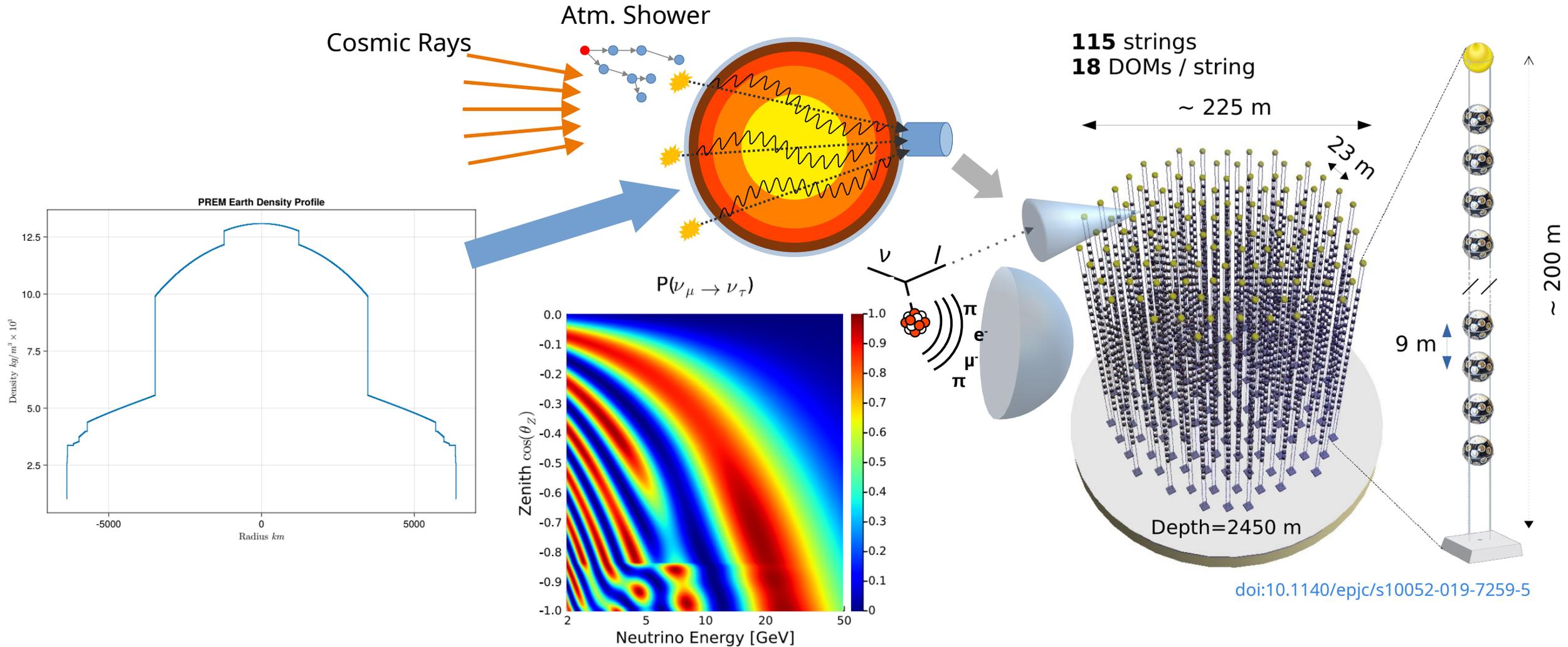




Neurthino

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JuliaHEP, 09 Nov 2023

Neutrino Oscillations



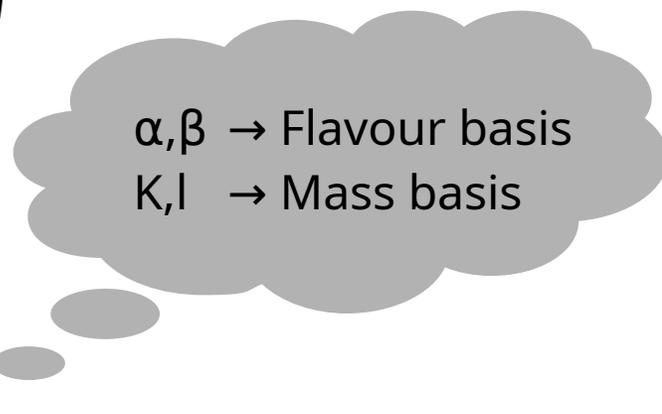
How it always starts: $i\partial_t |\psi\rangle = H |\psi\rangle$

