

Cumulants & Co.

Piotr Kalaczyński on behalf of Krzysztof Domino, Piotr Gawron, Łukasz Pawela, Dariusz Kurzyk, Tony Kelman, Alex Arslan and Kristoffer Carlsson



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Code presented here was developed at  IITIS in Gliwice, Poland

Based on: [arXiv:1301.7744v3](https://arxiv.org/abs/1301.7744v3)

Exploiting Symmetry in Tensors for High Performance: Multiplication with Symmetric Tensors
(M. D. Schatz, T. M. Low, R. A. van de Geijn, T. G. Kolda)

By:

- ❖ Krzysztof Domino
- ❖ Piotr Gawron
- ❖ Łukasz Paweła
- ❖ Tony Kelman
- ❖ Alex Arslan
- ❖ Kristoffer Carlsson

NOT by me!

Repos:

- ❖ github.com/iitis/Cumulants.jl → [arXiv:1701.05420v4](https://arxiv.org/abs/1701.05420v4)
- ❖ github.com/iitis/CumulantsFeatures.jl → [doi/10.1016/j.physa.2020.124995](https://doi.org/10.1016/j.physa.2020.124995),
[arXiv:1808.03513v1](https://arxiv.org/abs/1808.03513v1)
- ❖ github.com/iitis/CumulantsUpdates.jl → [doi/10.2478/amcs-2019-0015](https://doi.org/10.2478/amcs-2019-0015)

Me:



Just a humble technician in the Scientific Computing & IT Group at **ASTROCENT** in Warsaw, Poland
current focus: maintenance of python & C++ simulation codes,
excited to dive deeper into Julia 😊

What are cumulants anyway?

quantitative measures of the shape of the distribution

(an alternative to the moments)

Can be generated from a Taylor expansion:

$$H(t) = \sum_{d=1}^{\infty} \kappa_d \frac{(it)^d}{d!} = \underbrace{\mu}_{\kappa_1} it - \underbrace{\sigma^2}_{\kappa_2} \frac{t^2}{2} + \dots$$

which must be differentiated and evaluated at $t = 0$ to extract the cumulants κ_d
(hello Julia 😊)

d – order of the cumulant

The symbols may seem familiar for a good reason:

$\kappa_1 = \mu$: mean

$\kappa_3 \sim$ skewness (after rescaling)

$\kappa_2 = \sigma^2$: variance

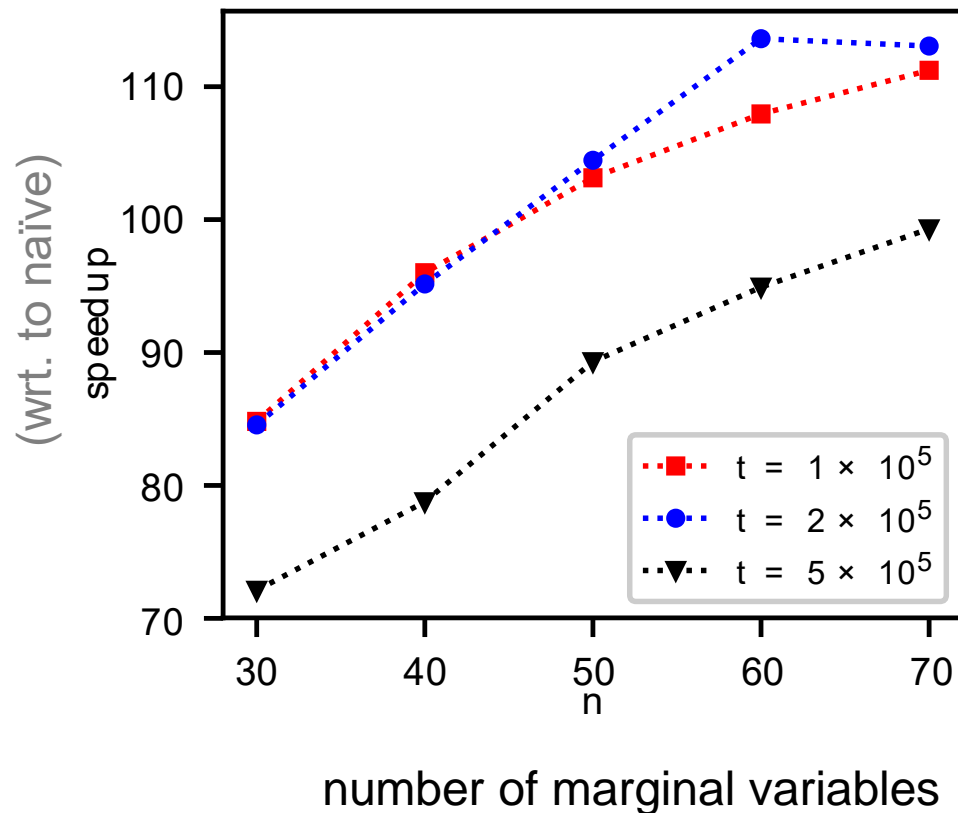
$\kappa_4 \sim$ kurtosis (after rescaling)

no special names for higher orders (that I know of ...)

