The Fluctuating Spacetime of Dark Matter

CATCH 22+2

Dublin, Ireland

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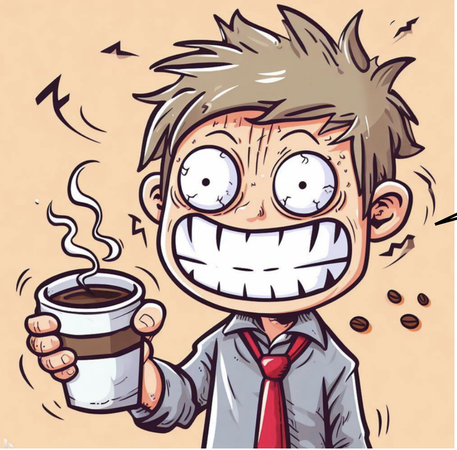
University of Florida

w/ Sarunas Verner

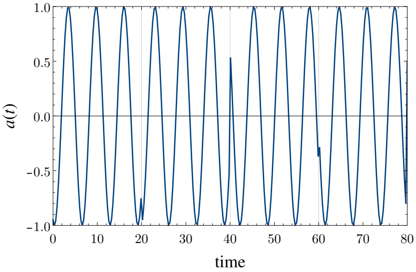
I think this coffee is giving me the jitters...



Maybe it's just dark matter?

Stress-Energy Tensor of Scalar Dark Matter

$$T_{\mu\nu} = \partial_\mu a \partial_\nu a - \eta_{\mu\nu} \left(\frac{1}{2} \eta^{\alpha\beta} \partial_\alpha a \partial_\beta a - \frac{1}{2} m^2 a^2 \right)$$



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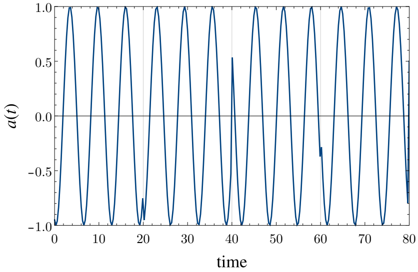
energy density (ρ)

$$\left[\begin{array}{c|ccc} T_{00} & T_{01} & T_{02} & T_{03} \\ \hline T_{10} & T_{11} & T_{12} & T_{13} \\ T_{2,0} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{array}\right]$$

momentum density (q^i)

pressure (p)

anisotropic stress (π_{ij})



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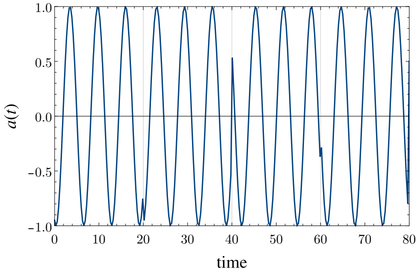

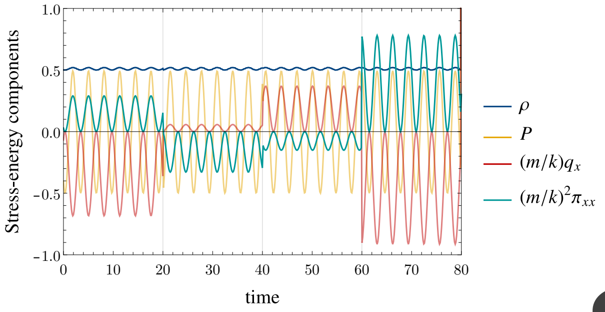

Spacetime Under Pressure

$$p \simeq \frac{1}{2} m^2 a_0^2 \cos(2mt)$$

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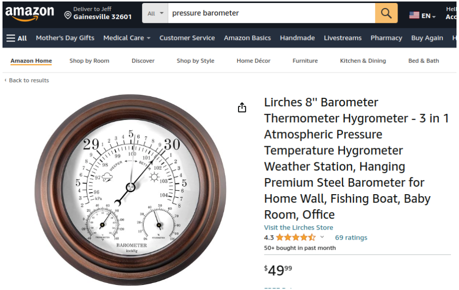

