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Transport equations for electroweak baryogenesis

Catch 22+2,

Dublin, 1-5.5.2024

1. Introduction
   - Overall EWBG-problem
   - Model building, Dark Sectors
2. Computing BAU
   - SC-method vs VIA-method
3. Moment expansion
   - Low moment approximations
   - Convergence issues with high moments
4. Beyond SC-limit
5. Conclusions

Kimmo Kainulainen,
University of Jyväskylä, Finland

1. Overall EWBG problem

Why $\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \times 10^{-10}$?

SM ok

==> BSM

==> BSM

At $T \approx$ 100 GeV

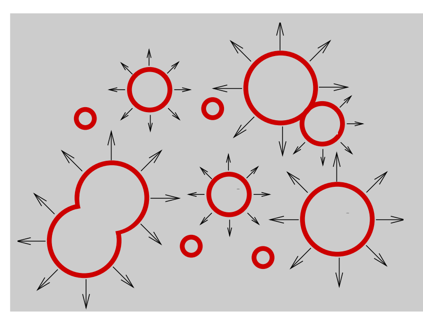
$$H \sim 10^{-14} T_{100}^2 \mathrm{GeV}$$
$$\Gamma \sim 10^{-5} T_{100} \mathrm{GeV}$$

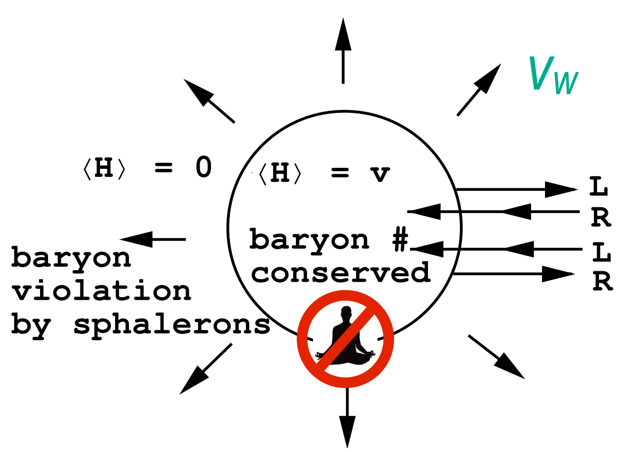
=> 

Need 1st order transtion

=> bubble passage time:

$$\frac{1}{t_w} \sim \frac{v_w}{L_w} \sim 10^{-5} T_{100}$$

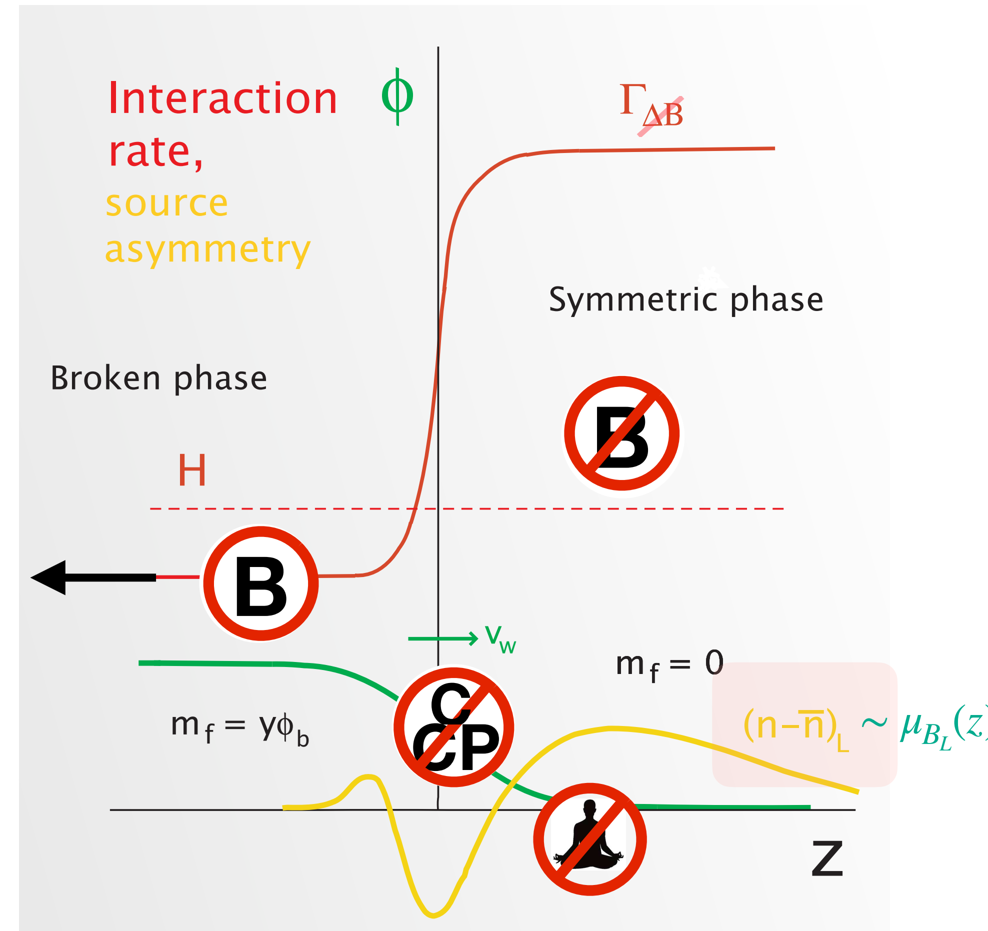




Sphaleron bound:

$$\frac{v_n}{T_n} > 1.1$$

1st order transition



Equilibrium: perturbative / nonperturbative

- B violation rate
- PT-parameters, $T_c, T_n, L, c_s, \ldots$

Transition strength, B-washout bound, GW-production,...

Out-of-equilibrium:

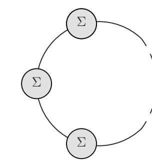
- CP-even perturbations $\delta f_{\rm even}$ ==> v_w, L_w, ...
- **CP-odd perturbations**, $\mu_{B_L}(z)$ ==> BAU

EWBG model building

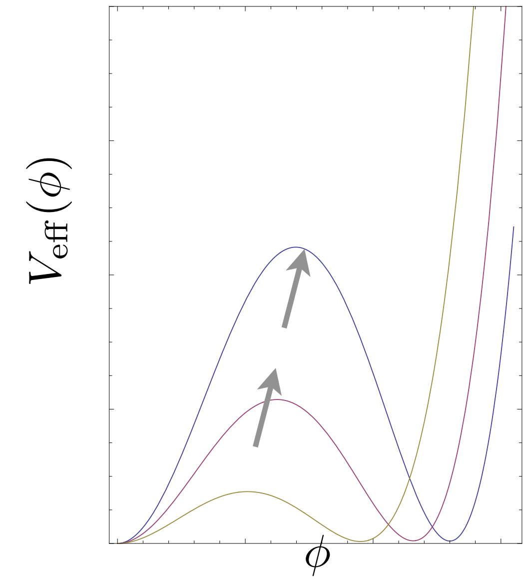
Transition strength

CP-violation

“SUSY cannot be disproved, only abandoned”



$$\delta V_{\rm eff} = -\sum_i \frac{T m_i^3(\phi, T)}{12\pi} + \dots$$



Anything relying on 1-step transition…

Issue: strong transition from loops requires very **large couplings**

- PT breaks down for $V_{\rm eff}$
- DR breaks down for 3d-lattice
- large eDM’s and nDM’s

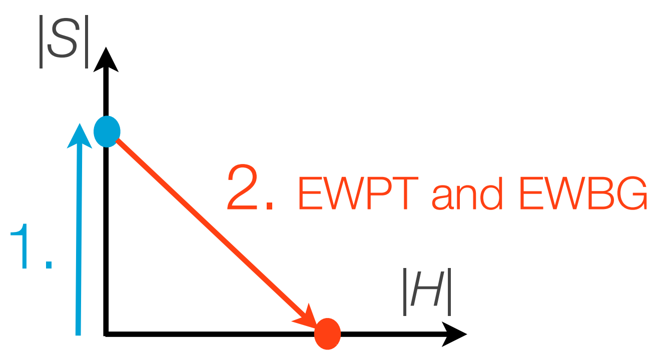
Some activity still observed eg. around nHDM models …

EWBG model building

Profumo, Ramsey-Musolf, Shaughnessy, JHEP 0708 (2007) 010
Inoue, Ovanesyan, Ramsey-Musolf, PRD93 (2016) 015013,
J.R.Espinosa, T.Konstandin, F.Riva, NPB854 (2012) 592
J.M.Cline, KK, JCAP 1301 (2013) 012
Laine, Rummukainen,
Cline, Moore, Quiros …,

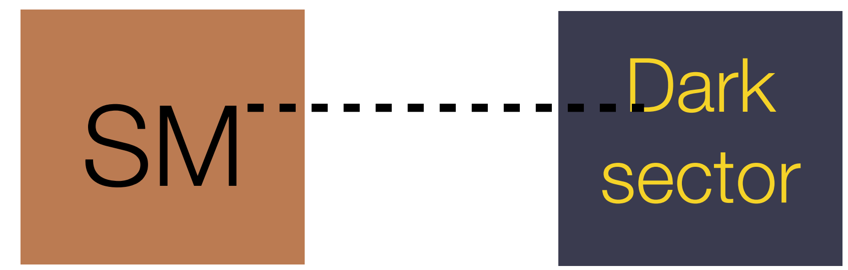
Two-step transitions:

$$V = \frac{1}{2}\lambda_{hs}h^2s^2 - (\mu_s^2 - c_sT^2)s^2 - (\mu_h^2 - c_hT^2)h^2 + ...$$



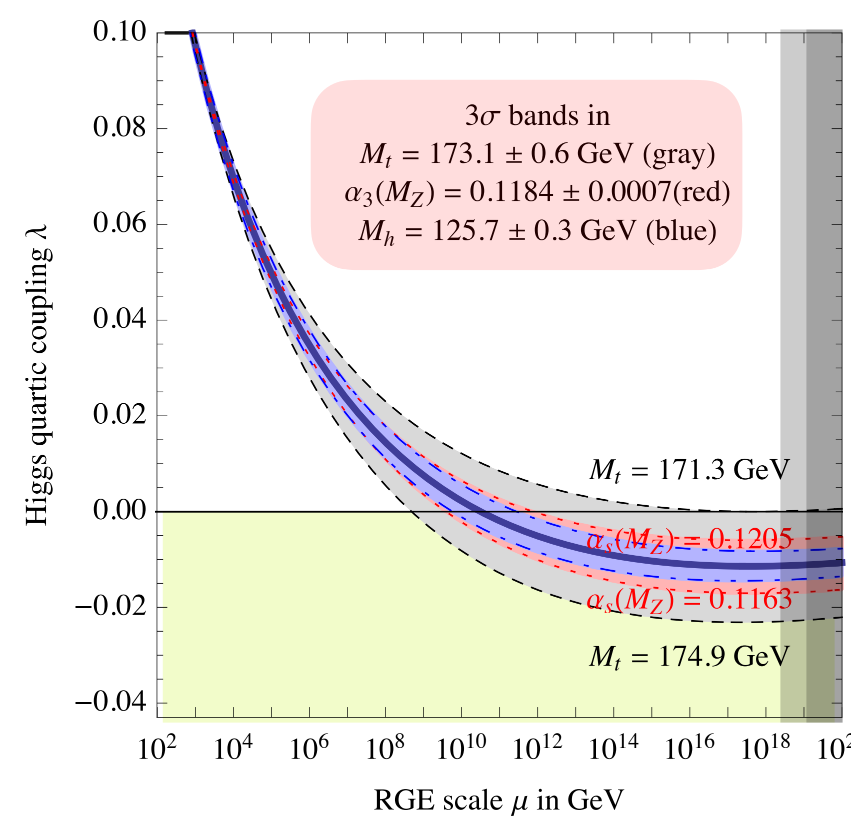
Works even for perturbative couplings

Portal models to Dark Sector(s)



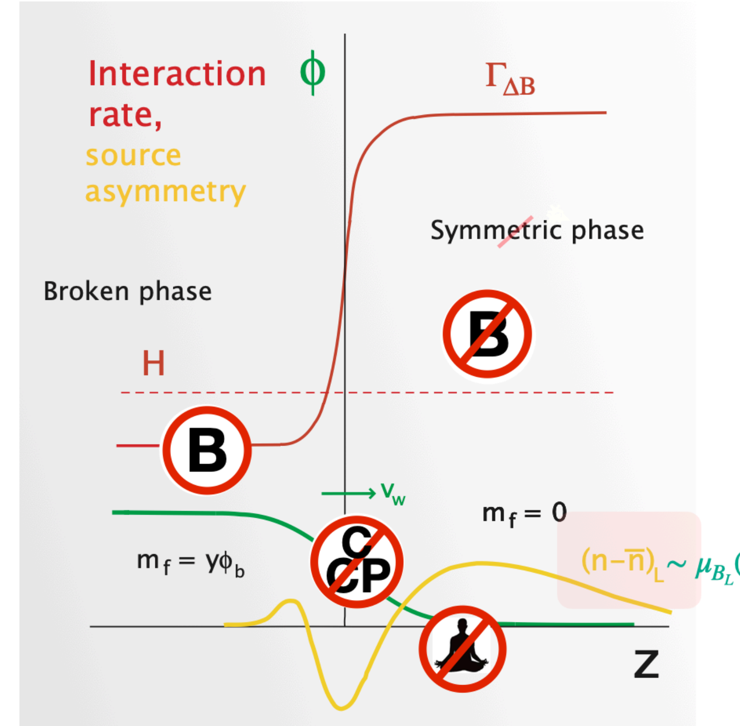
Low energy DS-models **motivated** by the near UV-completeness of the SM with little new physics beyond EW-scale

Shaposhnikov and Wetterich
Giudice etal, ..



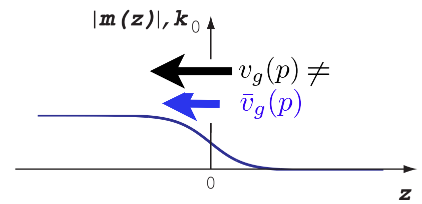
2. Computing B from EWBG

How to accurately model the CP-violating out-of-equilibrium interactions of particles with the expanding phase transition wall?



SC-method Boltzmann equation

$$v_{h\pm}\partial_z f_{h\pm} + F_{h\pm}\partial_{k_z} f_{h\pm} = \mathcal{C}_{h\pm}[f]$$



$$F_{h\pm} = -\frac{|m|^{2\prime}}{2\omega_{h\pm}} \pm h s_k \gamma_{||} \frac{(|m|^2\theta')'}{2\omega_0^2}$$

leading classical $\sim \hbar^0$ quantum $\sim \hbar$

WKB M.Joyce, T.Prokopec and N. Turok
J.M.Cline, M.Joyce and KK PLB417 (1998) 79; JHEP 0007 (2000) 018
J.M.Cline and KK, PRL85 (2000) 5519.

