Naturally small Yukawa couplings from asymptotic safety

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Based on

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in collaboration with
Abhishek Chikkaballi, Kamila Kowalska, Soumita Pramanick

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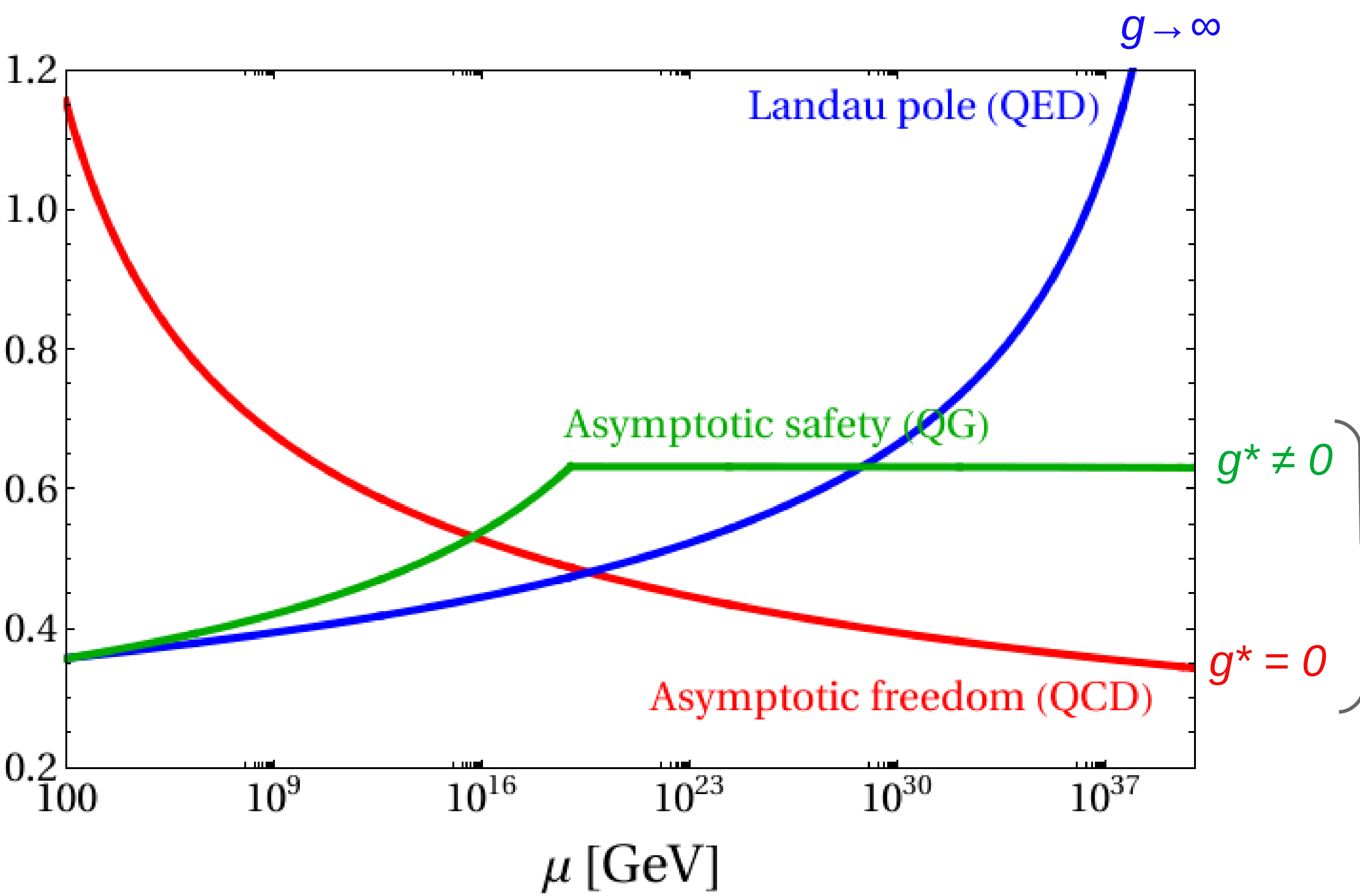
Renormalization group flow

Charge-screening by quantum fluctuations → *running* coupling constants, *g(μ)*



$$\beta_g = \frac{dg}{dt} = \frac{dg}{d\ln\mu}$$

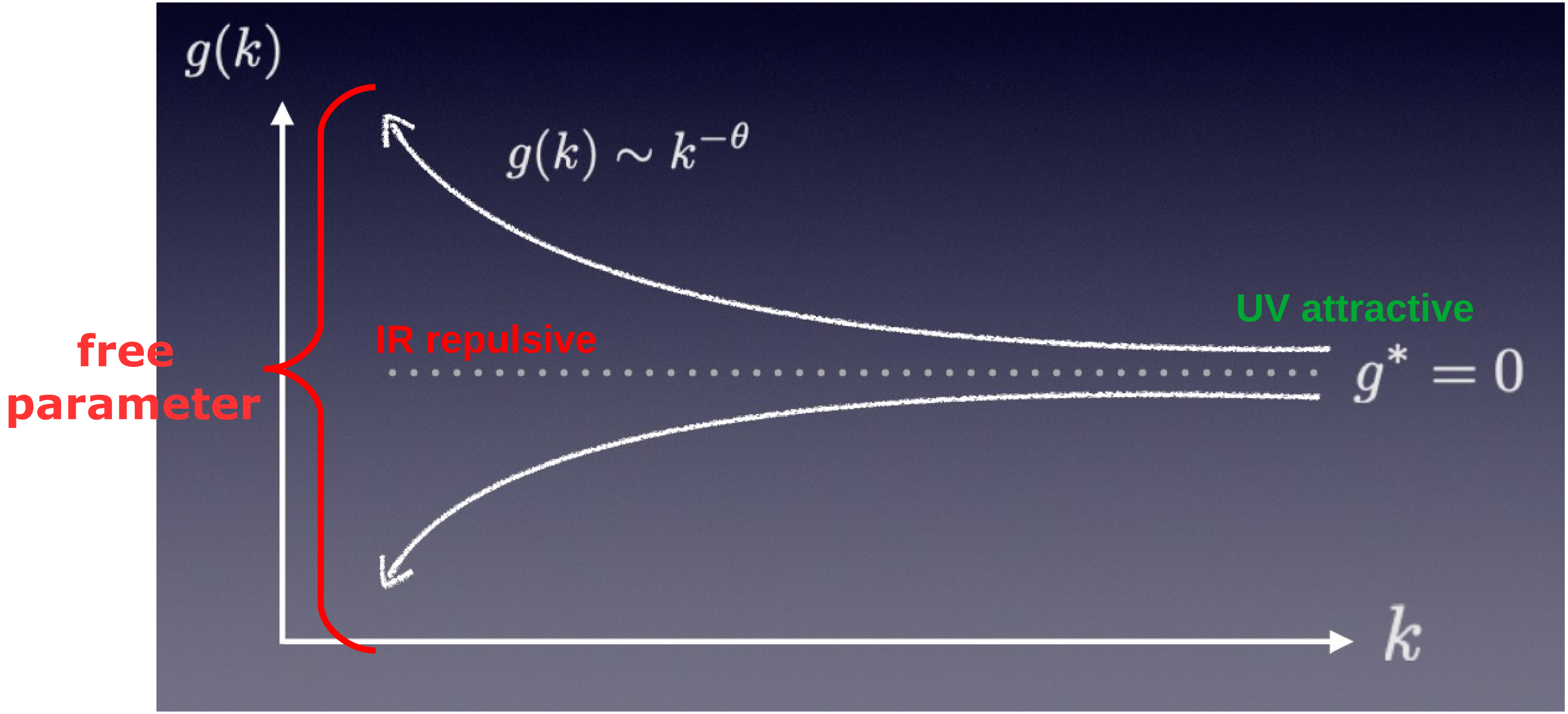
$$\beta_g(g^*) = 0$$

fixed point *g**
in the RG flow

Scaling properties of *g*

$$M_{ij} = \partial\beta_i / \partial\alpha_j |_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ > 0



