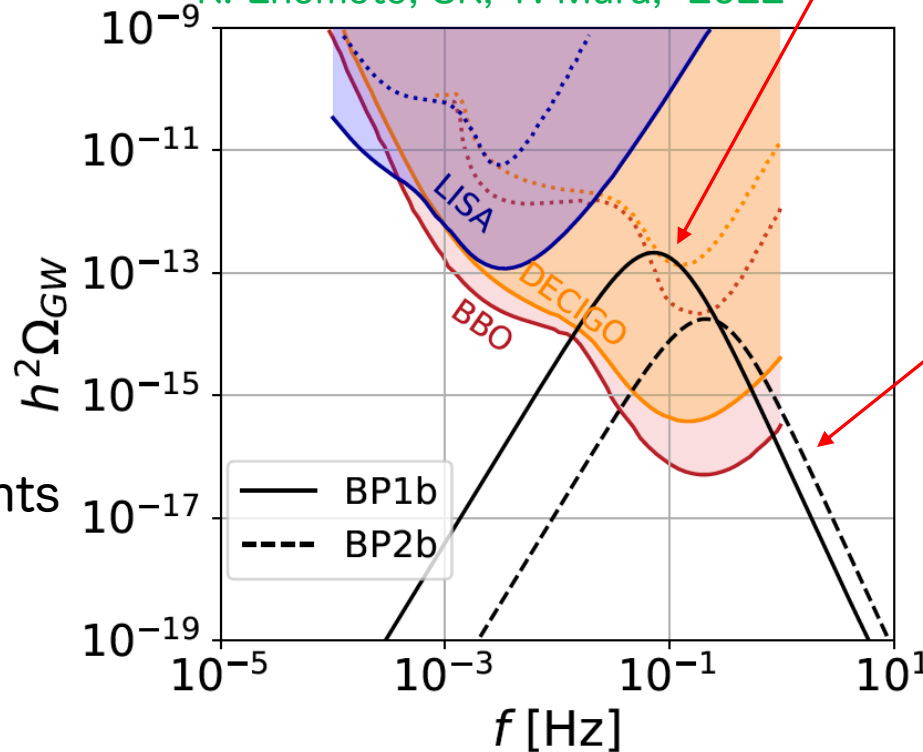


Aligned 2HDM:
viable scenario
of EWBG

K. Enomoto, SK, Y. Mura, 2022



K. Enomoto, SK, Y. Mura, 2022



GWs for benchmark
points of **BAU**

They may be tested
by future GW experiments

Dotted curves: Sensitivity Curve
M. Breitbach et al., arXiv: 1811.11175

Solid curves: $h^2 \Omega_{\text{PISC}}$ [SNR criterion]
J. Cline et al., arXiv: 2102.12490

In this talk

- We discuss how to test the strongly 1st OPT using HEFT
- Nearly aligned HEFT
(SM-like: assuming small mixing and deviation in Higgs couplings mainly comes from quantum effects of BSM)
- Simply EWPT can be described by parameters
 - κ_0 (d.o.f. of new particle)
 - Λ (mass of new particle),
 - r (non-decouplingness)

κ_0 : d.o.f of new particles with non-decoupling property

$$\kappa_0 = n_0 + 2 n_+ + 2 n_{++} + \dots$$

(n_0, n_+, n_{++} : d.o.f. of each neutral/charged particle)
- How we can test 1st OPT by future experiments?

Nearly aligned Higgs EFT

$$\mathcal{L}_{\text{naHEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}},$$

$$\mathcal{L}_{\text{BSM}} = \xi \left[-\frac{\kappa_0}{4} [\mathcal{M}^2(h)]^2 \ln \frac{\mathcal{M}^2(h)}{\mu^2} \right.$$

$$\left. + \frac{v^2}{2} \mathcal{F}(h) \text{Tr} [D_\mu U^\dagger D^\mu U] + \frac{1}{2} \mathcal{K}(h) (\partial_\mu h) (\partial^\mu h) \right.$$

$$\left. - v \left(\bar{q}_L^i U \left[\mathcal{Y}_q^{ij}(h) + \hat{\mathcal{Y}}_q^{ij}(h) \tau^3 \right] q_R^j + h.c. \right) - v \left(\bar{l}_L^i U \left[\mathcal{Y}_l^{ij}(h) + \hat{\mathcal{Y}}_l^{ij}(h) \tau^3 \right] l_R^j + h.c. \right) \right.$$

$$\left. + g^2 \mathcal{F}_W(h) \text{Tr} [\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] + g'^2 \mathcal{F}_B(h) \text{Tr} [\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}] \right.$$

$$\left. - gg' \mathcal{F}_{BW}(h) \text{Tr} [U \mathbf{B}_{\mu\nu} U^\dagger \mathbf{W}^{\mu\nu}] \right]$$

$$\xi = \frac{1}{16\pi^2} \quad U = \exp \left(\frac{i}{v} \pi^a \tau^a \right)$$

$$\mathcal{M}^2(h), \mathcal{F}(h), \mathcal{K}(h), \mathcal{Y}_\psi^{ij}(h), \hat{\mathcal{Y}}_\psi^{ij}(h)$$

arbitrary polynomials

To describe non-decoupling effects
we put a CW type structure (1-loop)

SK, R. Nagai (2021)

Buchalla, et al (2013)

naHEFT (for describing non-decoupling property)

(nearly aligned)

SK, R. Nagai (2021)

$$\mathcal{L}_{\text{naHEFT}} = \mathcal{L}_{\text{SM}} - \frac{\kappa_0}{64\pi^2} [\mathcal{M}^2(\varphi)]^2 \ln \frac{\mathcal{M}^2(\varphi)}{\mu^2}$$

$$\begin{aligned} \mathcal{M}^2(h) &= M^2 + \frac{\kappa_p}{2} \varphi^2 \\ &= M^2 + \frac{\kappa_p}{2} (h + v)^2 \end{aligned}$$

Three free parameters Λ , κ_0 , r

$$\Lambda = \sqrt{M^2 + \frac{\kappa_p}{2} v^2}, \quad \kappa_0, \quad r = \frac{\frac{\kappa_p v^2}{2}}{\Lambda^2} \quad \text{Non-decouplingness}$$

Mass of New particles d.o.f of new particles

$$\left\{ \begin{array}{ll} r \sim 0 \Rightarrow M^2 \gg \frac{\kappa_p}{2} v^2 & \text{Decoupling} \\ r \sim 1 \Rightarrow M^2 \ll \frac{\kappa_p}{2} v^2 & \text{Non-decoupling} \end{array} \right.$$

In the decoupling region ($M^2 \gg \kappa_p v^2$),

$$V_{\text{BSM}}(\varphi) \simeq \frac{\lambda_\Phi^3}{64\pi^2 M^2} \varphi^6 = \frac{1}{\Lambda^2} \varphi^6 \Rightarrow \text{SMEFT is a good approximation}$$

SMEFT is not good in the non-decoupling region ($M^2 < \kappa_p v^2$)

Higgs couplings in naHEFT

Nearly aligned case:

small mixing and Higgs couplings can deviate mainly by quantum corrections

Λ : mass of new particles

κ_0 : d.o.f. of new particles

r : non-decouplingness

$$\kappa_0 = n_0 + 2 n_+ + 2 n_{++} + \dots$$

$$b = (n_+ + 4n_{++})/3$$

$$\xi = 1/(4\pi)^2$$

$$F_{\text{SM}} = 6.492$$

$$G_{\text{SM}} = 11.65$$

$$\kappa_V = \kappa_f = 1 - \kappa_0 \frac{\xi}{6} \frac{\Lambda^2}{v^2} r^2,$$

$$\kappa_3 = 1 + \kappa_0 \frac{4\xi}{3} \frac{\Lambda^4}{v^2 m_h^2} \left[r^3 - \frac{m_h^2}{8\Lambda^2} r^2 (3 - 2r) \right],$$

$$\kappa_{\gamma\gamma}^2 \simeq \left| \kappa_V - \frac{br}{F_{\text{SM}}} \right|^2,$$