from [arXiv:1701.05420v4](https://arxiv.org/abs/1701.05420v4)

Storage of symmetric tensors is optimized wrt. the naïve storage scheme
(storing all elements)

Expected computational complexity reduction: by a factor $d!$



Here :

- ❖ Computation of the 4th cumulant (κ_4) tensor
- ❖ Actual speedup exceeds $d! = 4! = 24$

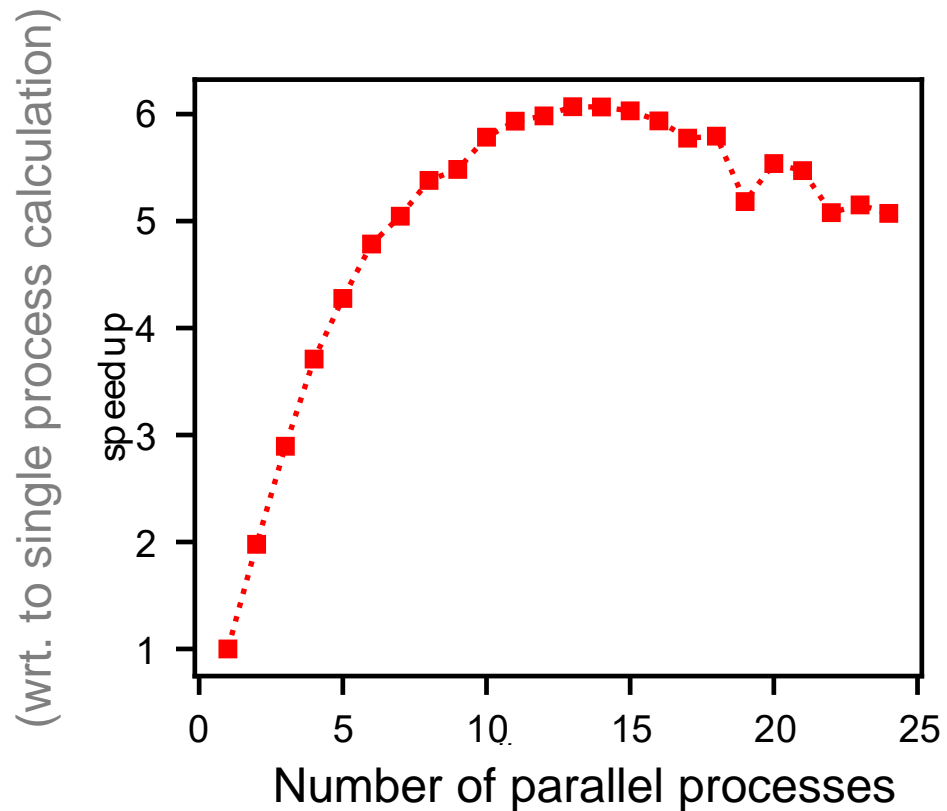
t – number of data samples

from [arXiv:1701.05420v4](https://arxiv.org/abs/1701.05420v4)