M.Yamada, Warsaw 08.10.2019

Relevant couplings are **free parameters**

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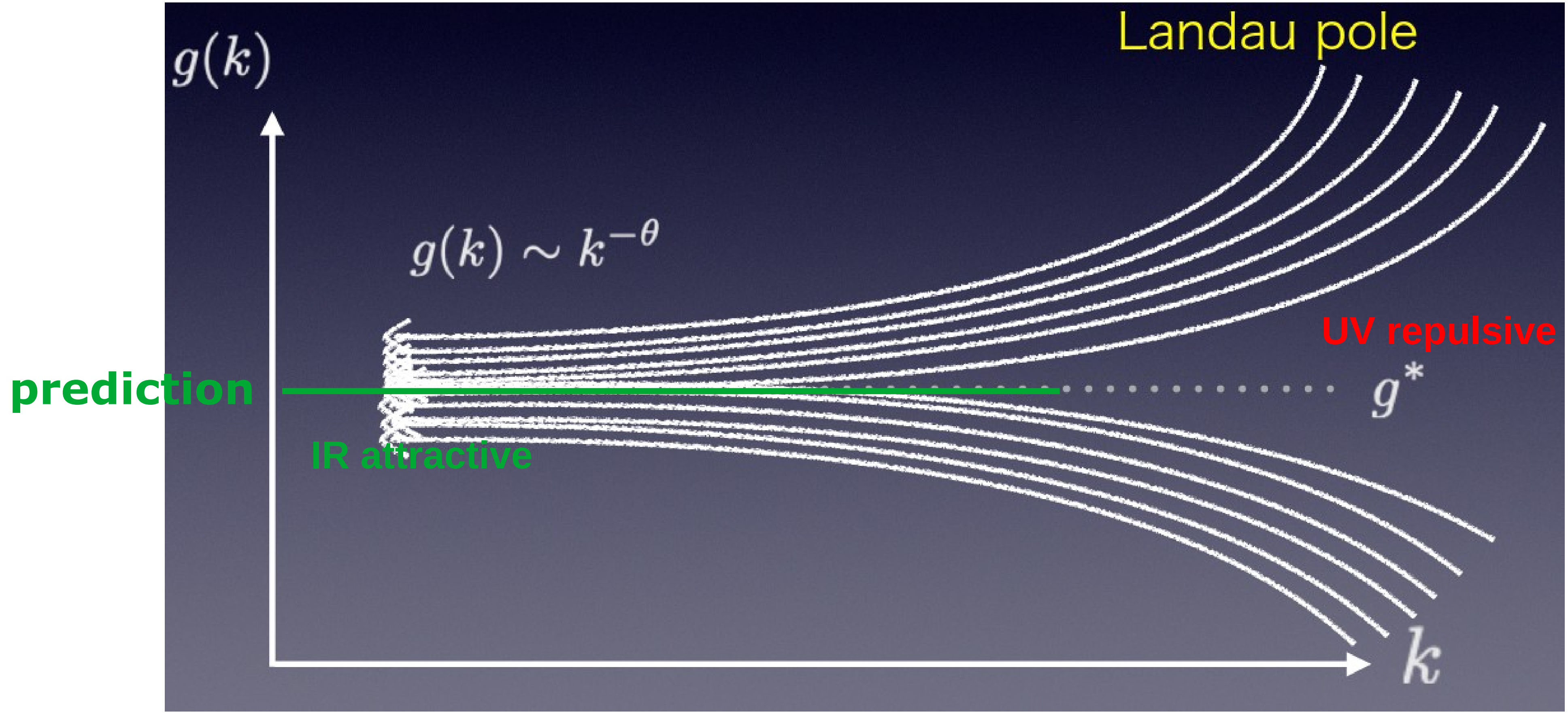

M.Yamada, Warsaw 08.10.2019

Irrelevant couplings provide **predictions**

Matter RGEs with quantum gravity

Robinson, Wilczek '06, Pietrykowski '07, Toms '08, Rodigast, Schuster '09, Zanusso et al. '09, Daum et al. '09,'10, Folkers et al. '11, Dona' et al. '13, Eichhorn et al. '16-17...

Trans-Planckian corrections of matter RGEs $k > M_{\rm Pl}$ (functional renormalization group)

Wetterich, Reuter, Saueressig, Percacci, Litim, Pawlowski, Eichhorn ...

SM gauge couplings

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} - f_g\, g_2$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 - f_g\, g_3$$

universal corrections depend on gravity fixed points

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}, \qquad f_y = -\tilde{G}^* \frac{96+\tilde{\Lambda}^*\left(-235+103\tilde{\Lambda}^*+56\tilde{\Lambda}^{*2}\right)}{12\pi\left(3-10\tilde{\Lambda}^*+8\tilde{\Lambda}^{*2}\right)^2}$$

e.g. A. Eichhorn, A. Held, 1707.01107
A. Eichhorn, F. Versteegen, 1709.07252

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_t$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{5}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_b \quad \ldots$$

... same for other quarks and leptons

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$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 + 3y_s^2 + 3y_c^2 - \frac{9}{4}g_2^2 - 8g_3^2 - \frac{5}{12}g_Y^2 + y_e^2 + y_\mu^2 + y_\tau^2\right) - f_y\, y_b = 0$$

... same for other quarks and leptons

get fixed points

Matter RGEs with quantum gravity

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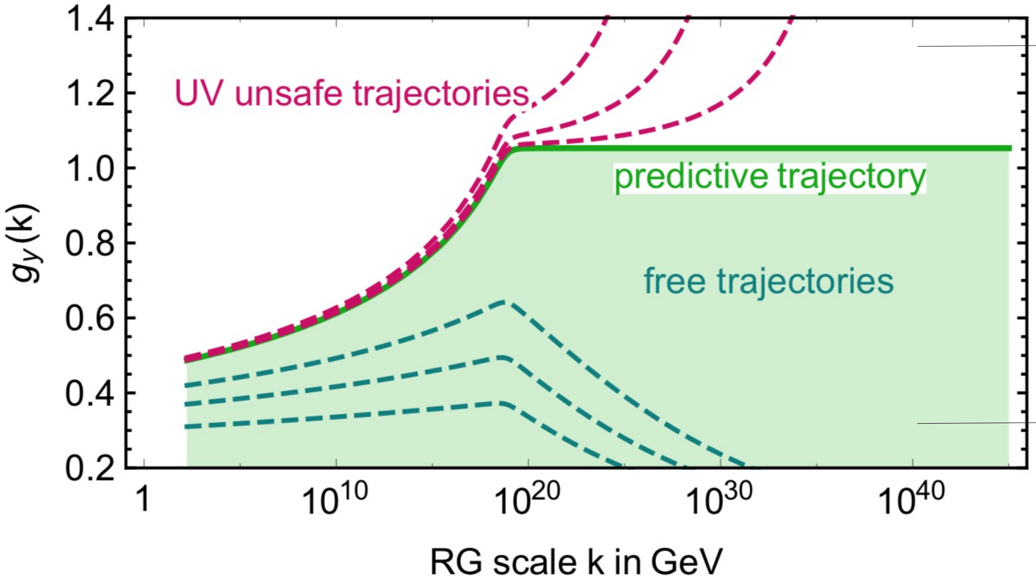
SM gauge couplings

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y = 0$$

A. Eichhorn, F. Versteegen, '17



$f_g < 0.0097$

$f_g \approx 0.0097$ matches SM value

$f_g > 0.0097$

euristic determination $f_{g,y}$ → universality of couplings → **predictions BSM**

Naturalness
with
asymptotic safety

Neutrino mass

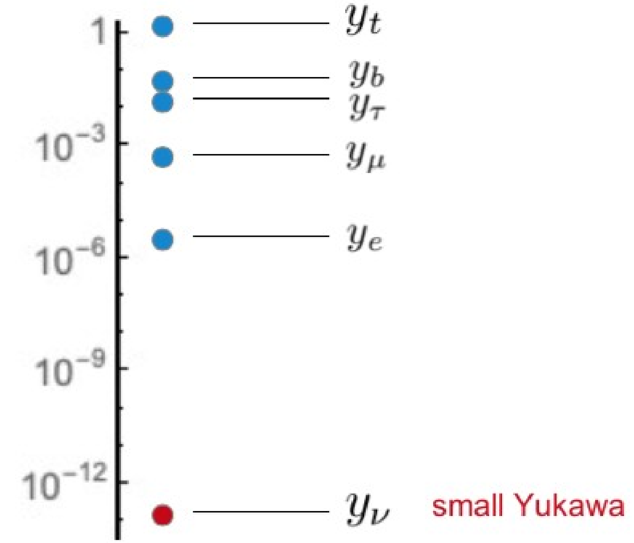
Neutrino masses are very small !

either Dirac neutrino ...

