

The Fluctuating Spacetime of Dark Matter

CATCH 22+2

Dublin, Ireland

Jeff Dror
University of Florida

w/ Sarunas Verner

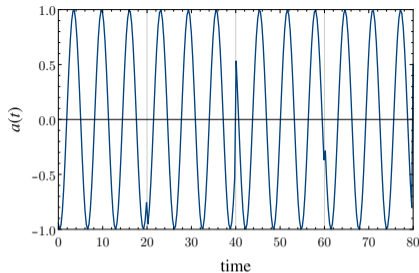
I think this
coffee is giving
me the jitters...



Maybe it's just
dark matter?

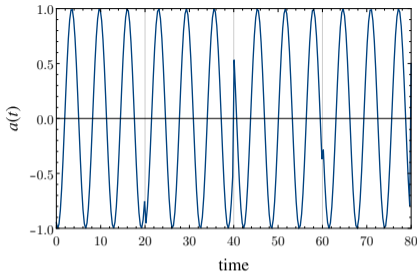
Stress-Energy Tensor of Scalar Dark Matter

$$T_{\mu\nu} = \partial_\mu a \partial_\nu a - \eta_{\mu\nu} \left(\frac{1}{2} \eta^{\alpha\beta} \partial_\alpha a \partial_\beta a - \frac{1}{2} m^2 a^2 \right)$$



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energy density (ρ)

T_{00}	T_{01}	T_{02}	T_{03}
T_{10}	T_{11}	T_{12}	T_{13}
$T_{2,0}$	T_{21}	T_{22}	T_{23}
T_{30}	T_{31}	T_{32}	T_{33}

momentum density (q^i)

pressure (p)

anisotropic stress (π_{ij})

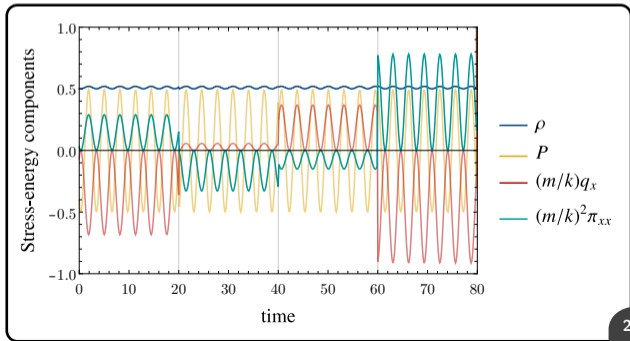
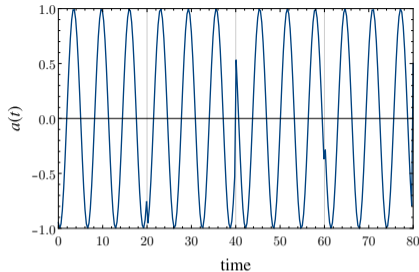
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energy density (ρ) ↙ ↘ pressure (p) ↙ ↘ anisotropic stress (π_{ij})

↖ momentum density (q^i)

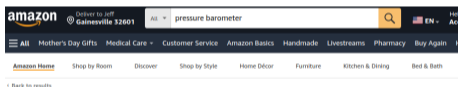


Spacetime Under Pressure

$$p \simeq \frac{1}{2} m^2 a_0^2 \cos(2mt)$$

Spacetime Under Pressure

$$p \simeq \frac{1}{2} m^2 a_0^2 \cos(2mt)$$
$$\simeq (5 \times 10^{-5} \text{Pa}) \cos(2mt)$$



The screenshot shows the Amazon website interface. At the top, the Amazon logo is on the left, and the delivery location is 'Gainesville 32601'. The search bar contains the text 'pressure barometer'. Below the search bar, there are navigation links for 'All', 'Mother's Day Gifts', 'Medical Care', 'Customer Service', 'Amazon Basics', 'Handmade', 'Livestreams', 'Pharmacy', and 'Buy Again'. At the bottom of the navigation bar, there are links for 'Amazon Home', 'Shop by Room', 'Discover', 'Shop by Style', 'Home Décor', 'Furniture', 'Kitchen & Dining', and 'Bed & Bath'. Below the navigation bar, there is a link that says '* Back to results'.