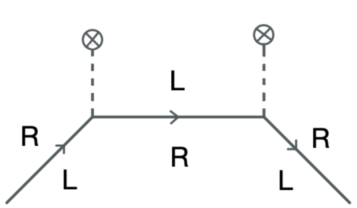
CTP KK, T.Prokopec, M.G.Schmidt and S.Weinstock,
JHEP 0106, 031 (2001); PRD66 (2002) 043502.
J.M.Cline, M.Joyce and KK, PLB417 (1998) 79; JHEP 0007 (2000) 018;
KK, JCAP 11 (2021) 11, 042.

VIA-method current divergence

$$\partial_\mu j^\mu_{\rm R}(x) \equiv S^{\rm CP}_{\rm R}(x) + S^{\not{C}\!\not{P}}_{\rm R}(x)$$

P.Huet, A.Nelson,

CTP A.Riotto,…
M.Carena etal., …
M.Ramsey-Musolf etal., …
M.Postma, J.van deVries; M. Wise, …

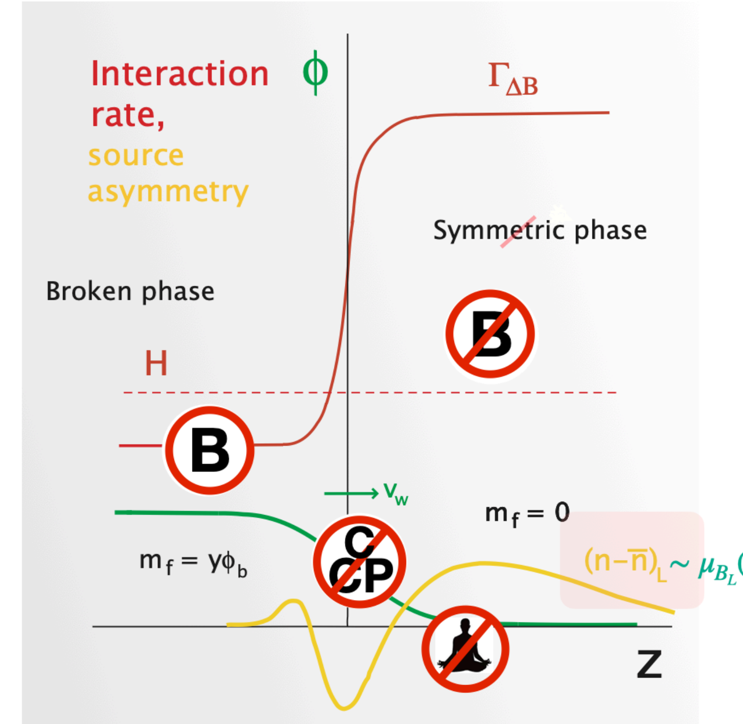


$$S^{\not{C}\!\not{P}}_{\rm R} = -v_w\gamma_w|m|^2\theta' \times I_\gamma$$

Identify j as a diffusion current
and employ Ficks law: $\mathbf{j} = -D\nabla n$

=> Diffusion equations

2. Computing B from EWBG



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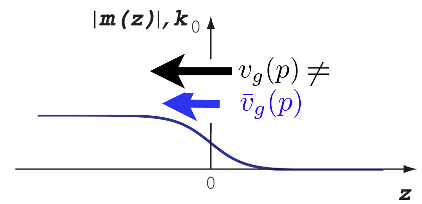

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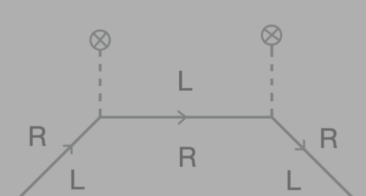
There is no such thing

K.K. JCAP 11 (2021) 11, 042.

Postma etal JHEP 12 (2022) 121



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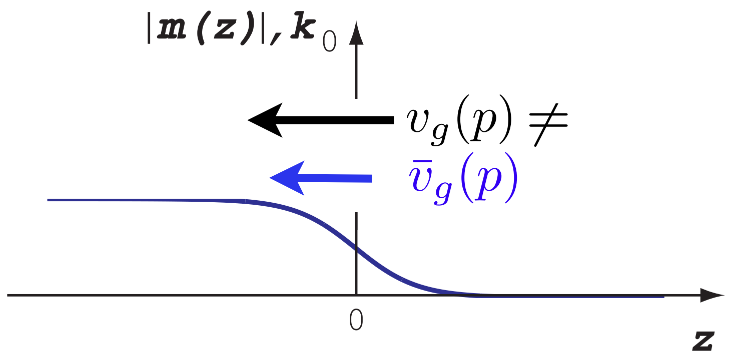
2.3 SC-BE equation

SC-equations describe classical motion under CP-violating force of QM-origin:

$$v_{h\pm}\partial_z f_{h\pm} + F_{h\pm}\partial_{k_z} f_{h\pm} = \mathcal{C}_{h\pm}[f]$$



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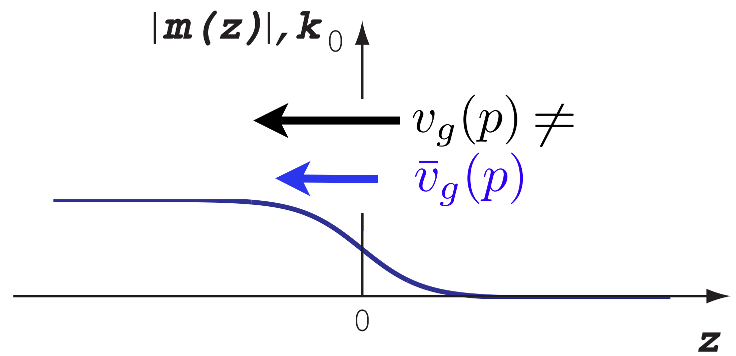

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CP-even

true energy

CP-odd, depends on *h*

consistent gradient expansion

$$\frac{k_z}{\omega_0}\partial_z \Delta f_h - \frac{|m|^{2\prime}}{2\omega_0}\partial_{k_z}\Delta f_h = \mathcal{S}_h + \mathcal{C}_h[f]$$

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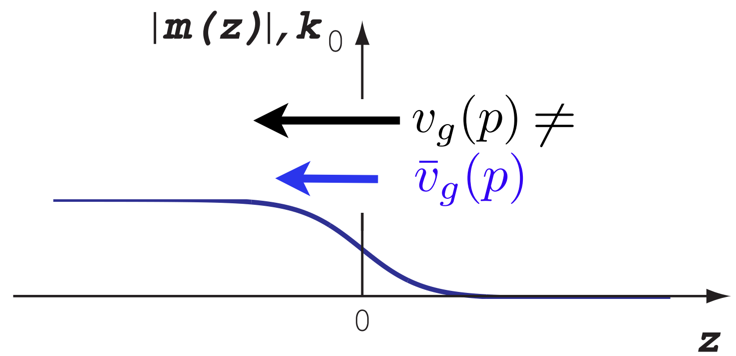

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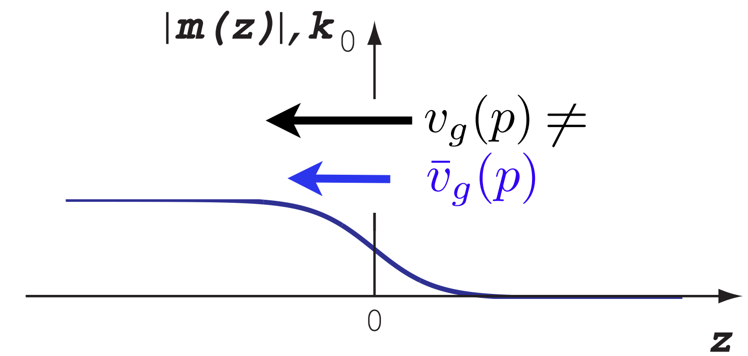

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WKB-quasiparticles

$$\mathcal{S}^{\gamma}_{qs\pm} = {\rm Re}\big[\mathcal{S}_{qs\pm}(\omega_{s\pm} + i\gamma_{s\pm})\big]$$

Only correction to source from damping

6

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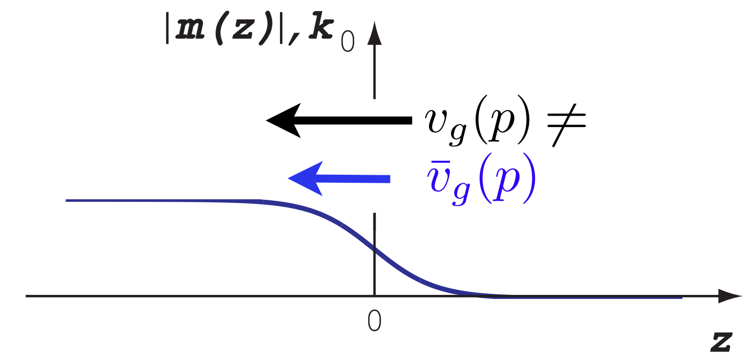

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pseudochemical potential

$\Delta f_h \equiv -\,\mu_h f'_{0w\pm} + \delta f_h$ with $\int \frac{{\rm d}^3 p}{(2\pi)^3}\delta f_h \equiv 0$

kinetic perturbation

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MB- and relaxation limits in C-term

$$-\frac{p_z}{\omega_{0i}} f'_{0wi}\,\partial_z\mu_{hi} + v_w\gamma_w\frac{|m_i|^{2\prime}}{2\omega_{0i}} f''_{0wi}\mu_{hi} + \frac{p_z}{\omega_{0i}}\partial_z\delta f_{hi} - \frac{|m_i|^{2\prime}}{2\omega_{0i}}\partial_{p_z}\delta f_{hi} = \mathcal{S}_{hi} + \Big(\sum_{a,k} s^a_{ik}\mu_k\Big)\,\Gamma^a_{{\rm inel},i} f_{{\rm MB}i}(p) - \Gamma_{{\rm T,i}}(p)\,\delta f_{hi}(p)$$

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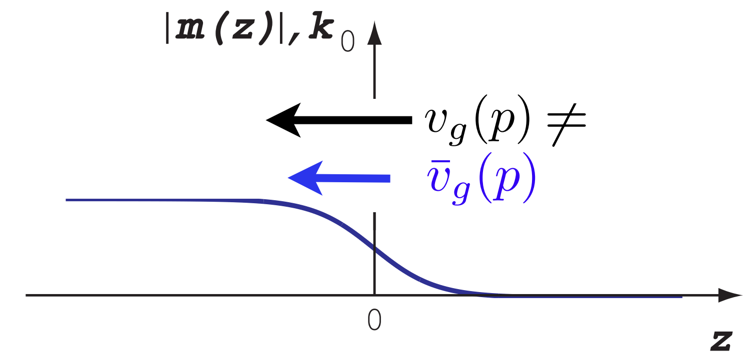

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Full problem consists of coupled set of SC BE's for all interacting species.

2.4 Solving the SC-BE

$$-\frac{p_z}{\omega_{0i}} f'_{0wi}\,\partial_z\mu_{hi} + v_w\gamma_w \frac{|m_i|^{2'}}{2\omega_{0i}} f''_{0wi}\mu_{hi} + \frac{p_z}{\omega_{0i}}\partial_z\delta f_{hi} - \frac{|m_i|^{2'}}{2\omega_{0i}}\partial_{p_z}\delta f_{hi} = \mathcal{S}_{hi} + (\sum_{a,k} s^a_{ik}\mu_k)\,\Gamma^a_{\mathrm{inel},i} f_{\mathrm{MB}i}(p) - \Gamma_{\mathrm{T,i}}(p)\,\delta f_{hi}(p)$$

SC-BE is a coupled set of partial differential equations (hard).

Direct solution of nPDE — Discretize p_z, $p_{\parallel}$, **or** turn into an integral equation — ~10000 coupled ODE's

S. De Curtis etal. JHEP 03 (2022) 163, arXiv:2401.13522v1, …

Integrated methods:

	Ansaz	Take moment integrals over BE's…
Fluid equations	$f(p_z\,.E;z) = [e^{\beta(z)(E+(v_w+v(z))p_z+\mu(z))}+1]^{-1}$	=> ODE's for $\beta(z)$, $v(z))$ and $\mu(z)$
Extended fluid equations	$f(p_z\,.E;z) = [e^{\beta(z)(E+(v_w+v(z))p_z+\mu(z))}+1]^{-1} + \sum_i a_i(z)F_i(p,z)$	=> ODE's for $\beta(z)$, $v(z))$ $\mu(z)$ and $a_i(z)$

Moore, Prokopec, Enqvist, Ignatius, Kajantie, Rummukainen, Bodeker, Espinosa, Konstandin, No, Servant Dorsch, Huber, Koczaczuck, Laurent, Cline, Garbrecht, Tamaris, De Curtis, Rose, Guiggiani, Muyor, Panico, Jinno, Cai, Wang, Sala, ….

Moment equations $f(p_z\,.E;z) = [e^{\beta(E+v_w p_z+\mu(z))}+1]^{-1} + \delta f$ with $\int d^3p\,\delta f \equiv 0$ => ODE's for $\mu(z)$ and $u_i^{(n)} = \langle (p_z/\omega_{i0})^n \delta f_i\rangle$

Most often used. How accurate?