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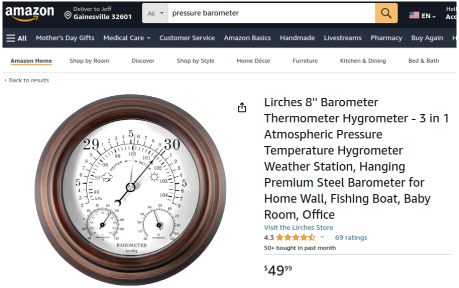

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

"Newtonian" gauge

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad h_{\mu\nu} = \begin{bmatrix} 2\phi & 0 & 0 & 0 \\ 0 & 2\psi & 0 & 0 \\ 0 & 0 & 2\psi & 0 \\ 0 & 0 & 0 & 2\psi \end{bmatrix}$$

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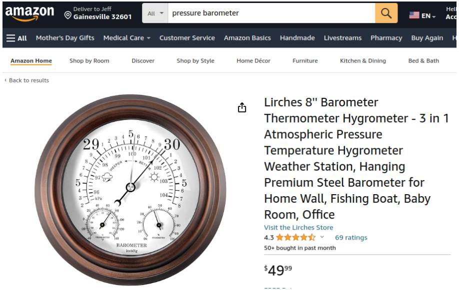

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$$G_{\mu\nu}(\phi, \psi) = 8\pi G T_{\mu\nu}(a)$$

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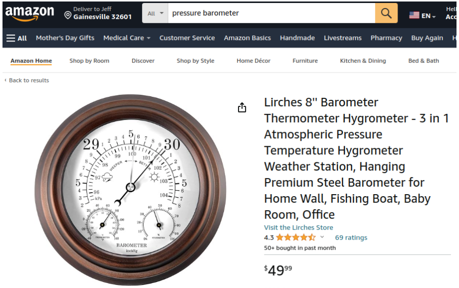

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$$G_{\mu\nu}(\phi, \psi) = 8\pi G T_{\mu\nu}(a)$$

$$\psi, \phi \supset \frac{\rho}{m^2 M_{\mathrm{Pl}}^2} \cos(2mt + \alpha)$$

[Khmelnitsky, Rubakov, '13]

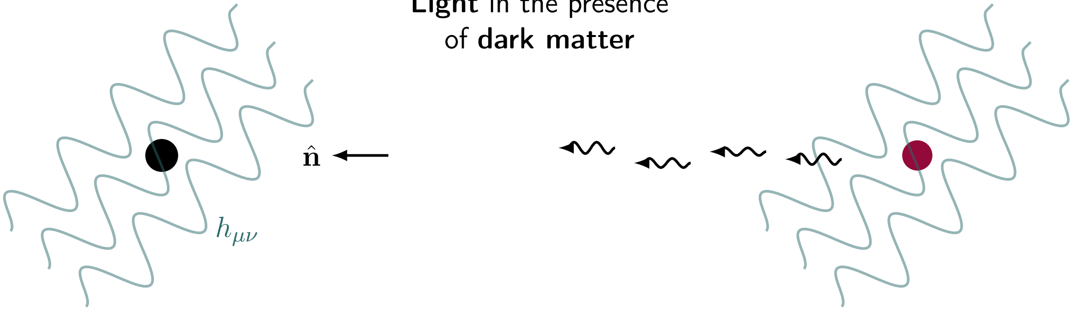
Opportunity for **gravitational** direct detection of dark matter

(signals akin to gravitational waves)

$$|h| \sim 3 \times 10^{-16} \left(\frac{\rho}{0.3 \text{ GeV/cm}^3} \right) \left(\frac{10^{-23} \text{ eV}}{m} \right)^2$$