RHN → Higgs mechanism → small Yukawa

$$\mathcal{L}_D = -y_\nu^{ij}\,\nu_{R,i}\,(H^c)^\dagger L_j + \text{H.c.}$$

$$m_\nu = \frac{y_\nu v_H}{\sqrt{2}}$$



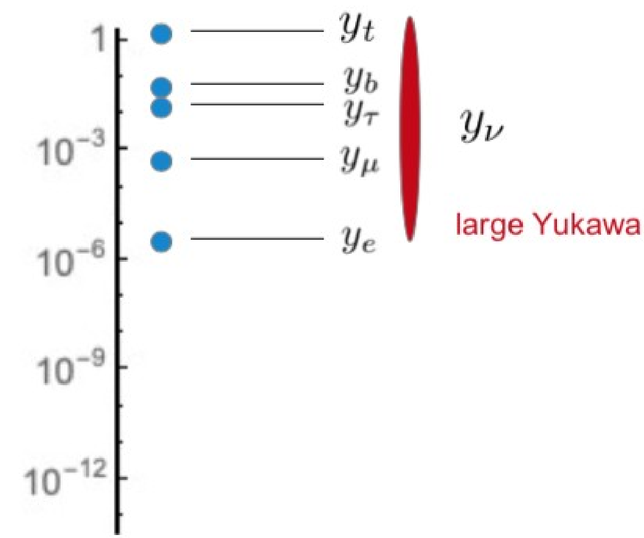
or Majorana neutrino ...

see-saw mechanism → large Yukawa

$$\mathcal{L}_M = \mathcal{L}_D - \frac{1}{2}M_N^{ij}\nu_{R,i}\nu_{R,j} + \text{H.c.}$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix} \qquad m_\nu = \frac{y_\nu^2 v_H^2}{\sqrt{2}M_N}$$

1 free parameter M_N



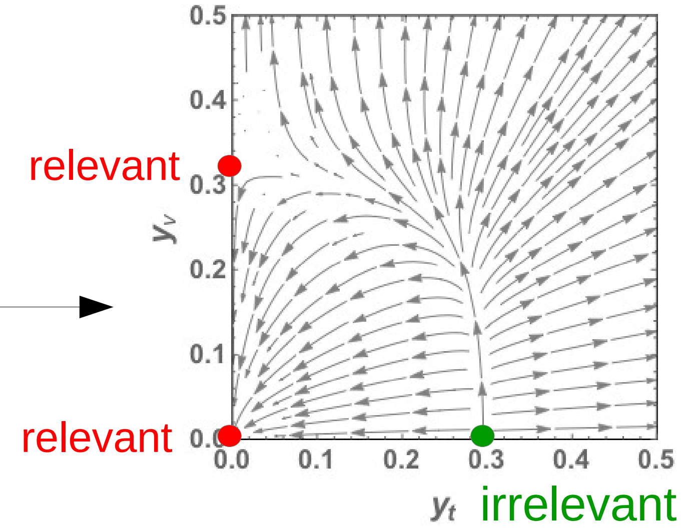
Fixed points SM + RHN

K. Kowalska, S. Pramanick, EMS '22

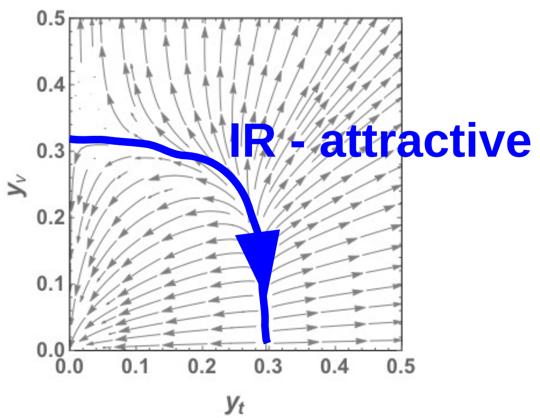
$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y \quad = 0$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t \quad = 0$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu \quad = 0$$



Arrows towards UV



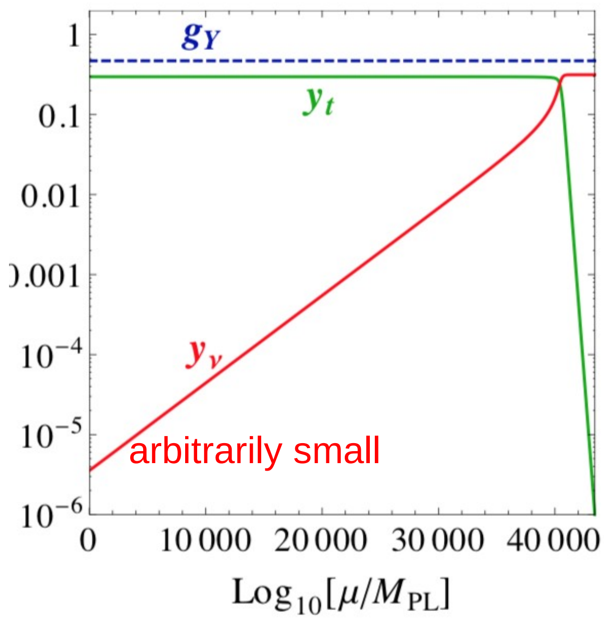
Integrate the curve:

$$y_\nu(t,\kappa) \approx \sqrt{\frac{16\pi^2(f_{\text{crit}} - f_y)}{e^{(f_{\text{crit}} - f_y)(16\pi^2\kappa - t)} + 5/2}}$$

$16\,\pi^2\kappa$ = Planck "distance" (e-folds)

(1 free parameter)

trans-Planckian running



No fine tuning

Neutrinos could be Dirac **naturally**

Also any feeble Yukawa coupling (BSM, freeze-in, ...)

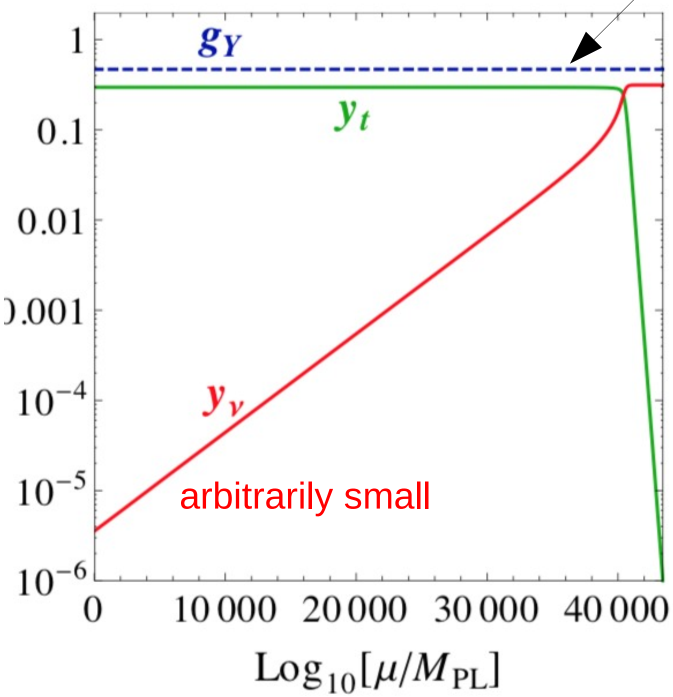
Connections to quantum gravity

In SMRHN + Gravity

f_g value linked to hypercharge gauge coupling

$$16\pi^2\theta_\nu \approx -\frac{2}{3}g_Y^{*2} + \frac{3}{2}y_t^{*2} < 0$$

trans-Planckian running



$$f_g \approx 0.0097$$

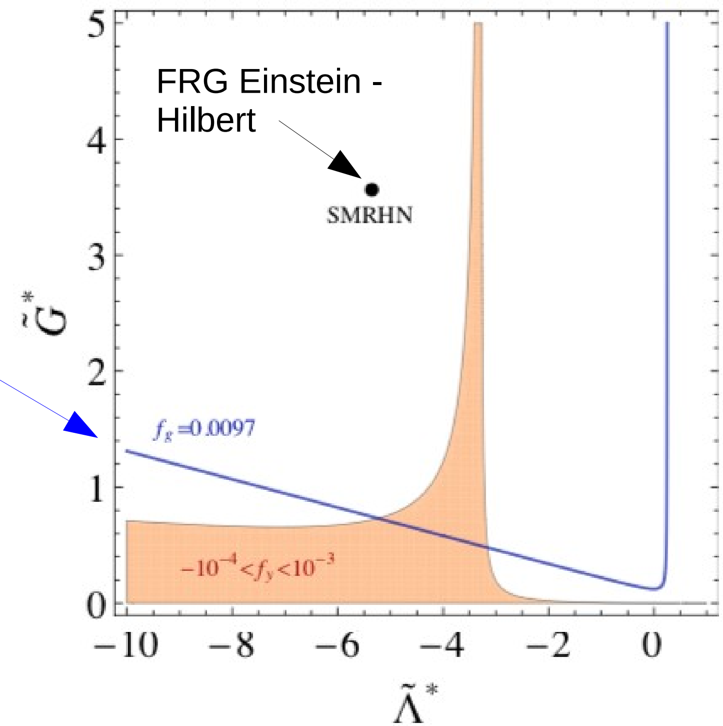
to match SM value

... It is a line in space of gravity fixed points

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}$$

Quantum gravity calculation should eventually match the blue line (difficult)

