

CATCH 22+2

Photo-production via anomaly in neutron stars and supernovae

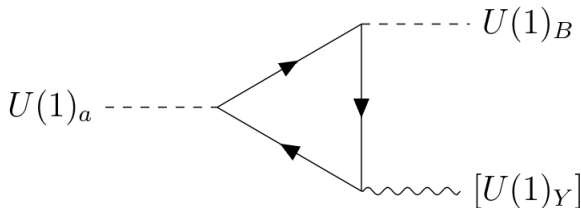
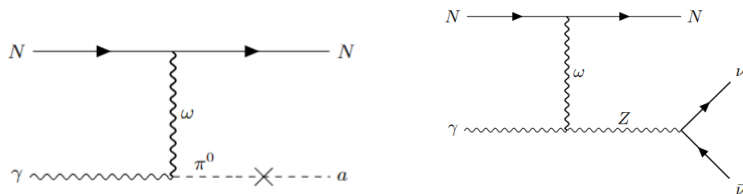
Miguel Vanvlasselaer
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VUB and IIHE

May 2024

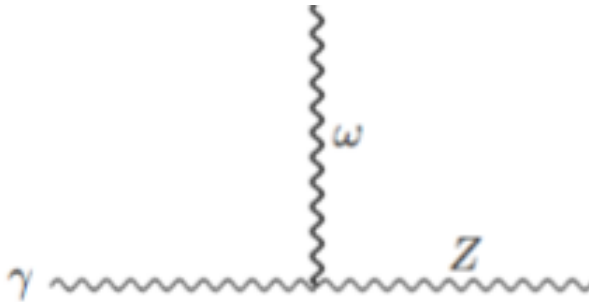
Wess-Zumino-Witten interactions

What are the Wess-Zumino-Witten (WZW) interactions ?



Computing the vertex

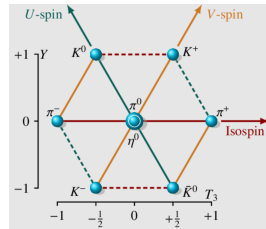
$$L \subset \omega \cdot Z \cdot F$$



Pion chiral lagrangian and WZW term

- Pion chiral lagrangian for the pseudo scalar mesons

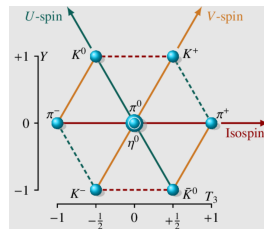
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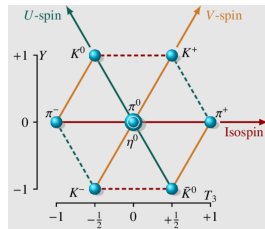
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$$D_\mu = \partial_\mu U - ir_\mu U + iU\ell_\mu , \quad r_\mu(\ell_\mu) = v_\mu \pm a_\mu ,$$

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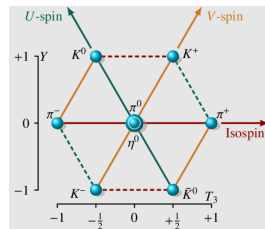
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- Problems! missing reactions: $K^+ K^- \rightarrow 3\pi$, $\pi^0 \rightarrow \gamma\gamma$. Too many symmetries
- Solution: Wess-Zumino-Witten **Phys. Lett. B 37 (1971) 95, Nucl. Phys. B 223 (1983) 422.**

$$S_{\text{WZW}} = \kappa \int_D d^5x \omega , \quad \omega = -\frac{i}{240\pi^2} \epsilon^{\mu\nu\rho\sigma\tau} \text{Tr} (U_\mu U_\nu U_\rho U_\sigma U_\tau) \quad U_\mu = U^\dagger \partial_\mu U$$

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set δA $\rightarrow \delta\omega = -e \delta A_\mu \hat{J}^\mu$,

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$$\hat{J}^\mu = \frac{1}{48\pi^2} \epsilon^{\mu\nu\rho\sigma\tau} \partial_\nu \text{Tr} [\{Q, U^\dagger\} \partial_\rho U U^\dagger \partial_\sigma U U^\dagger \partial_\tau U] .$$

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- Full expression for $S_{\text{WZW}}(U, A_\mu)$

$$\kappa \int_D d^5x \omega - \kappa e \int d^4x A_\mu J^\mu + \underbrace{\frac{ie^2}{24\pi^2} \int d^4x \epsilon^{\mu\rho\sigma\lambda} A_\rho (\partial_\mu A_\nu) [\text{Tr} (\{Q^2, U^\dagger\} \partial_\sigma U) - QUQ \partial_\sigma U^\dagger]}_{= \frac{ie^2}{48\pi^2} \frac{\pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}} \quad \text{We found the anomaly !}$$