Light without
dark matter

Light in the presence of **dark matter**



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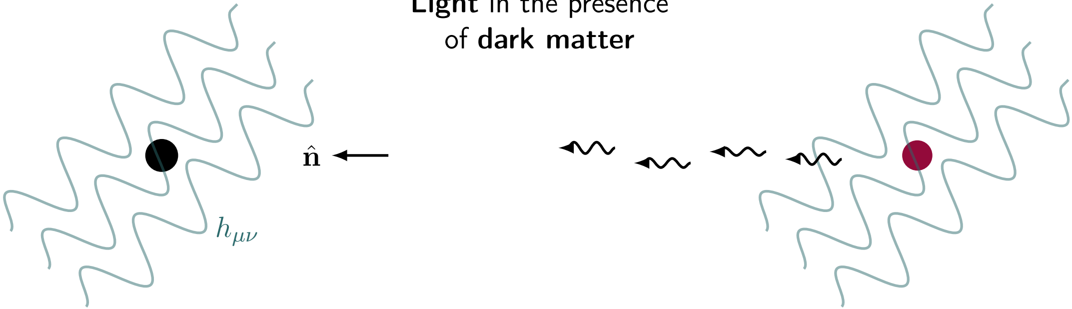

Three **gauge-dependent** effects:

The source and observer are wiggling

$x^\mu_{\mathrm{obs}}, v^\mu_{\mathrm{obs}}, x^\mu_{\mathrm{s}}, v^\mu_{\mathrm{s}}$

The photon along trajectory is wiggling

x^μ, P^μ

The observer reference frame is getting deformed

$\varepsilon^\mu_{\hat{\alpha}}$

Fundamental Equation

$$P_{\hat{\alpha}} = (\eta_{\mu\nu} + h_{\mu\nu}(0))(P^{\nu} + \delta P^{\nu}(0))(\varepsilon^{\mu}_{\hat{\alpha}} + \delta\varepsilon^{\mu}_{\hat{\alpha}}(0))$$

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$$\nu(t) = \frac{1}{2\pi}(P_{\hat{0}} + \delta P_{\hat{0}})$$

$$\hat{n}_i + \delta\hat{n}_i = \frac{P_{\hat{i}} + \delta P_{\hat{i}}}{P_{\hat{0}} + \delta P_{\hat{0}}}$$

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Frequency Redshift

$$\frac{\nu(t) - \nu(0)}{\nu(0)} = \psi(t, \mathbf{0}) - \psi(t_{\mathrm{em}}, \mathbf{x}_{\mathrm{em}})$$

$$+ \left[\mathbf{v}_{\mathrm{obs}}(t, \mathbf{0}) - \mathbf{v}_{\mathrm{source}}(t_{\mathrm{em}})\right] \cdot \hat{\mathbf{n}}$$

[Minor correction to
Khmelnitsky, Rubakov, '13]

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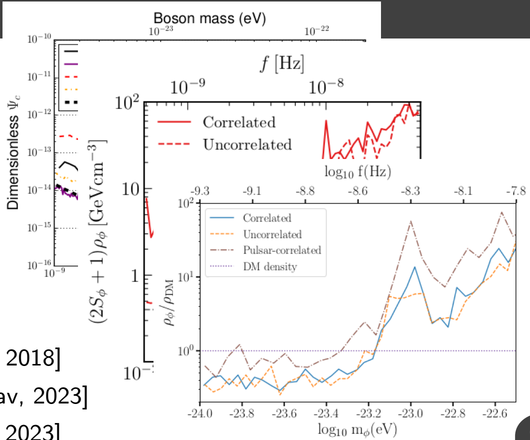
[PPTA, 2018]
[NANOGrav, 2023]
[EPTA, 2023]



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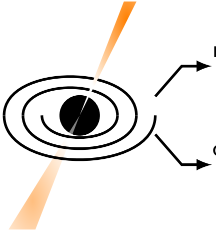
Astrometric
Deflection

$$\delta\hat{n}_{\hat{i}} \simeq \hat{n}_{\hat{i}}\,(\psi(0) + \mathbf{v}_{\rm obs}\cdot\hat{\mathbf{n}}) - \delta\epsilon^0_{\hat{i}} - \hat{n}^j\delta\epsilon^j_{\hat{i}})$$

$$\omega \equiv \frac{d(\delta\hat{\mathbf{n}}\cdot\hat{\theta})}{dt}$$

[JD, Verner]

