# Criterion for ultra-fast bubble walls: the impact of hydrodynamic obstruction (2401.05911)


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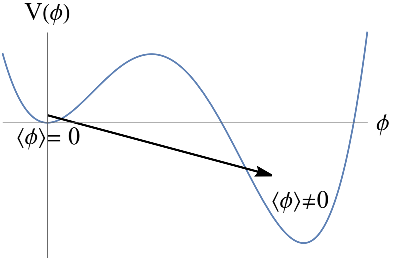

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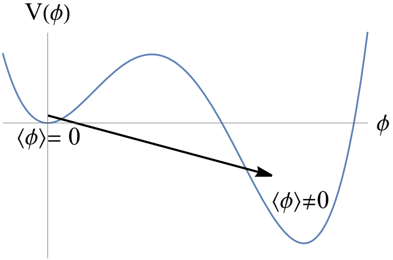


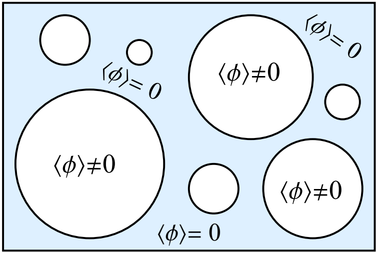

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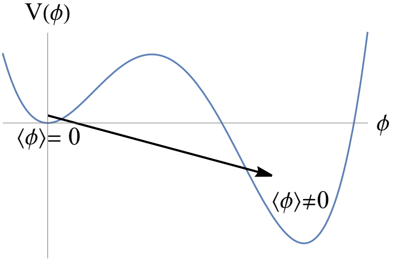
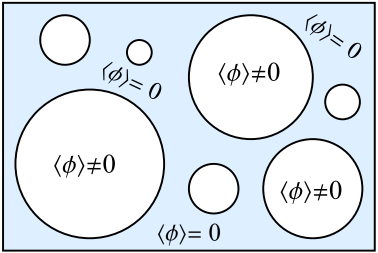
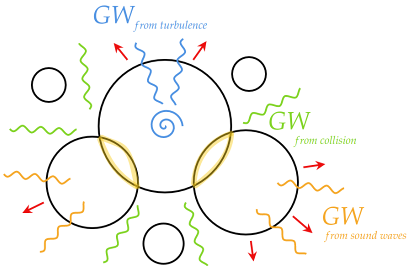



- Arises naturally in many BSM models
  → baryo/leptogenesis, dark matter, primordial black holes, . . .

# Parameters determining GW spectrum



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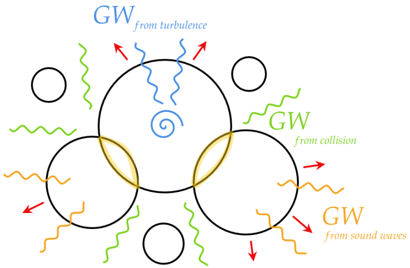


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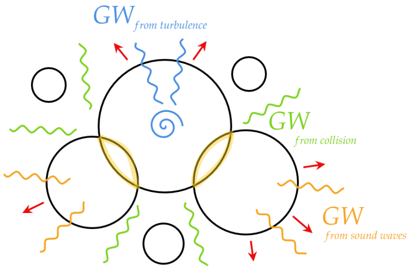


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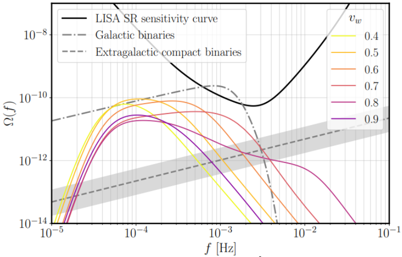


Gowling & Hindmarsh (2106.05984)

# Pressure in UR limit

- Integrating EoM of scalar field coupled to plasma over stationary wall profile:

$$\int_{-\delta}^{\delta} dz \left( \underbrace{\frac{dV_0(\phi)}{dz}}_{\mathcal{P}_{\text{driving}}} + \underbrace{(\partial_z \phi) \sum_i \frac{\partial m_i^2(\phi)}{\partial \phi} \int \frac{\partial^3 \mathbf{p}}{(2\pi)^3 2E_i} \left(f_i^{eq} + \delta f_i\right)}_{-\mathcal{P}_{\text{friction}>0}} \right) = 0 \quad (1)$$