A. Chikkaballi, K.Kowalska, EMS '23



FRG calculation following

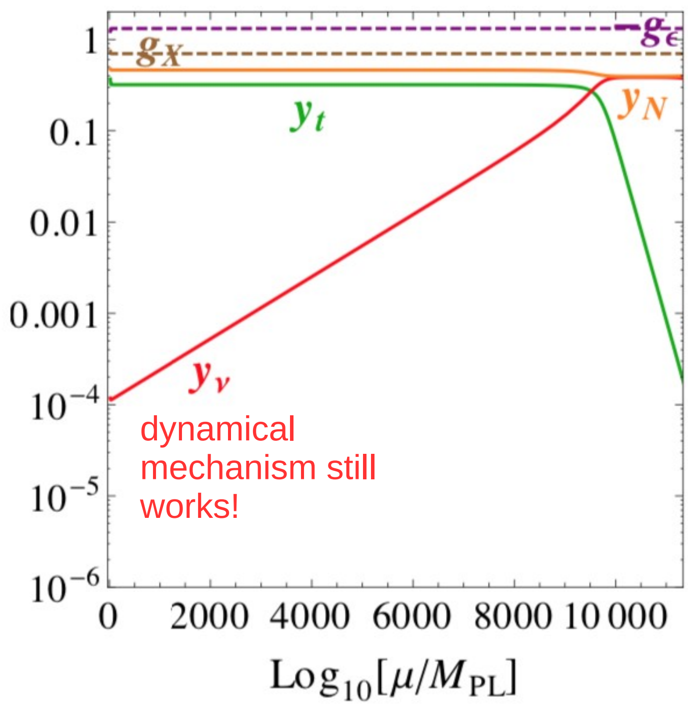
A. Eichhorn, F.Versteegen, 1709.07252

Connections to quantum gravity

In U(1)$_{B\text{-}L}$ + Gravity

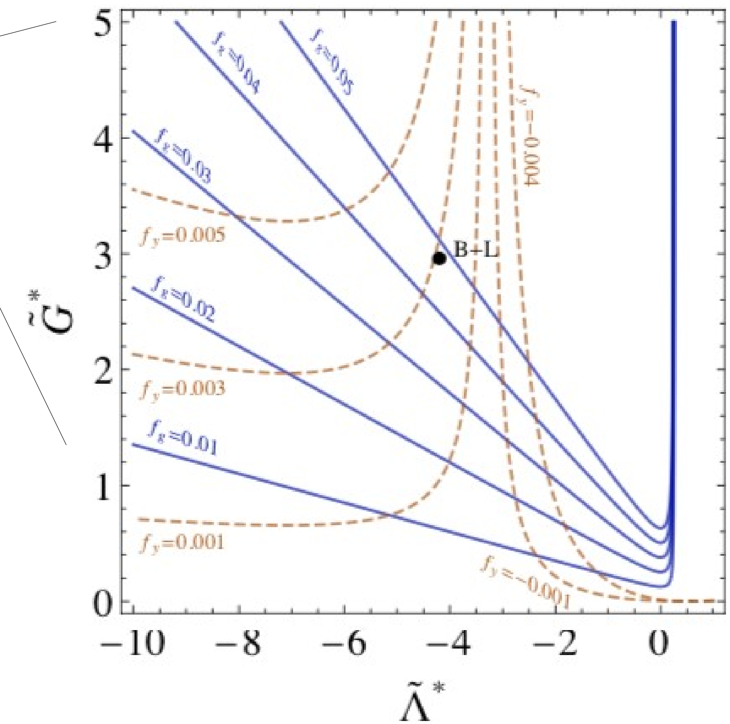
g_Y is *relevant* (free) … f_g value linked to $g_X = \frac{g_{B-L}}{\sqrt{1-\epsilon^2}}\,, \quad g_\epsilon = -\frac{\epsilon\, g_Y}{\sqrt{1-\epsilon^2}}$

trans-Planckian running



$\longrightarrow f_g =$ any

A. Chikkaballi, K.Kowalska, EMS '23



FRG calculation following

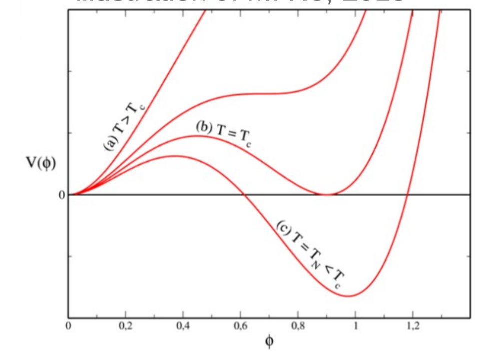
A. Eichhorn, F.Versteegen, 1709.07252

Quantum gravity calculation provides **predictions** for g_X, g_ϵ, and new Yukawa talks by M.Lewicki, S.King, O.Gould, more ...

Predictions *B-L*

Possible GW signatures in FOPT?

illustration J. M. No, 2015



Our predictions have strong discriminating features... may show up in GW amplitude!

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	g_X $(10^{5,7,9}$ GeV)	g_ϵ $(10^{5,7,9}$ GeV)	y_N $(10^{5,7,9}$ GeV)	
BP1	0.01	0.0005	0.10	−0.55	0.12	0.29, 0.29, 0.30	−0.26, −0.27, −0.28	0.16, 0.16, 0.16	Majorana
BP2	0.05	−0.005	0.70	−1.32	0.47	0.40, 0.41, 0.44	−0.52, −0.56, −0.61	0.42, 0.44, 0.45	
BP3	0.02	−0.0015	0.10	−0.75	0.0	0.12, 0.12, 0.12	−0.33, −0.35, −0.37	0.0	Dirac
BP4	0.03	−0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0	

$$\mathcal{L}_M = -y_N^{ij} S\, \nu_{R,i}\, \nu_{R,j} + \text{H.c.}$$

Well known signals if C-W is "conformal" ...

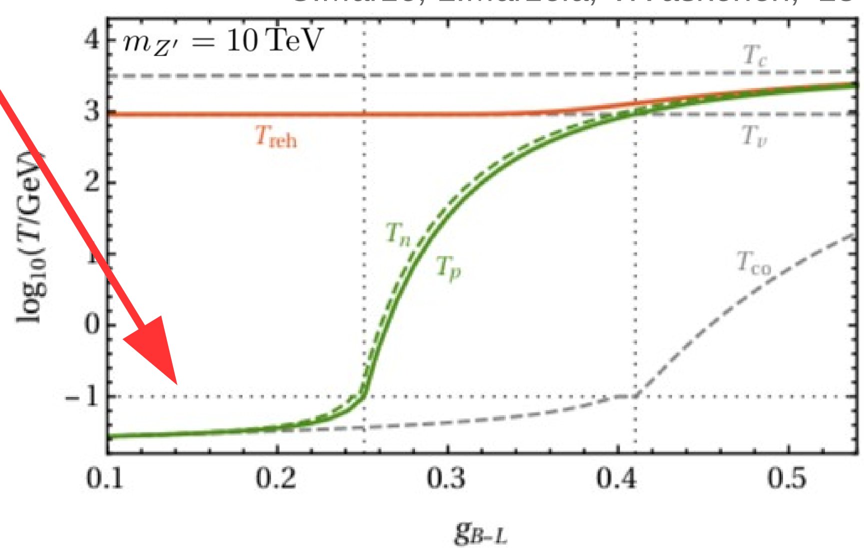
$$V_{\rm CW} = \frac{1}{2} m_S^2 \phi^2 + \frac{1}{4}\lambda_S \phi^4 + \frac{1}{128\pi^2}\left(20\lambda_S^2 + 96 g_X^4 - 48 y_N^4\right)\phi^4 \left(-\frac{25}{6} + \ln\frac{\phi^2}{k^2}\right)$$

But ...

… nucleation/percolation *T* is too low
… FOPT stop-condition not satisfied

NO GW SIGNAL HERE!