Astrometric Deflection

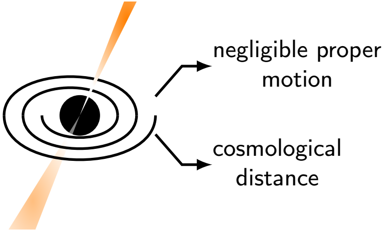


negligible proper motion

cosmological distance

Astrometric Deflection

$$\omega = \frac{\pi a_0^2}{M_{\rm Pl}^2} \hat{\theta} \cdot \mathbf{k} \sin(2(mt + \alpha))$$



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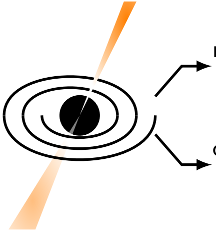

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(Probe tiny mass dark matter)

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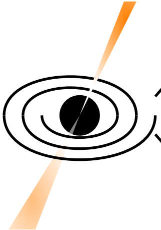

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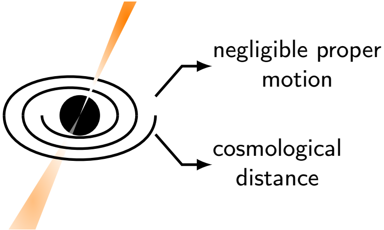

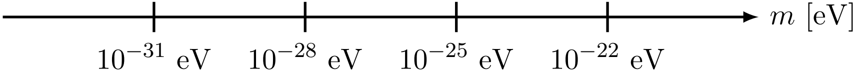

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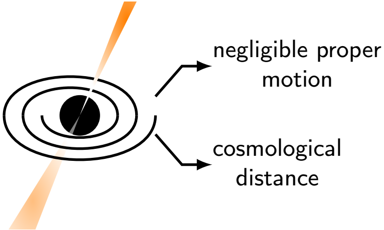

