Bubble nucleation for cosmological phase transitions

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CATCH22+2, Dublin
2 May 2024

Cosmological 1st-order phase transitions

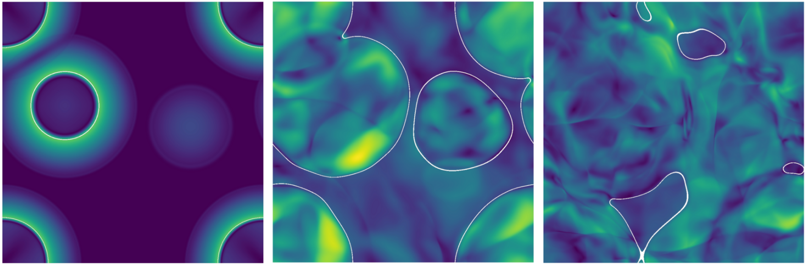
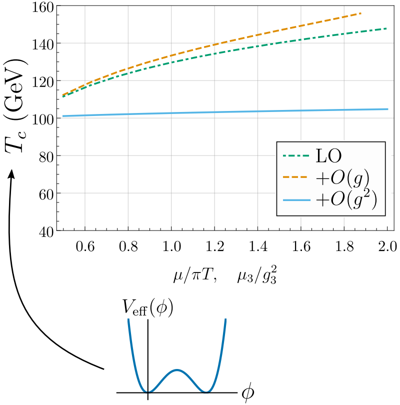


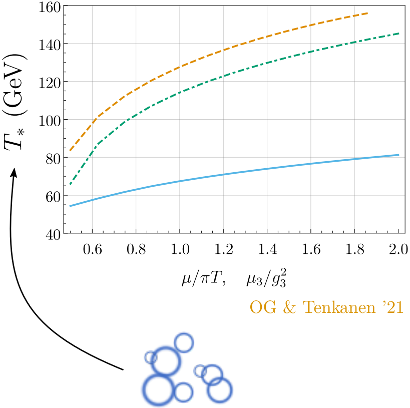
Figure: Cutting et al. arXiv:1906.00480.

- Universe supercools
- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows
- And creates gravitational waves: $\Box h_{ij}^{(\mathrm{TT})} \sim T_{ij}^{(\mathrm{TT})}$

Bubble nucleation uncertainties



critical temperature



OG & Tenkanen '21

percolation temperature

Gravitational wave spectrum

GW signal depends strongly on 4 phase transition quantities,

$$\Omega_{\mathrm{GW}} = F(T_*, R_*, \alpha_*, v_{\mathrm{w}}),$$

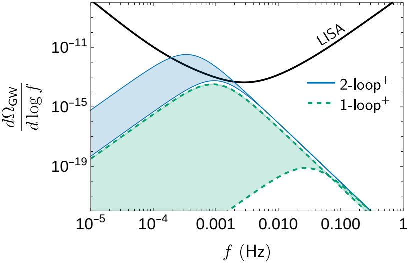
T_* : percolation temperature,

R_* : bubble radius,

α_* : transition strength,

v_{w} : bubble wall speed.

Each depends on the bubble nucleation rate.



Large uncertainties linked to predictions of nucleation rate.

OG & Tenkanen '21

The bubble nucleation rate

Perturbation theory for bubble nucleation

The bubble nucleation rate takes a semiclassical form,

$$\Gamma \sim A e^{-B},$$

with tree-level (B) and one-loop (A) contributions.

Solving for B

Computing B requires solving one nonlinear ODE

$$\frac{\mathrm{d}^2\phi_{\mathrm{b/F}}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi_{\mathrm{b/F}}}{\mathrm{d}r} - V'(\phi_{\mathrm{b/F}}) = 0.$$

$$B = \frac{1}{T}\left(E[\phi_{\mathrm{b}}] - E[\phi_{\mathrm{F}}]\right).$$

Think of B as the *internal energy* of a critical bubble, E_{b}/T.