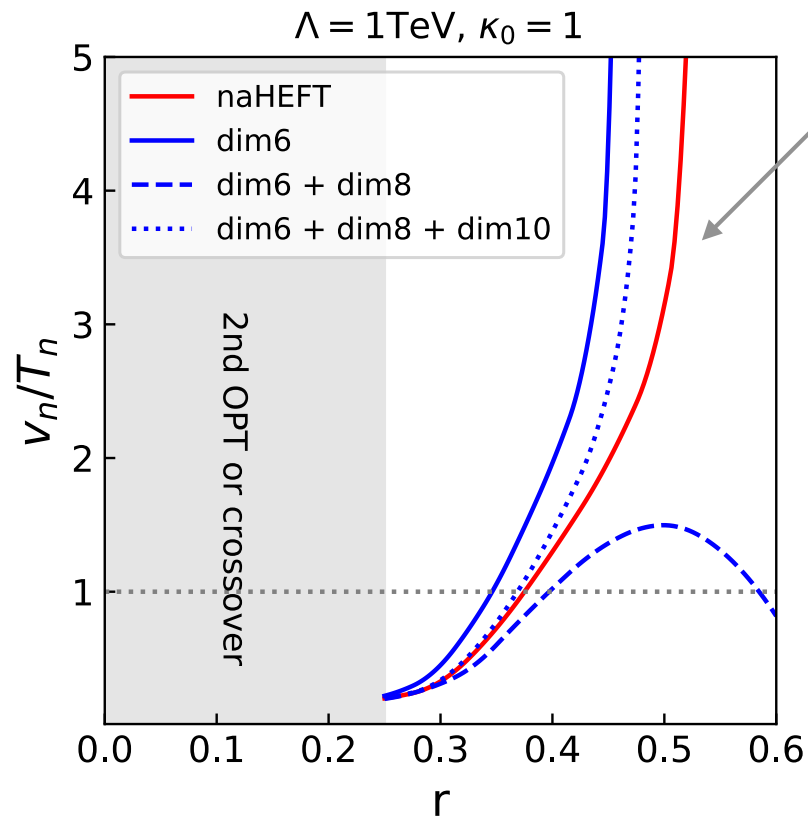
R. Florentino, S.K., M. Tanaka (2024)

naHEFT at finite temperature

SK, R. Nagai, M. Tanaka (2022)

$$V_{\text{EFT}} = V_{\text{SM}} + \frac{\kappa_0}{64\pi^2} [\mathcal{M}^2(\phi)]^2 \ln \frac{\mathcal{M}^2(\phi)}{\mu^2} + \frac{\kappa_0}{2\pi^2} T^4 J_{\text{BSM}} \left(\frac{\mathcal{M}^2(\phi)}{T^2} \right)$$

$$J_{\text{BSM}}(a^2) = \int_0^\infty dk^2 k^2 \ln \left[1 - \text{sign}(\kappa_0) e^{-\sqrt{k^2+a^2}} \right] \quad \mathcal{M}^2(\phi) = M^2 + \frac{\kappa_p}{2} \phi^2$$



Consistent with results in the SM with a singlet

[Kakizaki et al., PRD 92 (2015), Hashino et al., PRD 94 (2016)]

Large deviation in v_n/T_n exists b/w the SMEFT and naHEFT



SMEFT may not be appropriate when we discuss the strongly first order EWPT

$$r = \frac{\kappa_p v^2}{\Lambda^2}$$

$$r \sim 0 \Rightarrow M^2 \gg \frac{\kappa_p}{2} v^2 \quad \text{Decoupling}$$

$$r \sim 1 \Rightarrow M^2 \ll \frac{\kappa_p}{2} v^2 \quad \text{Non-decoupling}$$

Testing EW 1st OPT in nearly aligned case

Strongly 1st OPT ($\phi/T > 1$)

- Sensitive to the hhh coupling, and also (if charged BSM) to the $h\gamma\gamma$ coupling
 - HL-LHC \oplus ILC etc $\Delta \kappa_\gamma$ measured with 1% accuracy
 - HL-LHC (ILC1000) $\Delta \kappa_3$ measured with about 50% (10%)

- **Gravitational Waves** (with 10^{-3} to 10^{-1} Hz)
LISA, DECIGO, BBO, ...

- **Primordial Blackholes** ($M_{\text{PBH}} = 10^{-5} M_{\text{solar}}$ for EW 1st OPT) Liu et al (2021)
Hashino, SK, Takahashi (2021)
PBH may be formed at the 1st EWPT by the contrast of the PT time around T_c (depending on the Higgs potential) .

PBH searches by microlensing: Subaru HSC, OGLE, Prime, Roman, ...

Strongly 1st OPT

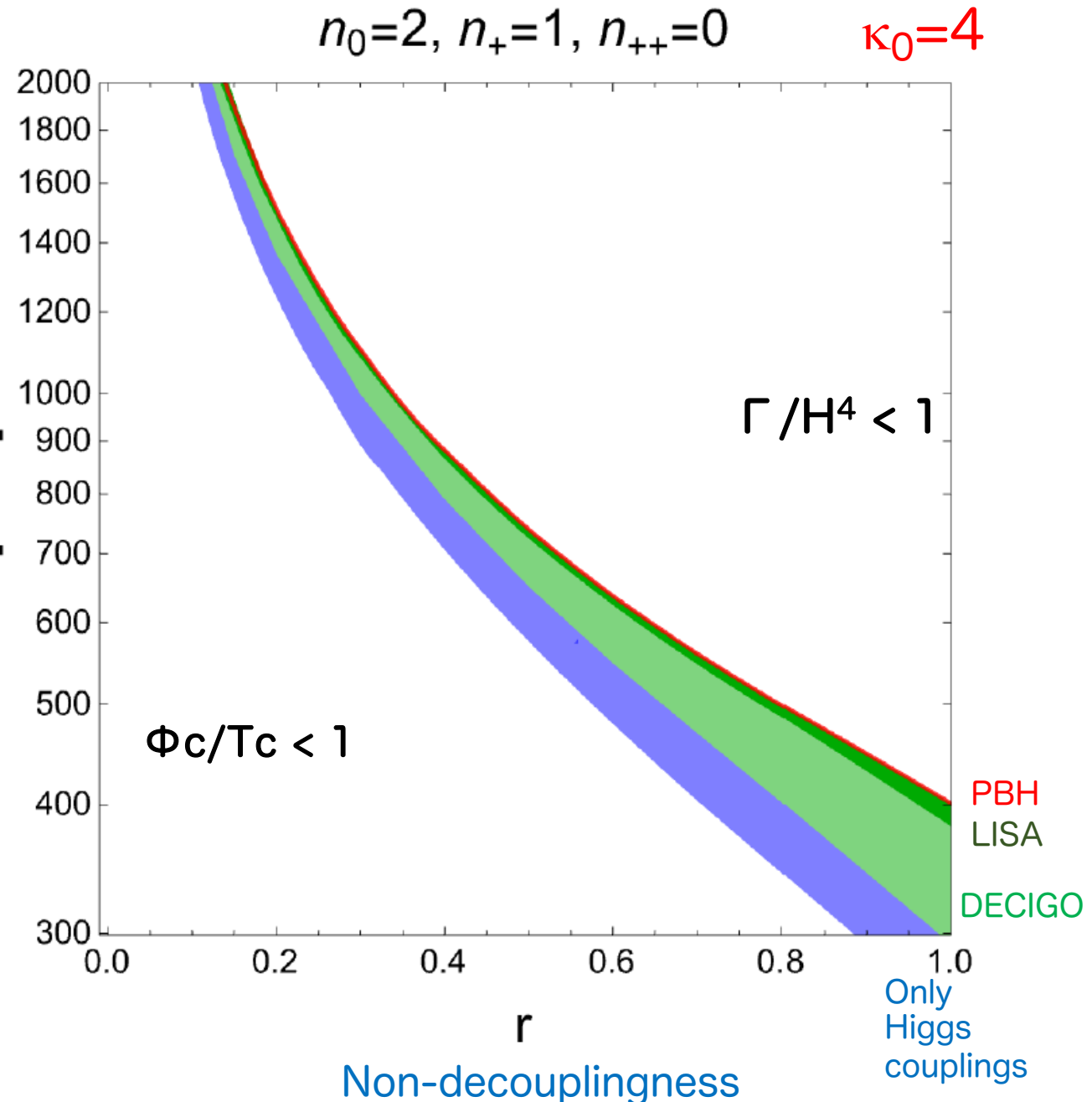
Colored region satisfies two conditions

- Sphaleron decoupling $\frac{\varphi_c}{T_c} \gtrsim 1$
- Bubble nucleation completion $\frac{\Gamma}{H^4} \Big|_{T=T_t} \gtrsim 1$

- PBH (Roman detectable $f_{\text{PBH}} > 10^{-4}$)
- GW (LISA detectable)
- GW (DECIGO detectable)
- Only Higgs couplings can test 1st OPT
($\Delta \lambda_3, \Delta \kappa_\gamma, \dots$)

Mass

Λ [GeV]