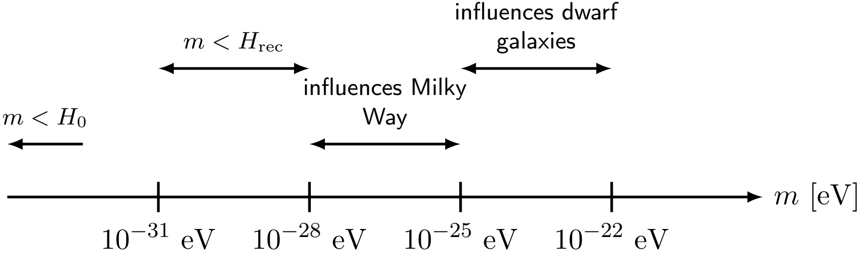

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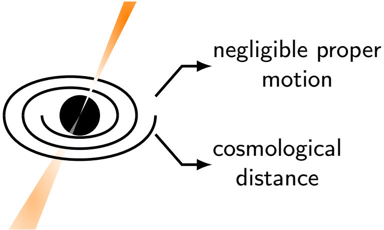
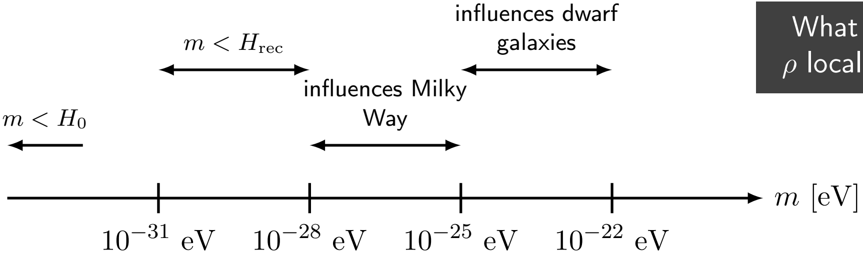
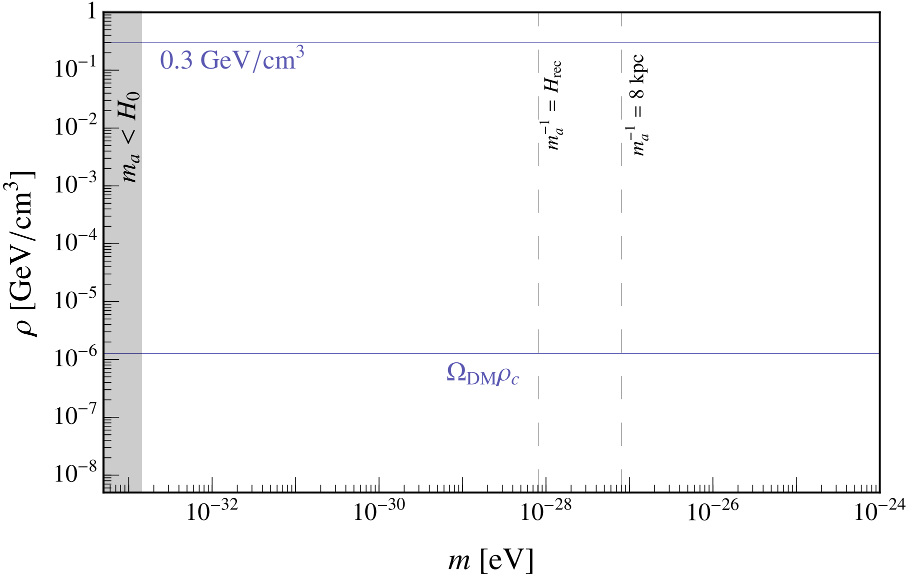




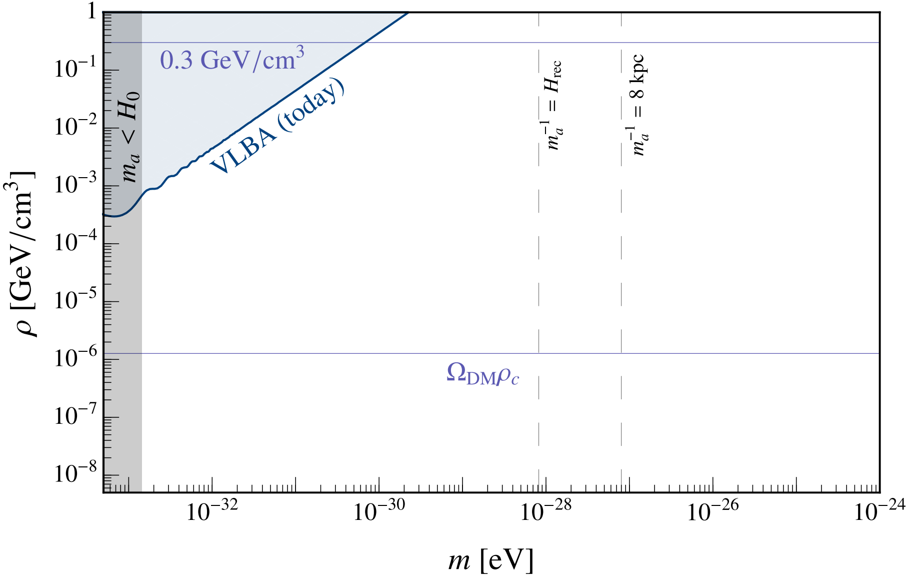
What is ρ locally?

[JD, Verner]



0.3 GeV/cm^3
$m_a < H_0$
$m_a^{-1} = H_{\rm rec}$
$m_a^{-1} = 8$ kpc
$\Omega_{\rm DM}\rho_c$
ρ [GeV/cm^3]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-32}
10^{-30}
10^{-28}
10^{-26}
10^{-24}
m [eV]

[JD, Verner]