Solving for A

Computing A requires solving an infinite number of linear ODEs

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}-\frac{l(l+1)}{r^2}+V''(\phi_{\mathrm{b/F}})\right]\psi_l^{\mathrm{b/F}}(r)=0.$$

$$A=\frac{\kappa_{\mathrm{dyn}}}{2\pi}\left(\frac{B}{2\pi}\right)^{3/2}\prod_{l=0}^{\infty}{}'\left|\frac{\psi_l^{\mathrm{F}}(\infty)}{\psi_l^{\mathrm{b}}(\infty)}\right|^{1/2}.$$

Think of A as the log entropy of fluctuations about this bubble,

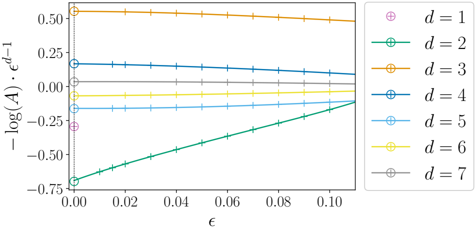
$$A\sim m^4 e^S,$$

so that minus the free energy appears in the exponent

$$\Gamma\sim m^4 e^{-(E_{\mathrm{b}}-ST)/T}.$$

BubbleDet

First public code for computing the one-loop term, A.



Ekstedt, OG, Hirvonen '23

An example, the thin wall limit for $d = 3$,

$$\Gamma \sim \underbrace{m^4 \exp\left(-\frac{0.55}{\epsilon^2}\right)}_{A} \underbrace{\exp\left(-\frac{3.3}{\epsilon^2\lambda}\right)}_{e^{-B}}$$

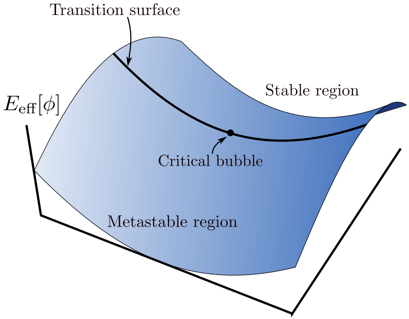
Munster & Rotsch '00, Matteini et al. 24

Out-of-equilibrium effects on nucleation

Nucleation is not just energy and entropy,

$$\Gamma \approx \frac{\kappa_{\text{dyn}}}{2\pi} m^3 e^{-(E_b - ST)/T}$$

Langer '69



and κ_{dyn} depends on dynamics of out-of-equilibrium particles Δf:

$$\kappa_{\text{dyn}}^2 \Delta\phi = (\nabla^2 - V''[\phi_b])\Delta\phi - \sum_a \frac{\mathrm{d}m_a^2}{\mathrm{d}\phi} \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2E} \Delta f,$$

Hirvonen '24

Do we really understand bubble nucleation?

- At least five different expressions for κ_{dyn}.

  Langer '69, Affleck '81, Linde '81

  Arnold & McLerran '87, Hirvonen '24

- Infrared divergences in κ_{dyn} for weak damping.

  Hangi et al. '90, Ekstedt '22

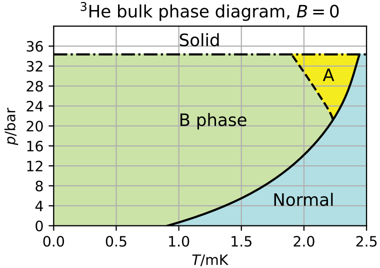
- Radical proposals:
  - additional saddlepoints
  - nucleation via intermediate solitons

  Tye & Wong '11

- How could we tell if we understand bubble nucleation?
  - analogue experiments
  - lattice simulations