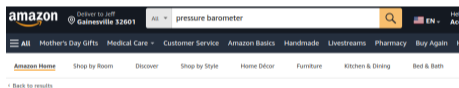
🏠 **Lirches 8" Barometer**
Thermometer Hygrometer - 3 in 1
Atmospheric Pressure
Temperature Hygrometer
Weather Station, Hanging
Premium Steel Barometer for
Home Wall, Fishing Boat, Baby
Room, Office
[Visit the Lirches Store](#)
4.3 ★★★★★ 69 ratings
50+ bought in past month

\$49⁹⁹

Spacetime Under Pressure

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

"Newtonian" gauge

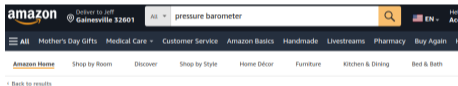
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2\phi & 0 & 0 & 0 \\ 0 & 2\psi & 0 & 0 \\ 0 & 0 & 2\psi & 0 \\ 0 & 0 & 0 & 2\psi \end{bmatrix}$$

Spacetime Under Pressure

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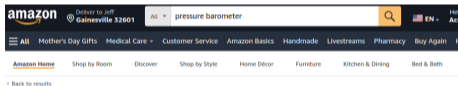
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$$G_{\mu\nu}(\phi, \psi) = 8\pi G T_{\mu\nu}(a)$$

Spacetime Under Pressure

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▣ Lirches 8" Barometer
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$$G_{\mu\nu}(\phi, \psi) = 8\pi G T_{\mu\nu}(a)$$

$$\psi, \phi \supset \frac{\rho}{m^2 M_{\text{Pl}}^2} \cos(2mt + \alpha)$$

[Khmelnitsky, Rubakov, '13]

Opportunity for **gravitational** direct detection of dark matter

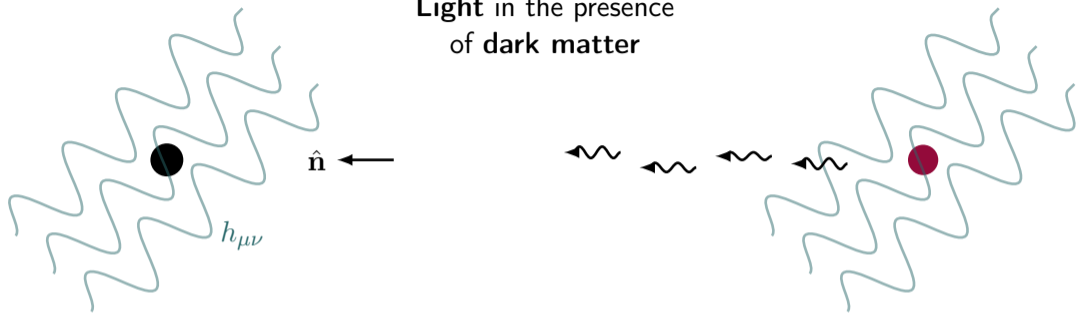
(signals akin to gravitational waves)

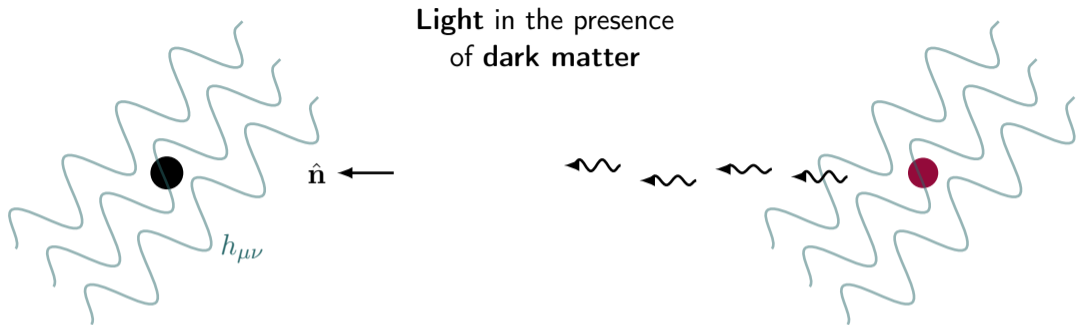
$$|h| \sim 3 \times 10^{-16} \left(\frac{\rho}{0.3 \text{ GeV/cm}^3} \right) \left(\frac{10^{-23} \text{ eV}}{m} \right)^2$$