1
10^{-1}
10^{-2}
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10^{-4}
10^{-5}
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ρ [GeV/cm^3]
10^{-32}
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m [eV]
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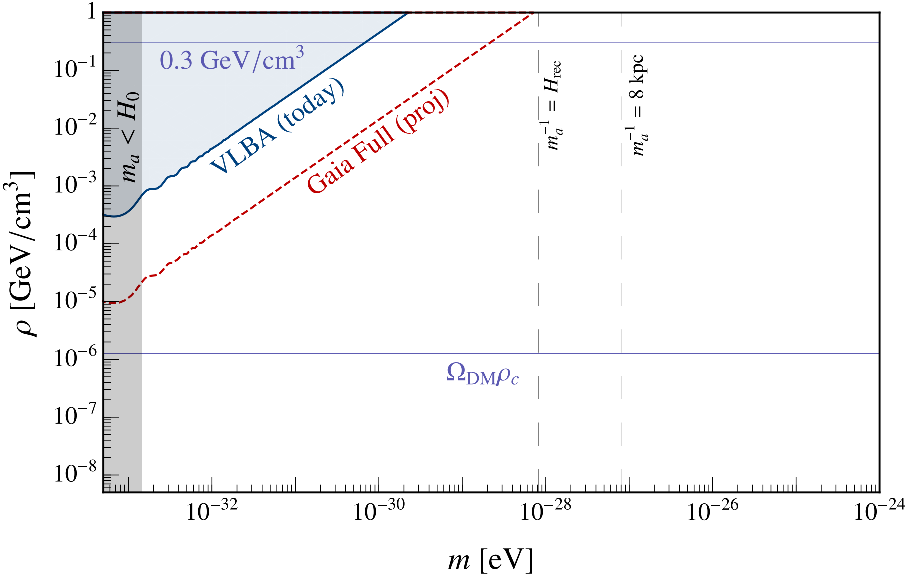
[JD, Verner]



1
10^{-1}
10^{-2}
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10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ρ [GeV/cm^3]
0.3 GeV/cm^3
$m_a < H_0$
VLBA (today)
Gaia Full (proj)
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10^{-32}
10^{-30}
10^{-28}
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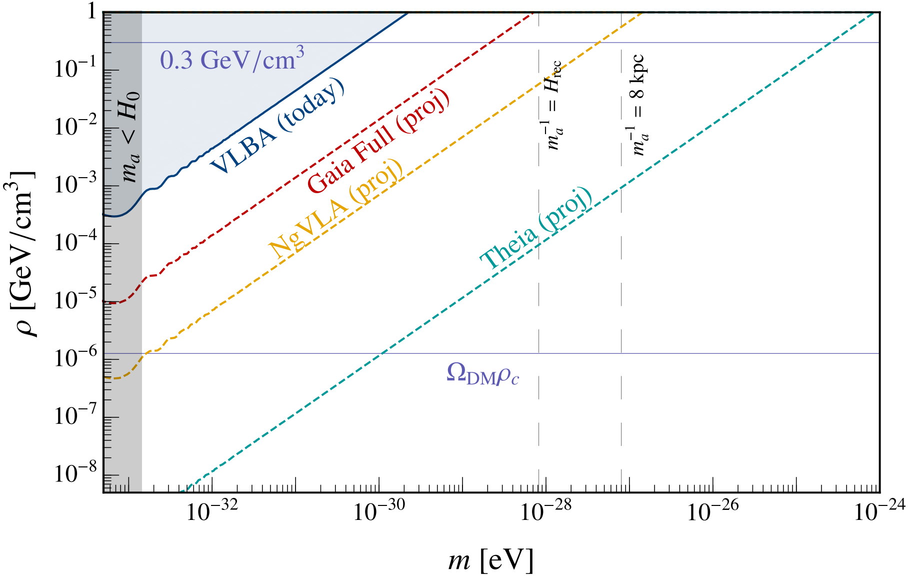
m [eV]

[JD, Verner]