C.Marzo, L.Marzola, V.Vaskonen, '18



Classical scale invariance vs. asymptotic safety

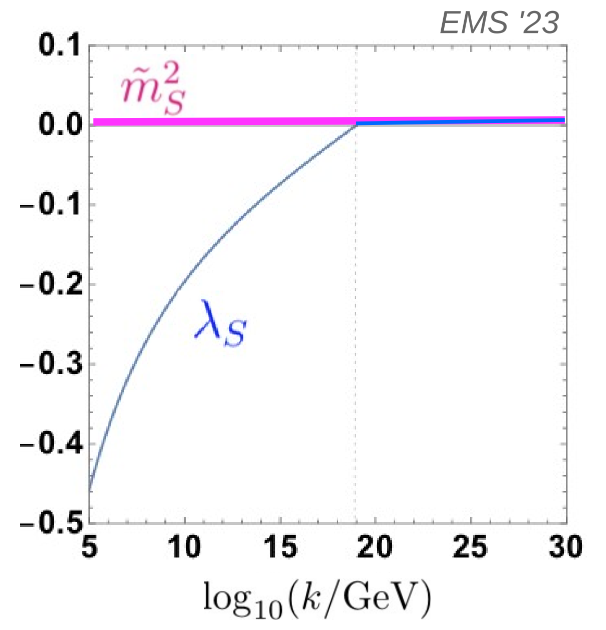
$$\frac{d\tilde{m}_S^2}{dt} \approx (-2 - f_\lambda)\,\tilde{m}_S^2$$

$$\frac{d\lambda_S}{dt} \approx -f_\lambda \lambda_S + \frac{6 g_X^{*4}}{\pi^2} + \ldots$$

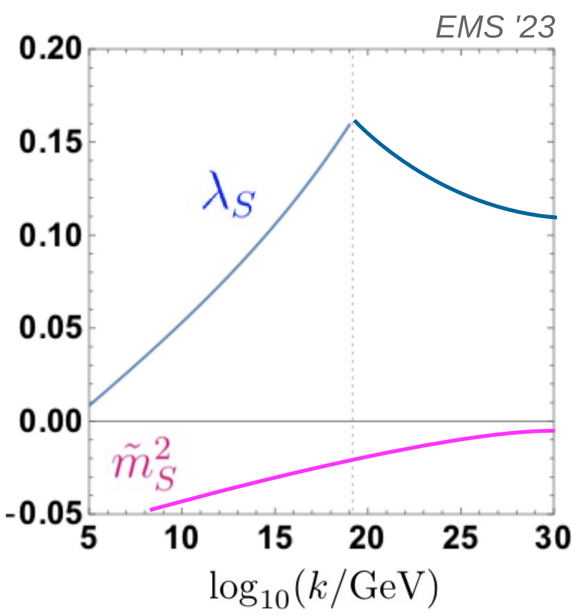
$\tilde{m}_S^{2*} = 0$ irrelevant

implies predictive $\lambda_S(t)$

potential destabilized!



viceversa...



$\lambda_S(t)$ consistent with C-W

implies $\tilde{m}_S^{2*} = 0$ relevant

tree-level mass is allowed

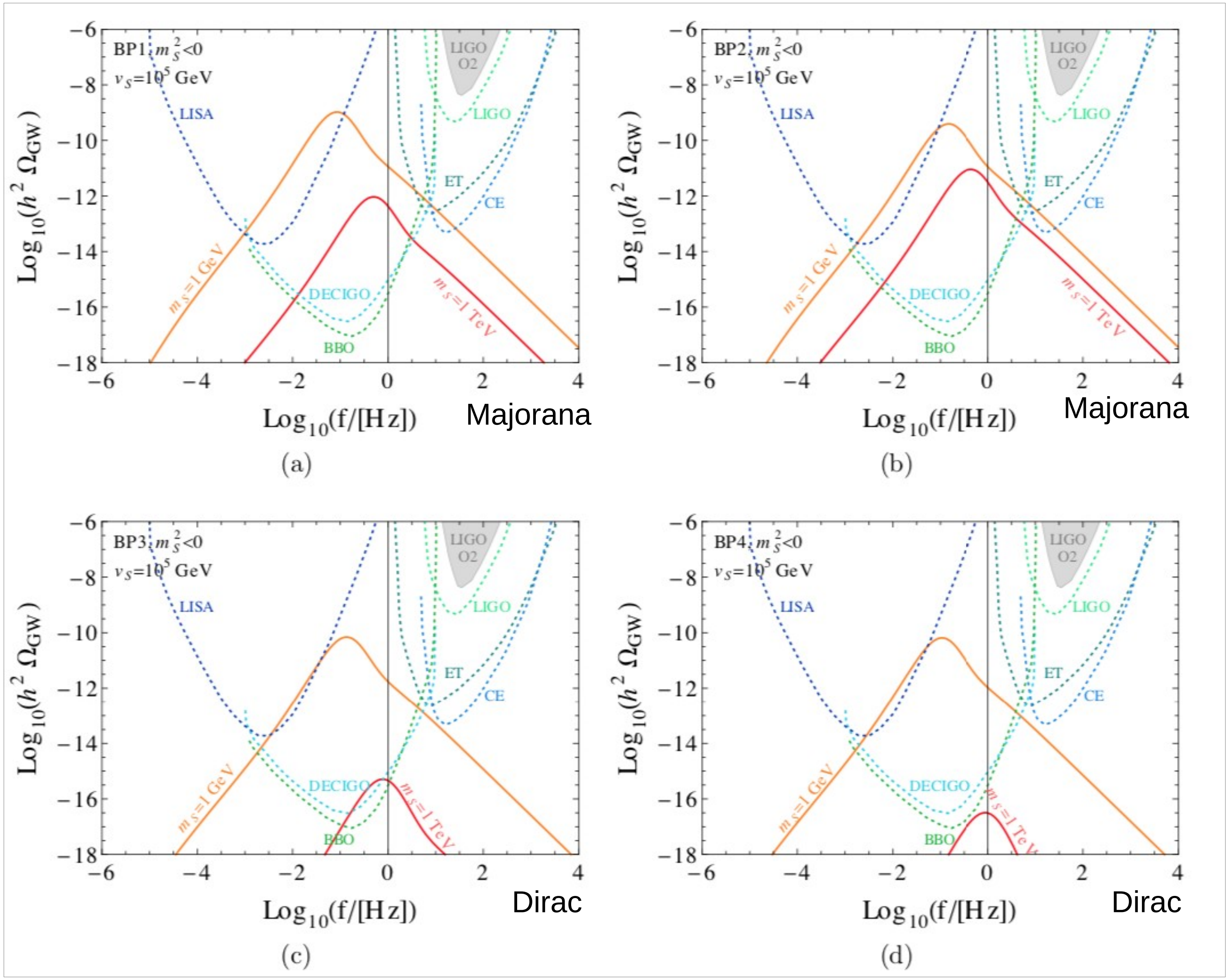
hence

No conformal potential

Gravitational waves

Signal with mass is interesting ...

A. Chikkaballi, K.Kowalska, EMS '23



... but discriminating features are washed out

To take home...

AS was used to make the neutrino (or other) Yukawa coupling **arbitrarily small dynamically**

Mechanism relies on an **irrelevant Gaussian fixed point** of the trans-Planckian RG flow of Yukawa coupling

In the SM + QG **some tension** between the FRG results and phenomenology, but perhaps not so in gauged *B-L*

Gravitational wave signatures from FOPTs

Majorana/Dirac discrimination via gravitational waves from FOPTs **not possible** in this scenario

Backup

Asymptotically safe gravity

Quantum gravity might feature interactive UV fixed points (functional renormalization group)

Reuter '96, Reuter, Saueressig '01, Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Zanusso *et al.* '09 … many more

EAA *e.g.* Einstein-Hilbert action

$$\Gamma_k = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,[-R(g)+2\Lambda]$$

FRG (Wetterich equation)

$$\partial_t\Gamma_k = k\,\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{1}{\Gamma_k^{(2)}+\mathcal{R}_k}\partial_t\mathcal{R}_k\right)$$

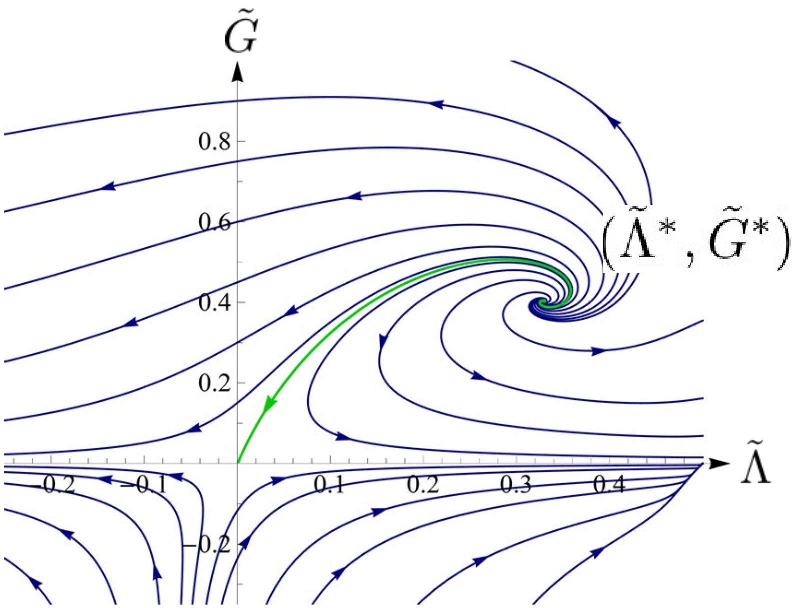
Beta functions of grav. couplings

$\tilde{G} = G(k)k^2$
$\tilde{\Lambda} = \Lambda(k)k^{-2}$
$t = \ln k$

$$\frac{d\tilde{G}}{dt} = \left[2+\tilde{G}\,\eta_1(\tilde{G},\tilde{\Lambda})\right]\tilde{G} = 0$$

$$\frac{d\tilde{\Lambda}}{dt} = -2\tilde{\Lambda}+\tilde{G}\,\eta_2(\tilde{G},\tilde{\Lambda}) = 0$$