Light without dark matter



Light in the presence
of dark matter





Three **gauge-dependent** effects:

The source and
observer are wiggling

$$x_{\text{obs}}^{\mu}, v_{\text{obs}}^{\mu}, x_{\text{s}}^{\mu}, v_{\text{s}}^{\mu}$$

The photon along
trajectory is wiggling

$$x^{\mu}, P^{\mu}$$

The observer reference
frame is getting
deformed

$$\epsilon_{\hat{\alpha}}^{\mu}$$

Fundamental Equation

$$P_{\hat{\alpha}} = (\eta_{\mu\nu} + h_{\mu\nu}(0))(P^{\nu} + \delta P^{\nu}(0))(\varepsilon_{\hat{\alpha}}^{\mu} + \delta\varepsilon_{\hat{\alpha}}^{\mu}(0))$$

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$$\hat{n}_i + \delta\hat{n}_i = \frac{P_{\hat{i}} + \delta P_{\hat{i}}}{P_{\hat{0}} + \delta P_{\hat{0}}}$$

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Frequency Redshift

$$\frac{\nu(t) - \nu(0)}{\nu(0)} = \psi(t, \mathbf{0}) - \psi(t_{\text{em}}, \mathbf{x}_{\text{em}}) \\ + [\mathbf{v}_{\text{obs}}(t, \mathbf{0}) - \mathbf{v}_{\text{source}}(t_{\text{em}})] \cdot \hat{\mathbf{n}}$$

[Minor correction to
Khmelnitsky, Rubakov, '13]

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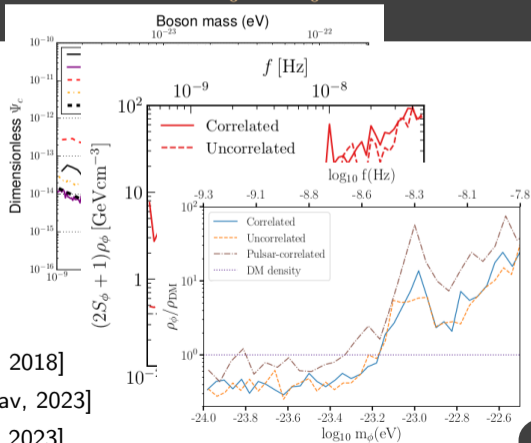
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[Minor correction to
Khmelnitsky, Rubakov, '13]

[PPTA, 2018]
[NANOGrav, 2023]
[EPTA, 2023]



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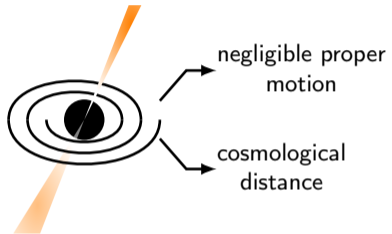
Astrometric
Deflection

$$\delta\hat{n}_{\hat{i}} \simeq \hat{n}_{\hat{i}} (\psi(0) + \mathbf{v}_{\text{obs}} \cdot \hat{\mathbf{n}}) - \delta\epsilon_{\hat{i}}^0 - \hat{n}^j \delta\epsilon_{\hat{i}}^j$$

$$\omega \equiv \frac{d(\delta\hat{\mathbf{n}} \cdot \hat{\boldsymbol{\theta}})}{dt}$$