Skeletons in the closet

Analogue experiments



- Clear disagreement for A-B transition in superfluid ^{3}He,

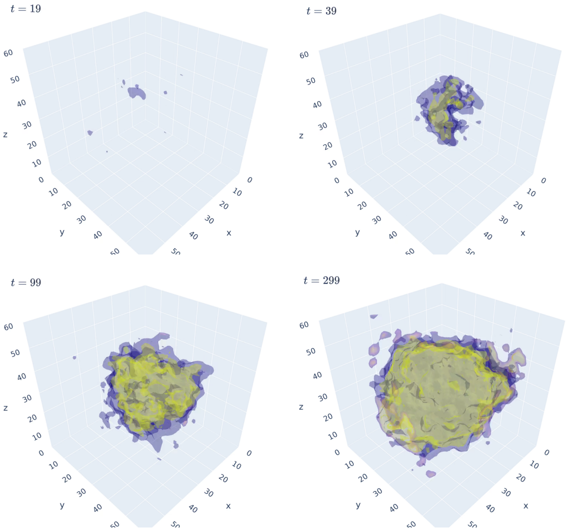
$$\frac{1}{\Gamma_{\text{experiment}}} \sim \text{few hours} \ll \text{age of universe} \ll \frac{1}{\Gamma_{\text{theory}}}.$$

Hindmarsh et al. '24

- Good agreement for rate in 1+1d ferromagnetic superfluid, taking entropy (A) as a fit parameter.

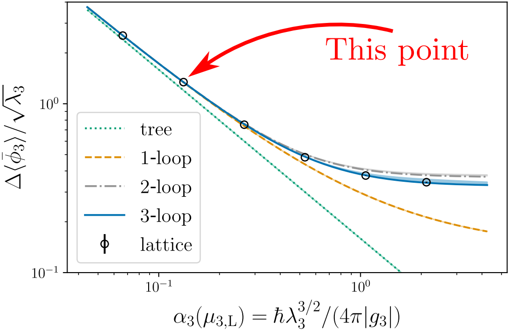
Zenesini et al. '23

Lattice simulations



OG, Güyer & Rummukainen '22

A super perturbative benchmark point



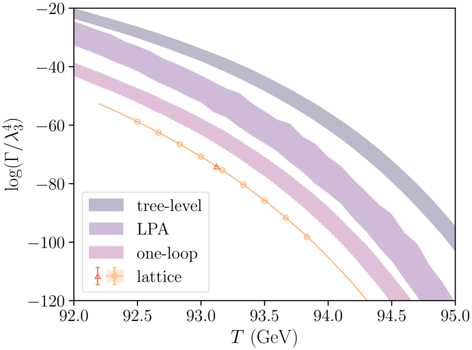
Perturbation theory converging *very* quickly for latent heat,

$$\underbrace{1.341(2)}_{\text{lattice}} \stackrel{?}{=} \underbrace{1.2}_{\text{tree}} + \underbrace{0.1378}_{\text{1-loop}} + \underbrace{0.0054}_{\text{2-loop}} - \underbrace{0.0016}_{\text{3-loop}} + \dots$$

$$\stackrel{\checkmark}{=} 1.34170(4)$$

OG '21

Benchmarking against the lattice



Qualitative agreement for log rate, but *way* worse than latent heat,

$$\underbrace{-74.09(5)}_{\text{lattice}} \stackrel{?}{=} \underbrace{-38.02}_{\text{tree}} - \underbrace{25.32}_{\text{1-loop}} + \dots$$

$$\stackrel{x}{=} -63(3)$$

OG, Kormu & Weir '24

Conclusions

- Nucleation rate $\rightarrow \Omega_{\text{GW}}$ predictions.
- Nucleation rate $\leftarrow$ energy, entropy & dynamics
- Can we get experiments, lattice and perturbation theory to agree?
- Are we missing something?

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Thanks for listening!

Backup slides

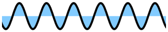
Real scalar model

A simple model,

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \sigma\phi + \frac{m^2}{2}\phi^2 + \frac{\kappa}{3!}\phi^3 + \frac{g^2}{4!}\phi^4 + J_1\phi + J_2\phi^2,$$

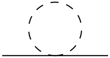
with only two relevant scales:

hard: $E \sim \pi T$ (nonzero Matsubara modes)



$$m_n^2 = m^2 + (n\pi T)^2 \text{ with } n \neq 0$$

soft: $E \sim gT$ (Debye screened)

$m^2_{\text{eff}} \sim$  $\sim g^2 T^2$