0.3 GeV/cm^3
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VLBA (today)
Gaia Full (proj)
NgVLA (proj)
Theia (proj)
$m_a^{-1} = H_{\rm rec}$
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ρ [GeV/cm^3]
m [eV]
1
10^{-1}
10^{-2}
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10^{-4}
10^{-5}
10^{-6}
10^{-7}
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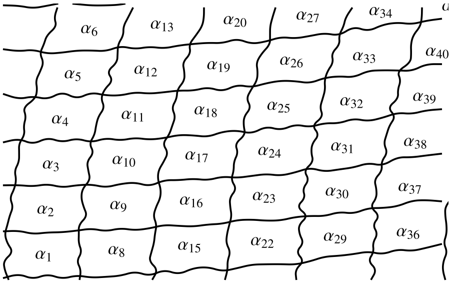
10^{-32}
10^{-30}
10^{-28}
10^{-26}
10^{-24}

Future Directions (time-pending)

Astrometry of Vector Dark Matter
[JD, Verner]



Astrometry of Vector Dark Matter
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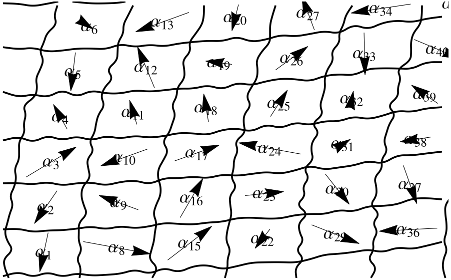

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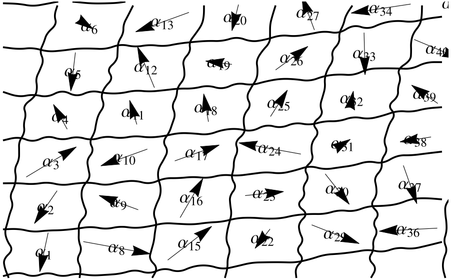
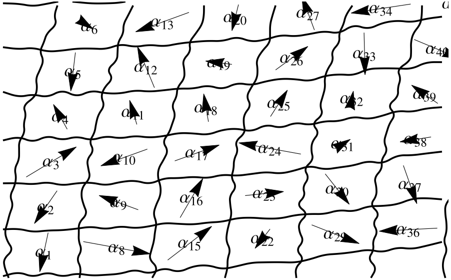


$$A_i = \hat{A}_i A \cos(mt + \alpha)$$

New contributions to T_{ij}

$$\frac{1}{2}(\partial_i \partial_j - \frac{1}{3}\nabla^2 \delta_{ij})(\phi - \psi) = 4\pi G m^2 A^2 \left(\hat{A}_i \hat{A}_j - \frac{1}{3}\delta_{ij}\right)$$

Astrometry of Vector Dark Matter
[JD, Verner]



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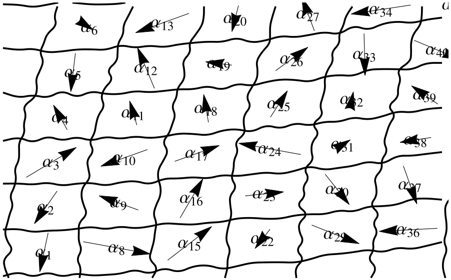
Scalar dark matter: $\phi \sim \psi$

Vector dark matter: $\phi \sim 10^6\psi$

Astrometry of Vector Dark Matter
[JD, Verner]



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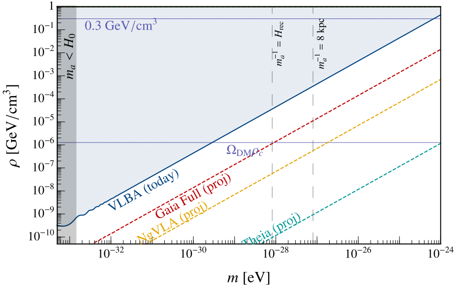
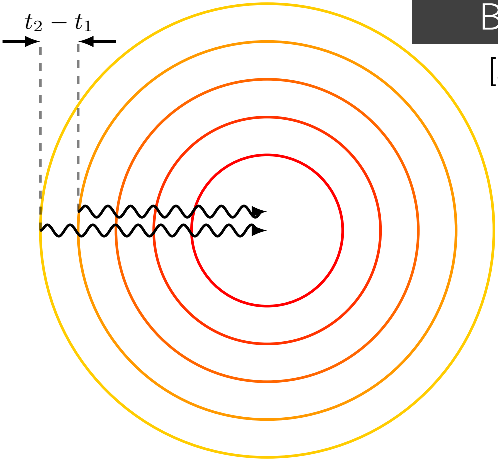
Scalar dark matter: $\phi \sim \psi$