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- Ballistic limit ($L_\mathrm{w} \ll \gamma_w L_\mathrm{MFP}$), $\delta f_i \to f_i^{eq}(p, T_n) - f_i^{eq}(z, p, T(z))$:

$$\Delta V_0 = \sum_i \frac{a_i n_i m_i^2 T_n^2}{48} = \mathcal{P}_{BM}$$

$a_i$ : DoF of species $i$

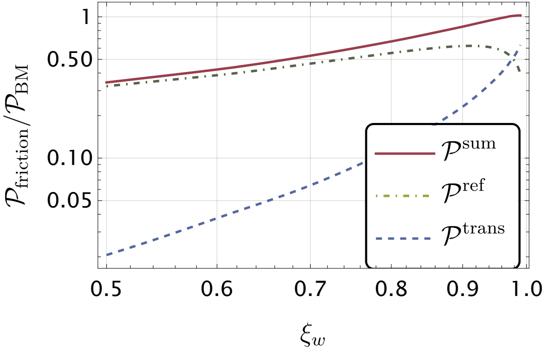
$$n_i = \begin{cases} 1 & \text{fermions} \\ 2 & \text{bosons} \end{cases}$$

# Bödeker & Moore criterion

- If pressure increases monotonously with wall velocity

$$\boxed{\Delta V_0 > \mathcal{P}_{BM}} \quad (2)$$

determines if bubble is runaway (Dine et al. 9203203)



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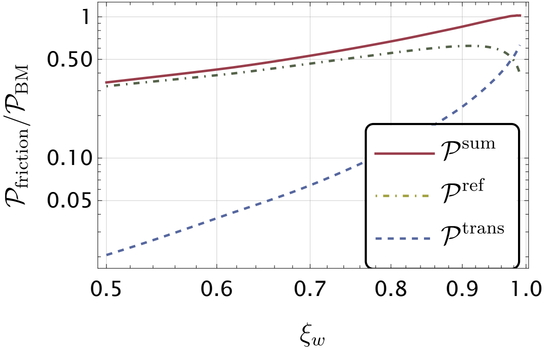


- Pressure from *hydrodynamical* effects will not be monotonously and gives extra criterion

# Expansion modes

- No out-of-equilibrium effects: $\delta f_i = 0 \Rightarrow$ lower bound on pressure

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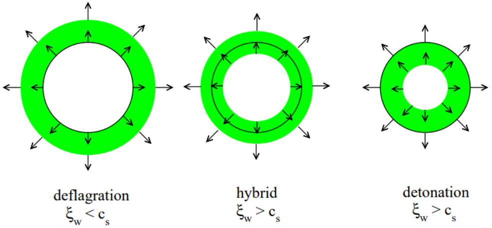
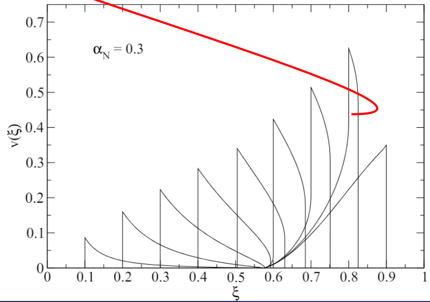
- Different expansion modes:




Jouguet velocity




$$\xi \equiv r/t$$

Espinosa, Konstandin, No & Servant (1004.4187)