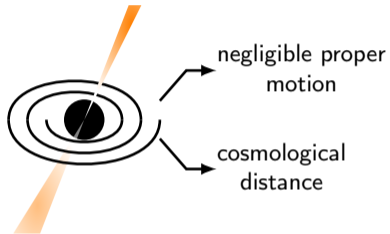
[JD, Verner]

Astrometric Deflection

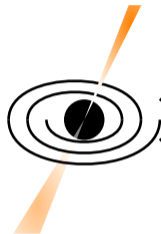


Astrometric Deflection

$$\omega = \frac{\pi a_0^2}{M_{\text{Pl}}^2} \hat{\theta} \cdot \mathbf{k} \sin(2(mt + \alpha))$$



Astrometric Deflection



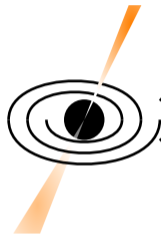
negligible proper motion

cosmological distance

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$$\left\{ \begin{array}{l} \rightarrow \frac{0.1 \mu\text{as}}{\text{yr}} \frac{v}{10^{-3}} \frac{10^{-30} \text{ eV}}{m} \frac{\rho}{0.3 \text{ GeV/cm}^3} \frac{\hat{k} \cdot \hat{\theta} \sin 2\alpha}{2} \end{array} \right.$$

Astrometric Deflection



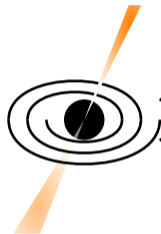
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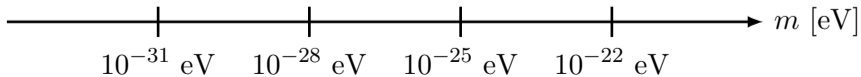


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cosmological distance

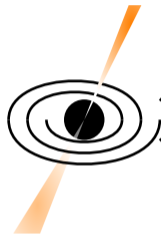
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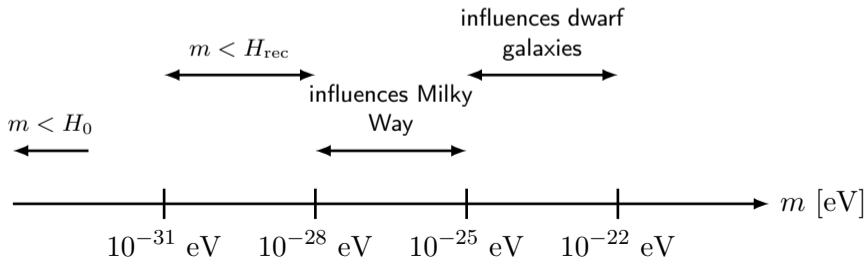
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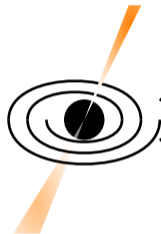
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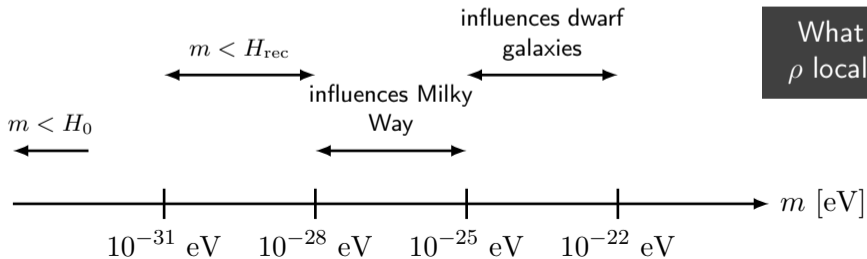


