Entanglement in flavored scalar scattering

Kamila Kowalska

National Centre for Nuclear Research (NCBJ)
Warsaw, Poland

in collaboration with

Enrico Sessolo

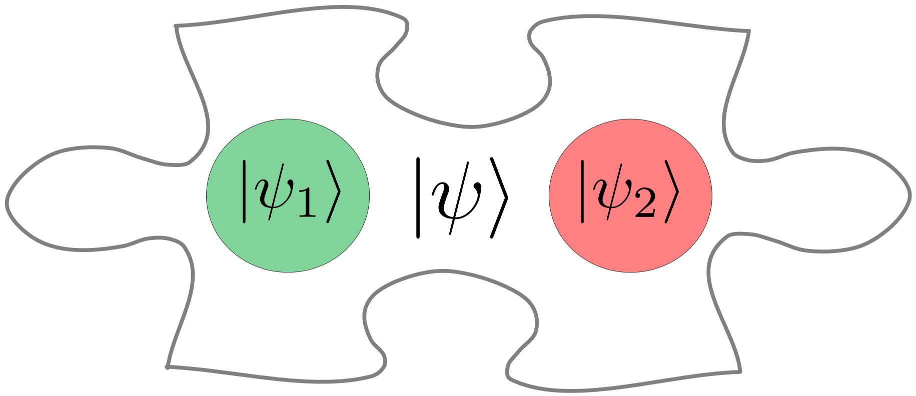
based on: arXiv: 2404.13743

CATCH 22+2, Dublin
04.05.2024

National Science Centre Poland

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

for mixed states:

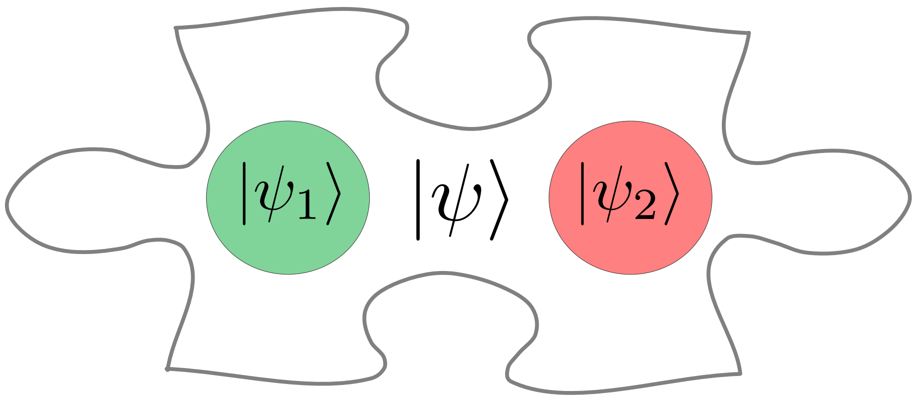
$$\rho \neq \sum_i p_i \rho_1^i \otimes \rho_2^i$$

entanglement = non-separability

Why should we care?

- Measured experimentally (Bell inequalities violation) Clauser et al., ‘72; Aspect et al., ‘82; Zeilinger et al., ‘98;
- Can be tested in colliders Afik, Muñoz de Nova, ‘21 and many following studies; ATLAS Collab., ‘23
- Quantum information dense coding (Bennett, Wiesner, ‘92), teleportation (Bennett et at., ‘93),key distrubution (Ekert, ‘91)
- Emergence of space and time Moreva et al., ‘13; Van Raamsdonk, ‘10, Ryu and Takayanagi, ‘06; Maldacena, Susskind, ‘13
- Emergence of symmetries Cervera-Lierta et al., ‘17; Fedida, Serafini, ‘23; Beane et al., ‘19; Liu et. al., ‘23; Carena et al., ‘23

Quantum entanglement



$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

for mixed states:

$$\rho \neq \sum_i p_i \rho_1^i \otimes \rho_2^i$$

entanglement = non-separability

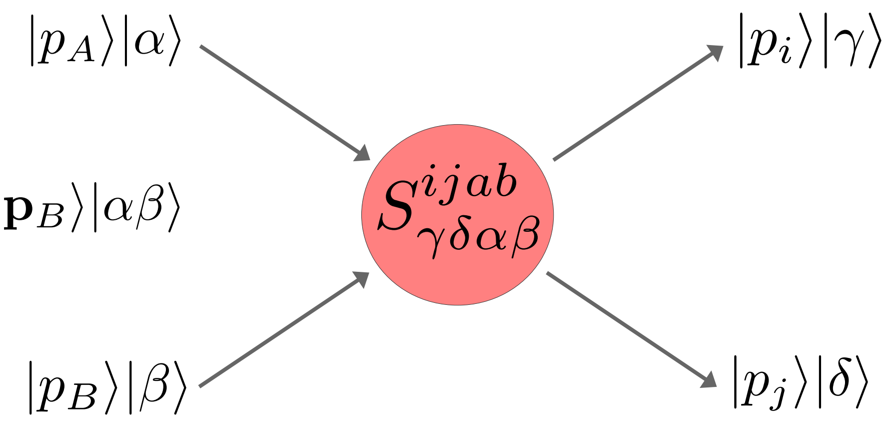
Why should we care?