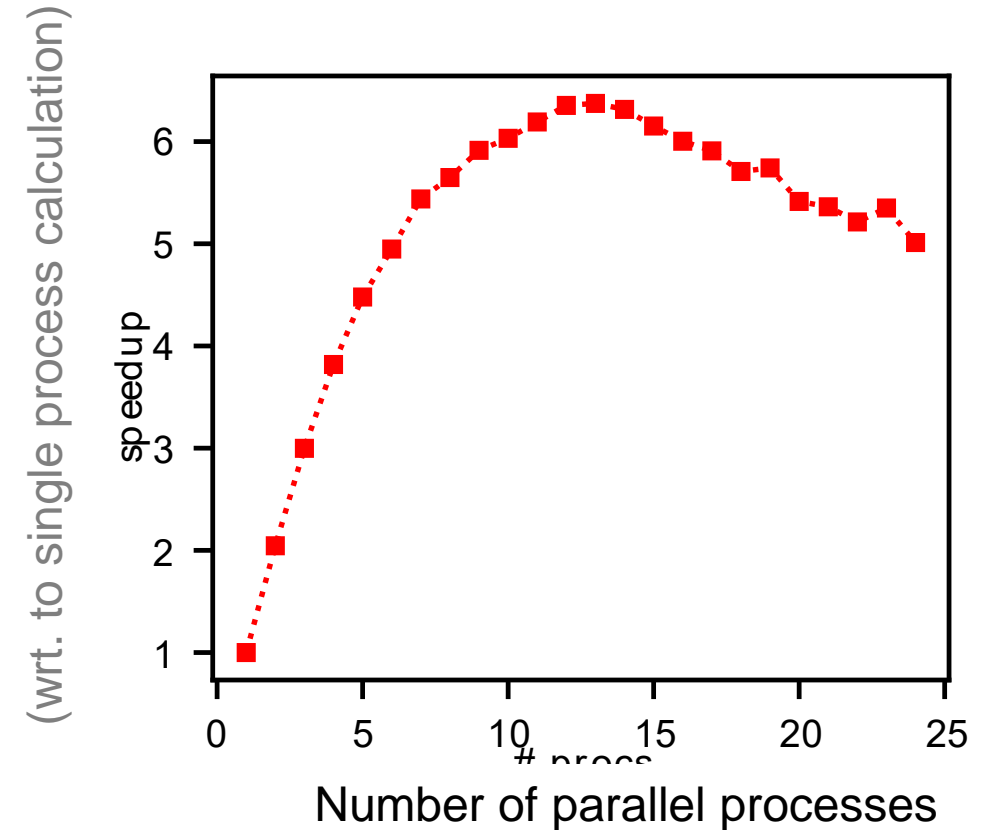
The code is not completely parallelisable, but can be accelerated by running multiple processes

speedup: with respect to single process calculation of cumulants

4-th order cumulant



5-th order cumulant



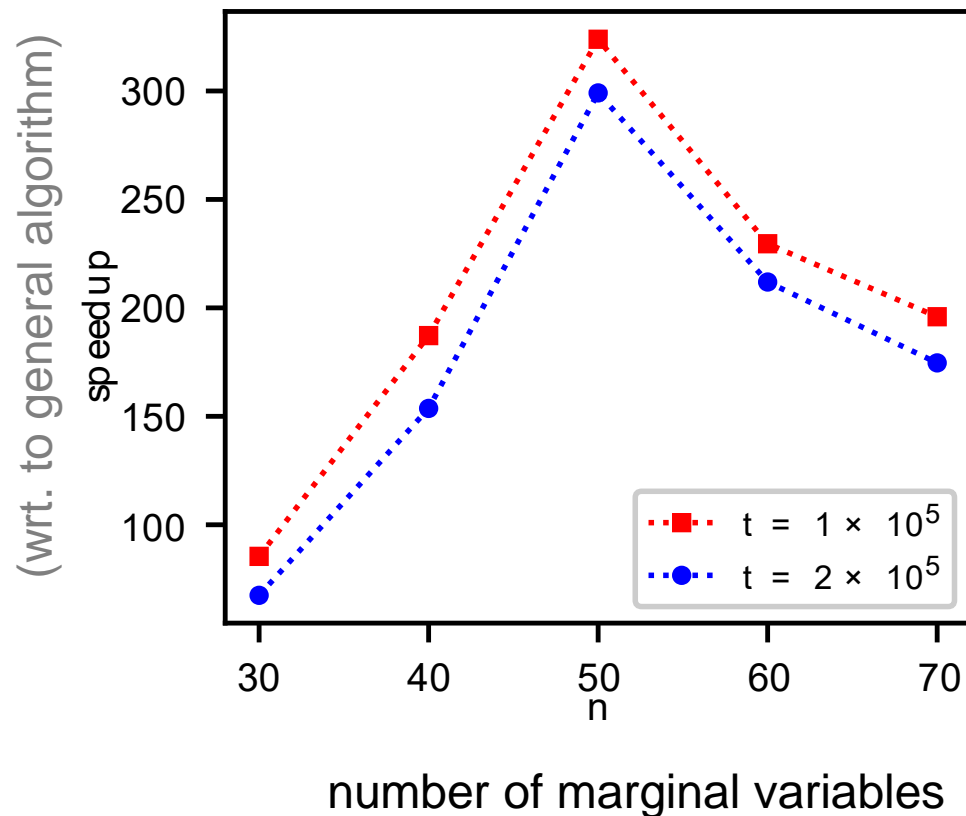
from [arXiv:1701.05420v4](https://arxiv.org/abs/1701.05420v4)