Strongly 1st OPT

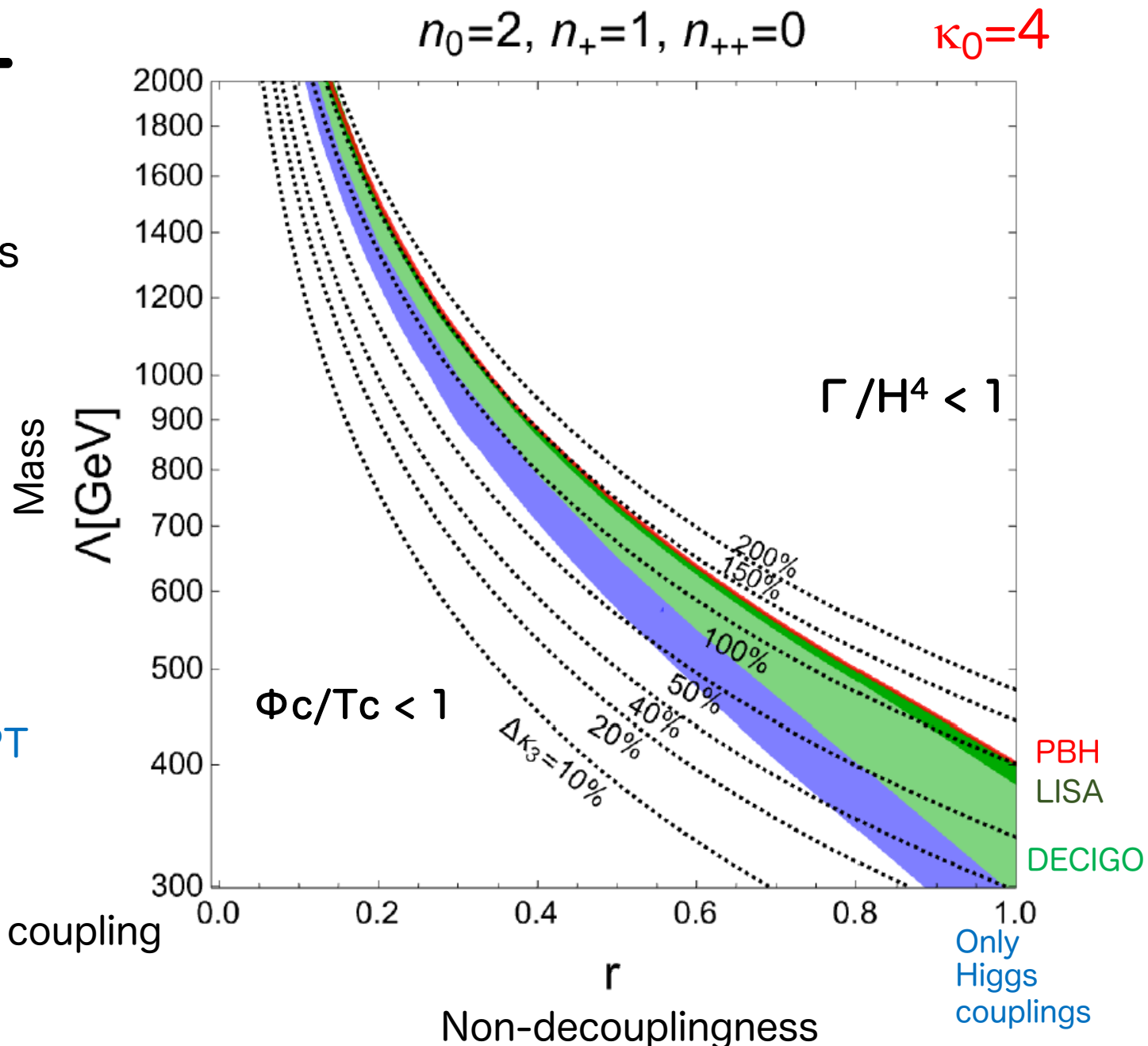
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$$\left\{ \begin{array}{l} \text{Sphaleron decoupling} \\ \text{Bubble nucleation completion} \end{array} \right. \quad \left\{ \begin{array}{l} \frac{\varphi_c}{T_c} \gtrsim 1 \\ \frac{\Gamma}{H^4} \Big|_{T=T_t} \gtrsim 1 \end{array} \right.$$

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Contour plots

Dotted curves: $\Delta \kappa_3$ deviation in the hhh coupling



Strongly 1st OPT

Colored region satisfies two conditions

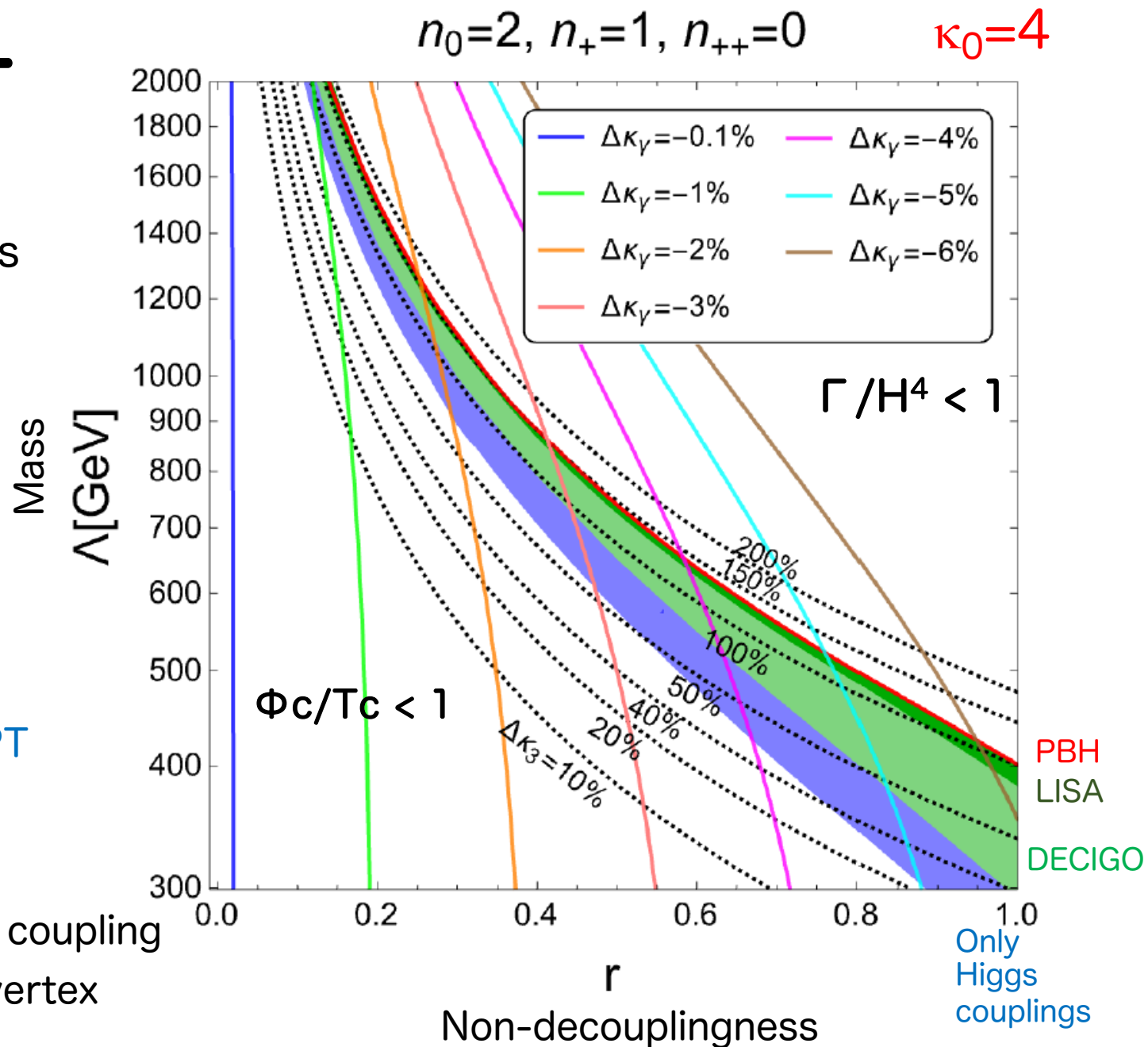
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Contour plots