Vector dark matter: $\phi \sim 10^6 \psi$

$\left(\begin{array}{c}\text{Improves detection} \\ \text{prospects by } \sim 10^6\end{array}\right)$





Cosmic Microwave Background

[JD, Kyriazis]

Cosmic Microwave Background

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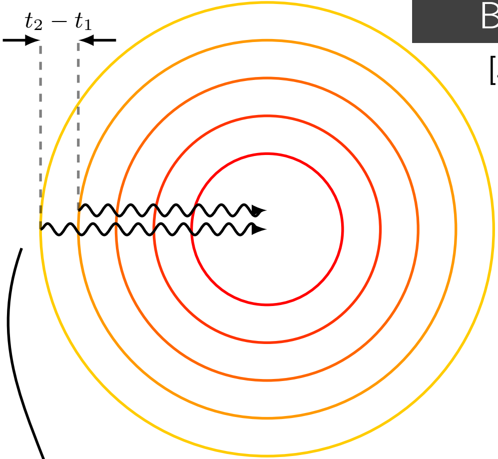

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$$z \propto \cos(2m(t_{\mathrm{rec}} - \mathbf{v} \cdot \hat{\mathbf{n}}(t - t_{\mathrm{rec}})))$$

Cosmic Microwave Background

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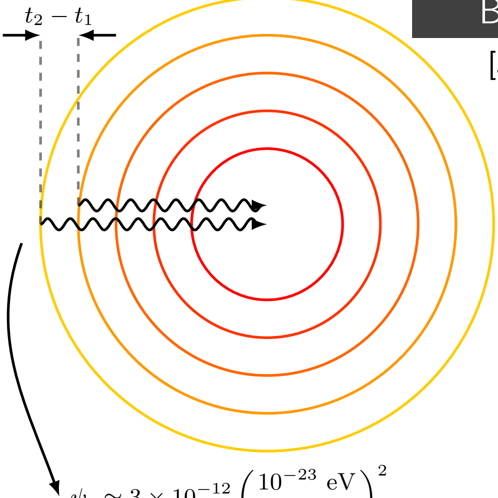

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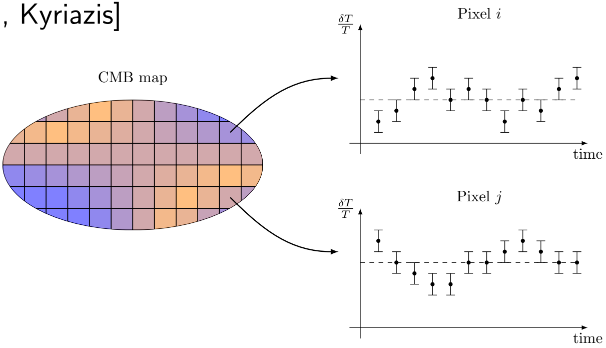

probe of DM velocity during recombination

The Fluctuating Spacetime of Dark Matter

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Bosonic dark matter has a time-varying space-time metric

Opportunity for gravitational direct detection

Astrometry is new application

More to come (vector dark matter, other observables, ...)

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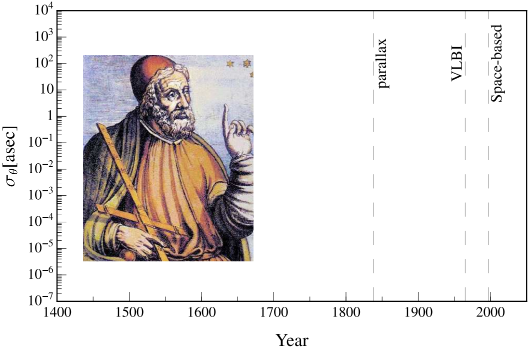

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