PMNS matrix: mass basis \leftrightarrow flavour basis

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & e^{i\delta} \sin \theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transition probability:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{k,l} \text{Re} \left[U_{\alpha k}^* U_{\beta k} U_{\alpha l} U_{\beta l}^* \right] \sin^2 \left(\frac{\Delta m_{kl}^2 L}{4E_\nu} \right) \\ \pm 2 \sum_{k,l} \text{Im} \left[U_{\alpha k}^* U_{\beta k} U_{\alpha l} U_{\beta l}^* \right] \sin^2 \left(\frac{\Delta m_{kl}^2 L}{2E_\nu} \right)$$



$\alpha, \beta \rightarrow$ Flavour basis
 $k, l \rightarrow$ Mass basis

Neutrino Oscillations – The math fun

Ok and what's the problem now ... seem pretty straight forward



Matter Hamiltonian:

$$H_{\alpha\beta}^m = H_{\alpha\beta}^0 + \delta_{ee}A$$

... which leads to new PMNS for every electron density

$$H_{ij}^m = \tilde{U} H_{\alpha\beta}^m \tilde{U}^\dagger$$

so depending on the resolution of matter composition:

→ many eigenvalues to find

→ ... and matrix multiplications to do

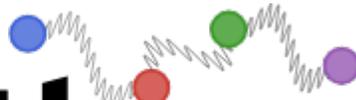
Mhmmmmmmm



Feasible problem, but annoying problem:
You want a package to do it for you

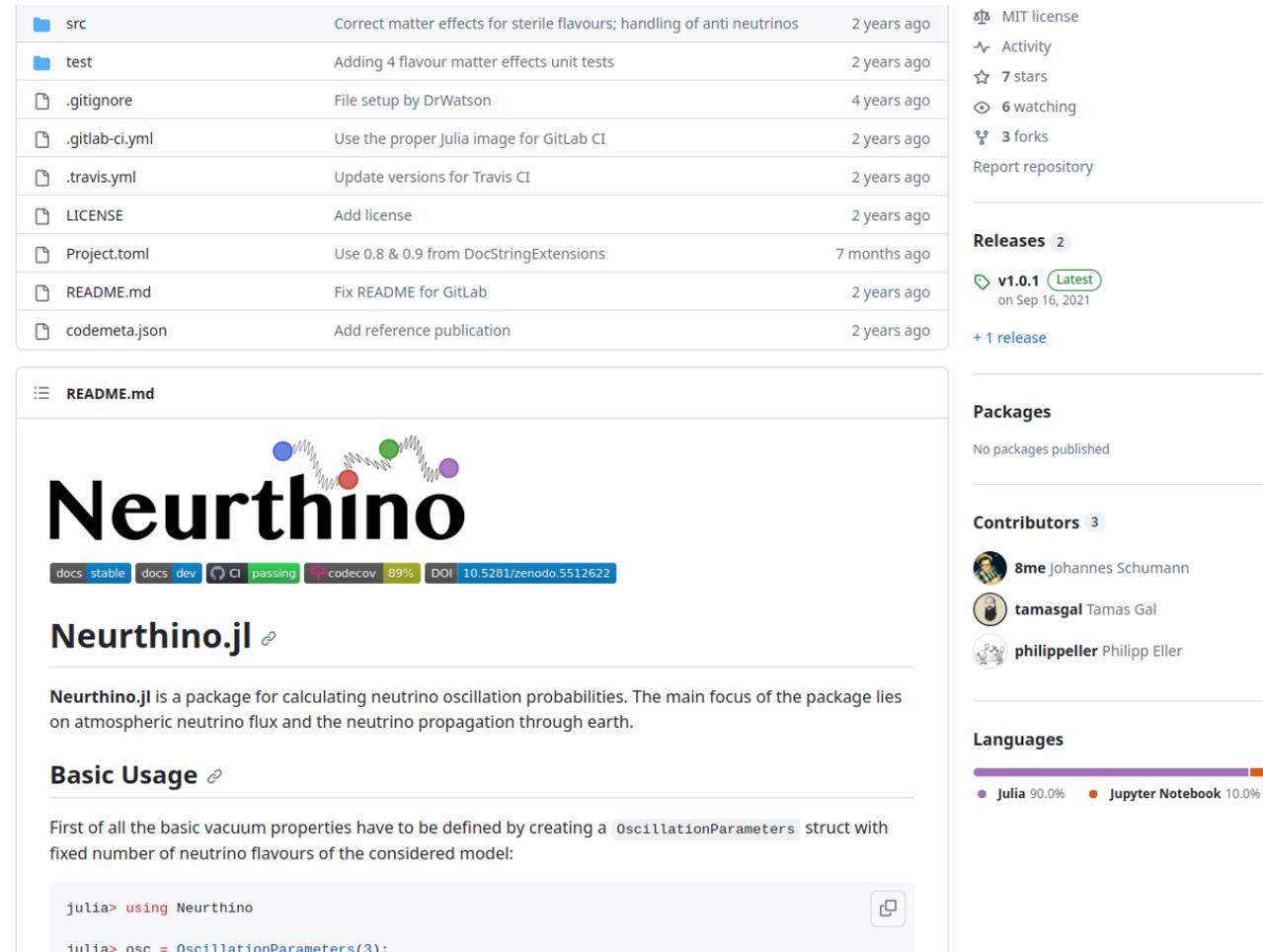
What you need to know about

Neurthino



- Neutrino oscillation probability calculator
→ n-Flavour modelling
- Includes **P**reliminary **R**eference **E**arth **M**odel

GitHub