Dotted curves: $\Delta \kappa_3$ deviation in the hhh coupling

Colored curves: $\Delta \kappa_\gamma$ deviation in the $h\gamma\gamma$ vertex



Strongly 1st OPT

Colored region satisfies two conditions

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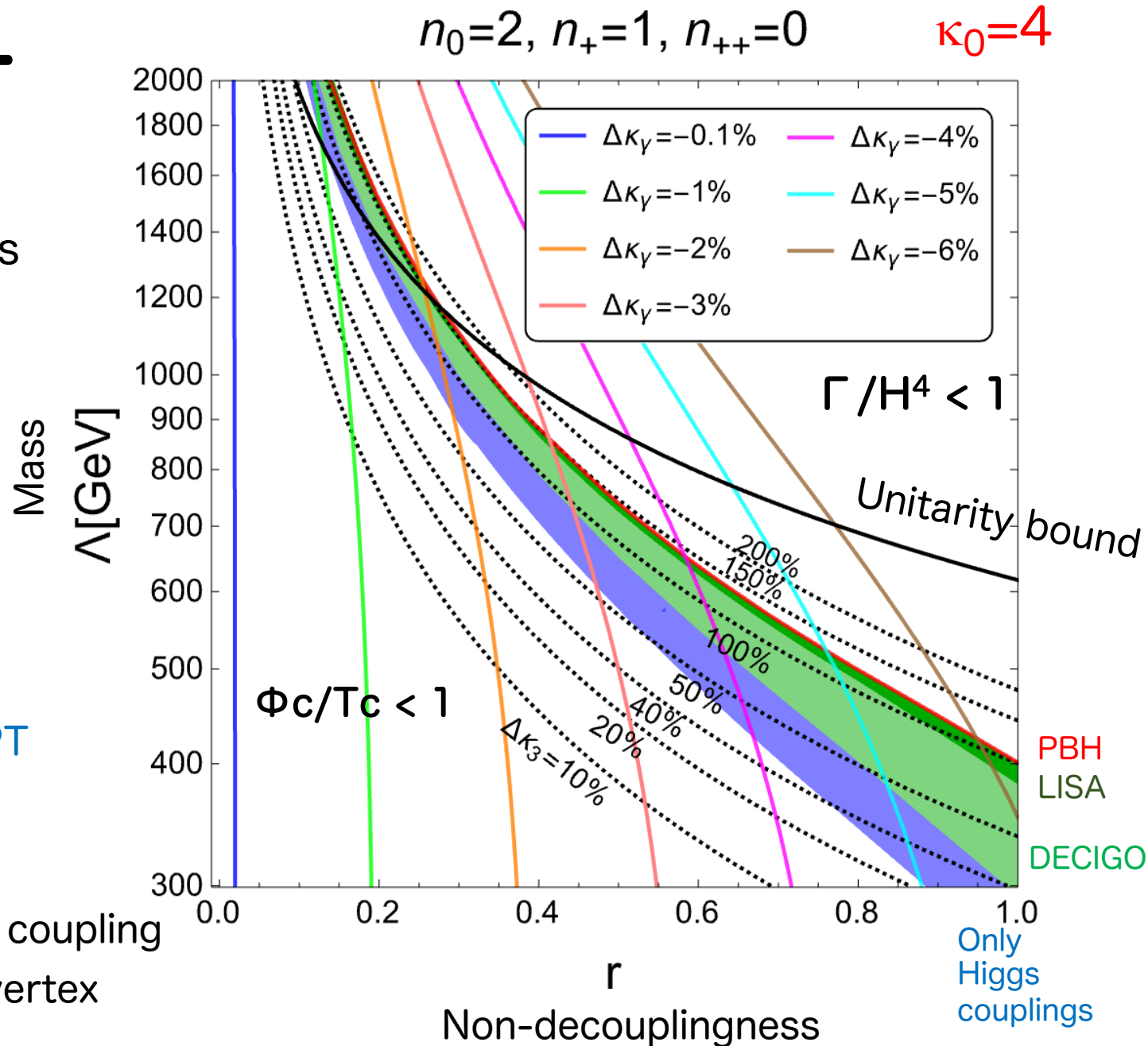
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Contour plots

Dotted curves: $\Delta \kappa_3$ deviation in the hhh coupling

Colored curves: $\Delta \kappa_\gamma$ deviation in the $h\gamma\gamma$ vertex

Black solid curve: unitarity bound



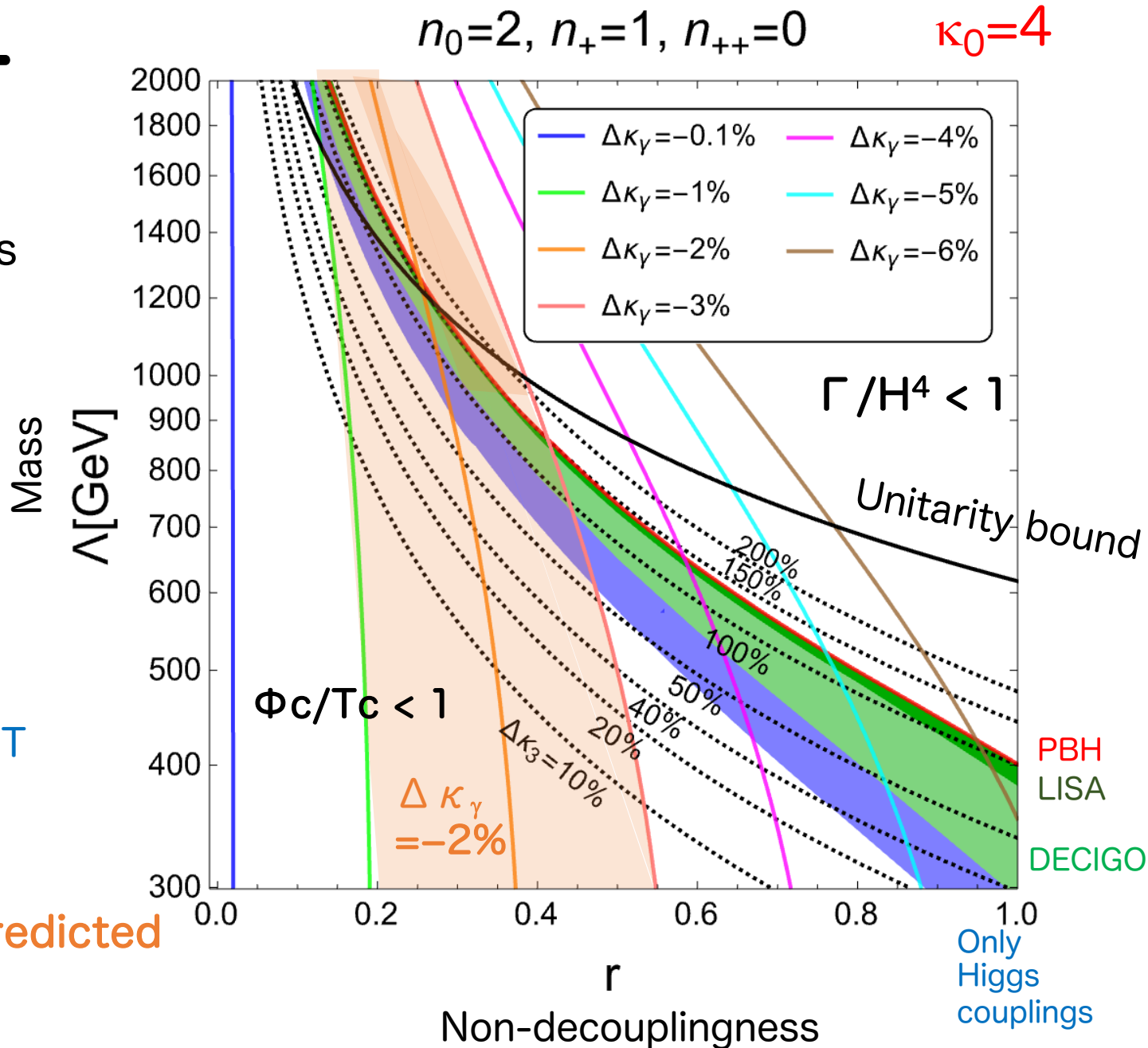
Strongly 1st OPT

Colored region satisfies two conditions

Sphaleron decoupling $\frac{\varphi_c}{T_c} \gtrsim 1$
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- PBH (Roman detectable $f_{\text{PBH}} > 10^{-4}$)
- GW (LISA detectable)
- GW (DECIGO detectable)
- Only Higgs couplings can test 1st OPT
($\Delta \kappa_3, \Delta \kappa_\gamma, \dots$)

If $\Delta \kappa_\gamma = -2\% \pm 1\%$ (LHC+ILC), predicted
 $150\% > \Delta \kappa_3 > 72\%$
 GW, PBH can also be used