3. Moment expansion

$$-\frac{p_z}{\omega_{0i}} f'_{0wi}\, \partial_z \mu_{hi} + v_w\gamma_w \frac{|m_i|^{2'}}{2\omega_{0i}} f''_{0wi}\mu_{hi} + \frac{p_z}{\omega_{0i}}\partial_z \delta f_{hi} - \frac{|m_i|^{2'}}{2\omega_{0i}}\partial_{p_z}\delta f_{hi} = \mathcal{S}_{hi} + (\sum_{a,k} s^a_{ik}\mu_k)\, \Gamma^a_{\mathrm{inel},i} f_{\mathrm{MBi}}(p) - \Gamma_{\mathrm{T,i}}(p)\, \delta f_{hi}(p)$$

Multiplying the SC BE with $(p_z/\omega_{0i})^\ell$ and integrating over momenta, one gets ***relativistic fluid equations***

$$\boxed{-D_{\ell+1}\xi'_h + u'_{h,\ell+1} + \frac{1}{2}v_w\gamma_w|x|^{2\prime}Q_\ell\xi_h + \frac{\ell}{2}|x|^{2\prime}\bar{R}u_{h,\ell} = \hat{\mathcal{S}}^w_{h,\ell} + \hat{\mathcal{C}}^w_{h\ell}.}$$

$$u_{h,\ell} \equiv \left\langle \frac{p_z^\ell}{\omega_0^\ell}\delta f_h \right\rangle, \qquad \xi_i \equiv \frac{\mu_i}{T} \quad \text{and} \quad x \equiv \frac{m}{T}. \qquad \langle X\rangle \equiv \frac{1}{N_1}\int \mathrm{d}^3p\, X \qquad N_1 \equiv -2\pi^2\gamma_w T^2/3$$

$$D_\ell \equiv \left\langle \left(\frac{p_z}{E}\right)^\ell f'_{0w} \right\rangle,$$

$$Q_\ell \equiv \left\langle \left(\frac{p_z^{\ell-1}}{2E^\ell}\right) f''_{0w} \right\rangle$$

$$Q^{8o}_\ell \equiv \left\langle \frac{s_\mathrm{p} p_z^{\ell-1}}{2E^\ell E_z} f'_{0w} \right\rangle,$$

$$Q^{9o}_\ell \equiv \left\langle \frac{s_\mathrm{p} p_z^{\ell-1}}{4E^{\ell+1}E_z} \left(\frac{1}{E} f'_{0w} - \gamma_w f''_{0w}\right) \right\rangle$$

$$K_\ell = \frac{1}{n}\int_p (\frac{p_z}{\omega_0})^\ell f_0(p)$$

$$\langle X\rangle \equiv \frac{1}{N_1}\int \mathrm{d}^3 pX$$

$$\bar{R} = \frac{\pi}{\gamma_w^2 \hat{N}_0}\int_m^\infty \mathrm{d}E \ln\left|\frac{p - v_w E}{p + v_w E}\right| f_0.$$

Relativistic auxiliary functions

3. Moment expansion

$$-\frac{p_z}{\omega_{0i}} f'_{0wi}\,\partial_z \mu_{hi} + v_w\gamma_w \frac{|m_i|^{2'}}{2\omega_{0i}} f''_{0wi}\mu_{hi} + \frac{p_z}{\omega_{0i}}\partial_z \delta f_{hi} - \frac{|m_i|^{2'}}{2\omega_{0i}}\partial_{p_z}\delta f_{hi} = \mathcal{S}_{hi} + (\sum_{a,k} s^a_{ik}\mu_k)\,\Gamma^a_{\mathrm{inel},i} f_{\mathrm{MBi}}(p) - \Gamma_{\mathrm{T,i}}(p)\,\delta f_{hi}(p)$$

Multiplying the SC BE with $(p_z/\omega_{0i})^\ell$ and integrating over momenta, one gets ***relativistic fluid equations***

$$\boxed{-D_{\ell+1}\xi'_h + u'_{h,\ell+1} + \frac{1}{2}v_w\gamma_w|x|^{2\prime}Q_\ell\xi_h + \frac{\ell}{2}|x|^{2\prime}\bar{R}u_{h,\ell} = \hat{\mathcal{S}}^w_{h,\ell} + \hat{\mathcal{C}}^w_{h\ell}.}$$

$$u_{h,\ell} \equiv \left\langle \frac{p_z^\ell}{\omega_0^\ell}\delta f_h \right\rangle, \qquad \xi_i \equiv \frac{\mu_i}{T} \quad \text{and} \quad x \equiv \frac{m}{T}. \qquad \langle X \rangle \equiv \frac{1}{N_1}\int \mathrm{d}^3 p\, X \qquad N_1 \equiv -2\pi^2\gamma_w T^2/3$$

where the source in ℓ:th equations is

$$\mathcal{S}_{h\ell} = -v_w\gamma_w h\Big[(|m|^2\theta')' Q^{8o}_\ell - |m|^{2\prime}|m|^2\theta' Q^{9o}_\ell\Big]$$

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Relativistic auxiliary functions

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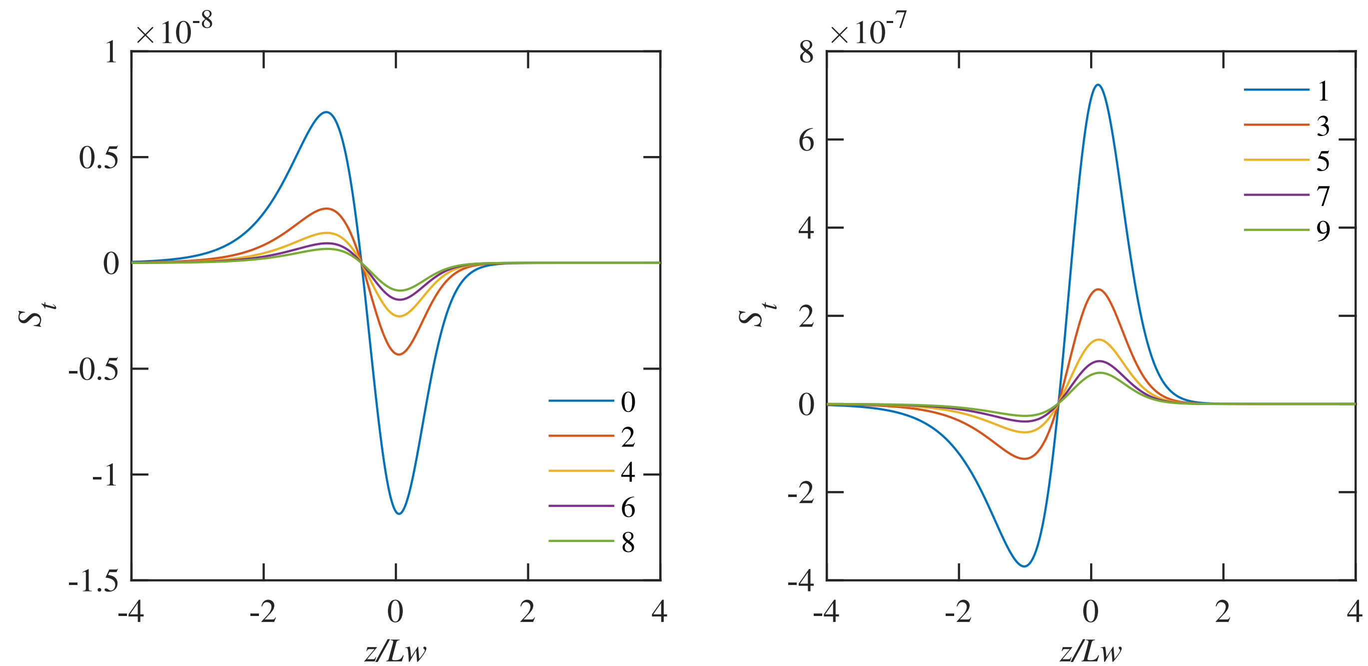
Relativistic auxiliary functions



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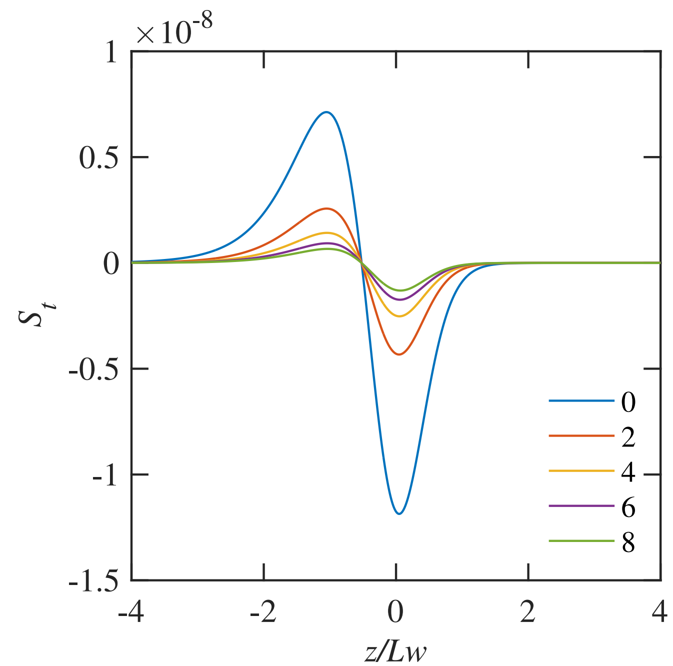

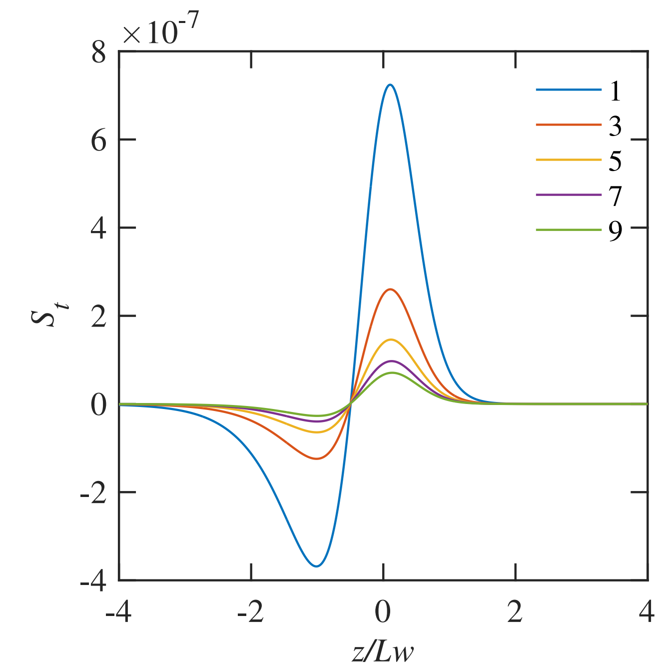

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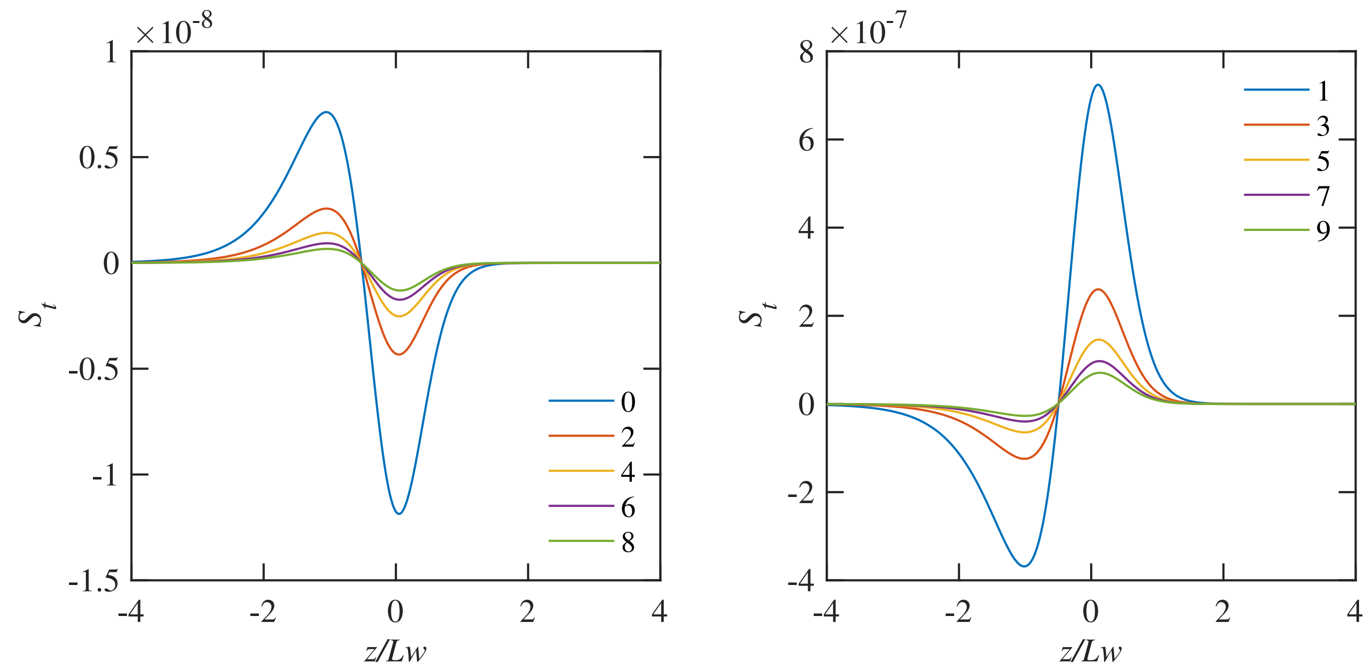

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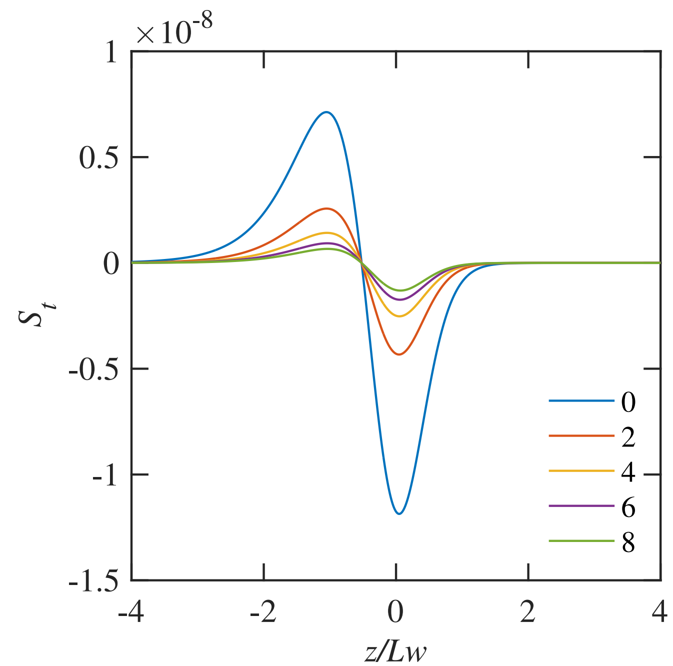
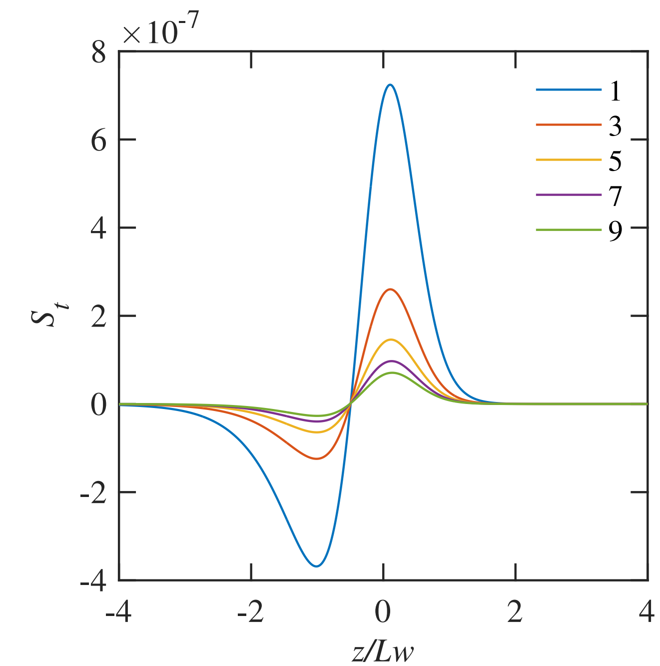
Relativistic auxiliary functions