# Hydrodynamic obstruction: A brief history

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- Balaji, Spannowsky & Tamarit (2010.08013):

$$\boxed{\partial_\mu (s u^\mu) = 0} \tag{3}$$

$\rightarrow$ Equivalent to total entropy conservation = LTE

# Hydrodynamic obstruction: A brief history

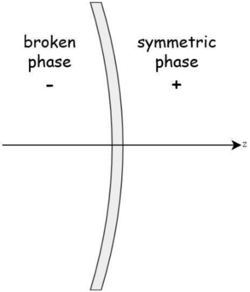
- Ai, Garbrecht & Tamarit (2109.13710): Assuming LTE

$$\begin{cases} \nabla_\mu T^{\mu\nu} = 0 \\ \partial_\mu (s u^\mu) = 0 \end{cases} \Leftrightarrow \begin{cases} \omega_+ \gamma_+^2 v_+ = \omega_- \gamma_-^2 v_- \\ \omega_+ \gamma_+^2 v_+^2 + p_+ = \omega_- \gamma_-^2 v_-^2 + p_- \\ \gamma_+ T_+ = \gamma_- T_- \end{cases} \quad (4)$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{plasma}} + T^{\mu\nu}_\phi$$

$\omega$ : enthalpy

$p$ : pressure

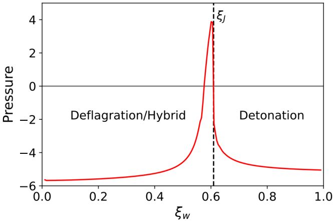



- 3 unknowns ($v_\pm$ & $T_-$) $\Rightarrow$ Solve numerically

# LTE pressure

- Pressure numerically :

$$\mathcal{P}_{\rm net} = -\Delta V_0 - \Delta V_T| + \bar{\mathcal{P}}_{\rm LTE}$$



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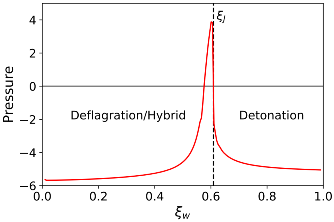



- However, at Jouguet, where pressure is maximal:

$$\bar{\mathcal{P}}^{\rm max}_{\rm LTE} = a_+ T_+^4 \left[\frac{4}{3}[\gamma(v_+)]^2 v_+ (v_+ - \frac{1}{\sqrt{3}})\right]$$

$$(-\Delta V_T)|_{\xi_w=\xi_J} = \frac{a_+ T_+^4}{3}\left[1 - \frac{4}{9}\gamma(v_+)^4 b\right]$$

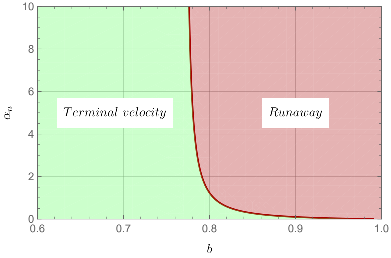
$$\Delta V_0 = a_+ T_+^4 \alpha_+$$

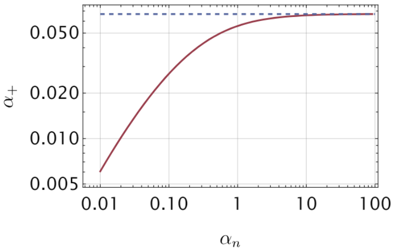
$a$ : Total DoF
$b \equiv \frac{a_-}{a_+}$
$\alpha_+ \equiv \frac{\Delta V_0}{a_+ T_+^4}$
$\alpha_n \equiv \frac{\Delta V_0}{a_n T_n^4}$

- $v_+(\alpha_+(\alpha_n)) \Rightarrow$ pressure depends only on $\alpha_n$ & $b$

# Infinitely efficient heating

- Analytically:



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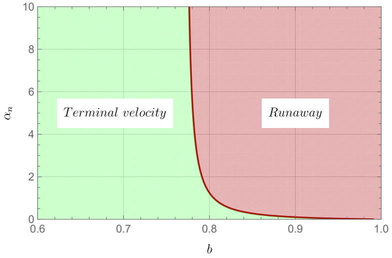