Repeat the procedure with non-abelian chiral subgroups

Nucl. Phys. B 223 (1983) 422: Witten, PhysRevD.30.594: Kaymakalan, Rajeev and Schechter

$$\begin{aligned}
 \Gamma_{WZW}(U, \mathcal{A}_L, \mathcal{A}_R) = & \Gamma_0(U) + \mathcal{C} \int \text{Tr} \left\{ (\mathcal{A}_L \alpha^3 + \mathcal{A}_R \beta^3) - \frac{i}{2} [(\mathcal{A}_L \alpha)^2 - (\mathcal{A}_R \beta)^2] \right. \\
 & + i(\mathcal{A}_L U \mathcal{A}_R U^\dagger \alpha^2 - \mathcal{A}_R U^\dagger \mathcal{A}_L U \beta^2) + i(d\mathcal{A}_R dU^\dagger \mathcal{A}_L U - d\mathcal{A}_L dU \mathcal{A}_R U^\dagger) \\
 & + i[(d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L) \alpha + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R) \beta] + (\mathcal{A}_L^3 \alpha + \mathcal{A}_R^3 \beta) \\
 & - (d\mathcal{A}_L \mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L) U \mathcal{A}_R U^\dagger + (d\mathcal{A}_R \mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R) U^\dagger \mathcal{A}_L U \\
 & \left. + (\mathcal{A}_L U \mathcal{A}_R U^\dagger \mathcal{A}_L \alpha + \mathcal{A}_R U^\dagger \mathcal{A}_L U \mathcal{A}_R \beta) + i \left[\mathcal{A}_L^3 U \mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger \mathcal{A}_L U - \frac{1}{2} (U \mathcal{A}_R U^\dagger \mathcal{A}_L)^2 \right] \right\}.
 \end{aligned}$$

$$\mathcal{L}_{WZW}^\pi \supset \frac{N_C}{48\pi^2} g_2^2 \tan \theta_W \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \mathcal{A}_\rho Z_\sigma$$

How to introduce vector mesons ?

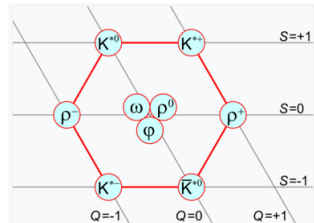
$$\mathcal{L} \subset \omega \cdot F \cdot \dots ??$$

Vector mesons not part of the U matrix ! How to introduce them ?

↓ [HHH: Phys. Rev. D 77 \(2008\) 085017](#)

Introduce a background field B with QN of ω, ρ, \dots $\delta B \rightarrow J^\mu$

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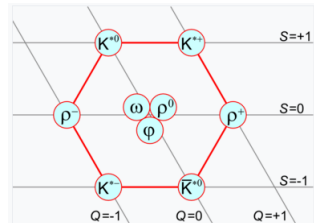
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$$\Gamma_c = -S_{\text{WZW}}^{\text{Bardeen}}(U = 1, e\mathcal{A}_\mu + g_\omega\omega_\mu) \quad (\text{Bardeen counterterms})$$

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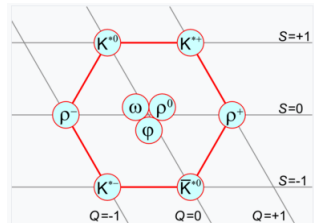
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- Non-vectorlike: Recipe is

$$S_{\text{full}} = S_{WZW}(U, A_L + B_L, A_R + B_R) - \Gamma_c$$



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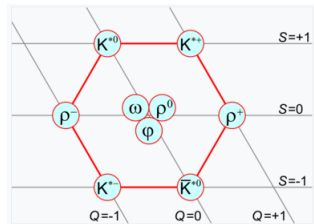
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Example: $\omega \rightarrow \pi_0 \gamma$ and $\Gamma_{\omega \rightarrow \pi_0 \gamma}$

- vertex $\gamma - \omega - Z$

$$\frac{N_C}{48\pi^2} g_2^2 g_\omega \tan \theta_W \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \omega_\rho Z_\sigma$$



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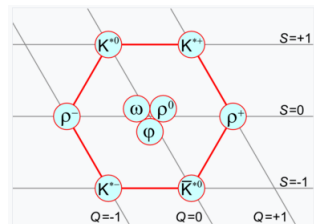
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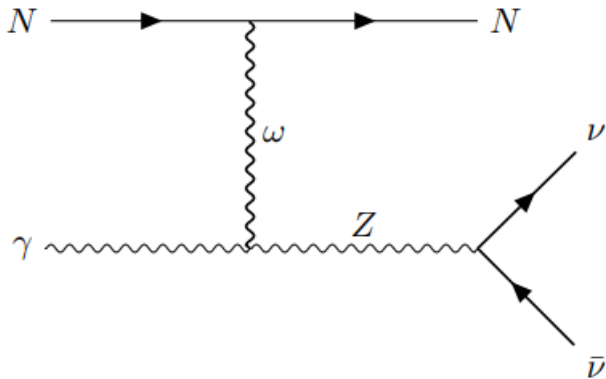
$$\frac{N_C}{48\pi^2} g_2^2 g_\omega \tan \theta_W \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \omega_\rho Z_\sigma$$

- vertex $\gamma - \omega - a$

$$\frac{\pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} = -2 \frac{\partial_\alpha \pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \mathcal{A}_\beta \rightarrow -2g_\omega \frac{\partial_\alpha \pi_0}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \omega_\beta \rightarrow -2g_\omega \theta_{a\pi_0} \frac{\partial_\alpha a}{f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \omega_\beta$$