Strongly 1st OPT

Colored region satisfies two conditions

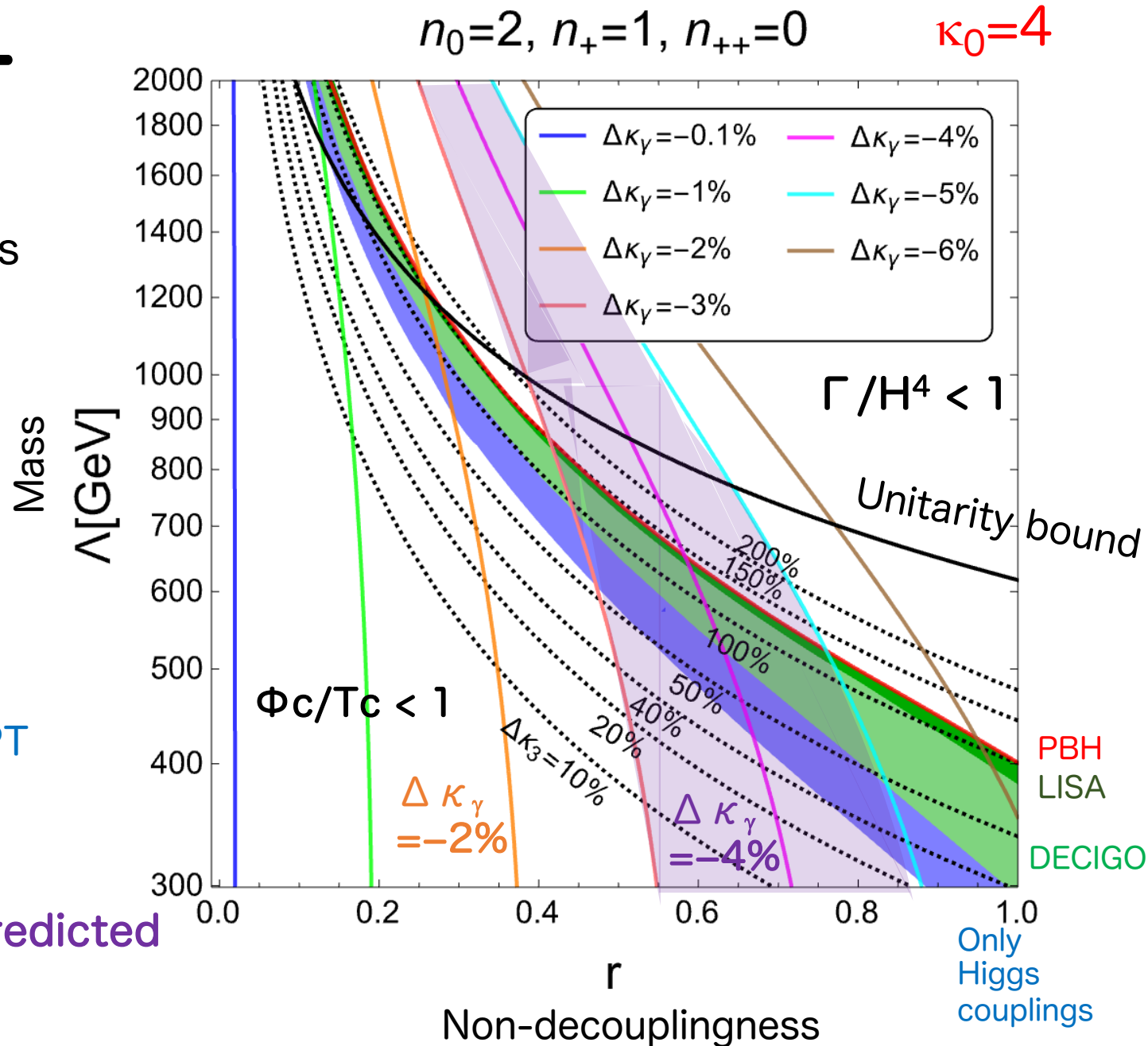
$$\left\{ \begin{array}{l} \text{Sphaleron decoupling} \\ \text{Bubble nucleation completion} \end{array} \right. \quad \left\{ \begin{array}{l} \frac{\varphi_c}{T_c} \gtrsim 1 \\ \frac{\Gamma}{H^4} \Big|_{T=T_t} \gtrsim 1 \end{array} \right.$$

- PBH (Roman detectable $f_{\text{PBH}} > 10^{-4}$)
- GW (LISA detectable)
- GW (DECIGO detectable)
- Only Higgs couplings can test 1st OPT
($\Delta \lambda_3, \Delta \kappa_\gamma, \dots$)

If $\Delta \kappa_\gamma = -4\% \pm 1\%$ (LHC+ILC), predicted

$$135\% > \Delta \kappa_3 > 55\%$$

GW, PBH can also be used

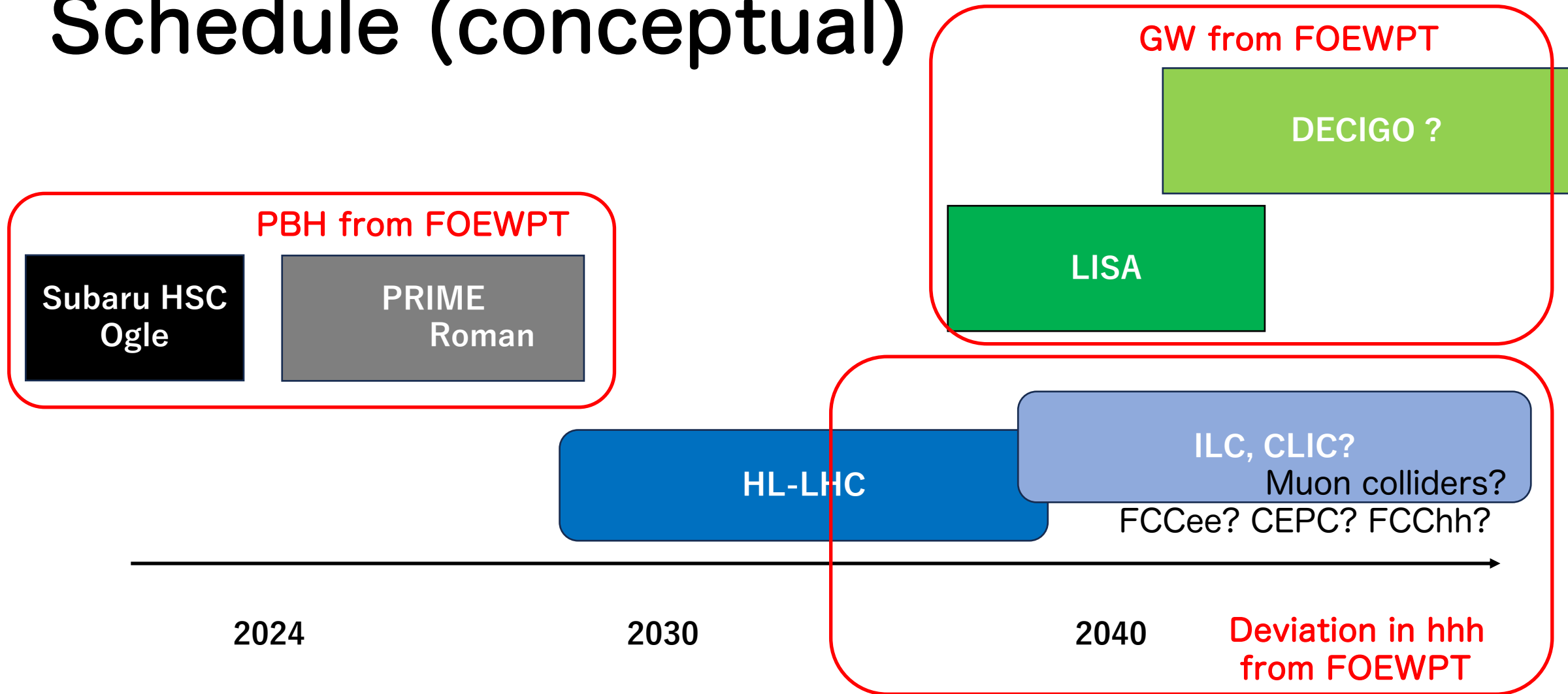


Summary

- Aspect of EWPT is a next global target
- Strongly 1st OPT is motivated by EW baryogenesis
- Non-minimal Higgs sector is required
- A simple mechanism for 1st OPT is the non-decoupling loop effect
- Described by the naHEFT
- Precision measurements of $h\gamma\gamma$ and hhh vertices, GW observation, searches for PBH provide a complementary probe of EW 1stOPT

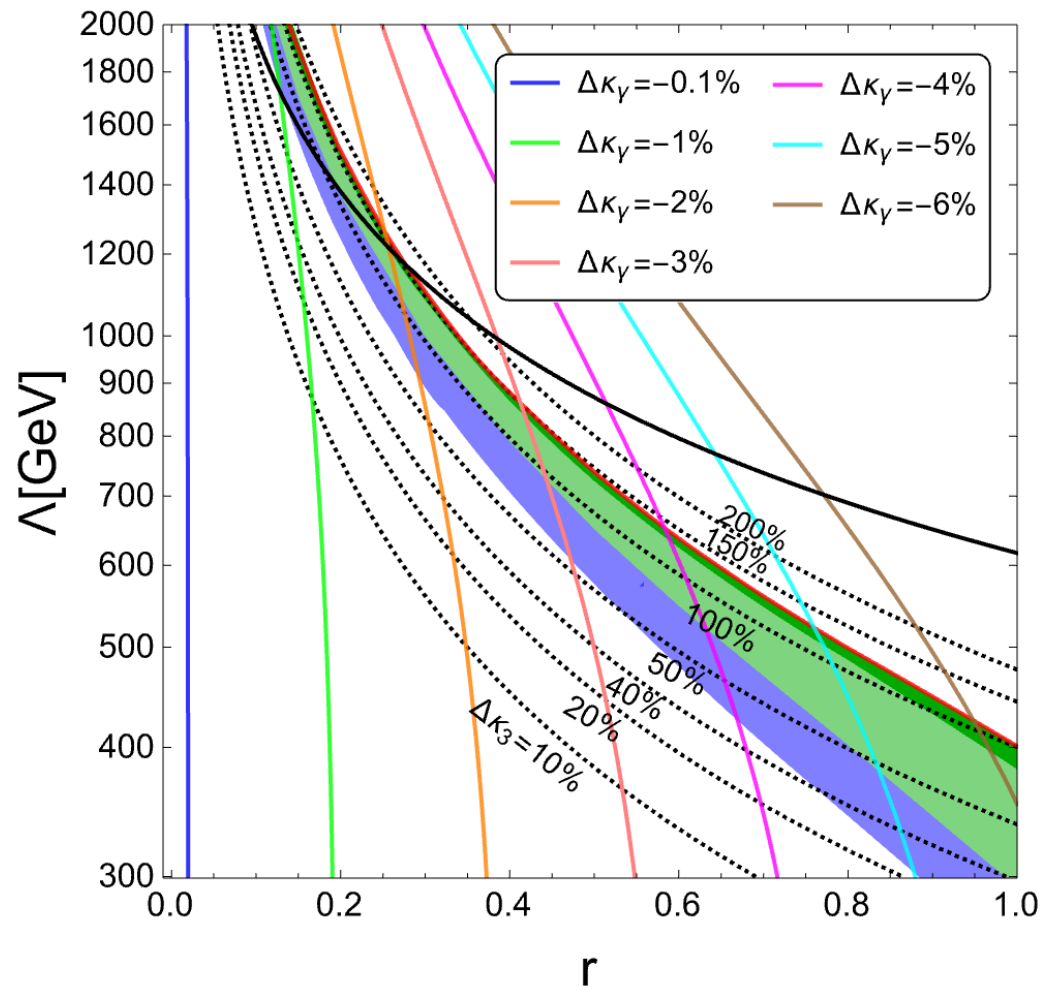
Thank you!

