Primordial black holes from stochastic inflation

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based on 2012.06551, 2111.07437, 2312.12911 with
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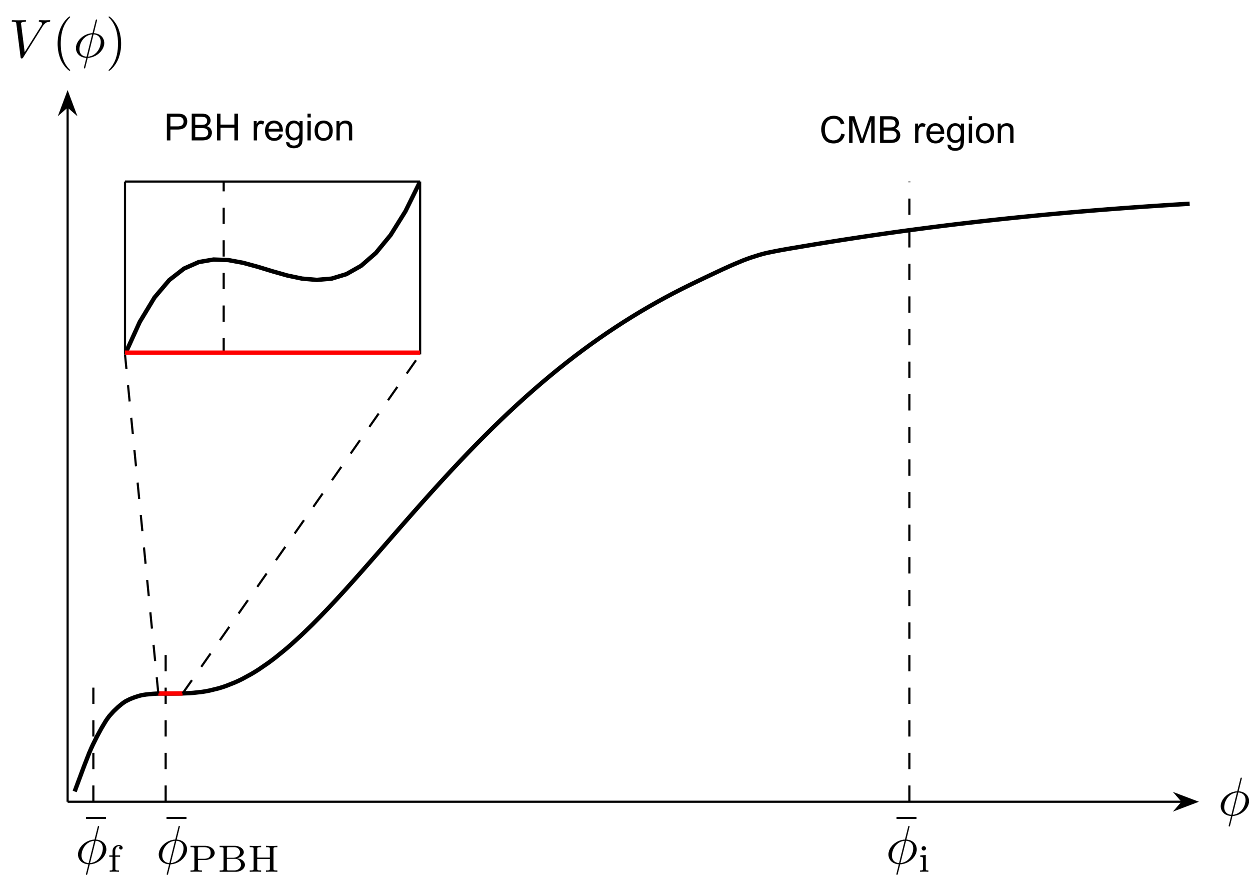
Dark matter without dark matter

- Primordial black holes (PBH) are a candidate for dark matter. (Hawking 1971)
- They could also seed supermassive black holes.
- Let's consider asteroid-mass PBHs as dark matter.