Computing the full interaction



Road to our Lagrangian

-

$$\mathcal{L}_{\text{WZW}} \supset \frac{N_C}{48\pi^2} g_2^2 g_\omega \tan \theta_W \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \omega_\rho Z_\sigma + \dots$$

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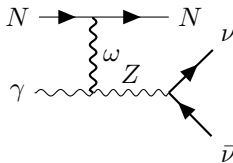
- vertex $\gamma N N a$

$$\mathcal{L}_{\text{int}} = C_A \frac{e N_c}{24\pi^2} \frac{g_\omega^2}{m_\omega^2} \epsilon_{\mu\nu\alpha\beta} \frac{\partial_\mu a}{f_a} F^{\nu\alpha} \bar{N} \gamma^\beta N$$

Summary of the results

- Photo-production of a neutrino pair: $\gamma N \rightarrow N \nu \bar{\nu}$

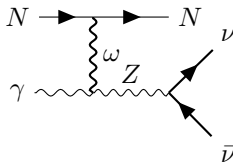
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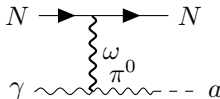
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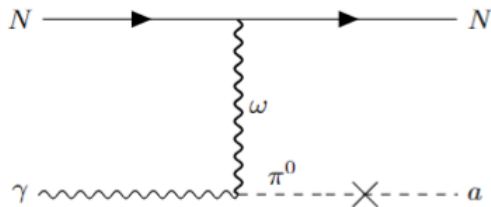
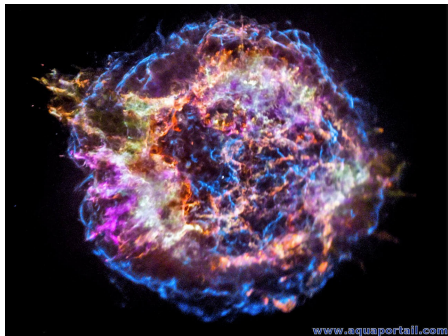
- Photo-production of axions: $\gamma N \rightarrow N a$

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Emission of axions from SN

Photo-production of axions in Supernovae



With Sabyasachi Chakraborty (IIT-Kanpur) and Aritra Gupta (IFIC Valencia): 2403.12169

The cooling argument in supernovae and SN1987A

- Neutrino received during SN1987A:

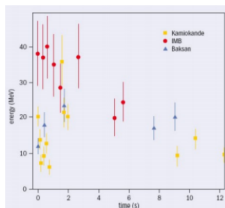
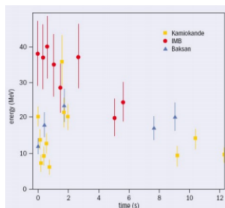


Figure: Credit:NirCam JWST

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- Raffelt bound:

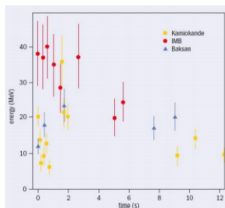
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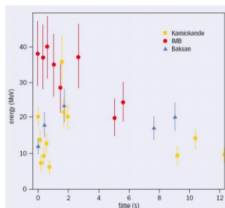
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- This is a coupling constraint !
- We need: Emissivity $Q =$, energy per unit of time and cc emitted



Figure: Credit:NirCam JWST

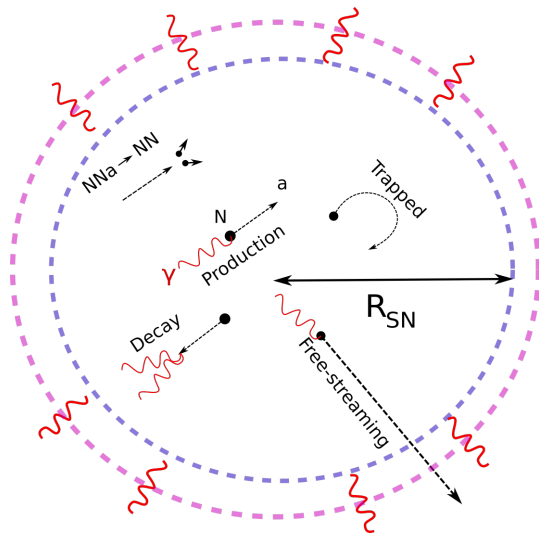
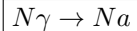
How to carry energy away from the SN?

Supernovae extreme medium:
 $\rho \sim \rho_0, \quad T \sim 30 - 50 \text{ MeV}$

bremsstrahlung and pion conversion



What about the photo-production ?