This talk: new insight on BSM

- Measured experimentally (Be[…] […]er et al., '72; Aspect et al., '82; Zeilinger et al., '98;
- Can be tested in colliders A[…] […]udies; ATLAS Collab., '23
- Quantum information dense codin[…] […] (Bennett et at., '93),key distrubution (Ekert, '91)
- Emergence of space and time Moreva et al., '13, van Raamsdonk, '10, Ryu and Takayanagi, '06; Maldacena, Susskind, '13
- Emergence of symmetries Cervera-Lierta et al., '17; Fedida, Serafini, '23; Beane et al., '19; Liu et. al., '23; Carena et al., '23

Entanglement in scattering

$$|\text{in}\rangle = \frac{1}{\sqrt{V}} \sum_{\alpha,\beta=1,2} a_{\alpha\beta} |\mathbf{p}_A \mathbf{p}_B\rangle |\alpha\beta\rangle$$



$$|\text{out}\rangle = S\,|\text{in}\rangle$$

Hilbert space: **momentum** + **flavor**

$$\mathcal{H}_{\text{tot}} = L^2(\mathbb{R}^6) \otimes \mathbb{C}^4$$

In perturbation theory:

$$S^{ijab}_{\gamma\delta\alpha\beta} = (\mathcal{I} + iT)^{ijab}_{\gamma\delta\alpha\beta}$$

$$= (2\pi)^6\, 4\, E_i E_j\, \delta^{ijab}_{\gamma\delta\alpha\beta} + (2\pi)^4 \delta^4(p_a + p_b - p_i - p_j)\, i\mathcal{M}_{\gamma\delta,\alpha\beta}(p_a, p_b \to p_i, p_j)$$

The final-state density matrix:

$$\rho = |\text{out}\rangle\langle\text{out}|$$

encodes all the properties of a quantum system **(entanglement)**

Perturbative density matrix

$$\rho = |\text{out}\rangle\langle\text{out}|$$

Shao et al., '08; Seki et al., '15; Peschanski, Seki, '16, '19; Carney et al., '16; Rätzel et al., '17; Fan et al., '17; Araujo et al., '19; Fan et al., '21; Fonseca et al., '22; Shivashankara '23; Aoude et al., '24

Properties:

$$1)\ \text{Tr}(\rho) = 1$$

⇒ unitarity of the *S*-matrix

optical theorem

$$\langle\text{out}|\text{out}\rangle = 1 + \Delta\left(i\sum_{\alpha\beta,\gamma\delta} a^*_{\alpha\beta}\mathcal{M}_{\alpha\beta,\gamma\delta}(p_A,p_B\to p_A,p_B)a_{\gamma\delta} + \text{c.c.}\right)$$
$$+\ \Delta\int\int\frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A+p_B-p_i-p_j)$$
$$\times\sum_{\alpha\beta,\rho\epsilon,\sigma\tau}\mathcal{M}_{\alpha\beta,\rho\epsilon}(p_A,p_B\to p_i,p_j)a_{\rho\epsilon}\mathcal{M}^*_{\alpha\beta,\sigma\tau}(p_A,p_B\to p_i,p_j)a^*_{\sigma\tau}$$

$$2)\ \text{Tr}(\rho^2)\begin{cases} = 1 & \text{pure state} \\ < 1 & \text{mixed state}\end{cases}$$

⇒ **needed different entanglement measures**

$$\Delta = \frac{(2\pi)^4\delta^4(p_A+p_B-p_A-p_B)}{4E_AE_B\left[(2\pi)^3\,\delta^3(0)\right]^2}$$

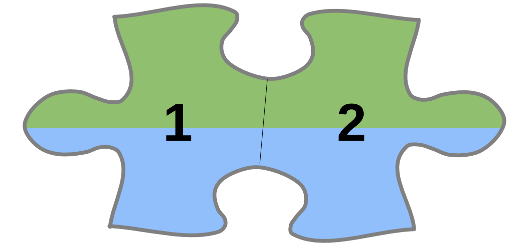
⇒

$$\Delta < \frac{1}{16\pi}$$

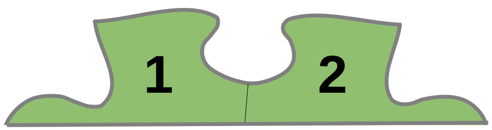
Only emerges with internal quantum numbers !!!