negligible proper motion

cosmological distance

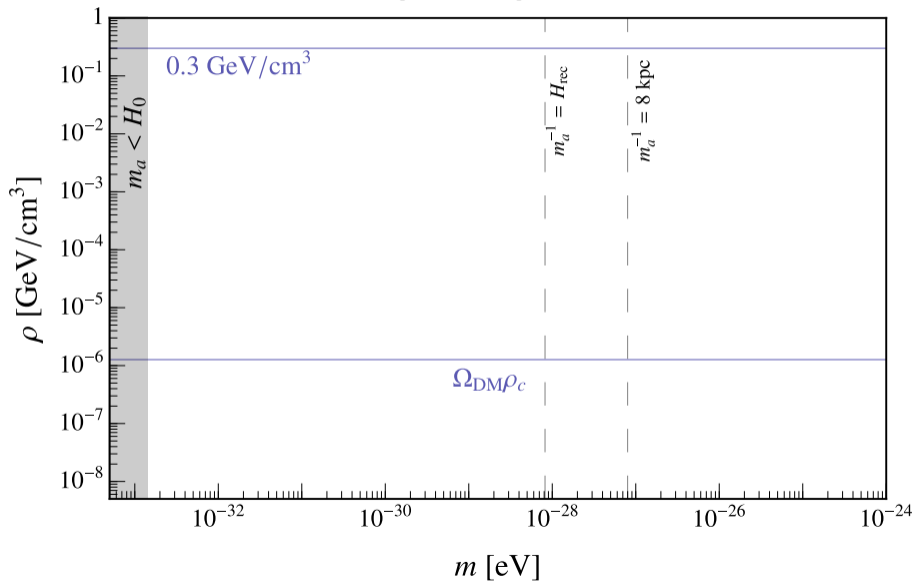
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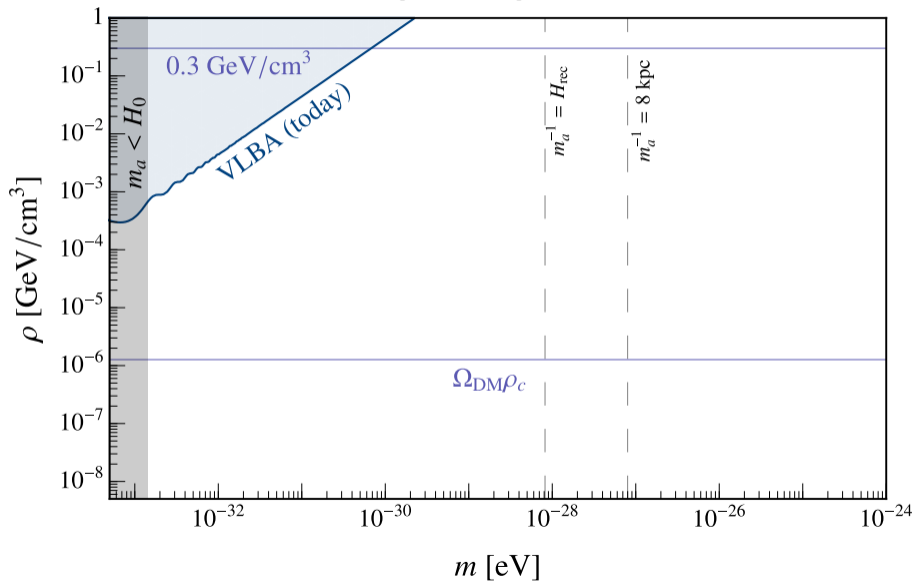