t – number of data samples

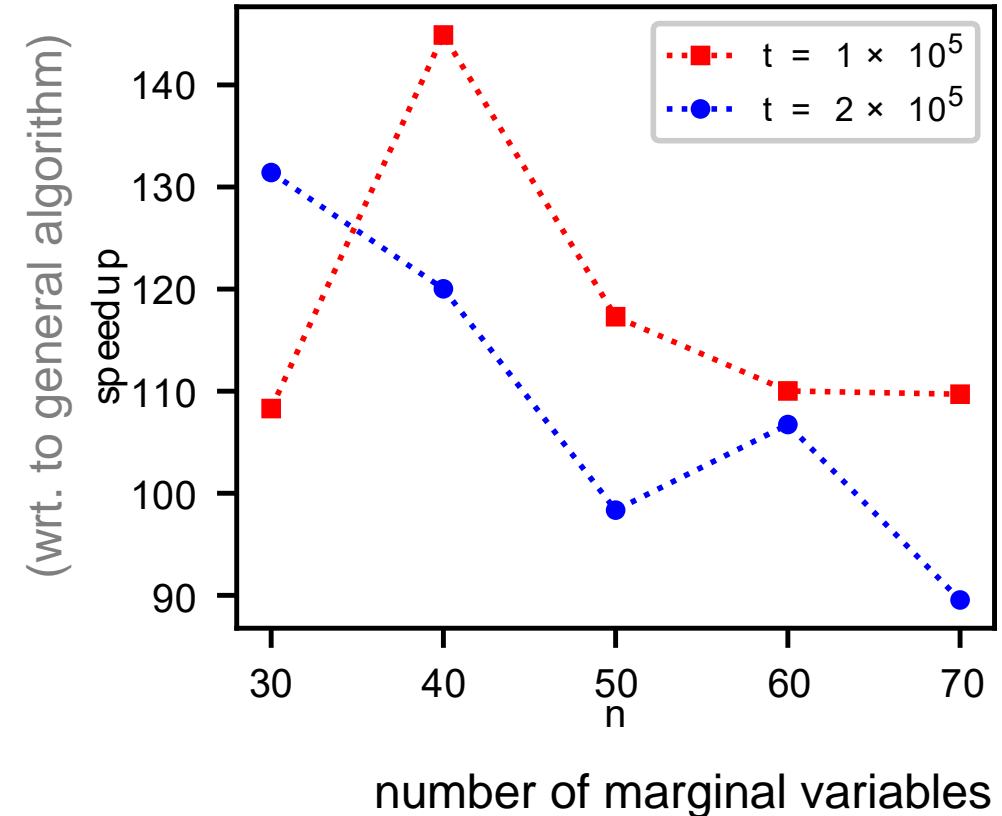
Speedup wrt. [state of the art](#):

- ❖ [Cumulants.jl](#) vs the general (arbitrary cumulant order) algorithm from [here](#)
- ❖ 4th cumulant tensor

Comparing with  reimplementation



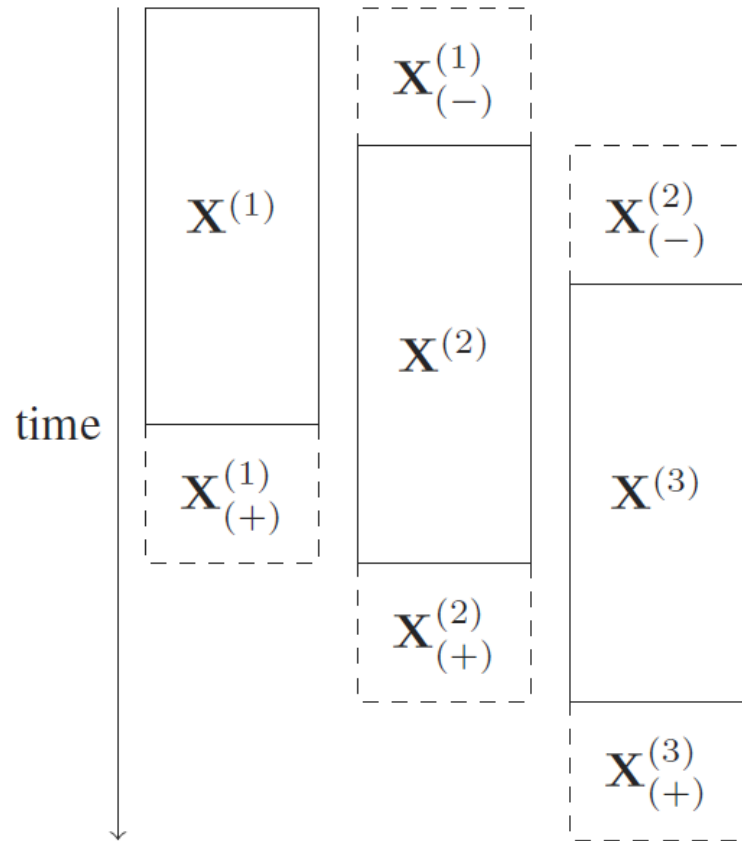
Comparing with  reimplementation



from [doi/10.2478/amcs-2019-0015](https://doi.org/10.2478/amcs-2019-0015)