Schedule (conceptual)

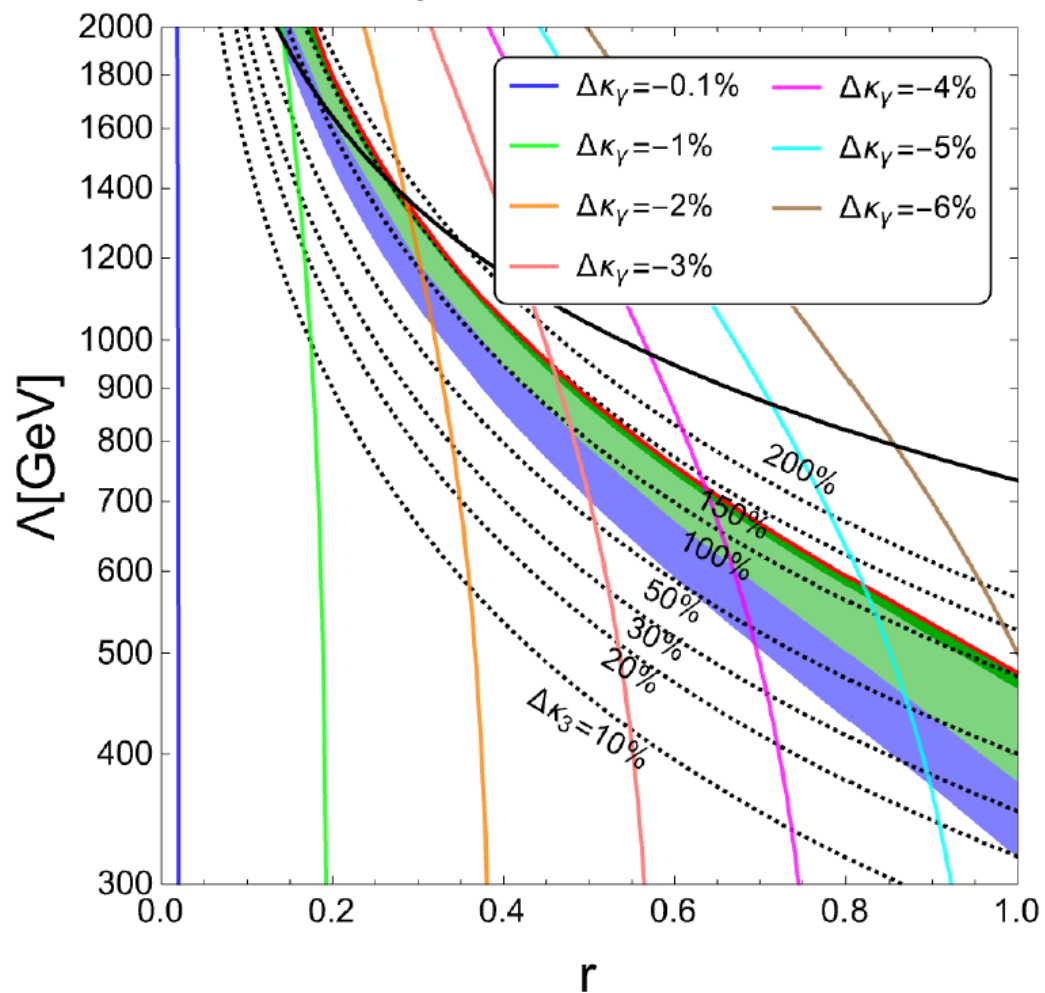


Complementary

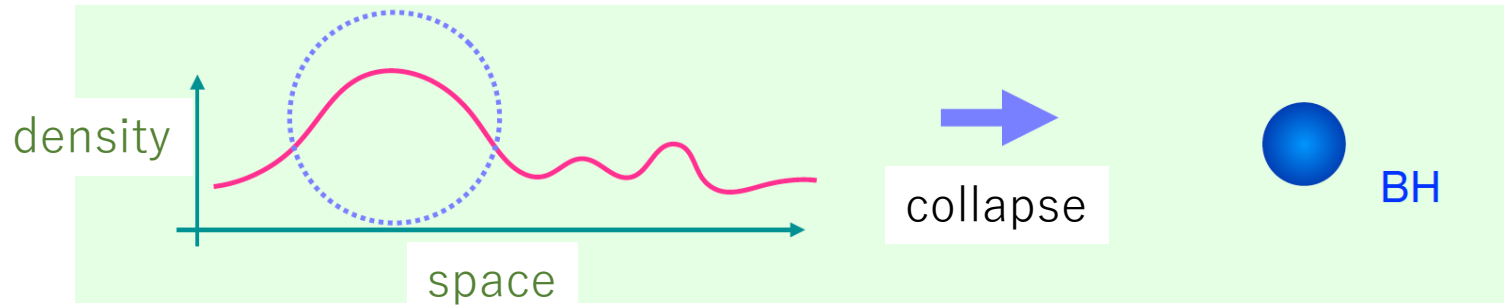
$n_0=2, n_+=1, n_{++}=0$



$n_0=0, n_+=1, n_{++}=0$



PBH Formation



Primordial black holes (PBH) : BHs formed before the star formation

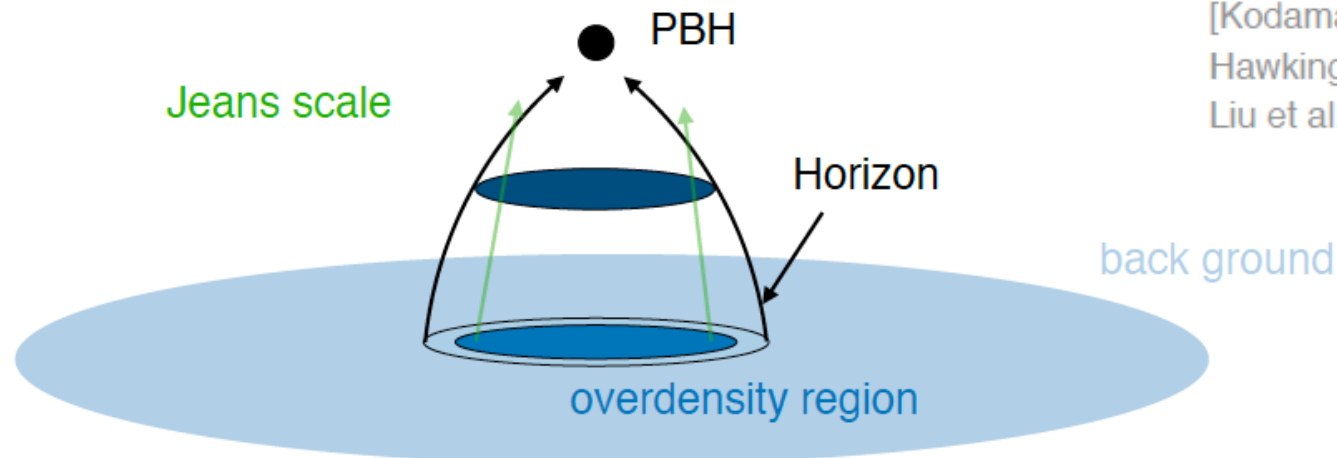
Condition for the PBH formation

$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_C$$

[Hawking, Mon. Not. Roy. Astron. Soc. 152 (1971),
Hawking and Carr, Mon. Not. Roy. Astron. Soc. 168 (1974),
Harada, Yoo and Kohri, PRD 88 (2013)]