The screenshot shows the GitHub repository for Neurthino. The repository contains several files and folders, including a `src` folder for source code, a `test` folder for unit tests, and various configuration files like `.gitignore`, `.gitlab-ci.yml`, `.travis.yml`, `LICENSE`, `Project.toml`, `README.md`, and `codemeta.json`. The repository is licensed under MIT and has 7 stars, 6 watchers, and 3 forks. The latest release is v1.0.1, published on September 16, 2021. The repository also includes a README.md file, which is shown in the preview below.

README.md

Neurthino

docs stable docs dev CI passing codecov 89% DOI 10.5281/zenodo.5512622

Neurthino.jl

Neurthino.jl is a package for calculating neutrino oscillation probabilities. The main focus of the package lies on atmospheric neutrino flux and the neutrino propagation through earth.

Basic Usage

First of all the basic vacuum properties have to be defined by creating a `OscillationParameters` struct with fixed number of neutrino flavours of the considered model:

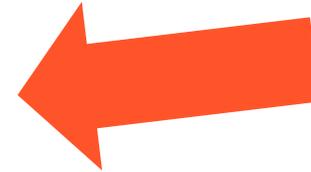
```
julia> using Neurthino
julia> osc = OscillationParameters(3);
```

Example

Set Oscillation Parameters (e.g. from NuFIT):

```

julia> using Neurthino
julia> osc = OscillationParameters(3);
julia> setθ!(osc, 1⇒2, 0.59);
julia> setθ!(osc, 1⇒3, 0.15);
julia> setθ!(osc, 2⇒3, 0.84);
julia> setδ!(osc, 1⇒3, 3.86);
julia> setΔm²!(osc, 2⇒3, -2.511e-3);
julia> setΔm²!(osc, 1⇒2, -7.41e-5);
    
```