Entanglement in final state

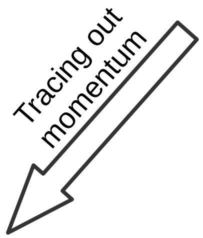
$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$

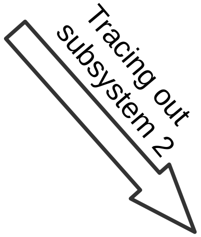


$\mathcal{H}_{\rm tot} = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^3) \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$



$\mathcal{H}_{\rm tot} = \mathbb{C}^2 \otimes \mathbb{C}^2$





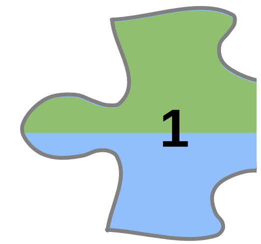
$$\rho^F = \mathrm{Tr}_p(\rho)$$

basis : $|\alpha\beta\rangle\langle\gamma\delta|$

- mixed state
- entanglement momentum/flavor:
  von Neuman entropy $S_N(\rho^F)$
- **entanglement if flavor space:**
  concurrence $C(\rho^F)$

$$\rho^1 = \mathrm{Tr}_2(\rho)$$

basis : $|\mathbf{p}_i\alpha\rangle\langle\mathbf{p}_\mathbf{j}\gamma|$



$\mathcal{H}_{\rm tot} = L^2(\mathbb{R}^3) \otimes \mathbb{C}^2$

- mixed state
- entanglement between 1 and 2:
  von Neuman entropy

See also: Aoude et al., ‘24

Entanglement monotones:

$S_N = 0,\ C = 0$ no entanglement

$S_N = 1,\ C = 1$ maximal entanglement

Definitions: $S_N(\rho^F) = -\mathrm{Tr}(\rho^F \log_2 \rho^F)$

$$C(\rho^F) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ: eigenvalues of $\rho^F(\sigma_y \otimes \sigma_y)\rho^{F*}(\sigma_y \otimes \sigma_y)$

Inert 2HDM in a nutshell

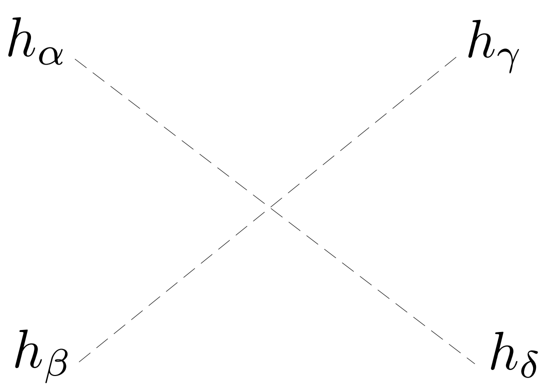
inert SU(2) doublets: $H_\alpha = \begin{pmatrix} h_\alpha^+ \\ h_\alpha^0 \end{pmatrix}_{Y=\frac{1}{2}} \qquad \alpha = 1,2$

scalar potential: $V(H_1,H_2) = \mu_1^2\, H_1^\dagger H_1 + \mu_2^2\, H_2^\dagger H_2 + \left(\mu_3^2\, H_1^\dagger H_2 + \text{H.c.}\right)$

$$+\,\lambda_1\,(H_1^\dagger H_1)^2 + \lambda_2\,(H_2^\dagger H_2)^2 + \lambda_3\,(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4\,(H_1^\dagger H_2)(H_2^\dagger H_1)$$

$$+\left(\lambda_5\,(H_1^\dagger H_2)^2 + \lambda_6\,(H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7\,(H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right)$$

contact interactions



$i\,\mathcal{M}_{\gamma\delta,\alpha\beta}$

$$i\mathcal{M}^{(0)}(h^0h^0 \to h^0h^0) = -i\begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & 4\lambda_5 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 4\lambda_5 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^+h^0 \to h^+h^0) = -i\begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & 2\lambda_5 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_6 & \lambda_4 & \lambda_3 & \lambda_7 \\ 2\lambda_5 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^+h^- \to h^+h^-) = -i\begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & \lambda_3+\lambda_4 \\ 2\lambda_6 & 4\lambda_5 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & 4\lambda_5 & 2\lambda_7 \\ \lambda_3+\lambda_4 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^0h^0 \to h^+h^-) = i\mathcal{M}^{(0)}(h^+h^- \to h^0h^0) = -i\begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & \lambda_3 \\ \lambda_6 & \lambda_4 & 2\lambda_5 & \lambda_7 \\ \lambda_6 & 2\lambda_5 & \lambda_4 & \lambda_7 \\ \lambda_3 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

Question: any constraints on λ from entanglement?