$\delta > \delta_C$ can be satisfied when the FOPT occurs

→ PBHs might be produced by the FOPT

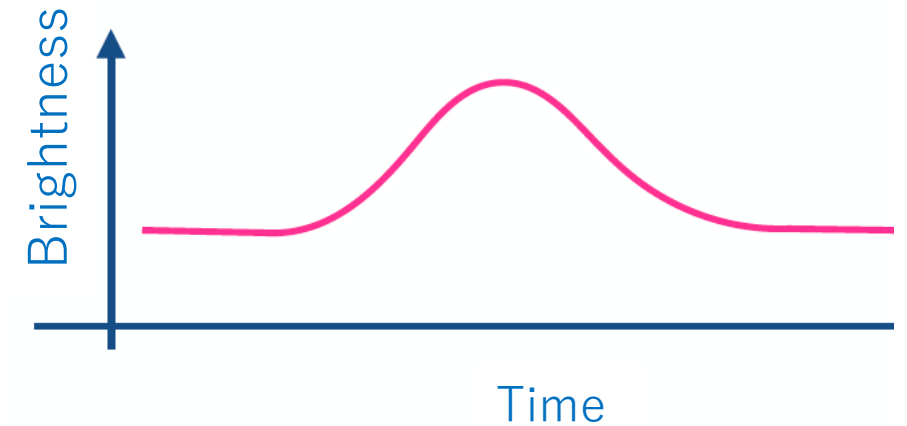
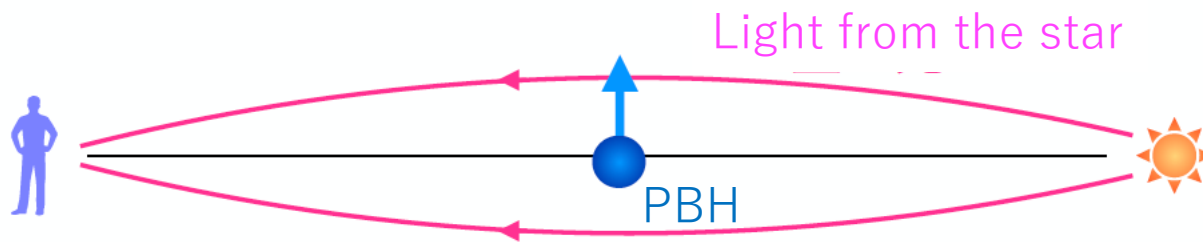


[Kodama, Sasaki and Sato, PTP 68 (1982);
Hawking, Moss and Stewart, PRD 26 (1982)
Liu et al., PRD105 (2022)]

Figure by Masanori Tanaka

PBH search

- Gravitational microlensing effect
- Brightness is up by passing PBH



Non-observation → constraint on the PBH abundance

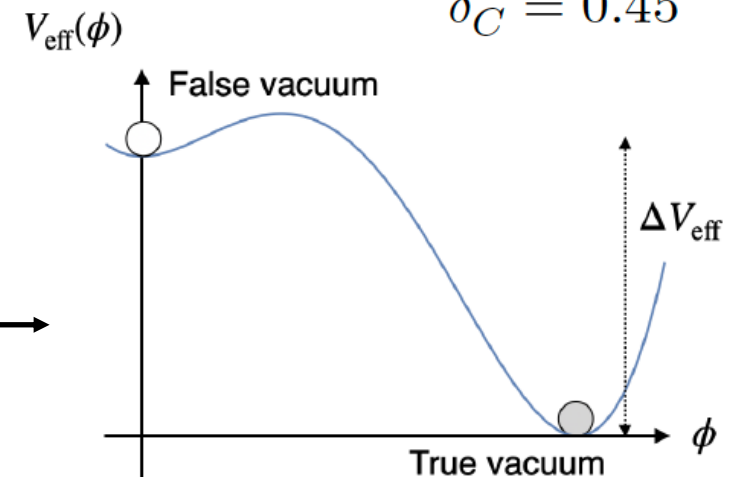
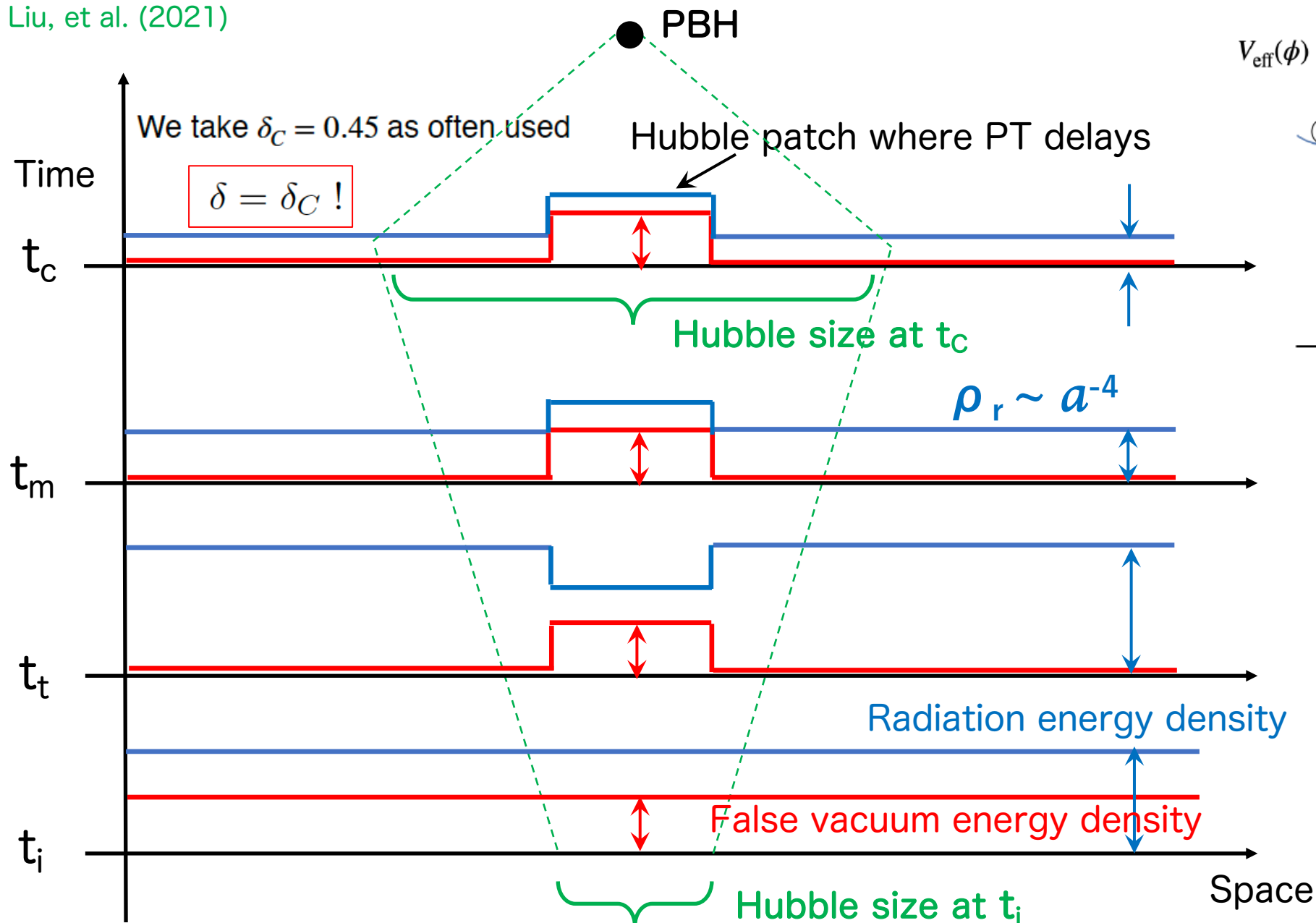
Subaru HSC, OGLE (Optical Gravitational Lensing Experiment)

PBH formation from the contrast

J. Liu, et al. (2021)

$$\delta = \frac{\rho_{\text{over}} - \rho_{\text{back}}}{\rho_{\text{back}}} > \delta_C$$