NuFIT 5.2 (2022)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)		
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
without SK atmospheric data	$\sin^2 \theta_{12}$	$0.303^{+0.012}_{-0.011}$	0.270 → 0.341	$0.303^{+0.012}_{-0.011}$	0.270 → 0.341
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	31.31 → 35.74	$33.41^{+0.75}_{-0.72}$	31.31 → 35.74
	$\sin^2 \theta_{23}$	$0.572^{+0.018}_{-0.023}$	0.406 → 0.620	$0.578^{+0.016}_{-0.021}$	0.412 → 0.623
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	39.6 → 51.9	$49.5^{+0.9}_{-1.2}$	39.9 → 52.1
	$\sin^2 \theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	0.02029 → 0.02391	$0.02219^{+0.00060}_{-0.00057}$	0.02047 → 0.02396
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	8.19 → 8.89	$8.57^{+0.12}_{-0.11}$	8.23 → 8.90
	$\delta_{CP}/^\circ$	197^{+42}_{-25}	108 → 404	286^{+27}_{-32}	192 → 360
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.41^{+0.21}_{-0.20}$	6.82 → 8.03	$7.41^{+0.21}_{-0.20}$	6.82 → 8.03
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.511^{+0.028}_{-0.027}$	+2.428 → +2.597	$-2.498^{+0.032}_{-0.025}$	-2.581 → -2.408
	with SK atmospheric data	$\sin^2 \theta_{12}$	$0.303^{+0.012}_{-0.012}$	0.270 → 0.341	$0.303^{+0.012}_{-0.011}$
$\theta_{12}/^\circ$		$33.41^{+0.75}_{-0.72}$	31.31 → 35.74	$33.41^{+0.75}_{-0.72}$	31.31 → 35.74
$\sin^2 \theta_{23}$		$0.451^{+0.019}_{-0.016}$	0.408 → 0.603	$0.569^{+0.016}_{-0.021}$	0.412 → 0.613
$\theta_{23}/^\circ$		$42.2^{+1.1}_{-0.9}$	39.7 → 51.0	$49.0^{+1.0}_{-1.2}$	39.9 → 51.5
$\sin^2 \theta_{13}$		$0.02225^{+0.00056}_{-0.00059}$	0.02052 → 0.02398	$0.02223^{+0.00058}_{-0.00058}$	0.02048 → 0.02416
$\theta_{13}/^\circ$		$8.58^{+0.11}_{-0.11}$	8.23 → 8.91	$8.57^{+0.11}_{-0.11}$	8.23 → 8.94
$\delta_{CP}/^\circ$		232^{+36}_{-26}	144 → 350	276^{+22}_{-29}	194 → 344
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$		$7.41^{+0.21}_{-0.20}$	6.82 → 8.03	$7.41^{+0.21}_{-0.20}$	6.82 → 8.03
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$		$+2.507^{+0.026}_{-0.027}$	+2.427 → +2.590	$-2.486^{+0.025}_{-0.028}$	-2.570 → -2.406

Simple Example

For a single case you are pretty much done:

```
julia> p = Pvv(osc, 1, 10000)
```

```
4-dimensional AxisArray{Float64,4,...} with axes:
```

```
 :Energy, [1.0]
```

```
 :Baseline, [10000.0]
```

```
 :InitFlav, NeutrinoFlavour[Electron, Muon, Tau]
```

```
 :FinalFlav, NeutrinoFlavour[Electron, Muon, Tau]
```

```
...
```

```
julia> p[Energy=1, Baseline=1, InitFlav=Muon, FinalFlav=Tau]
```

```
0.2592655682779296
```

The "Full" Example

First: Create PMNS matrix and Hamiltonian

```
julia> U = PMNSMatrix(osc)
```

```
3×3 Array{Complex{Float64}, 2}:
```

```
 0.82161+0.0im      0.550114+0.0im      -0.112505+0.0983582im
-0.301737+0.0608595im  0.601232+0.0407488im  0.736282+0.0im
 0.476688+0.0545516im -0.576975+0.0365253im  0.659968+0.0im
```

```
julia> H = Hamiltonian(osc)
```

```
3-element Array{Complex{Float64}, 1}:
```

```
-0.0008902666666666667 + 0.0im
-0.0008163666666666667 + 0.0im
 0.0017066333333333333 + 0.0im
```

The "Full" Example

Now one can determine the U and H for a density and energy:

```
julia> U_mat, H_mat = MatterOscillationMatrices(U, H, 1, 13);
```

```
julia> H_mat
```

```
3-element Array{Complex{Float64},1}:
```

```
-0.0008404901318507502 - 2.5459232191294903e-20im
```

```
9.078126149399635e-5 - 1.75151351027943e-20im
```

```
0.0017419062876598283 - 1.8741859435908039e-19im
```

```
julia> U_mat
```

```
3×3 Array{Complex{Float64},2}:
```

```
0.0358018-0.000158113im 0.970863+0.0im -0.178275+0.156083im
```

```
-0.662778+0.00661213im 0.157174+0.116074im 0.722845+0.0im
```

```
0.74793+0.0im 0.0917808+0.104043im 0.649115-0.00104331im
```