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Assumed a specific *factorization*

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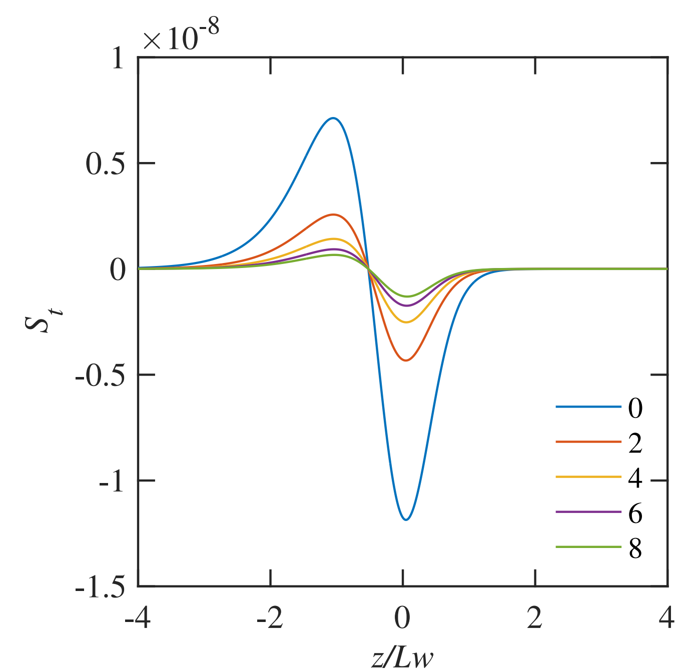
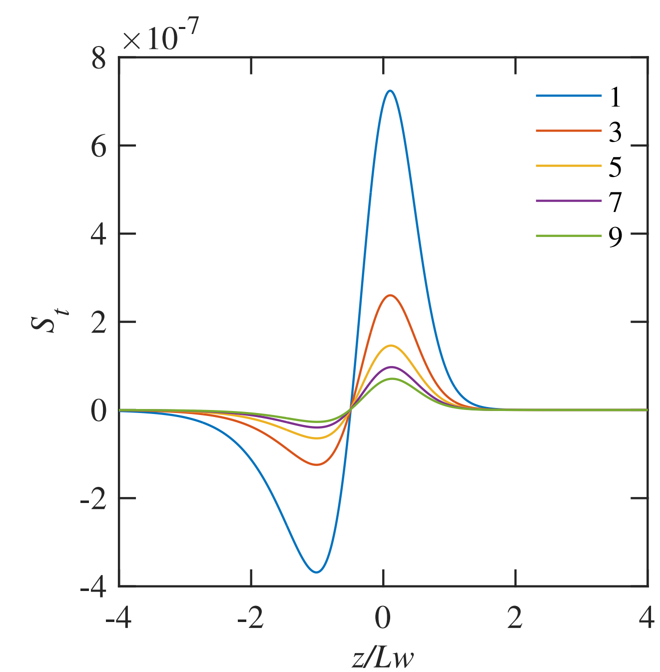




3.1 Benchmark model

We use a simple model with d=5 mass operator for top mass

Gripaios, J.R.Espinosa, T.Konstandin, Riva, 2012
J.M.Cline, K.K., ….

$$y_t h(z)\bar{t}_{\rm L}\left(1+i\frac{s(z)}{\Lambda}\right)t_{\rm R}+\text{H.c.},$$

with $h(z)=\frac{v_n}{2}\left(1-\tanh\frac{z}{L_w}\right),\qquad s(z)=\frac{w_n}{2}\left(1+\tanh\frac{(z-\delta_w)}{L_s}\right)$

$$v_n=\frac{1}{2}w_n=T_n,\qquad \Lambda=1\ \text{TeV},$$
$$L_w=L_s=\frac{5}{T_n},\qquad \delta_w=0,$$

$$\Gamma_{\rm sph}=1.0\times10^{-6}\ \text{T}\qquad \Gamma_y=4.2\times10^{-3}\ \text{T}$$
$$\Gamma_{\rm SS}=4.9\times10^{-4}\ \text{T}\qquad \Gamma_m=m_t^2/(63\ \text{T})$$
$$\Gamma_h=m_W^2/(50\ \text{T})$$

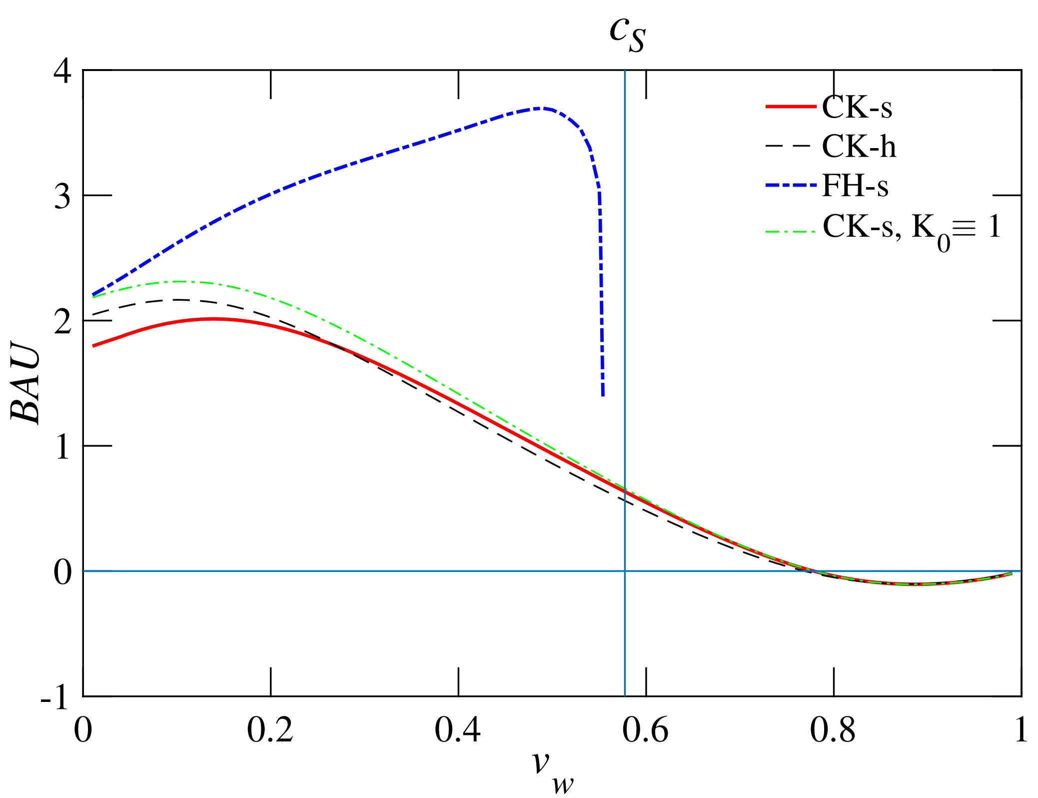
Include t_L, b_L, t_R and h_0 in the evolution equation network (chiral limit).

3.2 Two-moment results

J.M.Cline, K.K. PRD 101 (2020) 6, 063525

Relativistic equations do **not** display the spurious "sound speed limit" $v_w < c_s$ for the EWBG, found earlier with nonrelativistic equations of
L.Fromme and S.J.Huber, JHEP 03 (2007) 049.

The "sound speed limit" was also present in simple fluid ansaz approaches, but was removed in the extended fluid ansäze
G.C.Dorsh etal, JCAP 08 (2021) 020



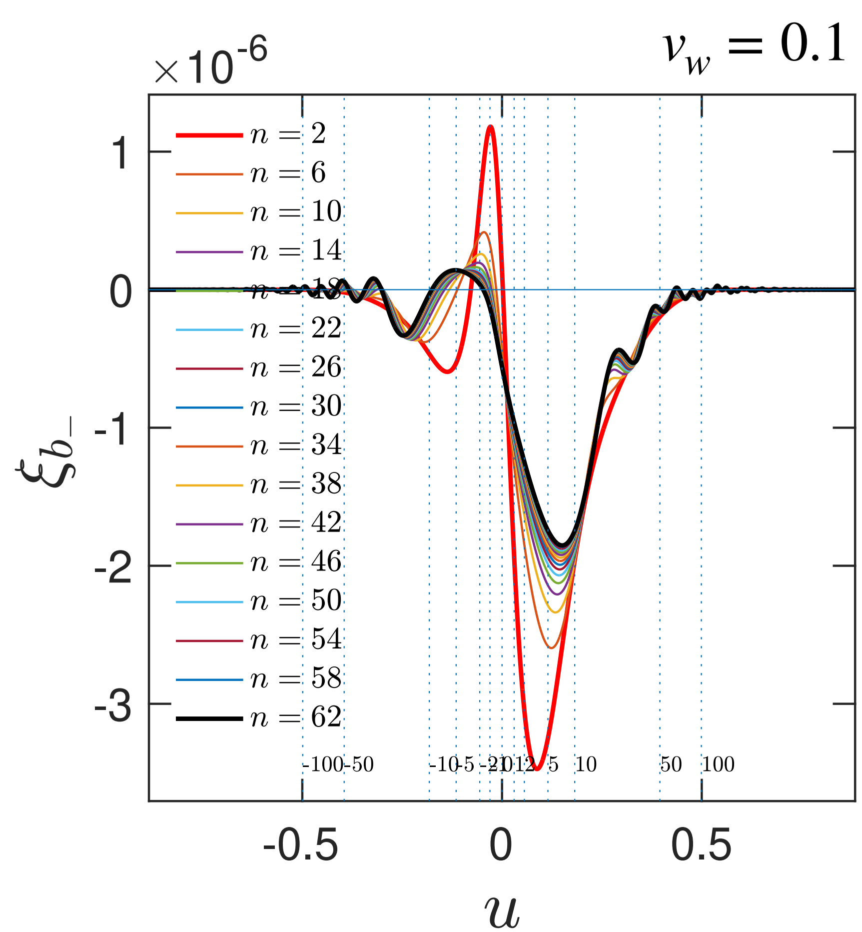
3.3 Higher moments

Examples of solutions for the bechmark model

K.K and N. Venkatesan, in preparation

$$\boxed{-D_{\ell+1}\xi_h' + u'_{h,\ell+1} + \frac{1}{2} v_w \gamma_w |x|^{2\prime} Q_\ell \xi_h + \frac{\ell}{2}|x|^{2\prime} \bar{R} u_{h,\ell} = \hat{S}^w_{h,\ell} + \hat{C}^w_{h\ell}.}$$

with t_L, b_L, t_R and h_0



$$zT = zT(u) = \frac{(zT)_* u}{(1 - |u| + \epsilon}$$

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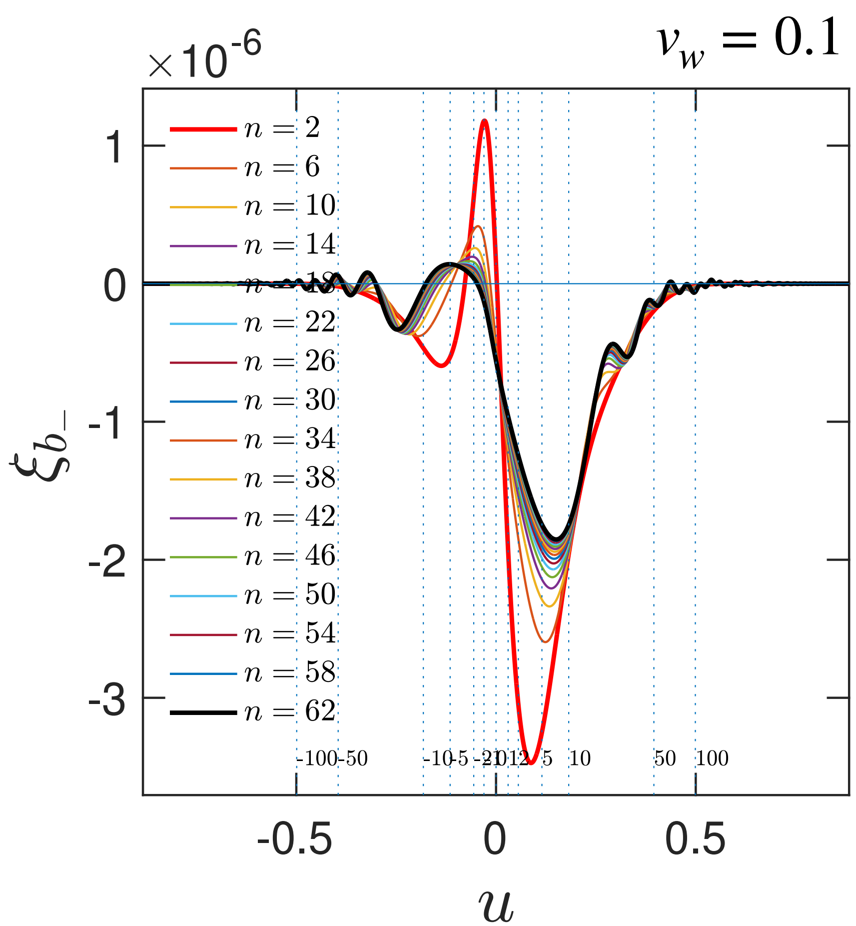