cf. Carena et al., ‘23

Entanglement creation

no entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}}|\mathbf{p}_A\mathbf{p}_B\rangle|11\rangle$

perturbativity preserved at order λ^2

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

with

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

generate entanglement between flavor and momentum

$h^0h^0 \to h^0h^0$

concurrence

$$C(\rho^F) = \sqrt{\frac{2\Delta}{\pi}}|\lambda_5|$$

generate entanglement between flavor qubits

$h^0h^0 \to h^0h^0$

Entanglement creation

no entanglement: $|\mathrm{in}\rangle = \frac{1}{\sqrt{V}}|\mathbf{p}_A\mathbf{p}_B\rangle|11\rangle$

perturbativity preserved at order λ^2

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

with

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$h^0h^0 \to h^0h^0$

generate entanglement between flavor and momentum

concurrence

$$C(\rho^F) = \sqrt{\frac{2\Delta}{\pi}}|\lambda_5|$$

generate entanglement between flavor qubits

$h^0h^0 \to h^0h^0$

$$\mathrm{Tr}\,(\rho^F)^2 = 1 - \frac{\Delta}{\pi}\left(2\lambda_5^2 + \lambda_6^2\right) + 16\,\Delta^2\left(2\lambda_5^2 + \lambda_6^2\right)$$

$$\mathrm{Tr}(\rho^F)^2 < 1 \Longrightarrow 0 < \theta_{1,2}$$

Entanglement creation

no entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$

perturbativity preserved at order λ^2

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

generate entanglement between flavor and momentum

with

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$h^0 h^0 \to h^0 h^0$

concurrence

$$C(\rho^F) = \sqrt{\frac{2\Delta}{\pi}} |\lambda_5|$$

generate entanglement between flavor qubits

$h^0 h^0 \to h^0 h^0$

$\Delta < \frac{1}{16\pi}$

perturbatively under control

$C < 1$

Entanglement creation

no entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}}|\mathbf{p}_A\mathbf{p}_B\rangle|11\rangle$

perturbativity preserved at order λ^2

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

generate entanglement between flavor and momentum

with

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$h^0h^0 \to h^0h^0$

concurrence

$$C(\rho^F) = \sqrt{\frac{2\Delta}{\pi}}|\lambda_5|$$

generate entanglement between flavor qubits

$h^0h^0 \to h^0h^0$

Repeating for 12, 21 and 22:

Momentum/flavor entanglers

$$\lambda_3\,,\lambda_4\,,\lambda_5\,,\lambda_6\,,\lambda_7$$

Two flavor entanglers

$$\lambda_3\,,\lambda_4\,,\lambda_5$$

Entanglement creation

no entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$

perturbativity preserved at order λ^2

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

generate entanglement between flavor and momentum

with

$$\theta_1 = 1 - \Delta \left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} \right) + 16\Delta^2 \left(\lambda_5^2 + \frac{\lambda_6^2}{2} \right)$$

$$\theta_2 = \Delta \left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi} \right) - 16\Delta^2 \left(\lambda_5^2 + \frac{\lambda_6^2}{2} \right)$$

$h^0 h^0 \to h^0 h^0$

concurrence

$$C(\rho^F) = \sqrt{\frac{2\Delta}{\pi}} |\lambda_5|$$

generate entanglement between flavor qubits

$h^0 h^0 \to h^0 h^0$

Repeating for 12, 21 and 22:

Momentum/flavor entanglers

$$\lambda_3 \, , \lambda_4 \, , \lambda_5 \, , \lambda_6 \, , \lambda_7$$

Two flavor entanglers

$$\lambda_3 \, , \lambda_4 \, , \lambda_5$$

Need to be zero to suppress entanglement

Entanglement transformation

maximal entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle \frac{1}{\sqrt{2}} (|11\rangle + |22\rangle)$

$S_N(\rho^F_{\text{in}}) = 0,\ C(\rho^F_{\text{in}}) = 1$

von Neuman entropy

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

with

$$\theta_1 = 1 - \Delta \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{4\pi}$$

$$\theta_2 = \Delta \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{4\pi}$$

concurrence

$$C(\rho^F) = \sqrt{1 - \Delta \frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{2\,\pi}}$$

Entanglement "flows"
from
the flavor to momentum space

Conclusions

- Entangelement may provide a **complementary way of constraining** interaction structure in New Physics models.

- 2HDM: flavor entanglement in $2 \rightarrow 2$ scalar scattering can be **created** by the couplings $\lambda_3,\lambda_4,\lambda_5$ from the basis flavor states.

- 2HDM: entanglement can be **transformed** by the couplings $\lambda_1,\lambda_2,\lambda_6,\lambda_7$ from the pure flavor entanglement to momentum/flavor entanglement.

- Upper bound of **momentum normalization** needed for physicality and unitarity.