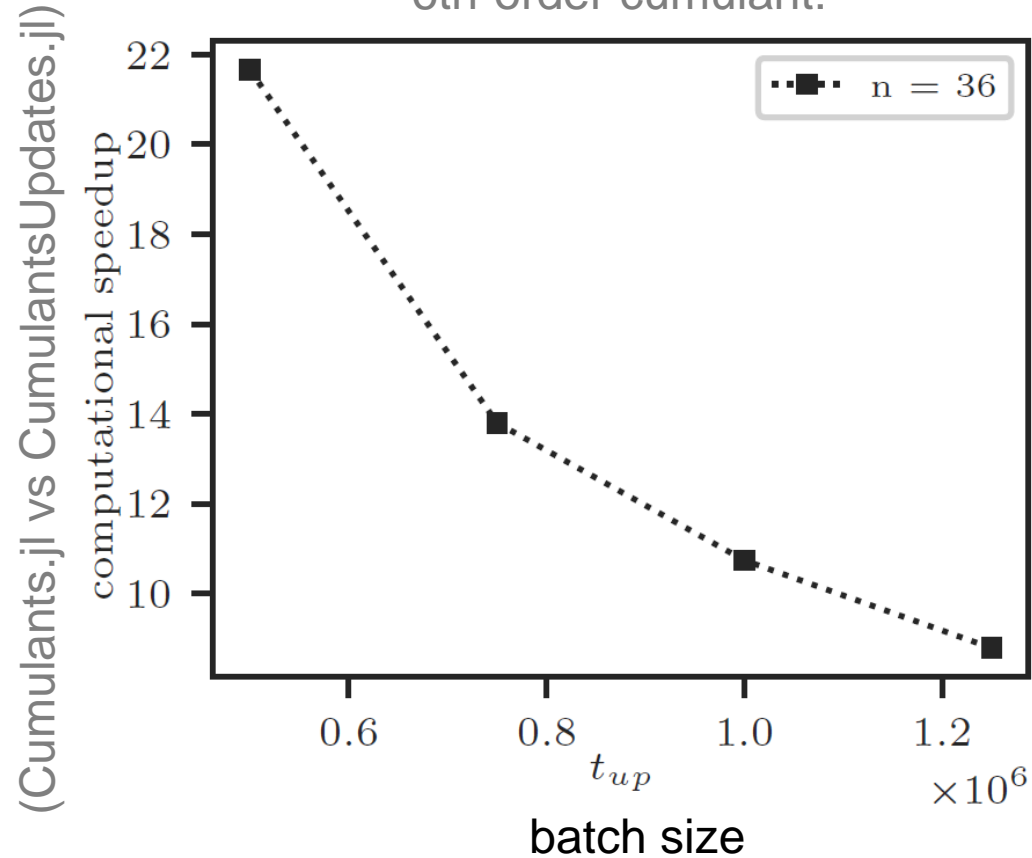
Cumulants can be computed online, in a sliding window of datastreams:

([CumulantsUpdates.jl](#))



... and this turns out to be even faster than [Cumulants.jl](#):

6th-order cumulant:



$t = 2.5 \cdot 10^7$ – number of data samples
 n – numer of marginal variables

Okay, where in HEP can this be used? (just some examples)

❖ At colliders:

- [arXiv:1109.0593v2](https://arxiv.org/abs/1109.0593v2)

Error Estimation for Moments Analysis in Heavy-Ion Collision Experiments
(Xiaofeng Luo)

- [arXiv:2305.13874v2](https://arxiv.org/abs/2305.13874v2)

Holographic study of higher-order baryon number susceptibilities at finite temperature and density
(Z. Li, J. Liang, S. He, L. Li)

- [arXiv:2303.13414v1](https://arxiv.org/abs/2303.13414v1)

Higher-order correlations between different moments of two flow amplitudes in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV
(ALICE Collaboration)

- [arXiv:2209.11940v2](https://arxiv.org/abs/2209.11940v2)