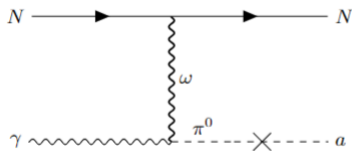


Emissivities of the $\gamma N \rightarrow Na$: main result

$$Q_{N\gamma \rightarrow Na} \approx \int \frac{dE_\gamma}{\pi^2} E_\gamma^2 f_\gamma(p_\gamma) \underbrace{n_B}_{\text{targets}} \underbrace{\sigma_{\gamma N \rightarrow Na}(E_\gamma)}_{\text{x-section}} \times \underbrace{E_\gamma}_{\text{energy escaping}}$$

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$$\frac{Q_{N\gamma \rightarrow Na}^{\text{WZW,ND}}}{10^{34} \text{ erg/s/cm}^3} \approx 5.6 C_A^2 g_{40}^4 T_{40}^8 \rho_{15} \left(\frac{10^9 \text{ GeV}}{f_a} \right)^2$$



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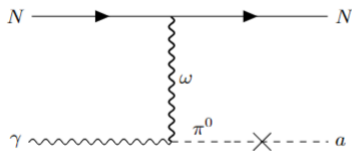
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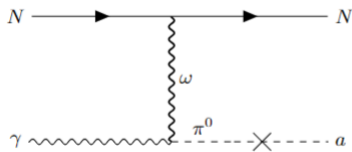
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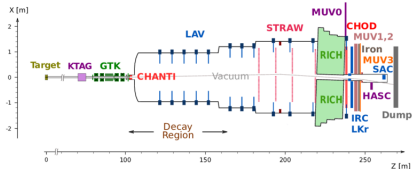
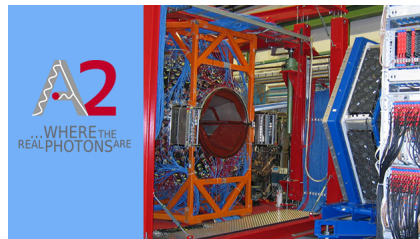
- ND computation holds for $T \gtrsim 30 \text{ MeV}$.



What is the value of g_ω ?

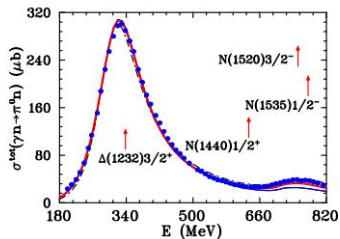
- Large range of theoretical prediction:
 $g_\omega \in 8 - 60!$

MAMI and NA60 collabs

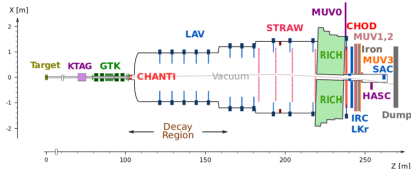
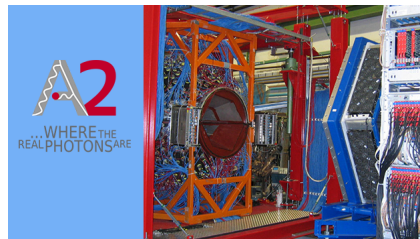


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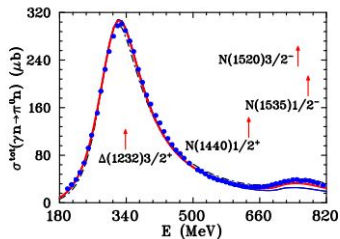


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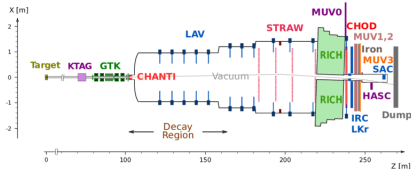
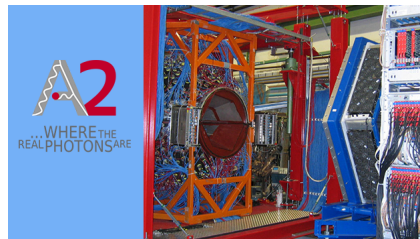
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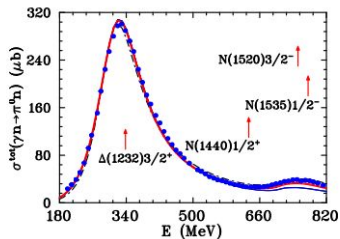
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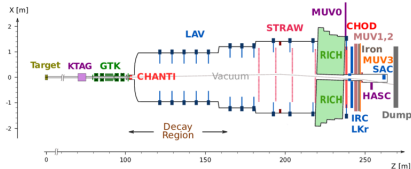
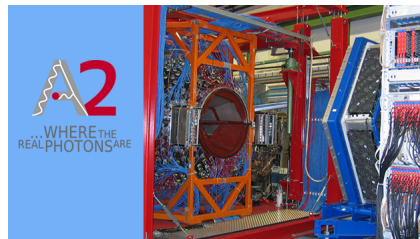
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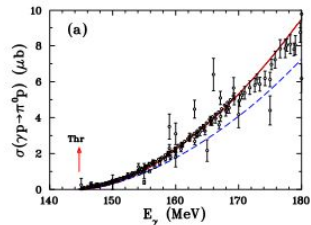
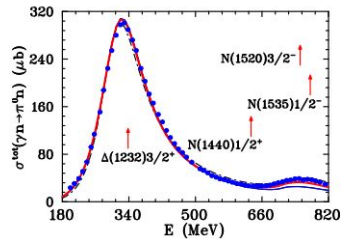
- Consistent with $g_\omega \sim 8 - 10$
- Data driven method available.