$$\delta_C = 0.45$$



False vacuum energy density

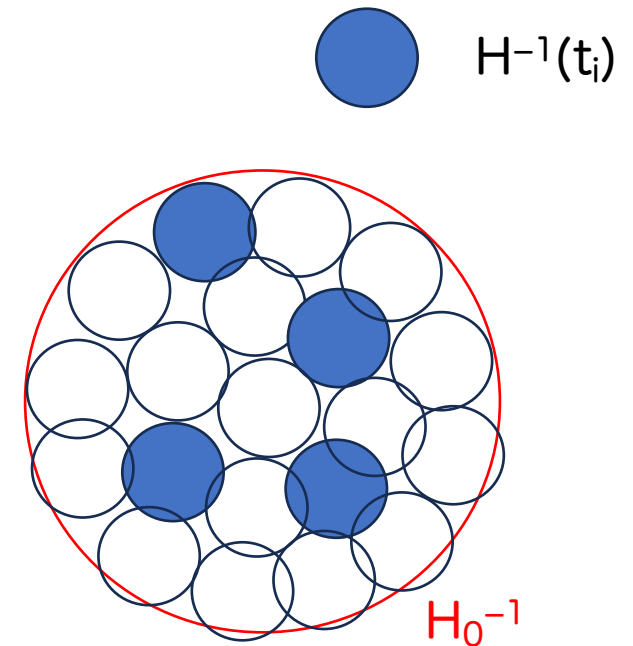
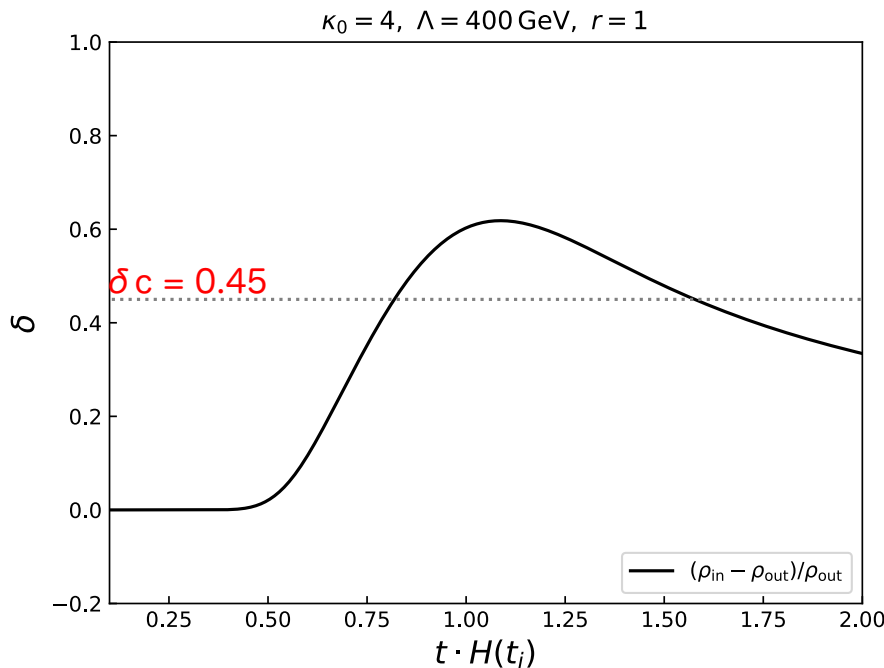
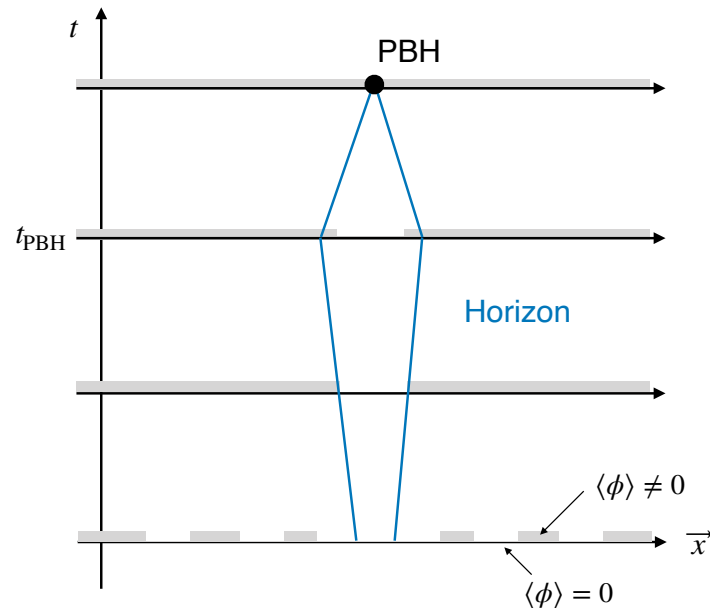
$$\rho_V \sim \text{constant} \sim F(t) \Delta V_{\text{eff}}$$

Radiation energy density

$$\rho_r \sim a^{-4}$$

How to calculate the fraction of PBH

1. Evaluate the possibility that the symmetry is not broken in a Hubble volume at t_{PBH}
2. Calculate how many Hubble patches at t_{PBH} are included in the present Hubble volume



$$f_{\text{PBH}}^{\text{EW}} \equiv \frac{\Omega_{\text{PBH}}^{\text{EW}}}{\Omega_{\text{CDM}}} \sim 1.49 \times 10^{11} \left(\frac{0.25}{\Omega_{\text{CDM}}} \right) \left(\frac{T_{\text{PBH}}}{100 \text{ GeV}} \right) P(t_{\text{PBH}})$$

$$P(t_n) = \exp \left[-\frac{4\pi}{3} \int_{t_i}^{t_n} \frac{a^3(t)}{a^3(t_{\text{PBH}})} H^{-3}(t_{\text{PBH}}) \Gamma(t) dt \right]$$

Prob. of Hubble patch of False

PBH from 1st order EWPT

K. Hashino, SK, T. Takahashi, 2021

K. Hashino, SK, T. Takahashi, M. Tanaka 2023

Mass of PBH from EWPT is determined by t_{PBH}

$$M_{\text{PBH}} \approx \frac{4\pi}{3} H^{-3}(t_{\text{PBH}}) \rho_c = 4\pi H^{-1}(t_{\text{PBH}})$$

$$M_{\text{PBH}} \sim 10^{-5} M_{\odot}$$

Microlensing observations

Subaru HSC <https://hsc.mtk.nao.ac.jp/ssp>

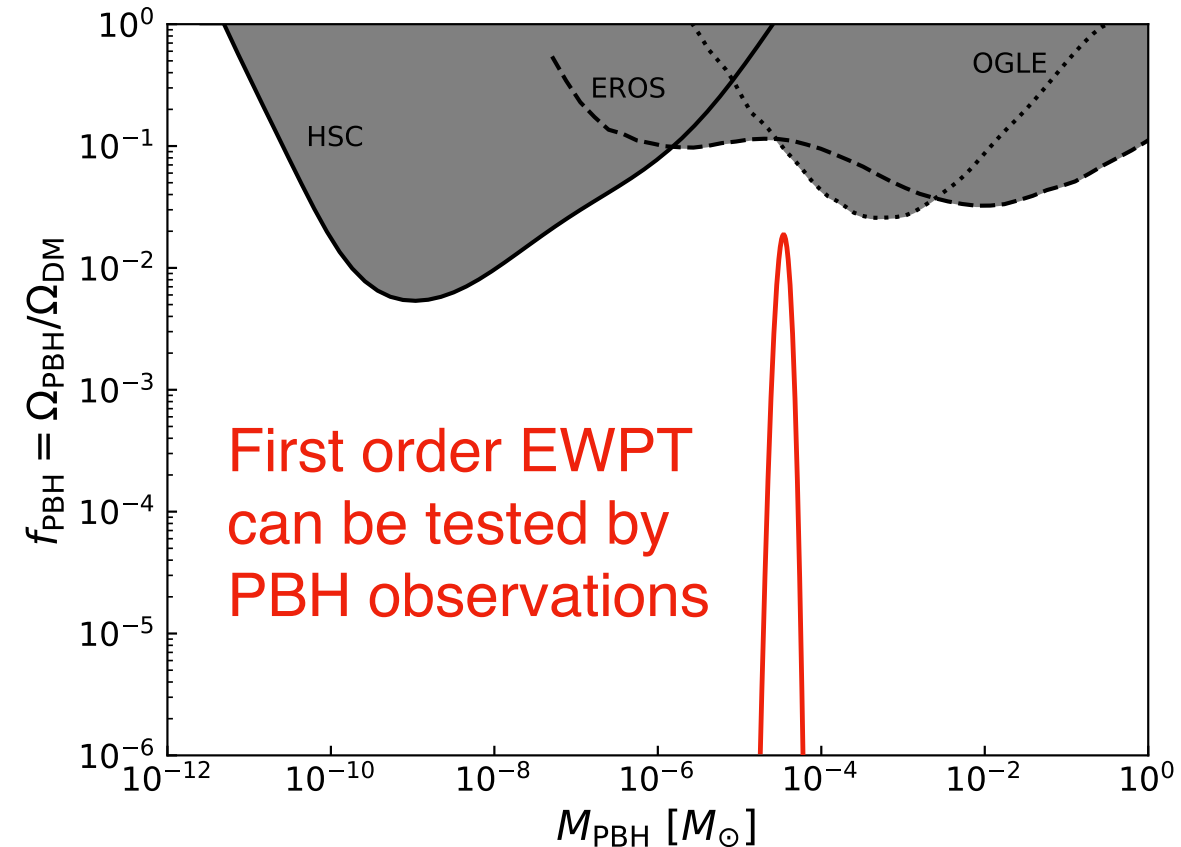
OGLE <http://ogle.astrouw.edu.pl/>

Future observations

PRIME 2023~ <http://www-ir.ess.sci.osaka-u.ac.jp/prime/index.html>

Roman 2026~ <https://roman.gsfc.nasa.gov>

f_{PBH} is constrained by 10^{-4}



Using near infrared rays:
sensitive to the microlensing
from center galaxy

Theory predictions on α - β plane

Non-dec. $r = 0.5$

DOF $\kappa_0 = 1, 4, 20$
for various Λ

New scale

Contours of f_{PBH}

$$10^{-4} < f_{\text{PBH}} < 1$$

PBH can be produced in this area

$r=0.5$