Higher-Order Cumulants and Correlation Functions of Proton Multiplicity Distributions in $\sqrt{s_{NN}} = 3$ GeV Au+Au Collisions at the RHIC STAR Experiment
(STAR Collaboration)

❖ In astrophysics:

- [arXiv:2204.05305v3](https://arxiv.org/abs/2204.05305v3)

Galaxy and halo angular clustering in LCDM and Modified Gravity cosmologies
(P. Drozda, W. A. Hellwing, M. Bilicki)

- [arXiv:2209.14810v2](https://arxiv.org/abs/2209.14810v2)

Magnetic helicity fluxes from triple correlators
(K. Gopalakrishnan, K. Subramanian)

❖ Lattice QCD:

- [arXiv:2305.10916v2](https://arxiv.org/abs/2305.10916v2)

Microscopic Encoding of Macroscopic Universality: Scaling Properties of Dirac Eigenspectra near QCD Chiral Phase Transition
(H. Ding, W. Huang, S. Mukherjee, P. Petreczky)

some analyses use cumulants up to 8th order



specialised algorithm (orders 1-4) NOT enough!

There is more if you're into ...

Quantum computing: [QuantumInformation.jl](#)

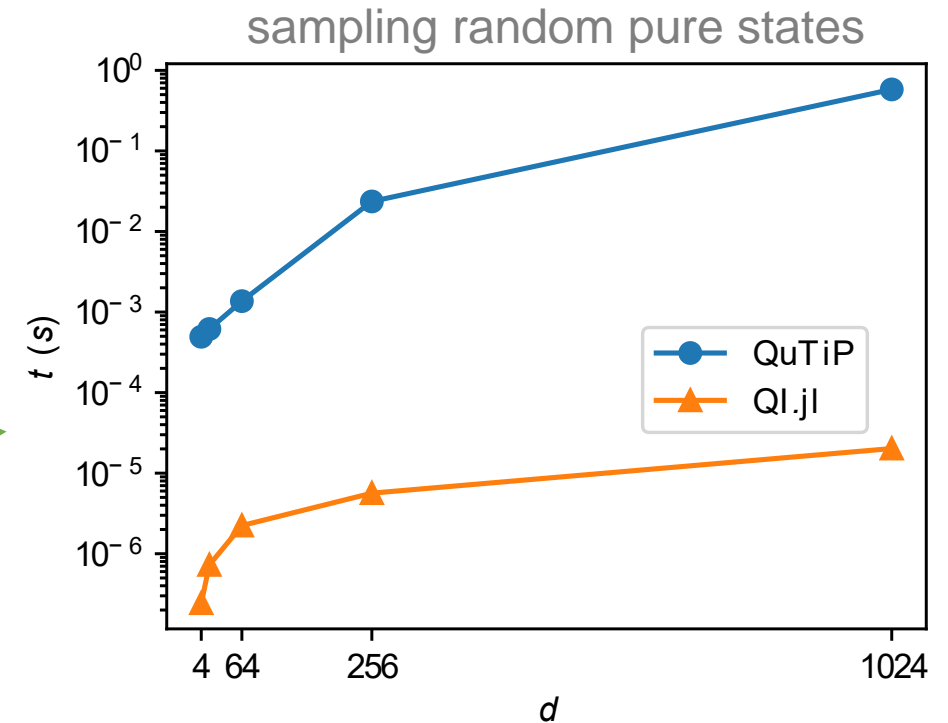
from [arXiv:1806.11464v3](#)

- ❖ natively works with bra-ket notation!

```
julia> ket(1,2)
2-element Array{Complex{Float64},1}:
 1.0 + 0.0im
 0.0 + 0.0im
```

```
julia> (1/sqrt(2)) * (ket(1,2) + ket(2,2))
2-element Array{Complex{Float64},1}:
 0.7071067811865475 + 0.0im
 0.7071067811865475 + 0.0im
```

- ❖ comes with many convenient implementations of states, transformations, etc.
- ❖ faster than [QuTiP](#) (python code)



Random matrix sampling on GPU: [MatrixEnsembles.jl](#)



To sum up:

- ❖ Need to compute high-order cumulants or just do it fast? Use [Cumulants.jl](https://github.com/maartenbreddius/Cumulants.jl) etc. 😊
- ❖ Want to learn more? See: [arxiv/1701.05420](https://arxiv.org/abs/1701.05420)
[doi/10.2478/amcs-2019-0015](https://doi.org/10.2478/amcs-2019-0015)

**Thank you
for your attention!**



My ideas for new projects:

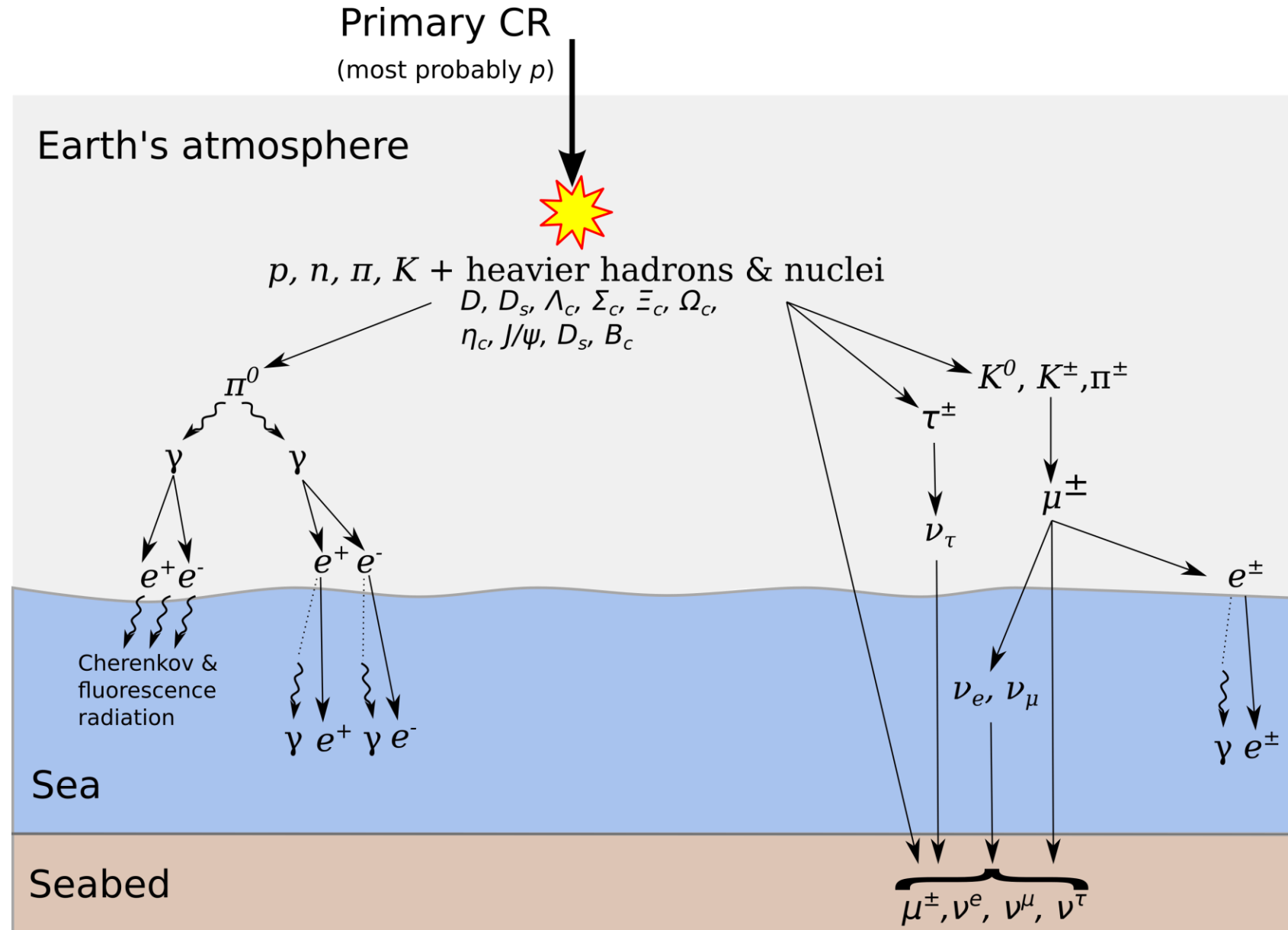
- ❖ CORSIKA alternative in Julia:
 - I'm aware of C++ rework (C8)
 - but ... maybe it can be more efficient? (looped differentiation and matrix operations under the hood ...)
 - and just easier (MUCH friendlier syntax)
 - healthy competition never hurts ;-)
- ❖ Simulation of acoustic signal from HE particles:
 - thermo-acoustic mechanism: known & measured
 - simulated, but nothing really open-source and general purpose, like CORSIKA (at least that I am aware of ...)
 - combined simulation of light & sound? (could allow for better event reco)