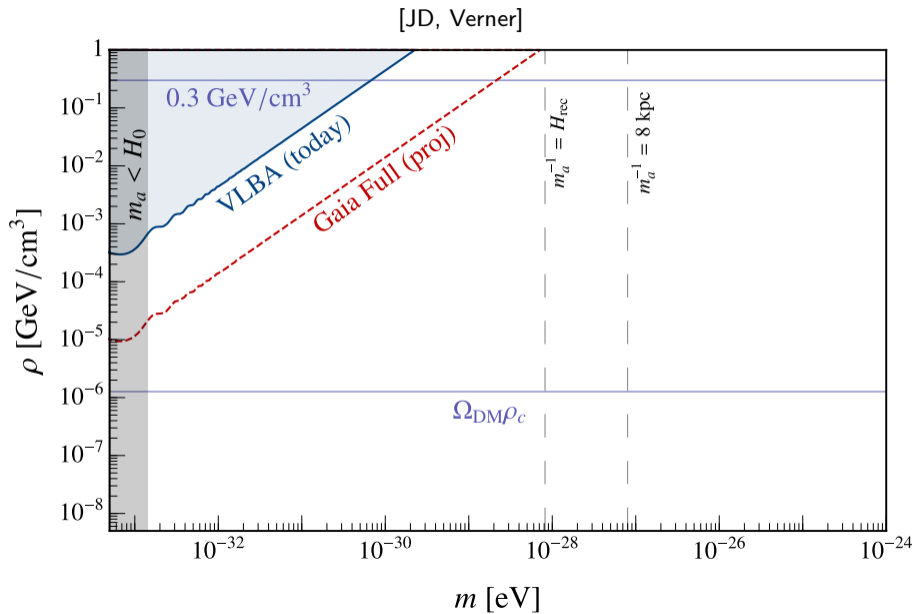
What is ρ locally?

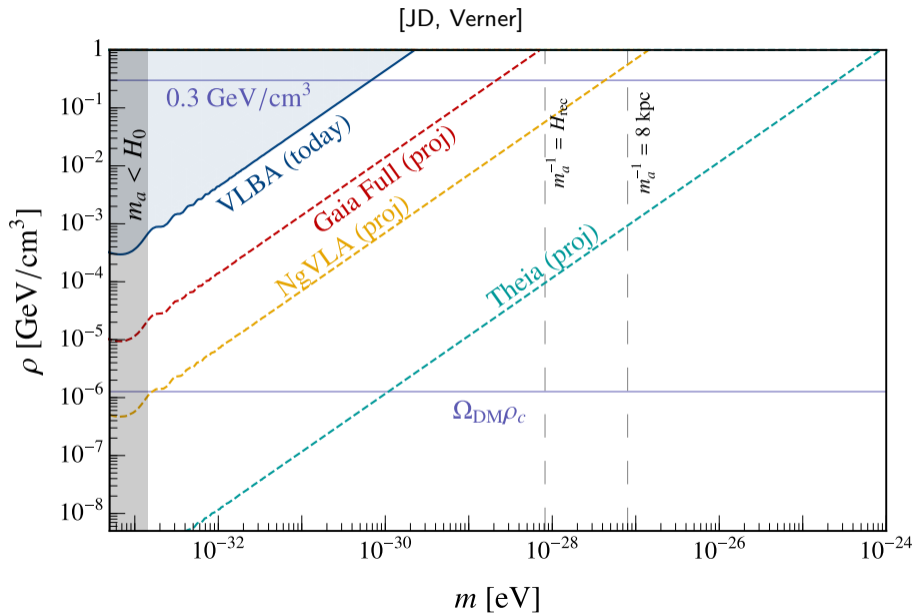
[JD, Verner]



[JD, Verner]







Future Directions (time-pending)



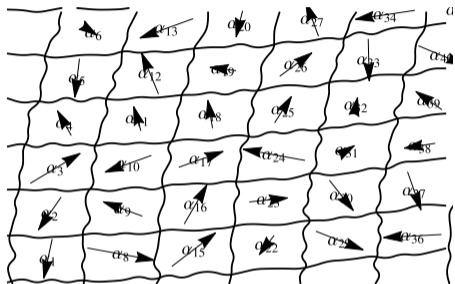
Astrometry of Vector Dark Matter

[JD, Verner]

α_6	α_{13}	α_{20}	α_{27}	α_{34}	α_{41}
α_5	α_{12}	α_{19}	α_{26}	α_{33}	α_{40}
α_4	α_{11}	α_{18}	α_{25}	α_{32}	α_{39}
α_3	α_{10}	α_{17}	α_{24}	α_{31}	α_{38}
α_2	α_9	α_{16}	α_{23}	α_{30}	α_{37}
α_1	α_8	α_{15}	α_{22}	α_{29}	α_{36}