MAMI and NA60 collabs



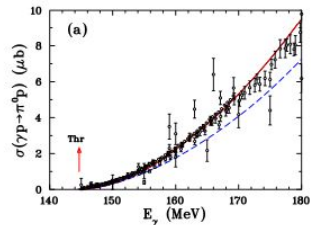
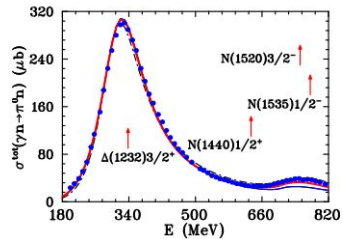
Data-driven photo production

- $\sigma_{\gamma N \rightarrow N a} \approx \frac{f_{\pi} C_A}{f_a} \sigma_{\gamma N \rightarrow N \pi_0}, \quad E > 145 \text{ MeV}$



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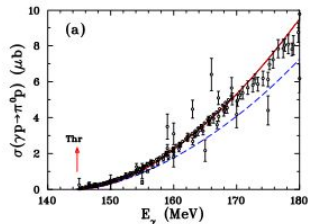
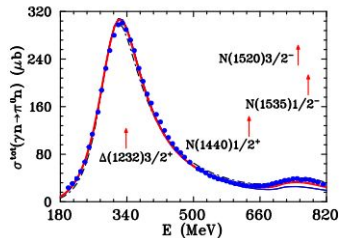
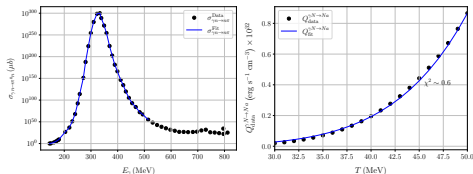
- $\sigma_{\gamma N \rightarrow N a} \approx \frac{f_{\pi} C_A}{f_a} \sigma_{\gamma N \rightarrow N \pi_0}, \quad E > 145 \text{ MeV}$
- $\sigma_{\gamma N \rightarrow N \pi_0}$ is measured!



Data-driven photo production

- $\sigma_{\gamma N \rightarrow Na} \approx \frac{f_\pi C_A}{f_a} \sigma_{\gamma N \rightarrow N\pi_0}$, $E > 145$ MeV
- $\sigma_{\gamma N \rightarrow N\pi_0}$ is measured!
- Axion emissivities:

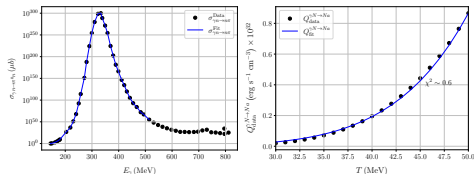
$$\frac{Q_{\text{data}}^{\gamma N \rightarrow Na}}{1.6 \times 10^{33} \text{ cm}^{-3} \text{ s}^{-1} \text{ erg}} \approx \left(\frac{C_A 10^9}{f_a / \text{GeV}} \right)^2 \times \rho_{15} T_{40}^{6.73}$$



Data-driven photo production

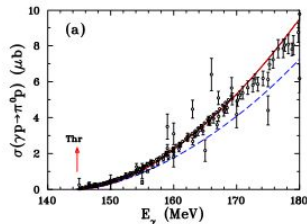
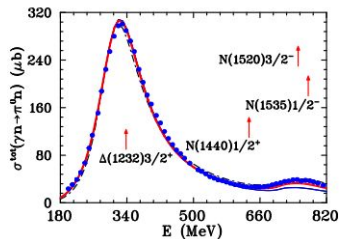
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- The data-driven piece dominates if $g_\omega < 20$:

$$Q_{N\gamma \rightarrow Na}^{\text{data,ND}} > Q_{N\gamma \rightarrow Na}^{\text{WZW}}$$

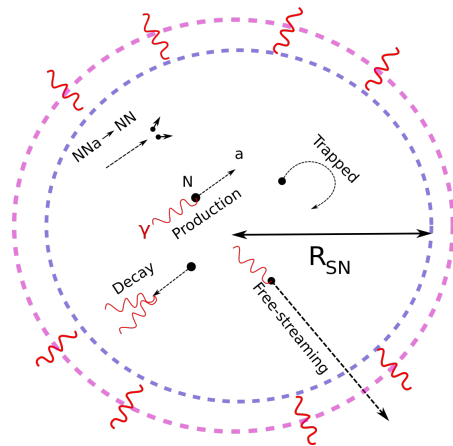


Emissivities of the $\gamma N \rightarrow Na$: further subtleties

- Absorption via $aN \rightarrow \gamma N$ or $NNa \rightarrow NN$:

$$L_a \approx \frac{1}{\Gamma_{aN \rightarrow \gamma N}} + \frac{1}{\Gamma_{NNa \rightarrow NN}}$$

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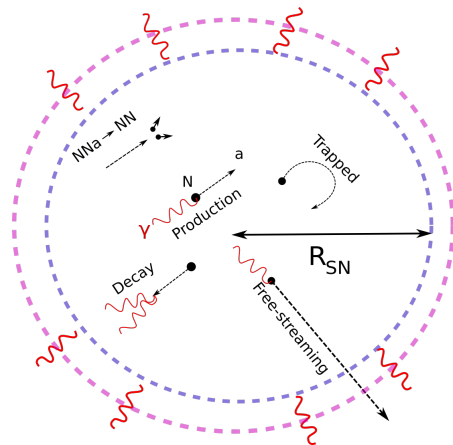
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- Lapse effects: $\alpha(M, r) \approx \sqrt{1 - 2M/r}$