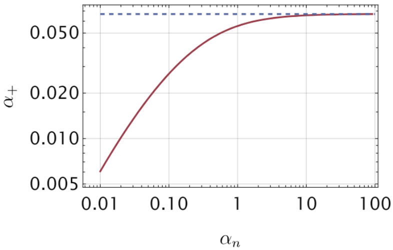


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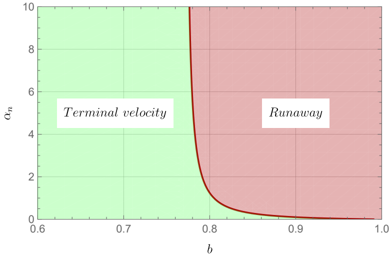
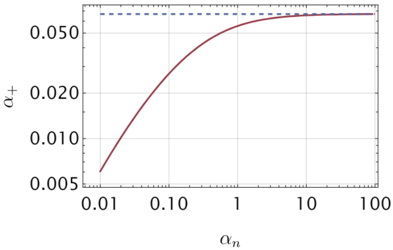
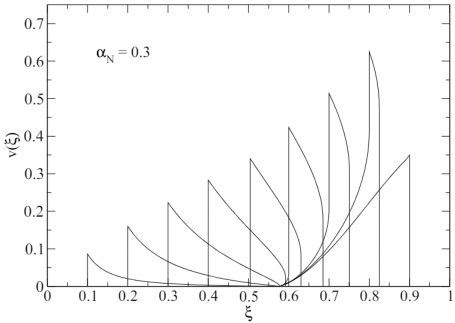


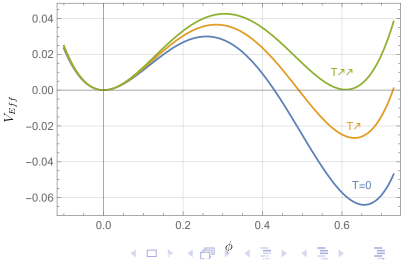

- Divergence at $b \approx 0.77$ when $\alpha_n \to \infty$
  - $\to$ Boundary depends heavily on DoF in and outside
- Saturation of $\alpha_+$
  - $\to$ Pressure independent of $\alpha_n$ for supercooled PT

# Unlimited efficient heating




Espinosa, Konstandin,
No & Servant (1004.4187)

- $\xi_w \to \xi_J \Rightarrow |\xi_{sw} - \xi_w| \searrow \Rightarrow T_+ \nearrow \Rightarrow \alpha_+ \searrow$
- Unphysical result
  - $\rightarrow$ Need for microphysics



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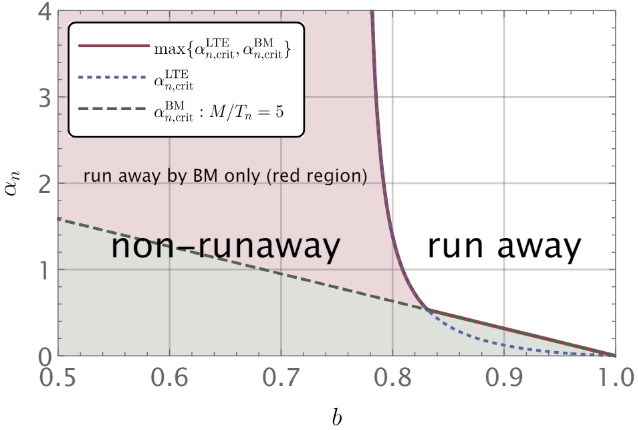
  - → Strongly interacting particles such that $R \gg L_{\text{MFP}}$

- Validity of expansion modes obtained in static picture
  - → Numerical simulations: Is adaptation to profile with new velocity faster than bubble acceleration?

# LTE vs. B & M




- If LTE picture holds, much more parameter space excluded to be runaway!

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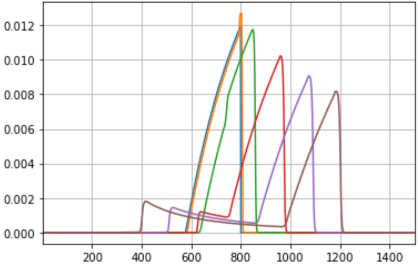

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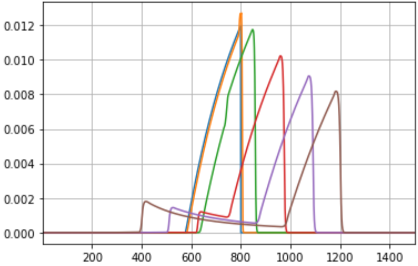


- Extra criterion on top of B & M criterion

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