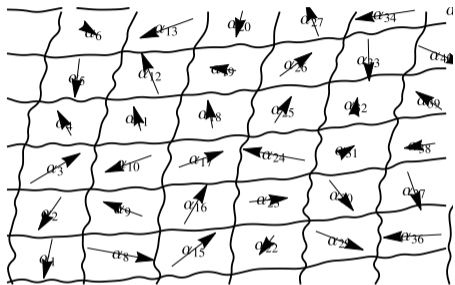
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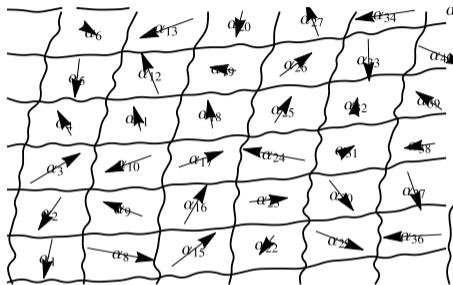
$$A_i = \hat{A}_i A \cos(mt + \alpha)$$

New contributions to T_{ij}

$$\frac{1}{2}(\partial_i \partial_j - \frac{1}{3} \nabla^2 \delta_{ij})(\phi - \psi) = 4\pi G m^2 A^2 \left(\hat{A}_i \hat{A}_j - \frac{1}{3} \delta_{ij} \right)$$

Astrometry of Vector Dark Matter

[JD, Verner]



Scalar dark matter: $\phi \sim \psi$

Vector dark matter: $\phi \sim 10^6 \psi$

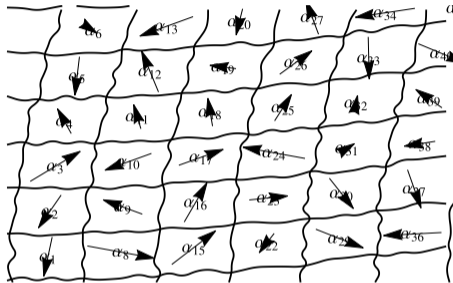
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Astrometry of Vector Dark Matter

[JD, Verner]



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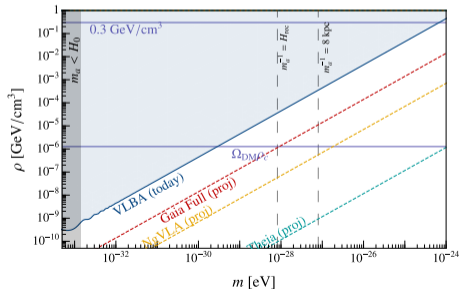
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Scalar dark matter: $\phi \sim \psi$

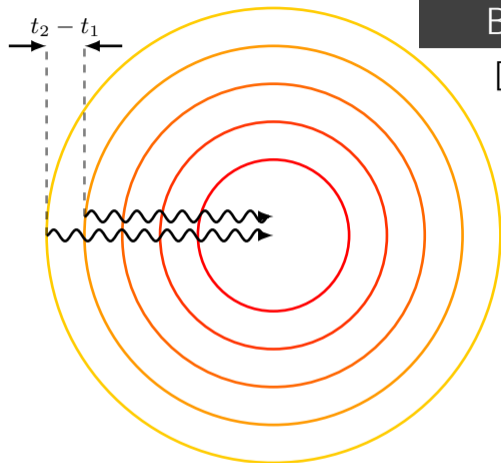
Vector dark matter: $\phi \sim 10^6 \psi$

(Improves detection prospects by $\sim 10^6$)



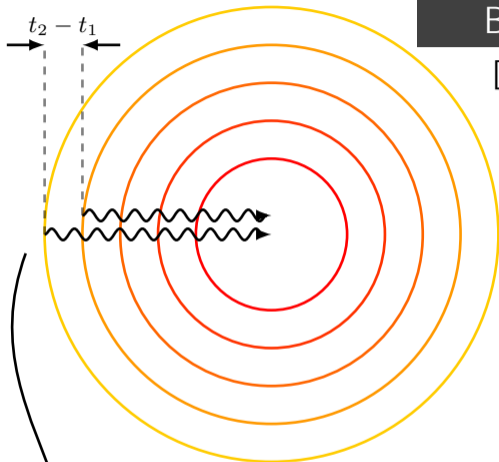
Cosmic Microwave Background

[JD, Kyriazis]