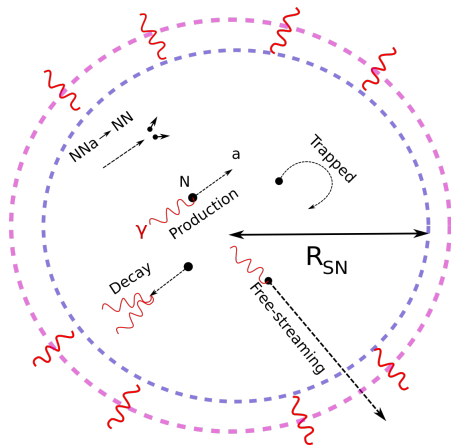
$$E_\infty = E_{\text{emission}} \times \alpha$$

$$n_a^\infty = n_{\text{emission}} \times \alpha$$

$$\Rightarrow Q_\infty = \alpha^2 Q_{\text{emission}}$$



Emissivities of the $\gamma N \rightarrow Na$: further subtleties when axions are massive

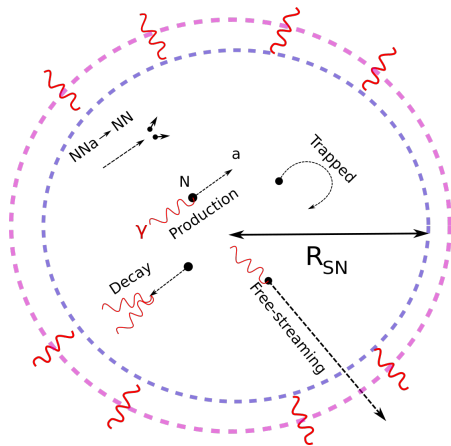


Massive axions regime

- Redshift effect:

$$Q \approx \int_{m_a/\alpha} dE...$$

Emissivities of the $\gamma N \rightarrow Na$: further subtleties when axions are massive



Massive axions regime

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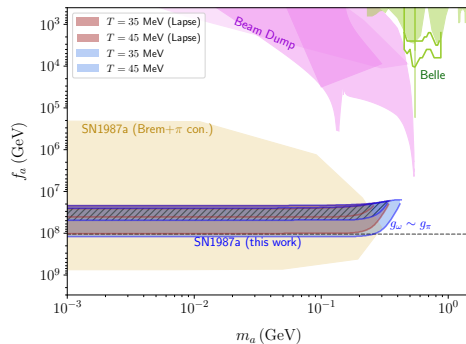
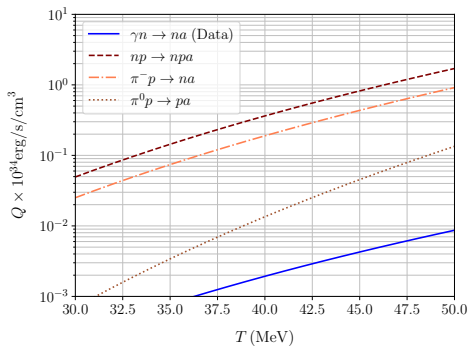
- Decay of the heavy axions:

$$L_{a \rightarrow \gamma\gamma} \approx \frac{4 \times 10^4 \text{ km}}{(G_{a\gamma\gamma}/10^{-9} \text{ GeV}^{-1})^2} \frac{E_a/100 \text{ MeV}}{(m_a/100 \text{ MeV})^4}$$

$$L_{a \rightarrow \gamma\gamma} > R_{SN}$$

Impact of the photo-production on axion constraints for KSVZ model

Contribution from photo-production to the emissivity



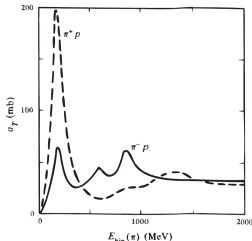
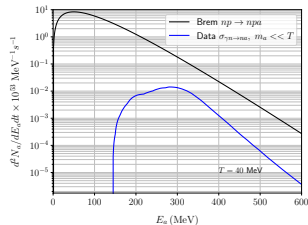
Can we observe the axions emitted from the supernovae

Bremsstrahlung peak: $E_a \sim 1.25T \sim 50 - 60$ MeV

Photo-production peak: $E_a \sim 6T \sim 250 - 300$ MeV

$$\frac{d^2 N_a}{dE_a dt} \approx C_f \rho_{15} \left(\frac{C_A 10^9}{f_a / \text{GeV}} \right)^2 g_{40}^4 \left(\frac{E_a}{\text{MeV}} \right)^6 e^{-E_a/T},$$

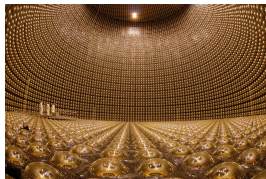
$$C_f = 4.6 \times 10^{42} \text{ MeV}^{-1} \text{ s}^{-1}$$



$$\sigma_{ap^+ \rightarrow N\pi^+} \approx 10^{-25} C_A^2 (f_\pi / f_a)^2 \text{ cm}^{-2}$$