Reuter, Saueressig, hep-th/0110054



2 relevant fixed points

... fixed points persist under the addition of gravity and matter interactions

Predictions from trans-Planckian AS

- **FRG calculation of f_g, f_y has very large uncertainties...**
  (truncation in number of operators, cut-off scheme dependence, etc.)
  Lauscher, Reuter '02, Codello, Percacci, Rahmede '07-'08, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Dona', Eichhorn, Percacci '13, Falls, Litim, Schroeder '18 ...

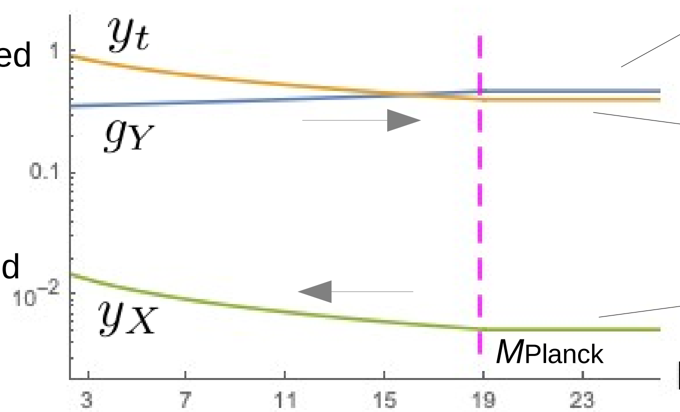
- **FRG calculation is *not required* to get predictions...**
  Wetterich, Shaposhnikov '09, Eichhorn, Held '18, Reichert, Smirnov '19; Alkofer *et al.* '20, Kowalska, EMS, Yamamoto '20, Kowalska, EMS '21, Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS '22 ...

… as the set of *irrelevant* couplings is overconstrained:

3 (or more) eqs (g_Y, y_t, y_X, ...)
2 unknowns (f_g, f_y)

measured in IR

predicted in IR



get fg

get f_y

Predict y_X
arbitrary gauge or Yukawa coupling (in SM or BSM)

AS leads to testable signatures ...

e.g. in flavor anomalies: *Kowalska, EMS, Yamamoto,* Eur.Phys.J.C 81 (2021) 4, 272
Chikkaballi, Kotlarski, Kowalska, Rizzo, EMS, JHEP 01 (2023) 164

in g-2 and DM: *Kowalska, EMS,* Phys. Rev. D 103, 115032 (2021)

… and neutrinos!
(this talk)

Lepton sector RGEs

$$\begin{aligned}\frac{dy_e}{dt} &= \frac{y_e}{16\pi^2}\left\{\frac{3}{2}y_e^2 - \frac{3}{2}\left[X y_{\nu1}^2 + Y y_{\nu2}^2 + (1-X-Y)y_{\nu3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\ &\quad \left. - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_e \qquad \text{(A.9)}\end{aligned}$$

$$\begin{aligned}\frac{dy_\mu}{dt} &= \frac{y_\mu}{16\pi^2}\left\{\frac{3}{2}y_\mu^2 - \frac{3}{2}\left[Z y_{\nu1}^2 + W y_{\nu2}^2 + (1-Z-W)y_{\nu3}^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\ &\quad \left. - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\mu \qquad \text{(A.10)}\end{aligned}$$

$$\begin{aligned}\frac{dy_\tau}{dt} &= \frac{y_\tau}{16\pi^2}\left\{\frac{3}{2}y_\tau^2 - \frac{3}{2}\left[(1-X-Z)y_{\nu1}^2 + (1-Y-W)y_{\nu2}^2 + (X+Y+Z+W-1)y_{\nu3}^2\right]\right.\\ &\quad \left. + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2 - \left(\frac{15}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_\tau \qquad \text{(A.11)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu1}}{dt} &= \frac{y_{\nu1}}{16\pi^2}\left\{\frac{3}{2}y_{\nu1}^2 - \frac{3}{2}\left[X y_e^2 + Z y_\mu^2 + (1-X-Z)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\ &\quad \left. - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu1} \qquad \text{(A.12)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu2}}{dt} &= \frac{y_{\nu2}}{16\pi^2}\left\{\frac{3}{2}y_{\nu2}^2 - \frac{3}{2}\left[Y y_e^2 + W y_\mu^2 + (1-Y-W)y_\tau^2\right] + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2\right.\\ &\quad \left. - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu2} \qquad \text{(A.13)}\end{aligned}$$

$$\begin{aligned}\frac{dy_{\nu3}}{dt} &= \frac{y_{\nu3}}{16\pi^2}\left\{\frac{3}{2}y_{\nu3}^2 - \frac{3}{2}\left[(1-X-Y)y_e^2 + (1-Z-W)y_\mu^2 + (X+Y+Z+W-1)y_\tau^2\right]\right.\\ &\quad \left. + y_e^2 + y_\mu^2 + y_\tau^2 + y_{\nu1}^2 + y_{\nu2}^2 + y_{\nu3}^2 - \left(\frac{3}{4}g_Y^2 + \frac{9}{4}g_2^2\right) + 3\left(y_t^2 + y_b^2\right)\right\} - f_y\, y_{\nu3} \qquad \text{(A.14)}\end{aligned}$$

$$\begin{aligned}\frac{dX}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)XZ + \frac{(y_{\nu3}^2-y_{\nu2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)X(1-X-Z) + \frac{(y_{\nu3}^2-y_{\nu2}^2)}{2}[(1-Y)(1-Z)-X(1-2Y)-W(1-X)]\right\}\\ &\quad + \left(\frac{y_{\nu1}^2+y_{\nu2}^2}{y_{\nu1}^2-y_{\nu2}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu1}^2+y_{\nu3}^2}{y_{\nu1}^2-y_{\nu3}^2}\right)\left\{(y_e^2-y_\tau^2)X(1-X-Y) + \frac{(y_\tau^2-y_\mu^2)}{2}[(1-Y)(1-Z)-X(1-2Z)-W(1-X)]\right\}\right] \qquad \text{(A.15)}\end{aligned}$$

$$\begin{aligned}\frac{dY}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_e^2+y_\mu^2}{y_e^2-y_\mu^2}\right)\left\{\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)] + (y_{\nu2}^2-y_{\nu3}^2)YW\right\}\right.\\ &\quad + \left(\frac{y_e^2+y_\tau^2}{y_e^2-y_\tau^2}\right)\left\{\frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[(1-Y)(1-Z)-W(1-X)-X(1-2Y)] + (y_{\nu2}^2-y_{\nu3}^2)Y(1-Y-W)\right\}\\ &\quad + \left(\frac{y_{\nu2}^2+y_{\nu1}^2}{y_{\nu2}^2-y_{\nu1}^2}\right)\left\{(y_e^2-y_\tau^2)XY + \frac{(y_\tau^2-y_\mu^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu2}^2+y_{\nu3}^2}{y_{\nu2}^2-y_{\nu3}^2}\right)\left\{(y_e^2-y_\tau^2)Y(1-X-Y) + \frac{(y_\mu^2-y_\tau^2)}{2}[W(1-X-2Y)+X-(1-Z)(1-Y)]\right\}\right] \qquad \text{(A.16)}\end{aligned}$$

$$\begin{aligned}\frac{dZ}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)XZ + \frac{(y_{\nu3}^2-y_{\nu2}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu1}^2-y_{\nu3}^2)Z(1-X-Z) + \frac{(y_{\nu2}^2-y_{\nu3}^2)}{2}[W(1-X-2Z)+X-(1-Y)(1-Z)]\right\}\\ &\quad + \left(\frac{y_{\nu1}^2+y_{\nu2}^2}{y_{\nu1}^2-y_{\nu2}^2}\right)\left\{\frac{(y_e^2-y_\tau^2)}{2}[(1-Y)(1-Z)-X-W(1-X)] + (y_\mu^2-y_\tau^2)ZW\right\}\\ &\quad \left. + \left(\frac{y_{\nu1}^2+y_{\nu3}^2}{y_{\nu1}^2-y_{\nu3}^2}\right)\left\{\frac{(y_\tau^2-y_e^2)}{2}[(1-Z)(1-Y)-W(1-X)-X(1-2Z)] + (y_\mu^2-y_\tau^2)Z(1-Z-W)\right\}\right] \qquad \text{(A.17)}\end{aligned}$$

$$\begin{aligned}\frac{dW}{dt} &= -\frac{3}{(4\pi)^2}\left[\left(\frac{y_\mu^2+y_e^2}{y_\mu^2-y_e^2}\right)\left\{(y_{\nu2}^2-y_{\nu3}^2)WY + \frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[W(1-X)+X-(1-Y)(1-Z)]\right\}\right.\\ &\quad + \left(\frac{y_\mu^2+y_\tau^2}{y_\mu^2-y_\tau^2}\right)\left\{(y_{\nu2}^2-y_{\nu3}^2)W(1-Y-W) + \frac{(y_{\nu3}^2-y_{\nu1}^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Z)]\right\}\\ &\quad + \left(\frac{y_{\nu2}^2+y_{\nu1}^2}{y_{\nu2}^2-y_{\nu1}^2}\right)\left\{(y_\mu^2-y_\tau^2)WZ + \frac{(y_\tau^2-y_e^2)}{2}[(1-X)W+X-(1-Y)(1-Z)]\right\}\\ &\quad \left. + \left(\frac{y_{\nu2}^2+y_{\nu3}^2}{y_{\nu2}^2-y_{\nu3}^2}\right)\left\{(y_\mu^2-y_\tau^2)W(1-Z-W) + \frac{(y_\tau^2-y_e^2)}{2}[(1-Y)(1-Z)-X-W(1-X-2Y)]\right\}\right]. \qquad \text{(A.18)}\end{aligned}$$

Couple of comments...

1. Asymp. safe SM full fit works (with normal ordering)

PMNS parametrization

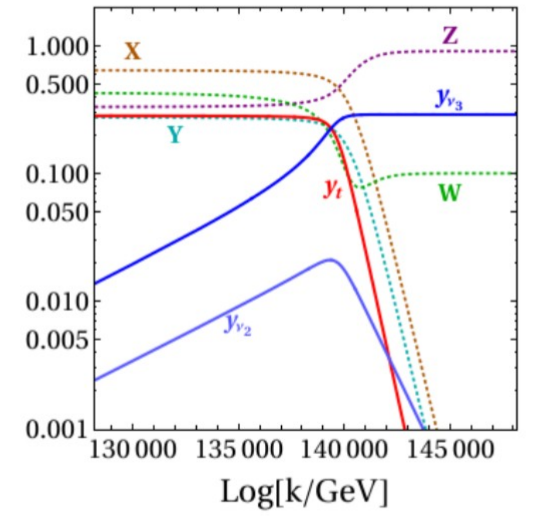
$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}$$

$$\theta_{12} = \arctan\sqrt{\frac{Y}{X}}$$
$$\theta_{13} = \arccos\sqrt{X+Y}$$
$$\theta_{23} = \arcsin\sqrt{\frac{1-W-Z}{X+Y}}$$

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}$$

PMNS fit

$X \in [0.64 - 0.71]$ $Y \in [0.26 - 0.34]$ $Z \in [0.05 - 0.26]$ $W \in [0.21 - 0.48]$



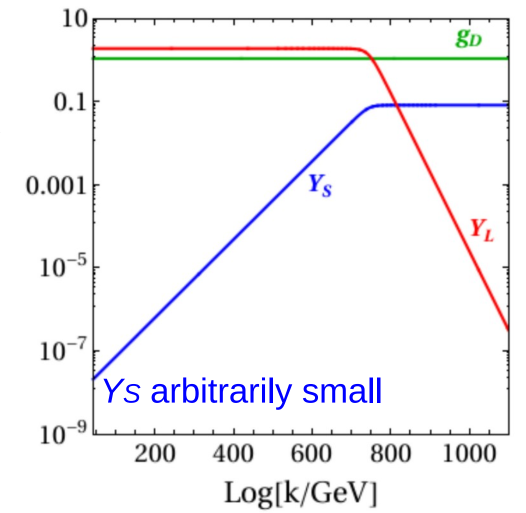
2. The mechanism is more generic than SM

e.g. dark gauge coupling g_D + Yukawa interactions

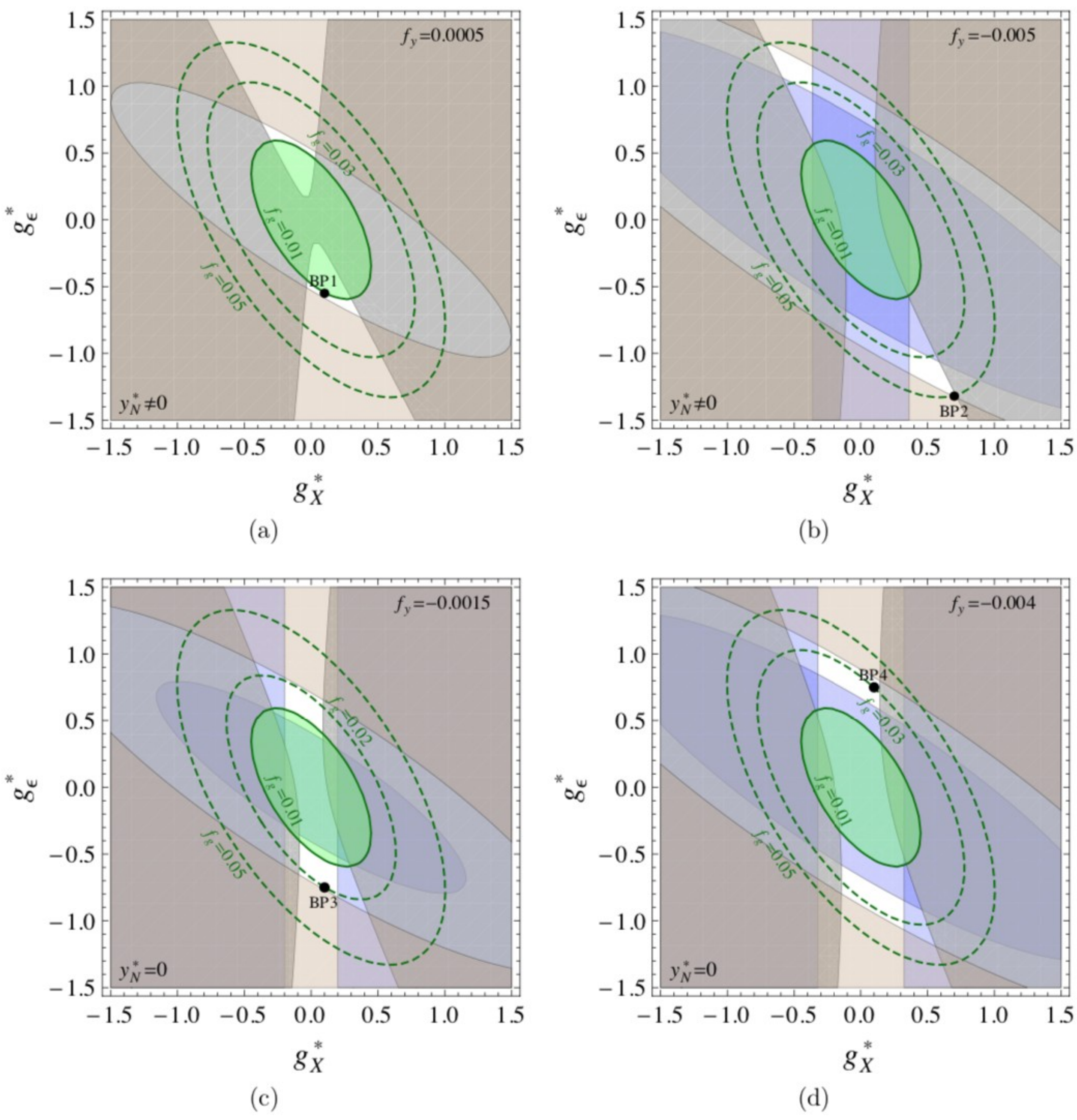
$$\mathcal{L} \supset Y_S\, \chi_R \Phi \chi_L + Y_L\, \psi_R \Phi \psi_L + \text{H.c.}$$

$$Q_\psi \gg Q_\chi$$ (dark abelian charge)

Can use it to justify freeze-in, feebly interacting models, etc...