Cosmic Microwave Background

[JD, Kyriazis]

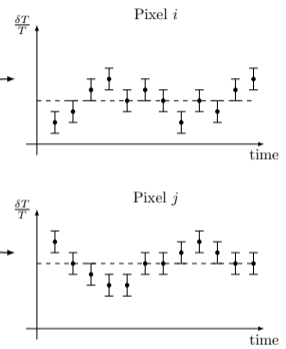
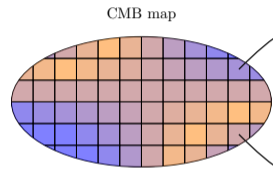
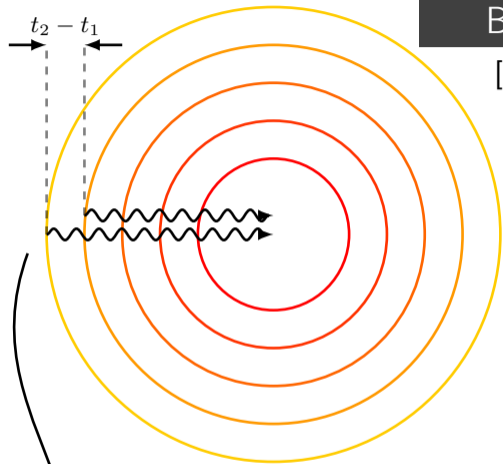


$$\psi_c \simeq 3 \times 10^{-12} \left(\frac{10^{-23} \text{ eV}}{m} \right)^2$$

$$z \propto \cos(2m(t_{\text{rec}} - \mathbf{v} \cdot \hat{\mathbf{n}}(t - t_{\text{rec}})))$$

Cosmic Microwave Background

[JD, Kyriazis]



probe of DM velocity during recombination

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$$z \propto \cos(2m(t_{\text{rec}} - \mathbf{v} \cdot \hat{\mathbf{n}}(t - t_{\text{rec}})))$$

The Fluctuating Spacetime of Dark Matter

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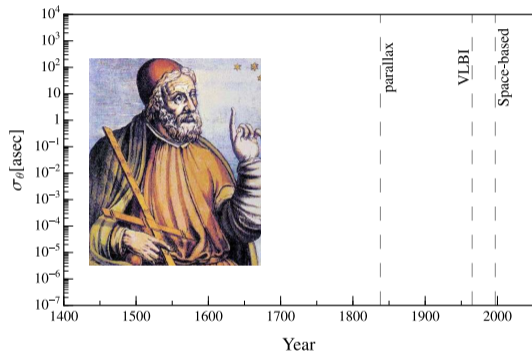
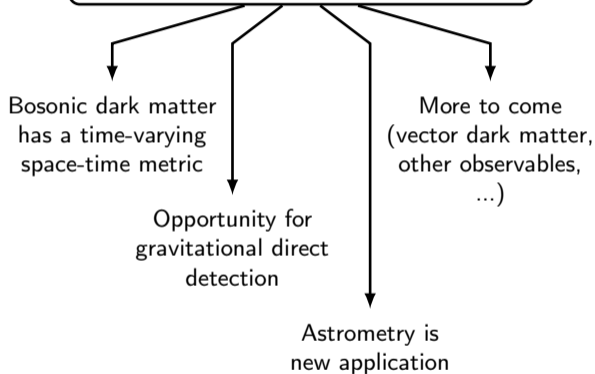
Bosonic dark matter
has a time-varying
space-time metric

Opportunity for
gravitational direct
detection

Astrometry is
new application

More to come
(vector dark matter,
other observables,
...)

The Fluctuating Spacetime of Dark Matter



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