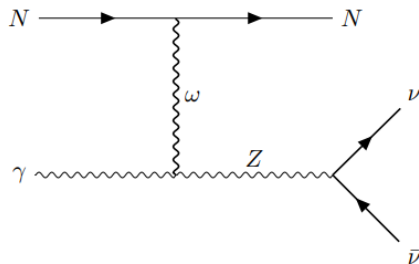
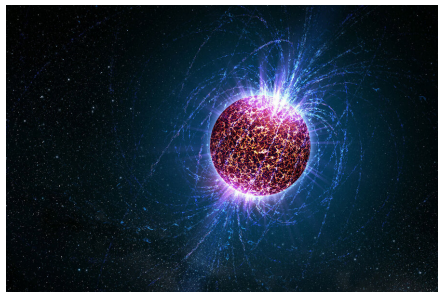
$$\frac{dN_\pi}{dt} \approx 6\rho_{15} C_A^4 \left(\frac{10^9}{f_a / \text{GeV}} \right)^4 g_{40}^4 T_{40}^7$$

$$\times \left(\frac{\text{Kpc}}{d} \right)^2 \left(\frac{M_{\text{detector}}}{\text{kton}} \right) \left(\frac{\text{g/mol}}{m_{H_2O}} \right)$$



Emission of neutrino from SN

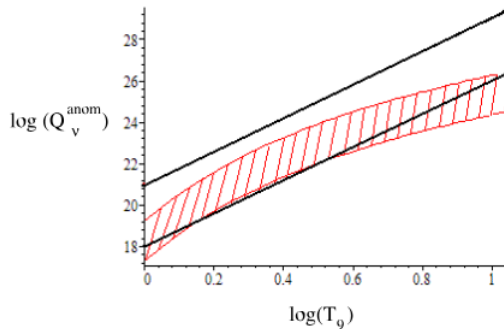
Photo-production of neutrino in NS



With Sabyasachi Chakraborty and Aritra Gupta: 2306.15872

WZW could contribution to NS cooling !

First computation of $N\gamma \rightarrow N\nu\bar{\nu}$ in [Harvey, Hill and Hill, arXiv:0708.1281]



But neglect the degeneracy effect ...

How do we compute the emissivity from a star ?

- Emissivity computation for $\gamma N \rightarrow NX$

$$Q_{\text{cooling}}^{N\gamma \rightarrow NX} \equiv \int \frac{d^3 p_\gamma g_\gamma f_\gamma(p_\gamma)}{(2\pi)^3 2E_\gamma} \int \frac{g_N d^3 p_{N_1} d^3 p_{N_2}}{(2\pi)^3 2E_{N_1} (2\pi)^3 2E_{N_2}} \prod_{X_i} \int \frac{(2\pi)^4 d^3 p_{X_i}}{(2\pi)^3 2E_{X_i}} \langle |\mathcal{M}|^2 \rangle \left(\sum_i E_{X_i} \right) f_N(E_{N_1}) (1 - f_N(E_{N_2})) \delta(E_1 - E_2 - Q_0) \delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q}). \quad (2)$$

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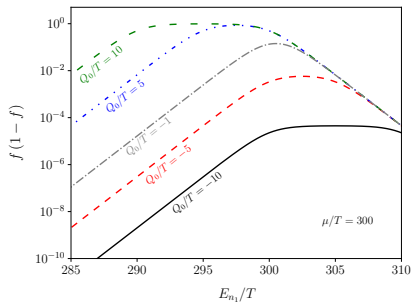
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- Degenerate case: $(1 - f_N(E_{N_2})) \ll 1$: $\xi \equiv \frac{p_F^2}{2\pi m_N T} \gg 1$
- Non-degenerate case: $(1 - f_N(E_{N_2})) \rightarrow 1$: $\xi \ll 1$

Degenerate computation



$$Q^{2 \rightarrow 3} = \frac{64 n_F g_\gamma}{4} \frac{g_\gamma}{3} \kappa^2 \int \frac{d^3 p_\gamma}{(2\pi)^3} \frac{f_\gamma}{2E_\gamma} |\vec{p}_\gamma|^2 \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6 2E_1 2E_2} E_1 E_2 (E_1 + E_2) S(q^\mu),$$

$$S(Q_0, q) = \frac{M_N^2 T}{\pi q} \frac{z}{1 - e^{-z}} \Theta(\mu - E_-).$$

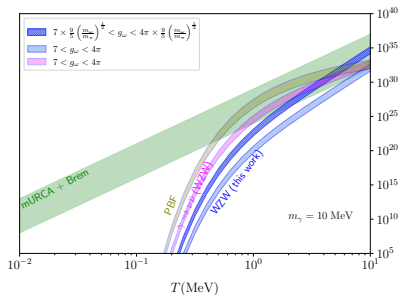
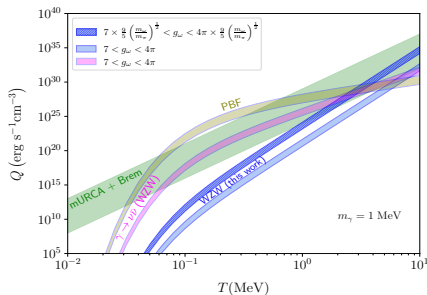
Standard cooling NS paradigm