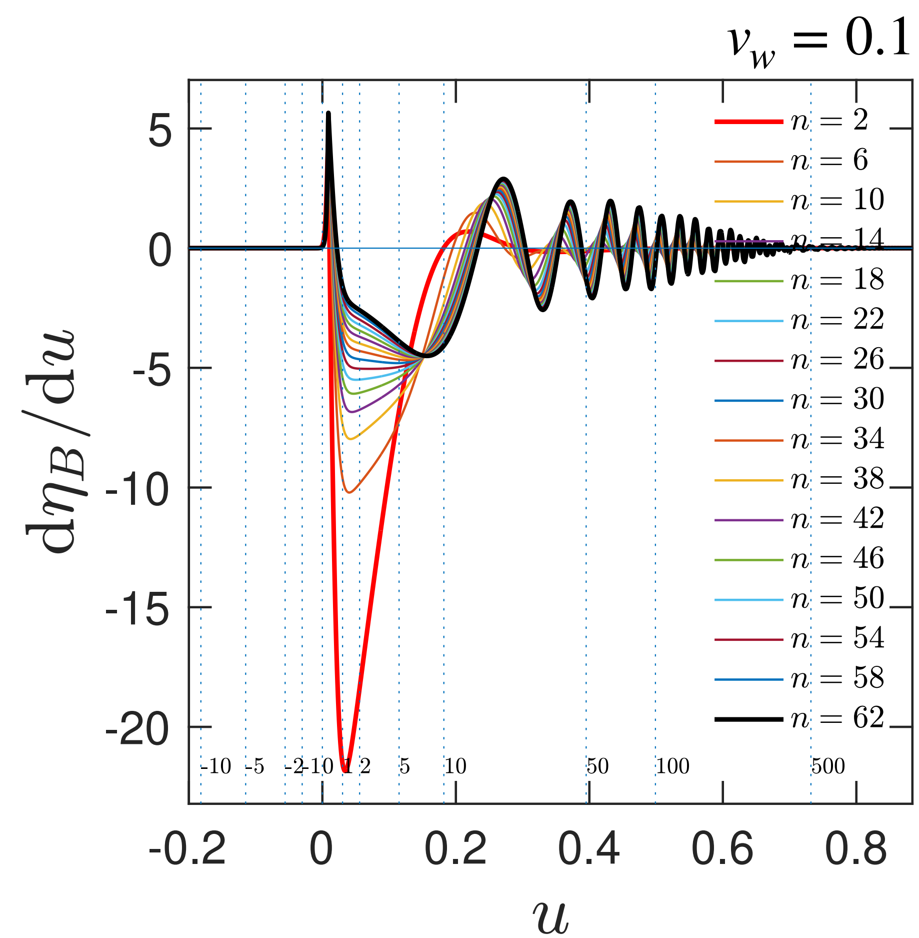

$$zT = zT(u) = \frac{(zT)_* u}{(1 - |u| + \epsilon}$$

$$\eta_B(v_w) = \frac{405\,\hat{\Gamma}_{\rm sph}}{4\pi^2 v_w\gamma_w g_*}\int d\hat{z}\,\xi_{B_{\rm L}}f_{\rm sph}\,e^{-45\Gamma_{\rm sph}|z|/4v_w\gamma_w} \equiv \int \mathrm{d}u\,\frac{\mathrm{d}\eta_{\rm B}}{\mathrm{d}u}$$

seed asymmetry

$$\xi_{B_{\rm L}} = \frac{1}{2}(1+4D_0^t)\xi_{t_-} + 2D_0^t\xi_{t_+} + \frac{5}{2}\xi_{b_-}$$

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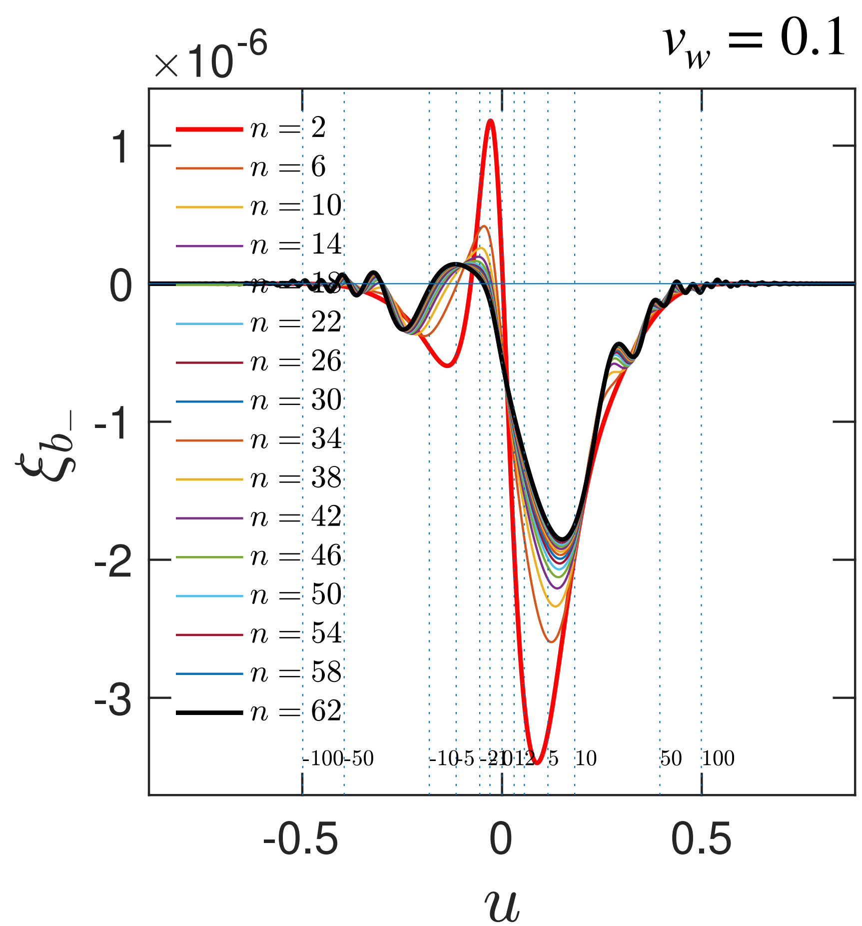
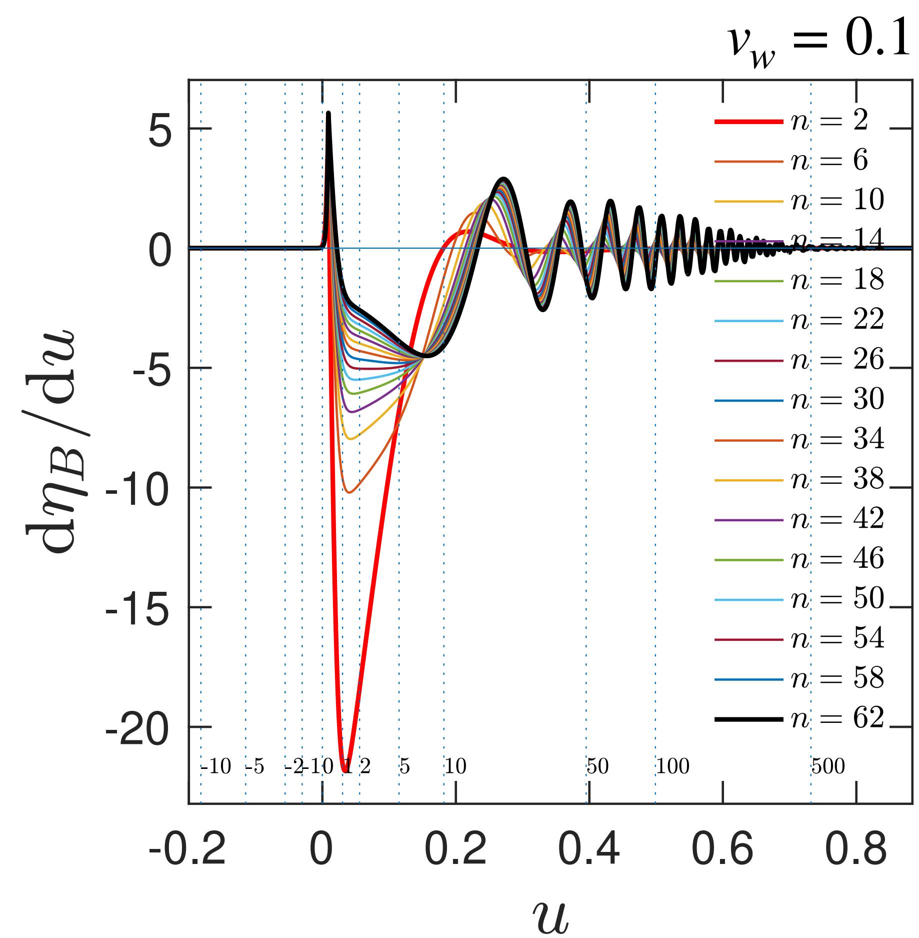
with t_L, b_L, t_R and h_0





$$zT = zT(u) = \frac{(zT)_* u}{(1 - |u| + \epsilon}$$

$$\eta_B(v_w) = \frac{405\,\hat{\Gamma}_{\rm sph}}{4\pi^2 v_w\gamma_w g_*}\int d\hat{z}\,\xi_{B_{\rm L}}f_{\rm sph}\,e^{-45\Gamma_{\rm sph}|z|/4v_w\gamma_w} \equiv \int \mathrm{d}u\,\frac{\mathrm{d}\eta_{\rm B}}{\mathrm{d}u}$$

seed asymmetry

$$\xi_{B_{\rm L}} = \frac{1}{2}(1+4D_0^t)\xi_{t_-} + 2D_0^t\xi_{t_+} + \frac{5}{2}\xi_{b_-}$$

Relatively nice convergence, but $\mathrm{d}\eta_{\rm B}/\mathrm{d}u$ shows weird oscillations for high moments

3.3.1 Cause of oscillations

is that the eigenfunctions of the inverse differential operator are oscillatory

$$\mathcal{W}' = \hat{\mathcal{A}}^{-1}\big(\hat{\mathcal{S}} + \hat{\mathcal{C}}[\mathcal{W}] - \hat{\mathcal{B}}[\mathcal{W}]\big)$$

$$\rightarrow \mathscr{A}^{-1}\Gamma\mathscr{W}$$

large, $(nN_s)^2$ constant real matrix

$$\hat{\mathcal{A}}^{-1} = \mathrm{diag}(\hat{\mathcal{A}}_1^{-1}, \hat{\mathcal{A}}_1^{-1}, ..., \hat{\mathcal{A}}_N^{-1})$$

$$\hat{\mathcal{A}}^{-1} = \frac{1}{\mathcal{D}_n}\begin{pmatrix} 0 & 0 & \cdots & \cdots & -R & 1 \\ \mathcal{D}_n & 0 & \cdots & \cdots & -RD_1 & D_1 \\ 0 & \mathcal{D}_n & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & -RD_{n-3} & D_{n-3} \\ \vdots & \vdots & \cdots & \mathcal{D}_n & -RD_{n-2} & D_{n-2} \\ 0 & 0 & \cdots & 0 & -D_n & D_{n-1} \end{pmatrix}$$

$$\mathcal{D}_n \equiv (-1)^n \det(\hat{\mathcal{A}}) = D_{n-1}R - D_n$$

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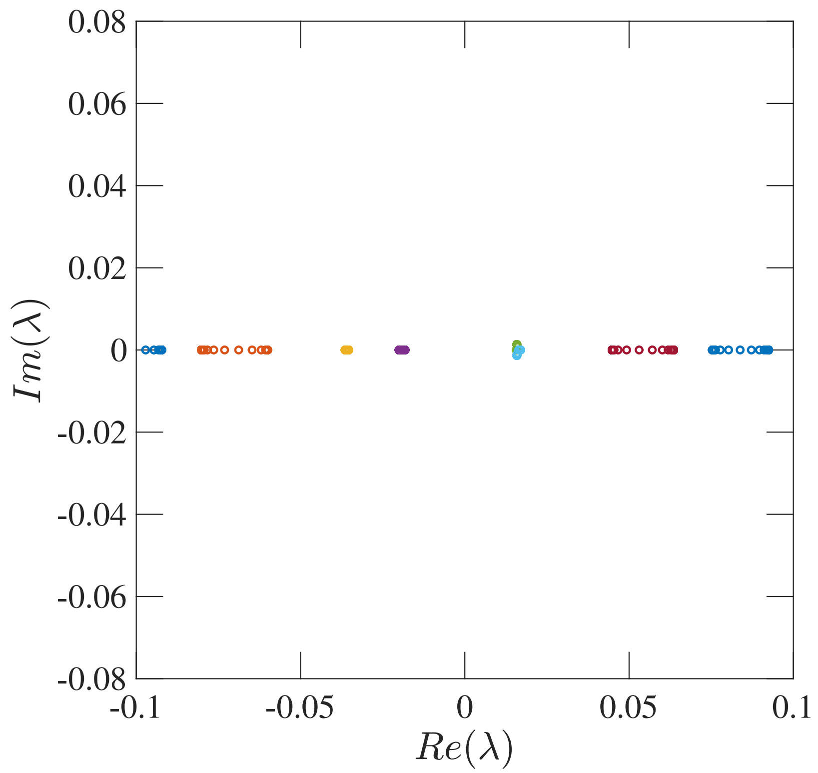
Eigenvalues are real only for the n=2 case



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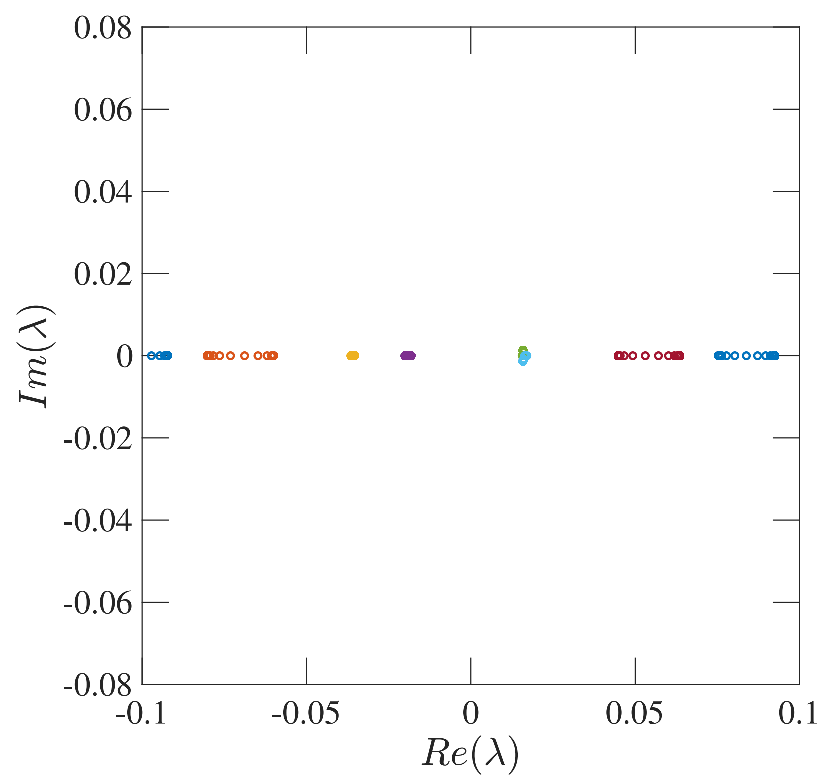
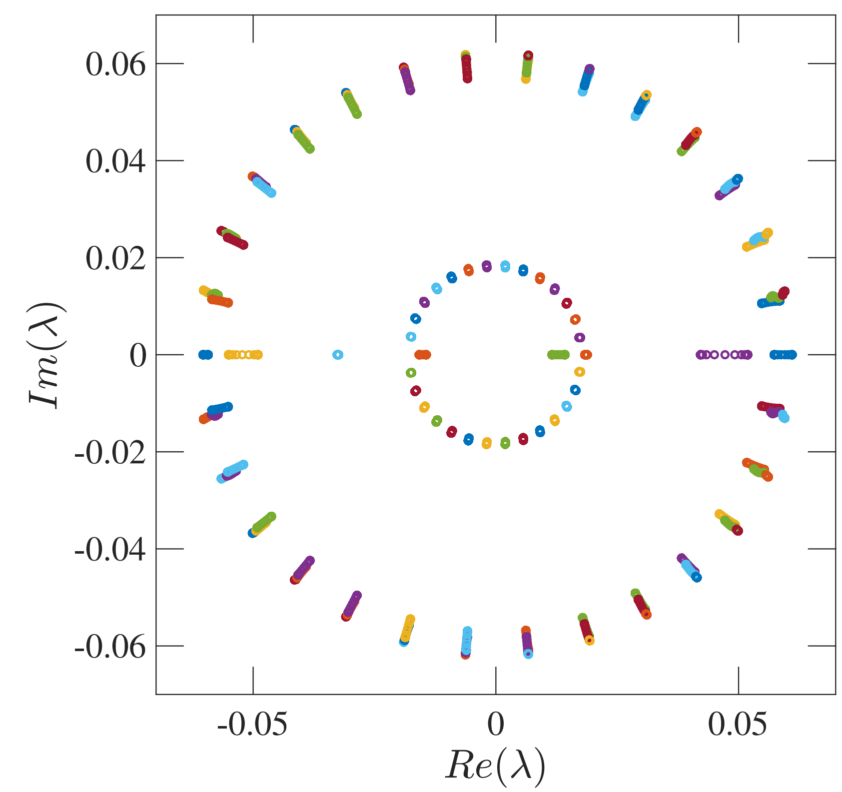