Backup

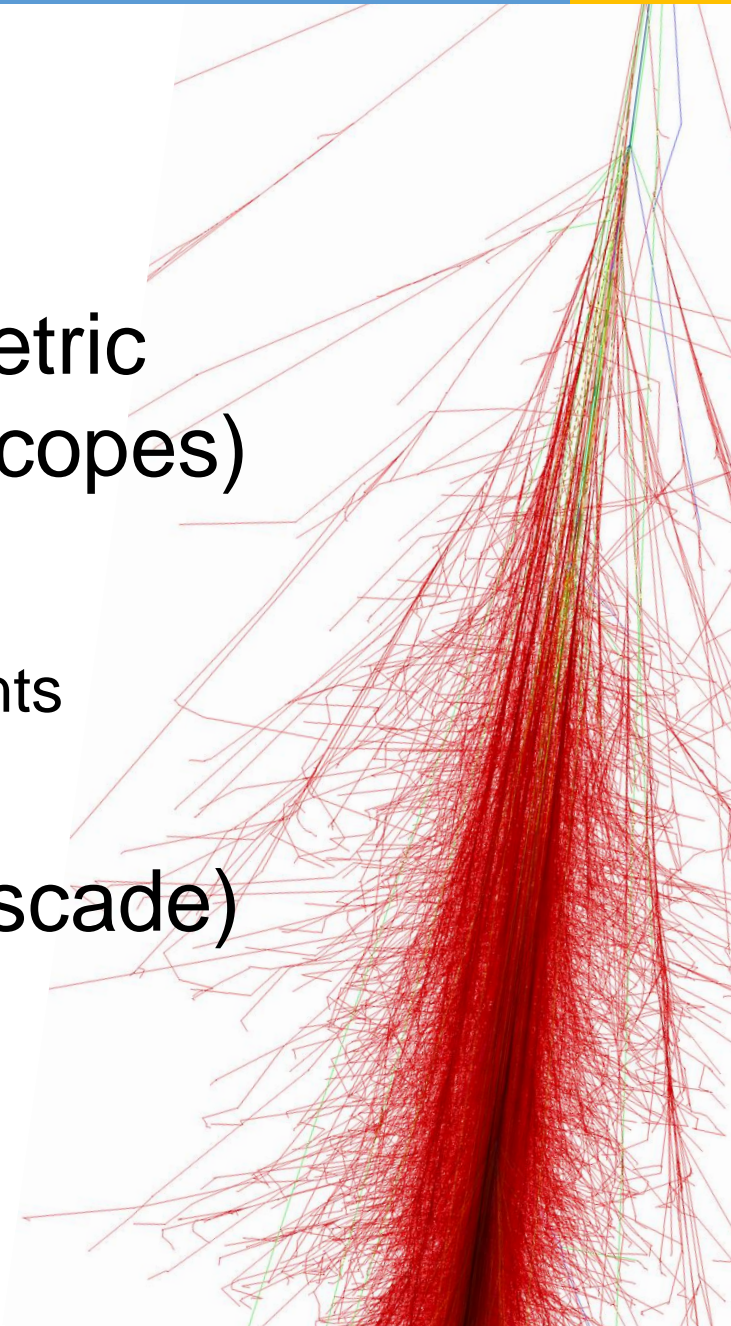
EAS:

- ❖ Caused by primary CR
- ❖ Typically start at $h \sim 30 - 40\text{km}$
- ❖ 3 main components:
 - electromagnetic (EM)
 - hadronic
 - muonic
- ❖ Simulated with CORSIKA



We have 2 options:

1. [MUPAGE](#) (atmospheric **MU**ons from **PA**rametric formulas: a fast **GE**nerator for neutrino telescopes)
 - developed for ANTARES
 - fast muon MC generator
 - based on parametric formulas and MACRO measurements
 - parameters can be freely tuned
2. [CORSIKA](#) (**CO**smic **R**ay **SI**mulations for **KA**scade)
 - developed for KASCADE
 - full simulation of air showers
 - customizable (models, primaries, etc.)



gSeaGen

Code for propagating muons and/or neutrinos to neutrino telescopes. Developed for KM3NeT, but applicable to other experiments

My work in this project:

- ❖ Implement processing of CORSIKA showers
- ❖ Speed, memory & storage optimization
- ❖ Rework of the geometry: no more flat Earth!
- ❖ Code maintenance

A paper by me under internal review ...

git.km3net.de/opensource/gseagen

Tech stack:

- ❖ C++
- ❖ ROOT
- ❖ PERL
- ❖ PROPOSAL
(github.com/tudo-astroparticlephysics/PROPOSAL)

Current devs:

- ❖ Carla Distefano
- ❖ Alfonso Andres Garcia Soto
- ❖ Piotr Kalaczyński
- ❖ Johannes Schumann
- ❖ Rodrigo Garcia
- ❖ Andrey Romanov