- mURCA: $nn \rightarrow npe\bar{\nu}_e$, $npe \rightarrow nn\nu_e$ ∴

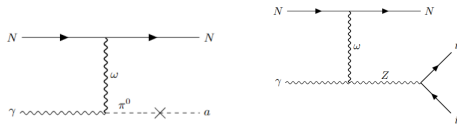
$$Q^{\text{mURCA}} \simeq 10^{26-29} \left(\frac{T}{1 \text{ MeV}} \right)^8 \text{ erg s}^{-1} \text{ cm}^{-3}$$

- Bremsstrahlung

$$Q^{\nu\text{-Brem}} \simeq 10^{24-28} \left(\frac{T}{1 \text{ MeV}} \right)^8 \text{ erg s}^{-1} \text{ cm}^{-3}$$

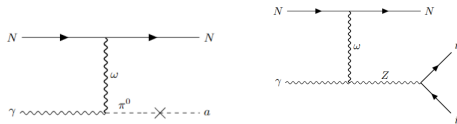


Conclusions and outlook



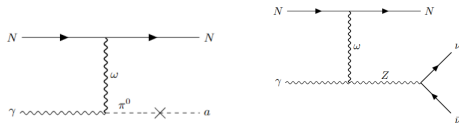
- Several new photo-production from anomaly: $\gamma N \rightarrow N\nu\nu$, $\gamma N \rightarrow Na$, ...ect.

Conclusions and outlook



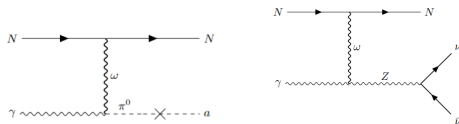
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Conclusions and outlook



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- In SN, $N\gamma \rightarrow \gamma\nu\nu$, likely always subdominant wtr to traditional channels.
- pheno of WZW: "Circulez, Y'a rien à voir"?



Better keep exploring: Harvey Hill arxiv:0712.1230

$$\begin{aligned}
 \Gamma_{AAB} &= C \int dZ \left[\frac{s_W^2}{c_W^2} \rho^0 + \left(\frac{3}{2c_W^2} - 3 \right) \omega - \frac{1}{2c_W^2} f \right] + dAZ \left[-\frac{s_W}{c_W} \rho^0 - \frac{3s_W}{c_W} \omega \right] + dZ [W^- \rho^+ + W^+ \rho^-] \frac{s_W^2}{c_W} \\
 &\quad - s_W dA [W^- \rho^+ + W^+ \rho^-] + (DW^+ W^- + DW^- W^+) \left[-\frac{3}{2} \omega - \frac{1}{2} f \right], \\
 \Gamma_{ABB} &= C \int Z \left\{ d\rho^0 \left[-\frac{3}{2c_W} \omega - \frac{s_W^2}{c_W} a^0 + \left(-\frac{3}{2c_W} + 3c_W \right) f \right] + d\omega \left[-\frac{3}{2c_W} \rho^0 + \left(-\frac{3}{2c_W} + 3c_W \right) a^0 - \frac{s_W^2}{c_W} f \right] \right. \\
 &\quad \left. + da^0 \left[\frac{s_W^2}{c_W} \rho^0 + \left(\frac{3}{2c_W} - 3c_W \right) \omega - \frac{1}{2c_W} f \right] + df \left[\left(\frac{3}{2c_W} - 3c_W \right) \rho^0 + \frac{s_W^2}{c_W} \omega - \frac{1}{2c_W} a^0 \right] \right\} \\
 &\quad + s_W dA (\rho^0 a^0 + 3\rho^0 f + 3\omega a^0 + \omega f + \rho^+ a^- + \rho^- a^+) - \frac{s_W^2}{c_W} dZ (\rho^+ a^- + \rho^- a^+) \\
 &\quad + \frac{3}{2} [W^+ D\rho^- + W^- D\rho^+] (-\omega + f) + \frac{3}{2} [W^+ (-\rho^- + a^-) + W^- (-\rho^+ + a^+)] d\omega \\
 &\quad + \frac{1}{2} [W^+ Da^- + W^- Da^+] (-3\omega - f) + \frac{1}{2} [W^+ (-3\rho^- - a^-) + W^- (-3\rho^+ - a^+)] df, \\
 \Gamma_{BBB} &= C \int 2 \left[(\rho^- f + \omega a^-) D\rho^+ + (\omega a^+ + \rho^+ f) D\rho^- + (\omega a^0 + \rho^0 f) d\rho^0 + (\rho^+ a^- + \rho^- a^+ + \omega f + \rho^0 a^0) d\omega \right], \\
 \Gamma_{AAAB} &= C \int iW^+ W^- Z \left[3c_W \omega + \left(c_W + \frac{1}{2c_W} \right) f \right], \\
 \Gamma_{AABB} &= C \int i \left\{ W^+ W^- \left[\frac{3}{2} (\rho^0 + a^0) \omega - \frac{1}{2} (\rho^0 - a^0) f \right] \right. \\
 &\quad \left. + W^+ Z \left[\left(\frac{3c_W}{2} - \frac{1}{c_W} \right) \rho^- f - \frac{3c_W}{2} \rho^- \omega - \frac{c_W}{2} a^- f + \frac{3c_W}{2} \omega a^- \right] \right\}
 \end{aligned}$$