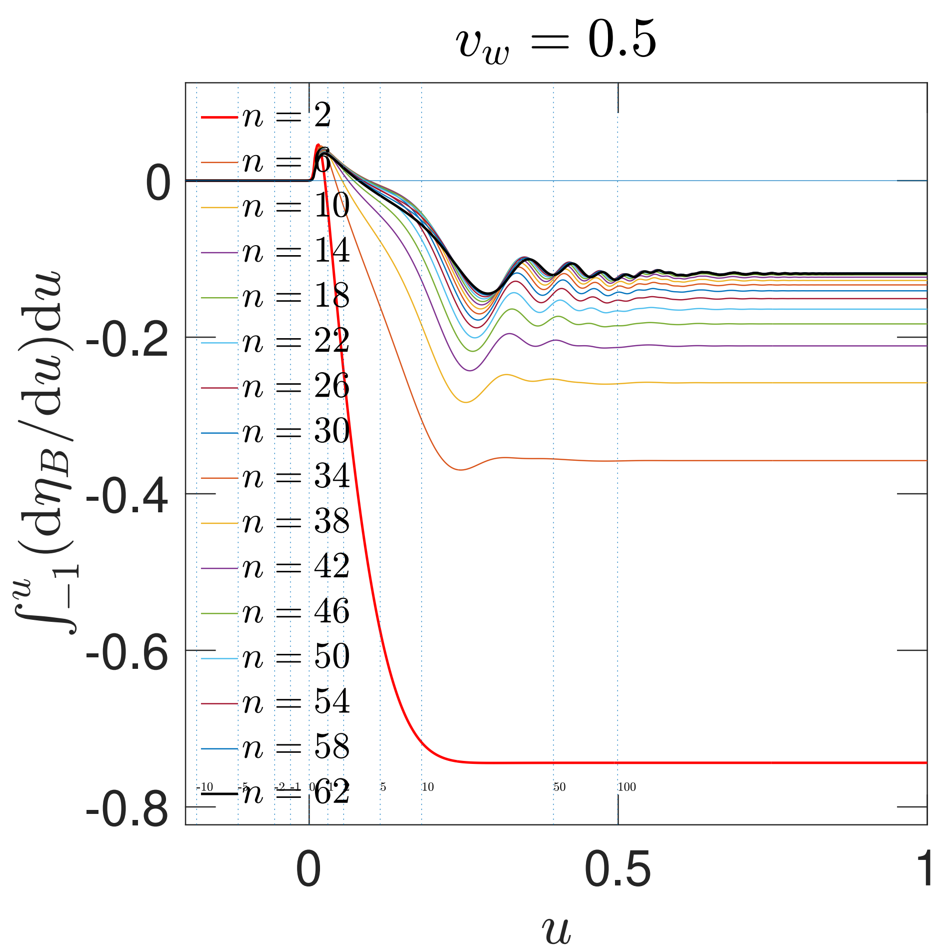
The larger the number of moments *n,* the more rapidly oscillating, slowly converging eigenfunctions emerge



=> moment equations become computationally more expensive and numerical accuracy more challenging to keep.

3.3.2 Higher moments, convergence

Oscillations do not affect the integrated baryon asymmetry very much however



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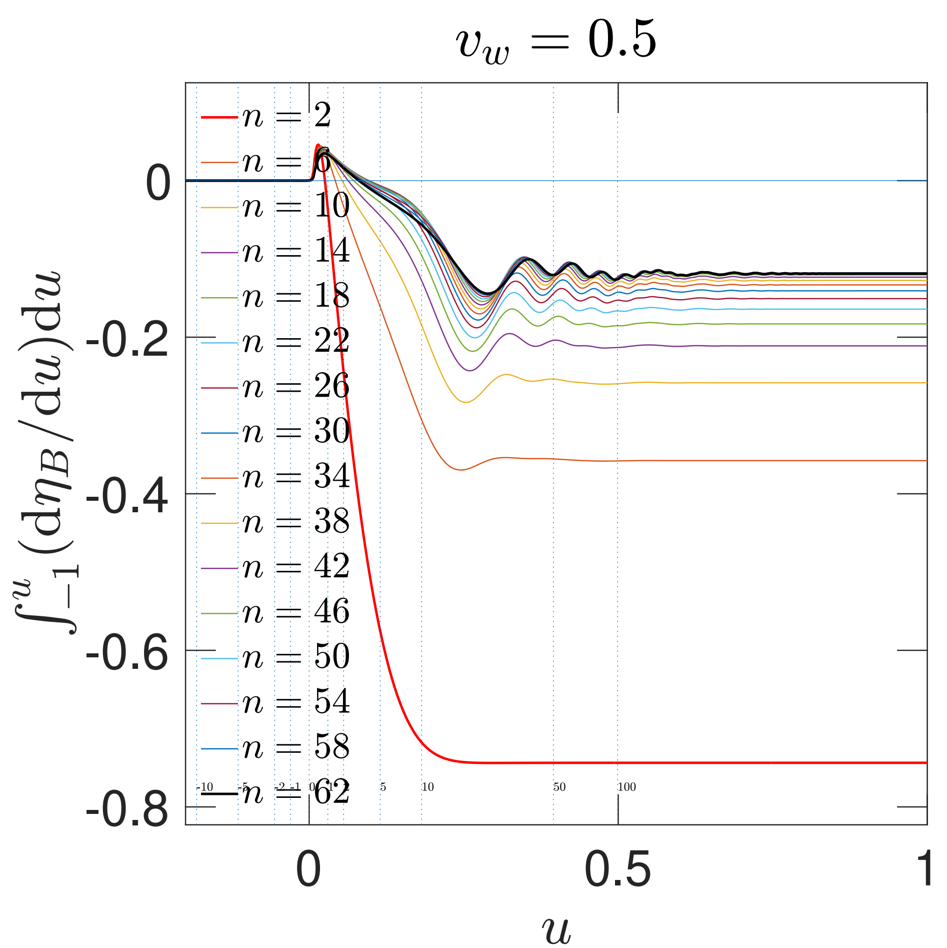

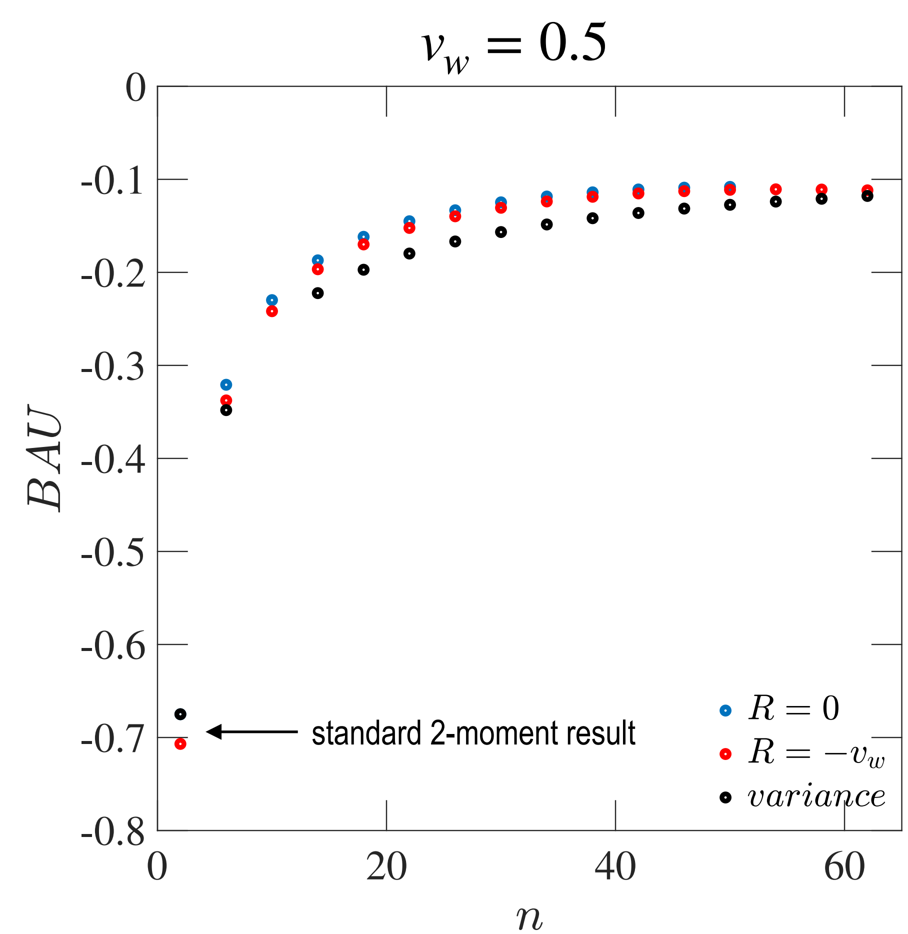

Truncations used:

constant $\quad u_{n+1} = R u_n \qquad u_\ell = \frac{1}{N_1}\int_p v_z^\ell \, \delta f$

variance $\quad \int_p (v_z - \langle v_z \rangle)^{n+1} \, \delta f = 0$

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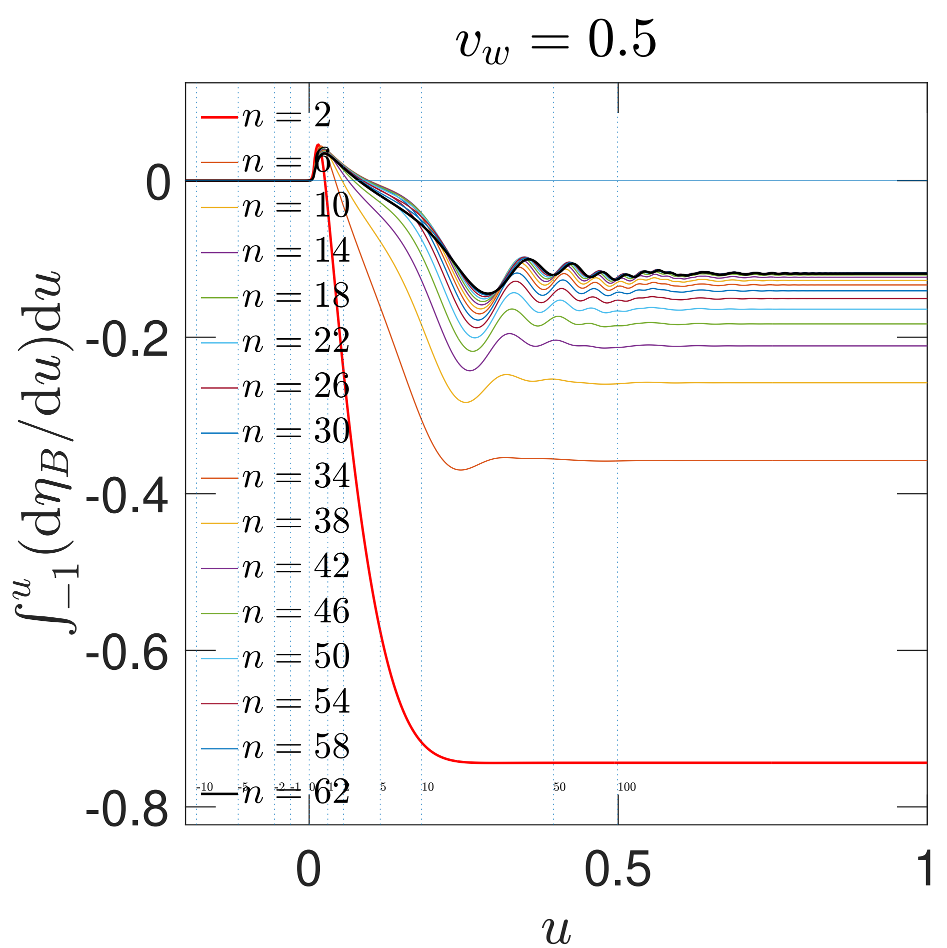
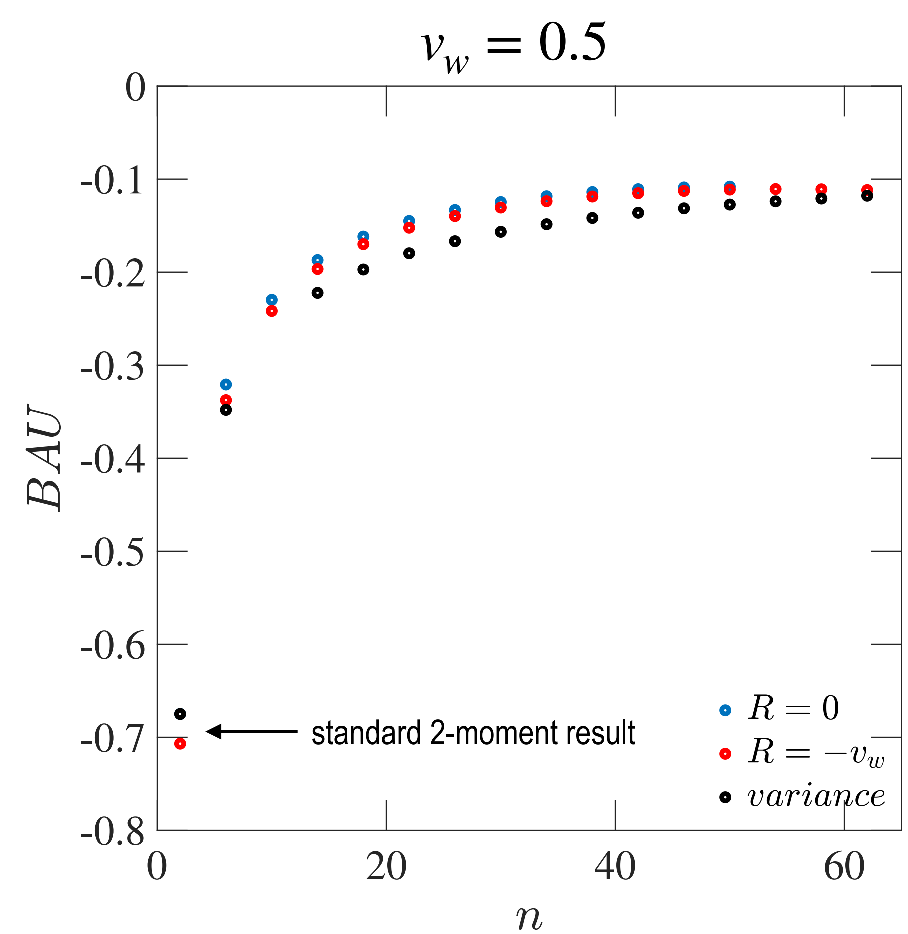




For reasonably large v_w the convergence is good and does not seem to depend on the truncation method.

Difference to 2-moment result is large however, and convergence requires a large number of momenta n ~ 50.

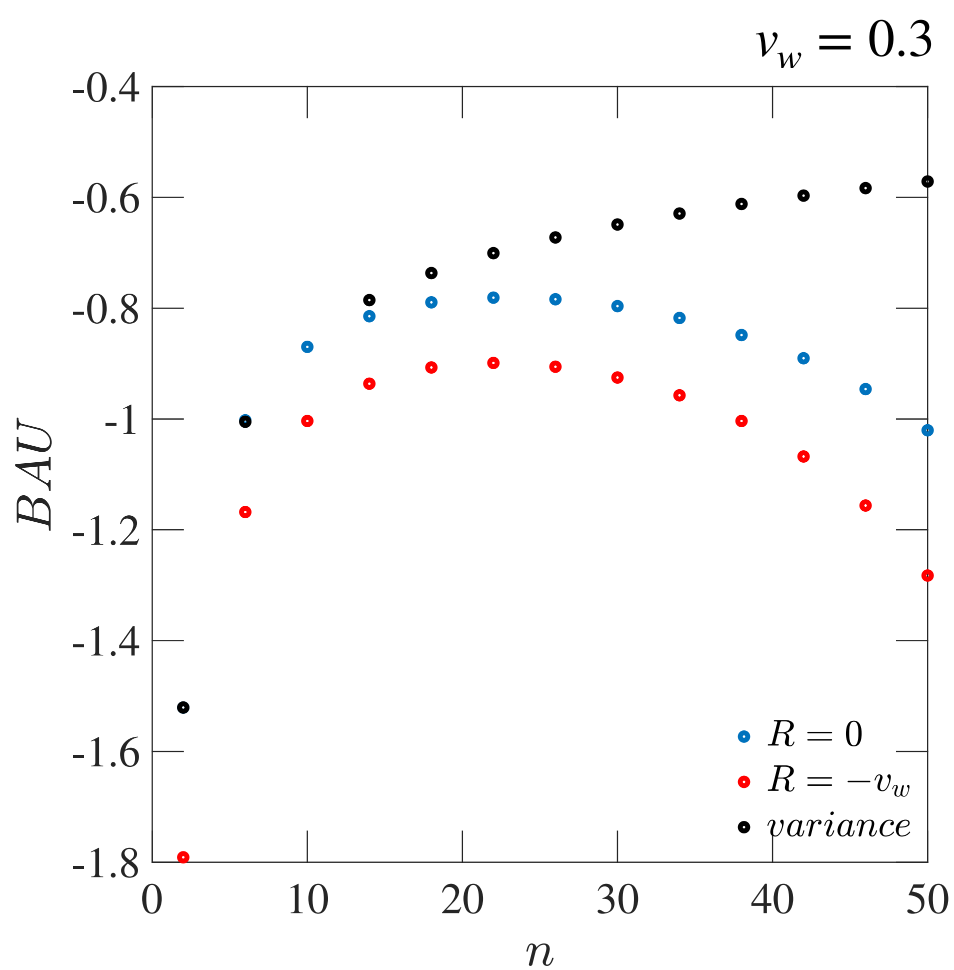
Truncations used:

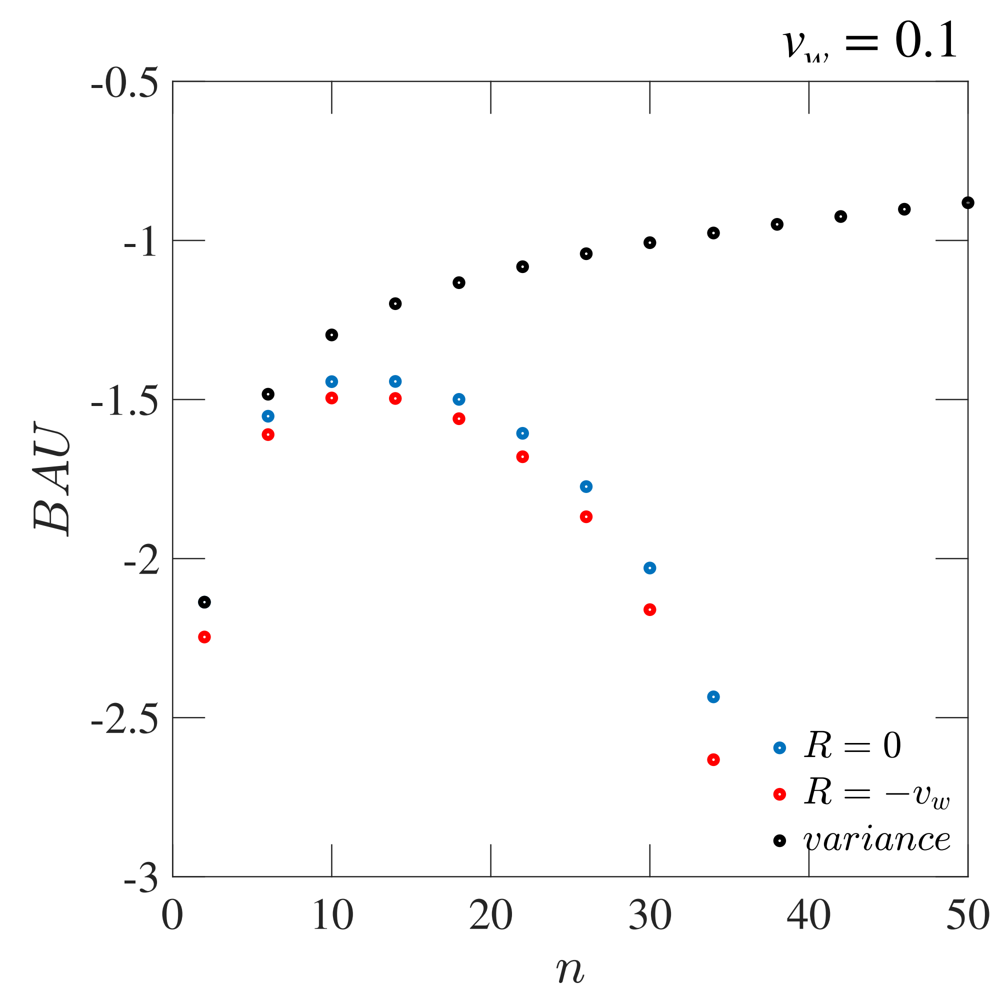
constant $\quad u_{n+1} = R u_n \qquad u_\ell = \dfrac{1}{N_1}\displaystyle\int_p v_z^\ell \, \delta f$

variance $\quad \displaystyle\int_p (v_z - \langle v_z \rangle)^{n+1} \, \delta f = 0$

3.3.2 Higher moments, convergence

For smaller v_w the convergence gets worse and results become more truncation dependent.





Perhaps, with some dose of optimistm, one could claim that results with n = 20 or so, are representative of the true BAU?
Or perhaps, not.

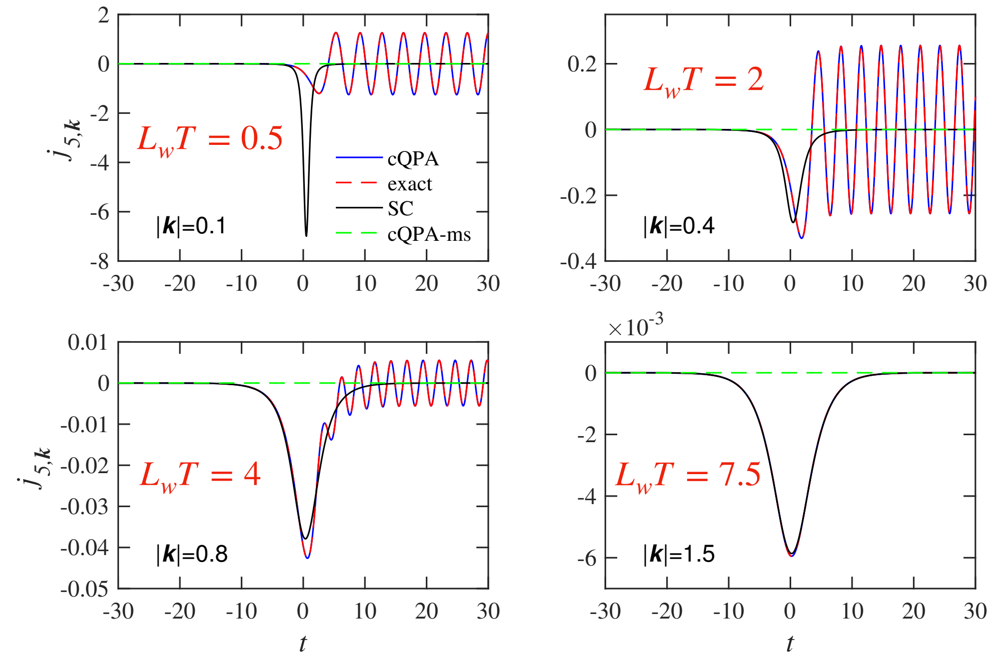
At any rate, low moment expansion results may not give a good estimate and tend to overestimate |BAU|
For precision calculation more advanced (tedious) methods (direct solution of SC-BE) are needed.

4. Beyond SC-limit

SC-BE's are based on gradient expansion of the full KB-equations. Formally **valid for $L_W >> 1/T$.**

They seem to remain quite accurate down to $L_W \sim (2\text{-}3)/T$

Very strong transitions, that could also source GW's, typically lead to very sharp walls: $L_W \sim 1/T$



H.Jukkala, K.K, O.Koskivaara, JHEP01(2020)012

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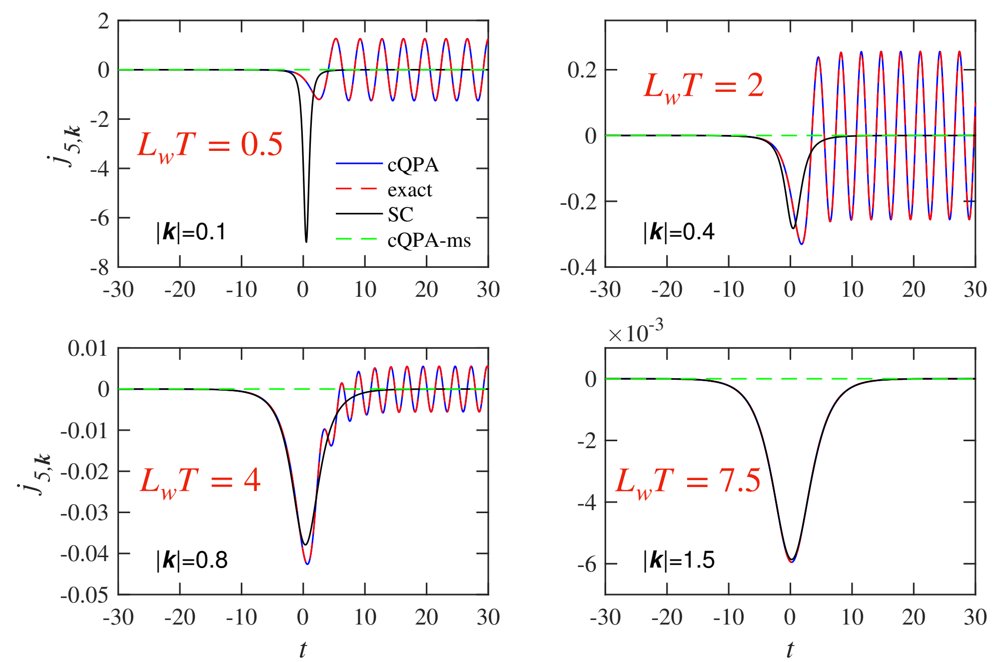

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In this regime quantum reflection becomes imporant and fully QM-treatment becomes necessary

Problem was studied in the 90's in the collisionless limit, and with phenomenological introduction of decoherence.