The "Full" Example

Second: create paths

```
julia> zenith = acos.(range(-1, stop=0, length=200));
```

```
julia> paths = Neurthino.prepath(zenith, 2.5, samples=100,  
discrete_densities=0:0.1:14);
```

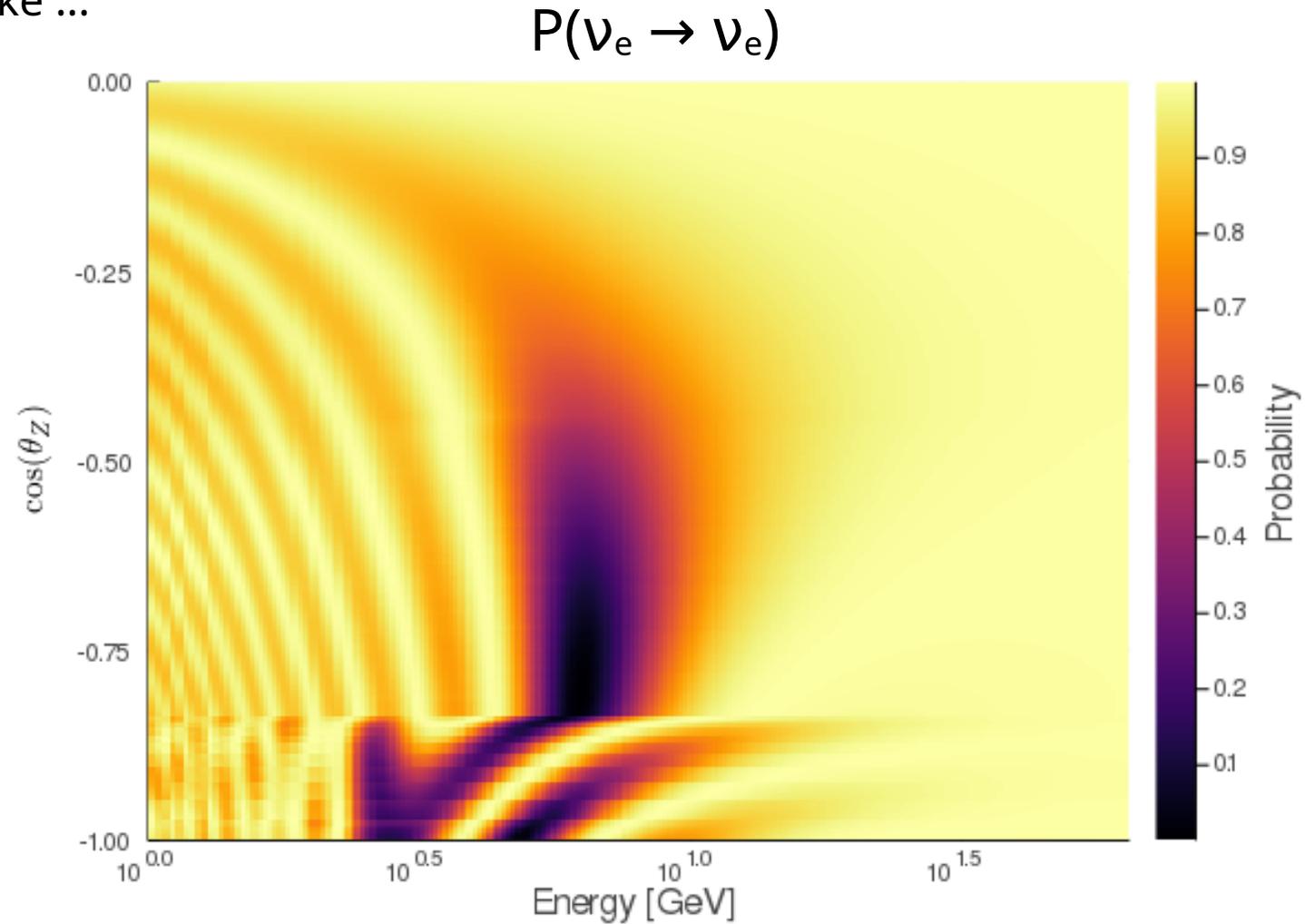
Third: Calculate probabilities

```
julia> energies = 10 .^ range(0, stop=2, length=200);
```