Facilitating collapse

- PBHs require large (~1) fluctuations on small scales.
- They could be generated by the same process as the small (~10^{-5}) fluctuations on large scales.
- Simplest and most successful scenario is inflation.
  - The curvature perturbation (in slow-roll) is $\zeta = -H\delta\phi/\dot{\phi} \sim H^2/\dot{\phi}$
  - The slower the field, the larger the perturbations.

Potential for asteroid-mass PBHs



Making black holes

- During inflation, ***k***-modes stretch to super-Hubble scales and freeze.
- After inflation, they cross back inside the Hubble radius and start evolving.
- If a Hubble patch is overmassive enough, it collapses into a PBH.
  - PBH mass is close to the Hubble patch mass.

Slow therefore stochastic

$$\ddot{\phi} + 3H\dot{\phi} = -V' + \xi$$

- Because the field moves slowly, kicks are large and stochastic effects are important.

- Stochastic effects increase PBH production:
  1. Patches with large fluctuations are more likely: distribution tail is exponential, not Gaussian.
  2. Individual patches are choppy.

All inflation is stochastic inflation

- Inflaton evolution is stochastic when the coupling of short- and long-wavelength modes is taken into account. (Starobinsky 1986)

- As modes become super-Hubble and classicalise, they change the background in which shorter modes evolve.

- Amplitude of every ***k***-mode is independently drawn from a Gaussian distribution, so the background is subject to Gaussian white noise.

One step at a time

- We solve the coupled background + perturbation evolution numerically:
  - Coarse-grain a mode, recalculate the background and perturbations at every timestep.
  - Continue until inflation ends, record the total number of e-folds N.
  - Repeat 10^{11} times to gather statistics.
  - Find distribution P(ΔN), where $\Delta N = N - \bar{N} = \zeta$.
- We also have a simplified treatment. (Tomberg: 2210.17441, 2304.10903)

From the inflaton potential to the compaction function

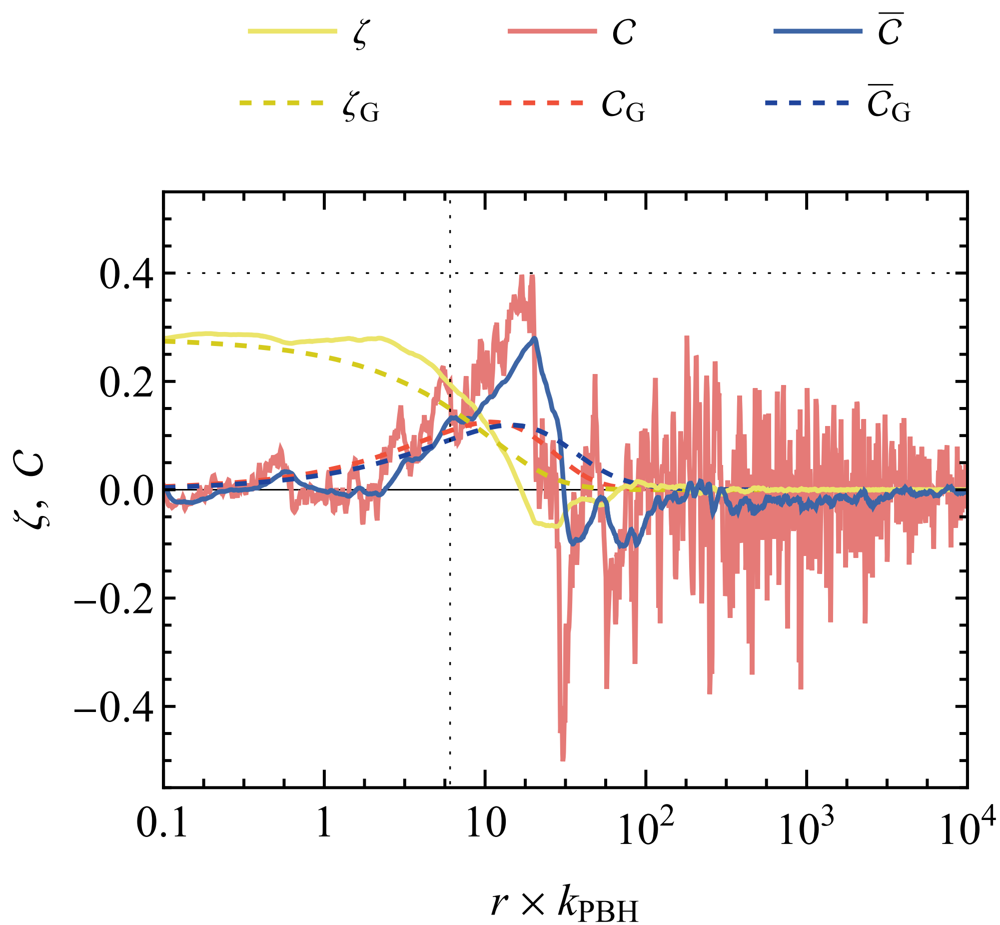
- We construct the spatial profile in each patch via inverse Fourier transform:

$$\zeta(r) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3 k \zeta_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{d}k k^2 \zeta_k \frac{\sin(kr)}{kr}$$

- According to simulations, PBH formation is determined by the maximum value of the compaction function $\mathcal{C}(r)=2G_N\Delta M(r)$, where $\Delta M(r)$ is mass excess. (Shibata and Sasaki: gr-qc/9905064)

- We go from the inflaton potential to the compaction function.

One realisation



Tail and spikes

- Taking into account the stochastic tail enhances PBH abundance by $\sim 10^5$ compared to the Gaussian case.

- Also taking into account the stochastic nature of the individual patches naively enhances PBH abundance by an extra factor of $\sim 10^9$.

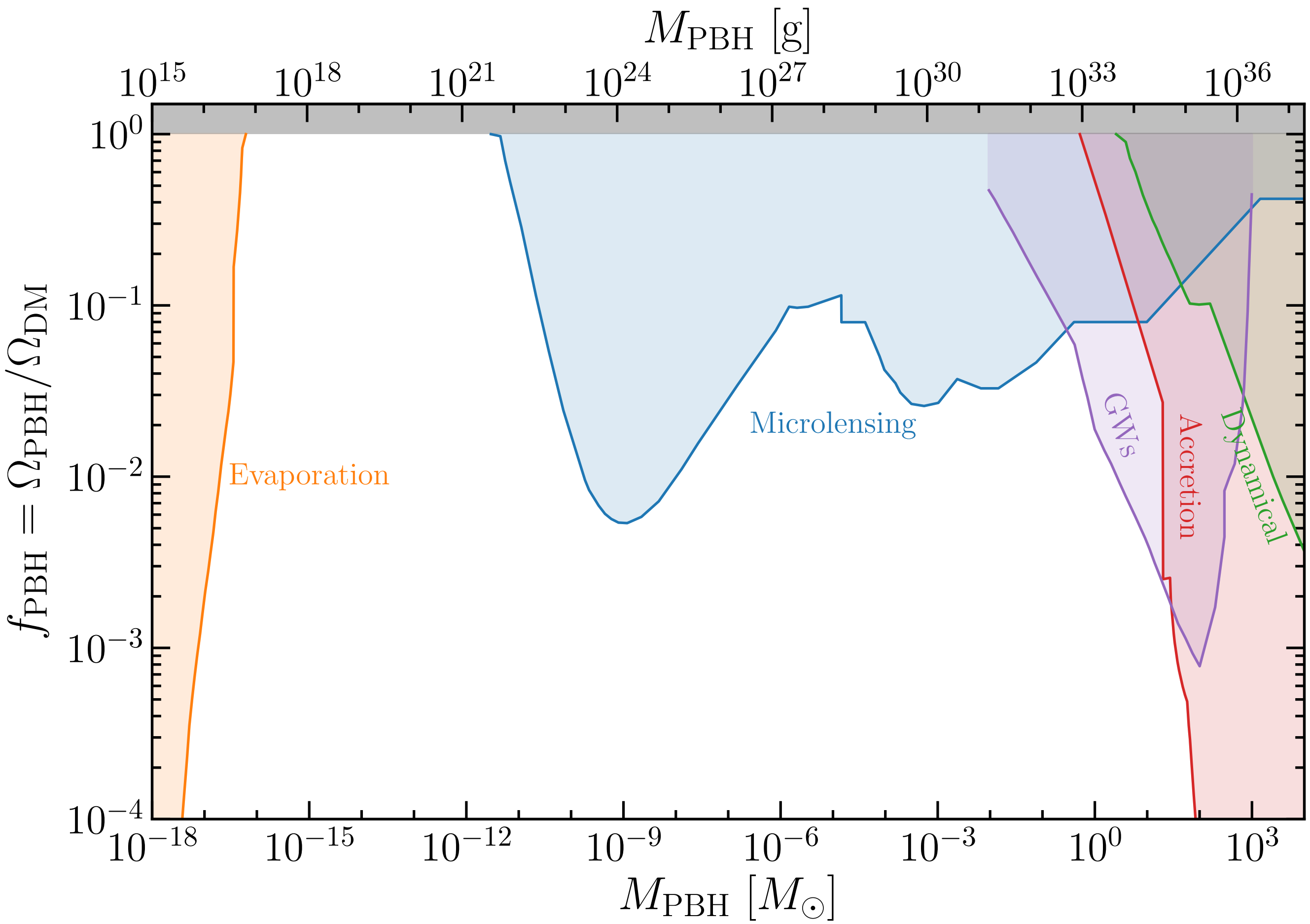
- Caveat: once inside Hubble radius, spikes lead to large pressure gradients which smoothen the profile.
  - Next step: redo PBH formation simulations.

Spiking the conclusions

- PBHs can constitute all dark matter.
- Generating PBHs requires (in single-field models) slowing down the inflaton.
- Stochastic effects can change PBH abundance by many orders of magnitude.
  1. Lead to an exponential tail for the perturbations.
  2. Make individual patches choppy.
     - Unclear how much pressure gradients wash away: redo PBH formation simulations.

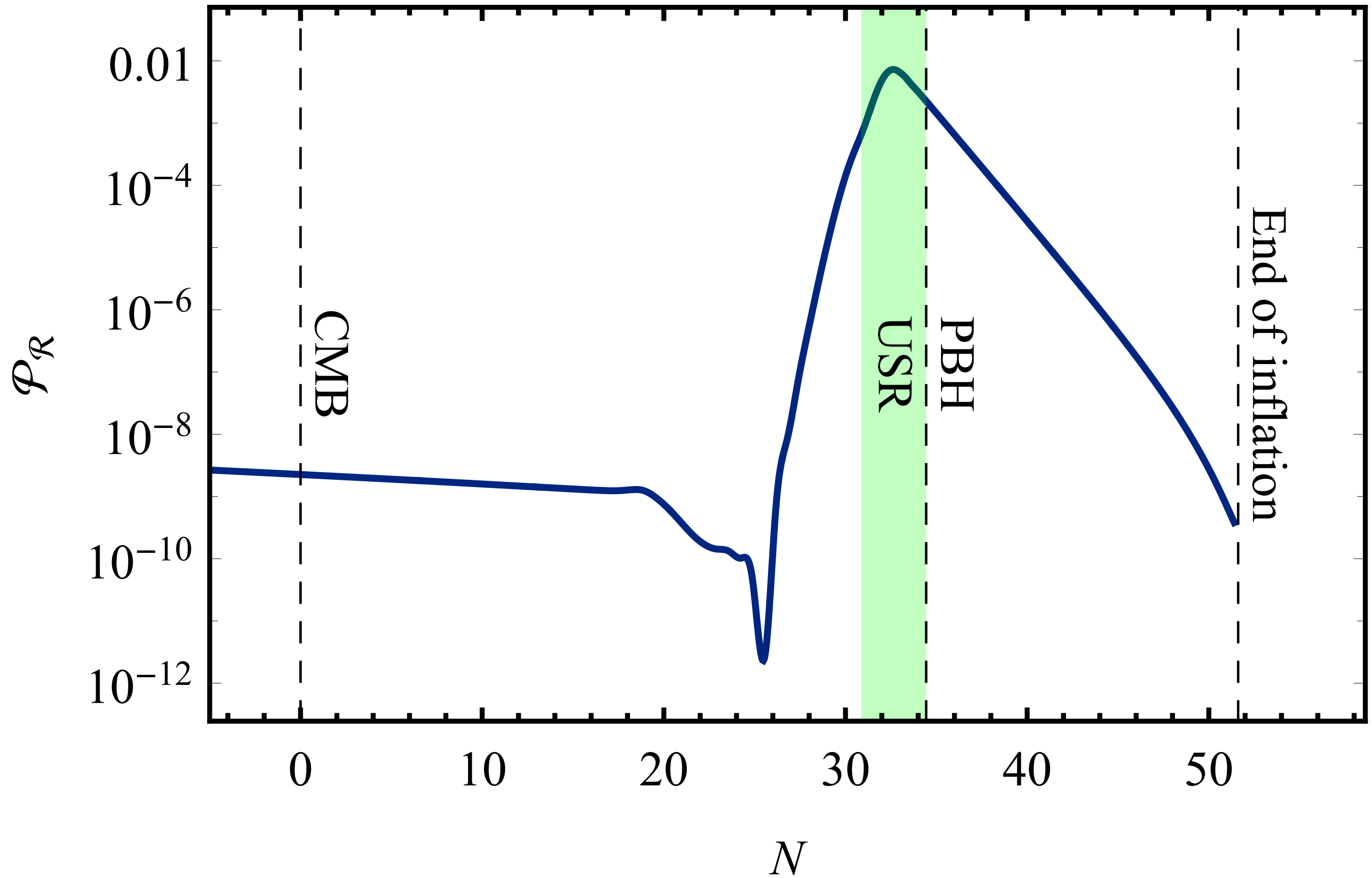
Backup slides

Asteroid-mass or Planck scale

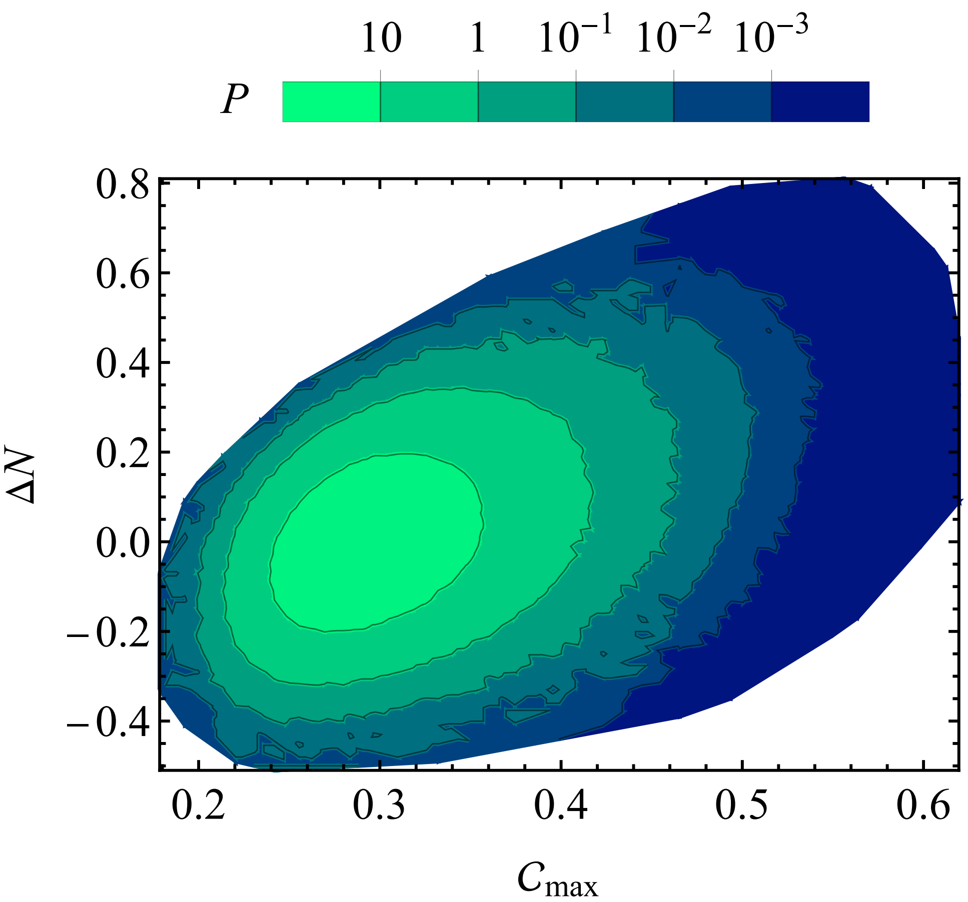


Green and Kavanagh: 2007.10722

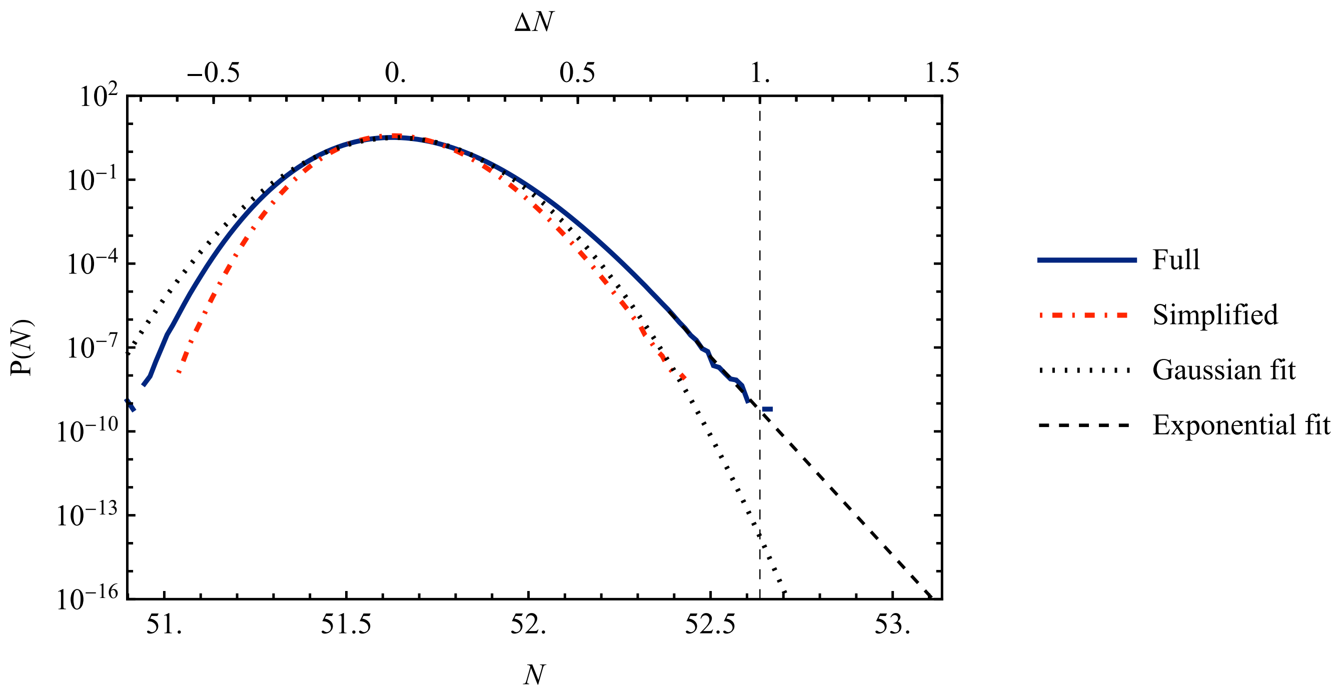
Curvature power spectrum



Correlation of max(ζ) and max($\mathcal{C}$)



P(Δ*N*=ζ)



- Stochastic kicks generate exponential tail.
  (Pattison, Vennin, Assadullahi, Wands: 1707.00537; Ezquiaga, Garcia-Bellido, Vennin: 1912.05399)
- This enhances PBH abundance by $\sim 10^5$.