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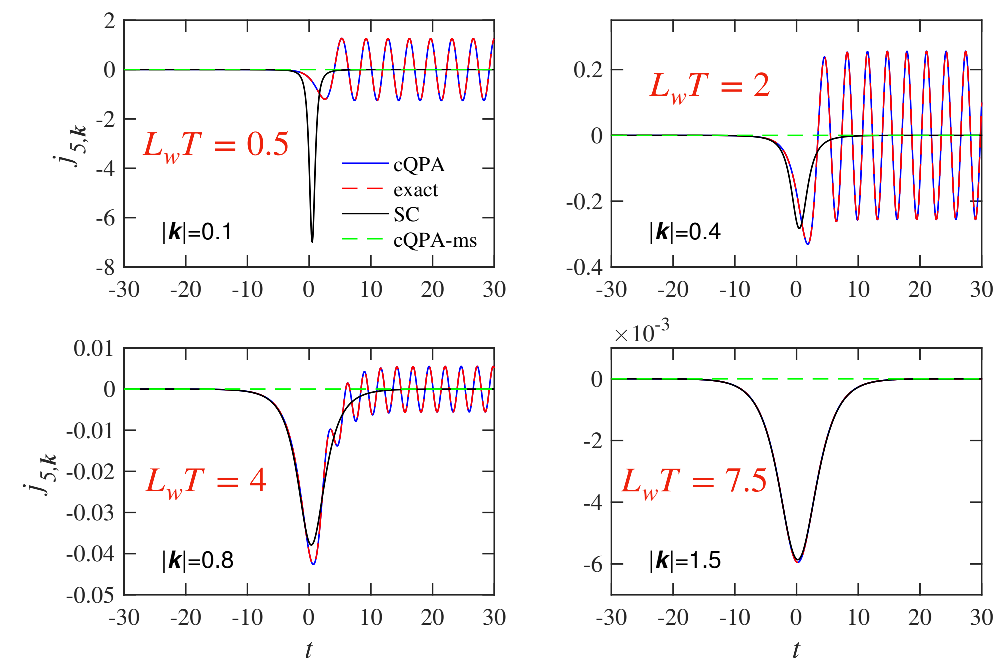


H.Jukkala, K.K, O.Koskivaara, JHEP01(2020)012

The main issue is how to treat collisions of (with) particles that are coherent mixtures of left- and right going states?

The problem can be handled by the methods introduced in H.Jukkala, KK, O.Koskivaara JHEP09(2021)119 (resonant leptogenesis)
KK, H.Parkkinen, JHEP 02 (2024) 217 (general neutrino QKE's)

Tehcnically, the (necessary) numerical solution of the momentum dependent QKE-equation netrowrks will be … tough.

5. Conclusions

- EWBG still a viable solution in some setups involving **Dark Sectors**

- We've come a long way from first attempts in 90's to the current consistent QKE's for the EWBG-problem.

- The only one method in town **to compute BAU** in the EWBG is the **semiclassical method.**

  The extension of SC-method to include thermal corrections to one-loop order.
  Basically: $\mathcal{S}^{\gamma}_{qs\pm} = \mathrm{Re}\big[\mathcal{S}_{qs\pm}(\omega_{s\pm} + i\gamma_{s\pm})\big]$: no VIA sources exist.

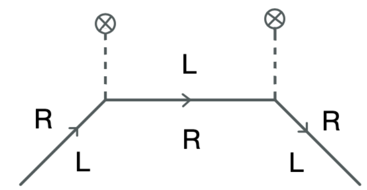
- ***Solving*** the SC-BE in the **moment expansion** straightforward, but **convergence can be an issue**

  Direct solution to SC-BE to test the **accuracy** of moment and polynominal expansions?

- **Extension** of the SC-formalism to the **quantum regime** doable in the **cQPA** context

Extra slides

VIA-method



$\Rightarrow \Sigma^{<}(x,y)_{\mathrm{VIA}}$ = nonlocal memory integral

Problem 1. **Foundational error**

Mass is a singular self-energy function, which contributes to Σ_{H} and *not* to $\Sigma^{<}$.

$$\partial_\mu j_5^\mu(x) = -\lim_{y\to x}\mathrm{Tr}[\gamma^5(m(x)+m(y))S^{<}(x,y)]$$

the true singular mass correction to axial vector current divergence

$$+\,2\mathrm{Re}\int \mathrm{d}^3\boldsymbol{w}\int_{t_{\mathrm{in}}}^{x_0}\mathrm{d}w_0\,\mathrm{Tr}\big[\gamma^5\big(\Sigma^{>}(x,w)S^{<}(w,x)-\Sigma^{<}(x,w)S^{>}(w,x)\big)\big]$$

true scattering terms = nonlocal memory integrals

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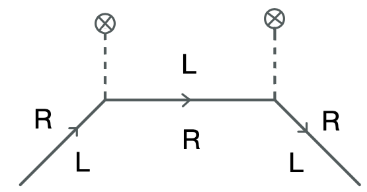

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$\Downarrow$

Problem 2. **Pinch singularity**

Using symmetries of the integrals one can show that the CP-odd VIA-source is

$$S_{\rm R}^{\not{C}\!P} = -v_w\gamma_w|m|^2\theta' \times I_\gamma \qquad \text{where} \qquad I_\gamma = 8\pi^2\int \frac{\mathrm{d}^4 k}{(2\pi)^4} k^2 \,[\mathrm{sgn}(k_0)\delta_\gamma(k^2)]^2 f'(k_0)$$

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Pinch singularities are familiar in CTP-formalism and **signal the need for resummation**…

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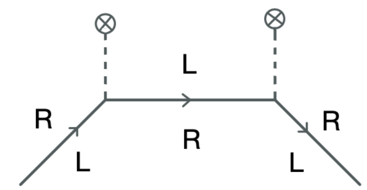

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Problem 3. further **math issues**

I_γ may be, and was regulated *eg* by a finite width γ and thermal masses (though, depending on the regulator, it can have an arbitrary value)

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Familiar result in VIA-literature

$$+\,r\mathrm{Re}\sum_{n=0}^{\infty}\sum_{\pm\pm'}\Big(g^{\pm}_{\boldsymbol{k}L}(-k_0)B^{(2)\pm'}_{\boldsymbol{k}R}(-k_0)+g^{\pm'}_{\boldsymbol{k}R}(k_0)B^{(2)\pm}_{\boldsymbol{k}L}(k_0)\Big)_{k_0=i\omega_n}\Big\}$$

from missed poles of the f'-function

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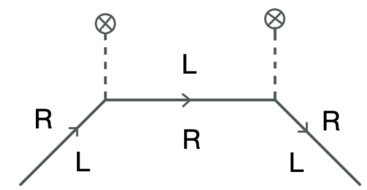

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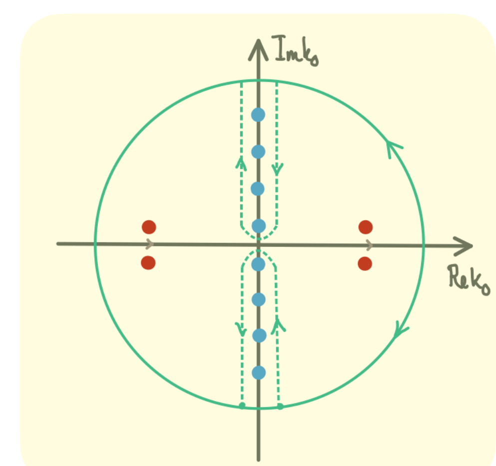
Actually s=-1, but this contribution **diverges.**

VIA: put s=0 by hand "renormalization"

True solution: a contribution from f'-poles cancel this term.



SC-method extended to thermal quasiparticles does not show VIA-term: no VIA-source exists
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One must be careful to correctly identify the quantities to which it is applied. Consider the vector current divergence equation (VCDE):

$$\partial_z j^z_{h\pm} = \partial_z \underbrace{\int \frac{\mathrm{d}^3 k}{(2\pi)^3} v_{h\pm}}_{\text{first velocity moment}} \underbrace{(f^{h\pm}_{\mathrm{FD}} - \mu_h f'_{0w} + \delta f_{h\pm})}_{f_{h\pm} \approx f_{0w} - \mu_h f'_{0w} + \delta f_{h\pm}} = C_{h1}$$

kinetic perturbation

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Three distinct parts can be identified:
KK, JCAP 11 (2021) 11, 042.

Source (drag): $\partial_z j^z_{h\pm,\mathrm{force}} \equiv \int \frac{\mathrm{d}^3 k}{(2\pi)^3} v_w \gamma_w F_{h\pm} f'_{0w} = \underbrace{-\mathcal{S}^n_{h1\pm}}_{\text{source}}$

Advection: $\partial_z j^z_{h\pm,\mathrm{adv}} \approx -v_w \partial_z \delta n_{h\pm}$

Diffusion: $\partial_z j^z_{h\pm,\mathrm{diff}} \equiv \partial_z \int \frac{\mathrm{d}^3 k}{(2\pi)^3} v_{h\pm} \delta f_{h\pm} \overset{\mathrm{FL}}{\equiv} -D\partial_z^2 \delta n_{h\pm}$

$\mathbf{j}_{\mathrm{diff}} = -D\partial_z \delta n$

=> $-v_w \delta n'_h - D\delta n''_h = \mathcal{S}^n_{h1} + \mathcal{C}^n_{h1}$

SC-method extended to thermal quasiparticles does not show VIA-term: no VIA-source exists
KK, JCAP 11 (2021) 11, 042.

This was later confirmed by Postma etal. (who found yet another problem within VIA, **see Mariekes talk**!)
M.Postma, G.White, J.van de Vis, JHEP 12 (2022) 121.

Problem 4. Naive use of Fick's law

Fick's law is a phenomenological relation, which *connects* the *diffusive flux* to the *rate of change of the concentration*.
One must be careful to correctly identify the quantities to which it is applied. Consider the vector current divergence equation (VCDE):

$$\partial_z j^z_{h\pm} = \underbrace{\partial_z \int \frac{\mathrm{d}^3k}{(2\pi)^3} v_{h\pm}}_{\text{first velocity moment}} (\underbrace{f^{h\pm}_{\mathrm{FD}} - \mu_h f'_{0w} + \delta f_{h\pm}}) \quad = \quad C_{h1}$$

$$f_{h\pm} \approx f_{0w} - \mu_h f'_{0w} + \underbrace{\delta f_{h\pm}}_{\text{kinetic perturbation}}$$

Three distinct parts can be identified:
KK, JCAP 11 (2021) 11, 042.

Source (drag): $\partial_z j^z_{h\pm,\mathrm{force}} \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3} v_w \gamma_w F_{h\pm} f'_{0w} = -\underbrace{\mathcal{S}^n_{h1\pm}}_{\text{source}}$

Advection: $\partial_z j^z_{h\pm,\mathrm{adv}} \approx -v_w \partial_z \delta n_{h\pm}$ => $-v_w \delta n'_h - D\delta n''_h = \mathcal{S}^n_{h1} + C^n_{h1}$

Diffusion: $\partial_z j^z_{h\pm,\mathrm{diff}} \equiv \partial_z \int \frac{\mathrm{d}^3k}{(2\pi)^3} v_{h\pm} \delta f_{h\pm} \overset{\mathrm{FL}}{\equiv} -D\partial_z^2 \delta n_{h\pm}$

$\mathbf{j}_{\mathrm{diff}} = -D\partial_z \delta n$

VCDE = 0th moment of the SC-equation: $\partial_z j^z_{h\pm} - C^n_{h1} = 0 \Leftrightarrow \int_{\mathbf{k}} (v_{h\pm}\partial_z f_{h\pm} + F_{h\pm}\partial_{k_z} f_{h\pm} - \mathscr{C}_{h\pm}[f]) = 0$

AVCDE = 1st moment of the SC-equation: $\partial_z j^{5z}_{h\pm} - C^n_{5h1} = 0 \Leftrightarrow \int_{\mathbf{k}} \frac{k_z}{\omega}(v_{h\pm}\partial_z f_{h\pm} + F_{h\pm}\partial_{k_z} f_{h\pm} - \mathscr{C}_{h\pm}[f]) = 0$

DE can be *derived* from the (first 2) SC moment equations.
J.M.Cline, K.K. PRD 101 2020) 6, 063525,

No collisional sources

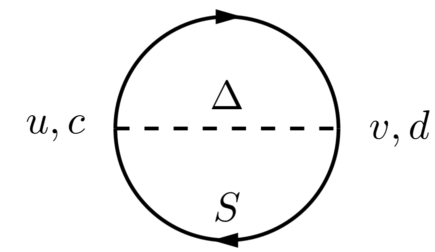
An early claim in SC-literature was that WKB-corrections to collision terms cause additional sources, of similar structure to the VIA:

$$\propto v_w \gamma_w |m|^2 \theta' y^2$$

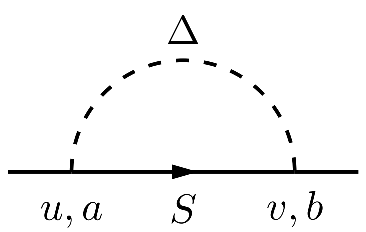
(it enters diffusion equations differently from VIA, though).
Troublingly, this source was created also by *equilibrium distributions.*

T.Prokopec, M.G.Schmidt and S.Weinstock,
Annals Phys. 314, 208 (2004), Annals Phys. 314, 267 (2004)

Contribution to 2PI-action



Contribution to self-energy



$$\Gamma_2 = y^2 \int_{\mathcal{C}} \mathrm{d}^4u \mathrm{d}^4v \sum_{cd} \mathrm{Tr}\big[S^{cd} P_R S^{dc} P_L\big] \Delta^{cd}$$

$$\Sigma^{ab}(u,v) = -iab\delta\Gamma_2/\delta S^{ba}(v,u)$$

The problem was that at gradient level, PSW used, to write $g^{s<}_{33,\mathrm{th}}$ in terms of $g^{s<}_{00,\mathrm{th}}$, using wrong KMS-identities:

$$g^{s<,>}_{33,\mathrm{th}} \to -\frac{k_z}{k_0} g^{s<,>}_{00,\mathrm{th}} - s\frac{|m|^2\theta'}{2k_0\tilde{k}_0}\partial_{k_z} g^{s<,>}_{00,\mathrm{th}} \quad \text{and} \quad \overset{\text{“KMS”}}{\partial_{k_z} g^{>}_{00,\mathrm{th}} \to \partial_{k_z}(e^{\beta p_0} g^{<}_{00,\mathrm{th}})} \quad \text{with} \quad p_0 = \gamma_w(k_0 + v_w k_z)$$

These actually ***break*** the KMS-condition ==> PSW collision term was not pushing *towards* equilibrium, but *away* from it.

The correct KMS-condition must be set at the matrix, not component level, where it states that

$$S^{<}_{\mathrm{th}} \equiv -2if^{<}_{\mathrm{th}}\mathcal{A} \quad \text{and} \quad S^{>}_{\mathrm{th}} \equiv -2if^{>}_{\mathrm{th}}\mathcal{A}$$

non-trivial structure comes from the equation for $\mathscr{A}_s$

This implies a symmetric relation $g^{s<,>}_{33,\mathrm{th}} = -2if^{<,>}_{0w} a^s_{33} = -2if^{<,>}_{0w}\Big(\frac{k_z}{k_0}a^s_{00} - s\frac{|m|^2\theta'}{2k_0\tilde{k}_0}\partial_{k_z} a^s_{00}\Big)$

As a result thermal equilibrium holds and the **C-source term vanishes.**

Related: **one can use zeroth order DR's** (in gradients) **in SC collision integrals** (to the order we computed the source).