Benchmark points of *B-L*



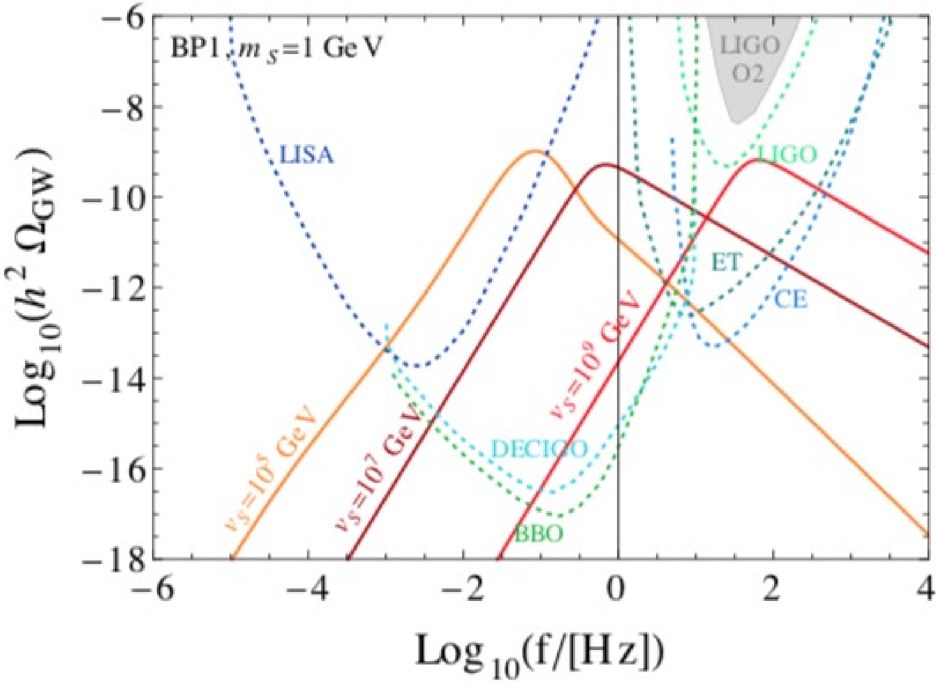
Gravitational wave signal

$$\alpha = \frac{\Delta V(T) - T\frac{\partial \Delta V(T)}{\partial T}}{\rho_R(T)}\bigg|_{T_p} \qquad \frac{\beta}{H_*} = T_p \frac{\mathrm{d}(S_3/T)}{\mathrm{d}T}\bigg|_{T_p} \qquad T_{\rm rh} = T_p[1+\alpha(T_p)]^{1/4}$$

$$\begin{aligned}
h^2\Omega^{\rm peak}_{\rm coll} &= 1.67\times 10^{-5}\,\kappa^2_{\rm coll}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11 v_w}{0.42+v_w^2}\right)\\
h^2\Omega^{\rm peak}_{\rm sw} &= 2.65\times 10^{-6}\,\kappa^2_{\rm sw}\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3}\\
h^2\Omega^{\rm peak}_{\rm turb} &= 3.35\times 10^{-4}\,\kappa^{3/2}_{\rm turb}\left(\frac{\alpha}{1+\alpha}\right)^{3/2}\left(\frac{v_w}{\beta/H_*}\right)\left(\frac{100}{g_*}\right)^{1/3},
\end{aligned}$$

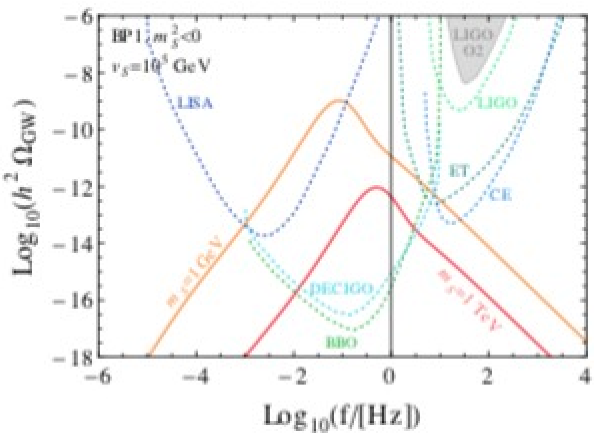
$$\begin{aligned}
f^{\rm peak}_{\rm coll} &= 1.65\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right)\left(\frac{0.62 v_w}{1.81-0.1v_w+v_w^2}\right)\\
f^{\rm peak}_{\rm sw} &= 1.90\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right)\\
f^{\rm peak}_{\rm turb} &= 2.70\times 10^{-5}\,{\rm Hz}\left(\frac{v_w}{\beta/H_*}\right)^{-1}\left(\frac{100}{g_*}\right)^{-1/6}\left(\frac{T_*}{100\ {\rm GeV}}\right). \qquad \text{(C.13)}
\end{aligned}$$

GWs at different scales



Details of BP1 and BP2

BP1



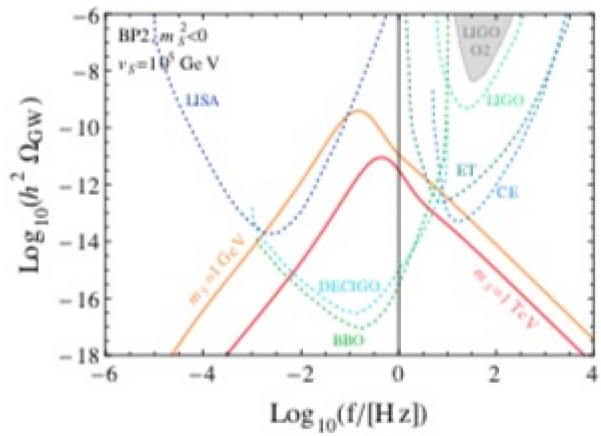
$m_S = 1\ GeV : \alpha = 10^{10}\ ,\ \beta = 49.8$

$T_p = 14.6\ GeV$

$m_S = 1\ TeV : \alpha = 0.27\ ,\ \beta = 185$

T_p ~ 10 TeV

BP2



$m_S = 1\ GeV : \alpha = 10^{11}\ ,\ \beta = 78.9$

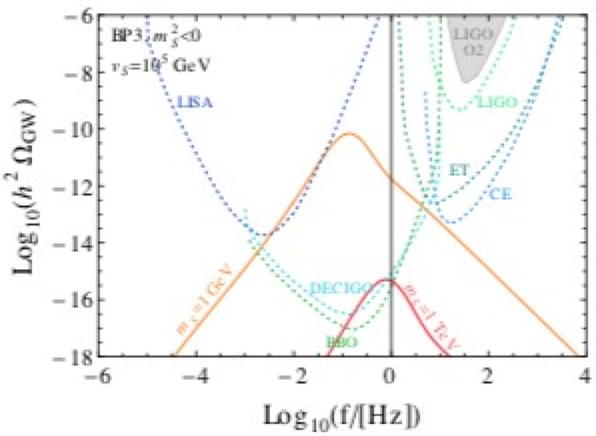
$T_p = 8\ GeV$

$m_S = 1\ TeV : \alpha = 0.88\ ,\ \beta = 187$

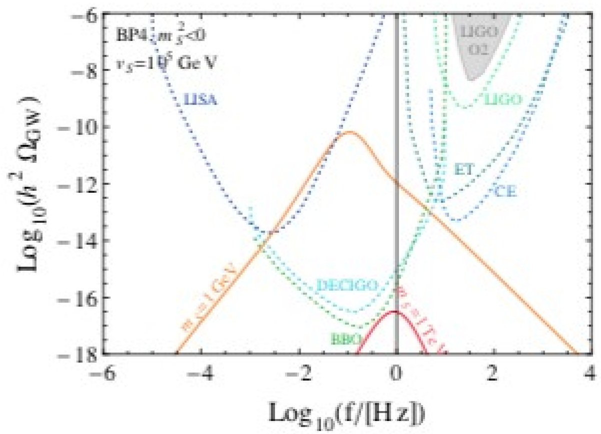
T_p ~ 10 TeV

Details of BP3 and BP4

BP3



BP4



$m_S = 1\ GeV : \alpha = 10^9\,,\ \beta = 189$

$T_p = 10.04\ GeV$

$m_S = 1\ GeV : \alpha = 10^8\,,\ \beta = 201$

$T_p = 11.5\ GeV$

$m_S = 1\ TeV : \alpha = 0.02\,,\ \beta = 227$

T_p ~ 10 TeV

$m_S = 1\ TeV : \alpha = 0.01\,,\ \beta = 229$

$T_p =$ ~ 10 TeV