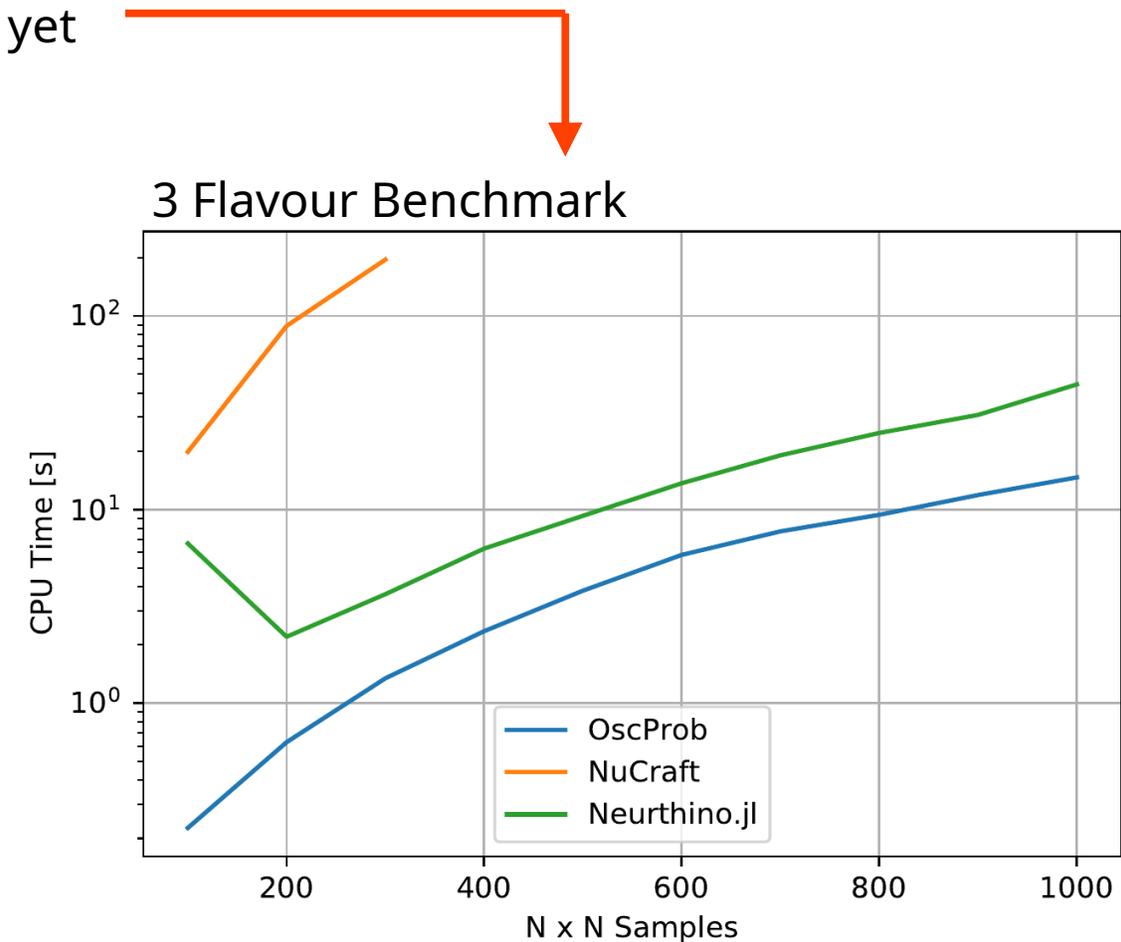
```
julia> prob = Pvv(U, H, energies, paths);
```

The "Full" Example

What does it look like ...



- Working package for neutrino oscillations
 - out of the box n-Flavour without any optimizations yet
 - Includes earth matter density model (PREM)
 - ... open to new ones if you have one ;)
 -
- Outlook / ToDo:
 - Use Liouville-von Neumann formulation
 - some effects (e.g. decoherence not fully covered)
 - Interpolation of oscillation parameters in matter
 - GPU option for determining eigenvalues





Thank You

- Matter Oscillation Parameters Interpolation:

