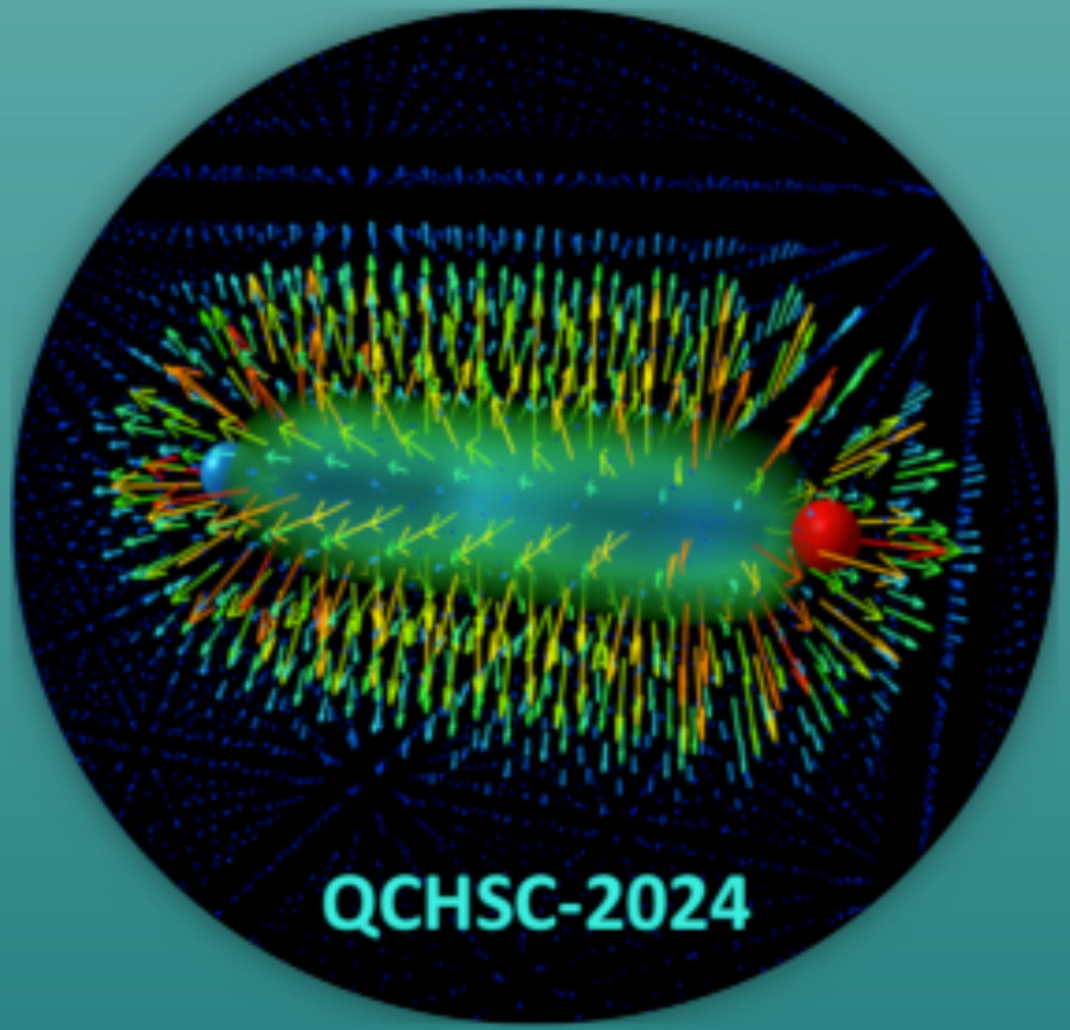
STUDYING EFFECTIVE STRING THEORY USING DEEP GENERATIVE MODELS

ELIA CELLINI



22/08/2024

Quark Confinement and the Hadron Spectrum

Cairns

Based On:

M. Caselle, E.C., A. Nada

- JHEP 02 (2024) 048, arxiv:2307.01107
- In prep., arxiv:2409.XXXX

OUTLINE

EFFECTIVE STRING THEORY

EFFECTIVE STRING THEORY

Correlators of Polyakov loops modelled in terms of **string partition functions**:

$$\langle P(0)P^{\dagger}(R)\rangle \sim \int DXe^{-S_{EST}[X]} \equiv Z_{EST}$$

The main choice for S_{EST} is the **Nambu-Goto (NG)** action:

$$S_{NG} = \sigma \int d\xi^{2}\sqrt{g}$$

- **Anomalous** at quantum level → **effective, large-distance description** of Yang-Mills theories (**low-energy universality theorem)**.
- Works only up to order $1/R^{5}$ → first order approximation of a more general theory → **Beyond Nambu-Goto (BNG)**
  [Aharony and Komargodski; 1302.6257], [Brandt and Meineri; 1603.06969],[Caselle; 2104.10486]

OBSERVABLES: Z

Main method: **zeta-function regularization**

This talk: **High Temperature (HT)** regime $R \gg L$

Partition function → directly associated with the **interquark potential.** Well known at all the order.

$$-\log Z = \sigma R L \sqrt{1 - \frac{\pi}{3\sigma L^2}} + \cdots$$

OBSERVABLES: WIDTH σw^2

$\underline{\sigma w^2}$: correlation function that measure the **density of the chromoelectric flux tube**:

$$\sigma w^2 = \langle X(\tau, R/2) X(\tau, R/2) \rangle$$

In the HT regime, for $\sigma \to \infty$:

$$\sigma w^2 = \frac{R}{4L} + \cdots$$

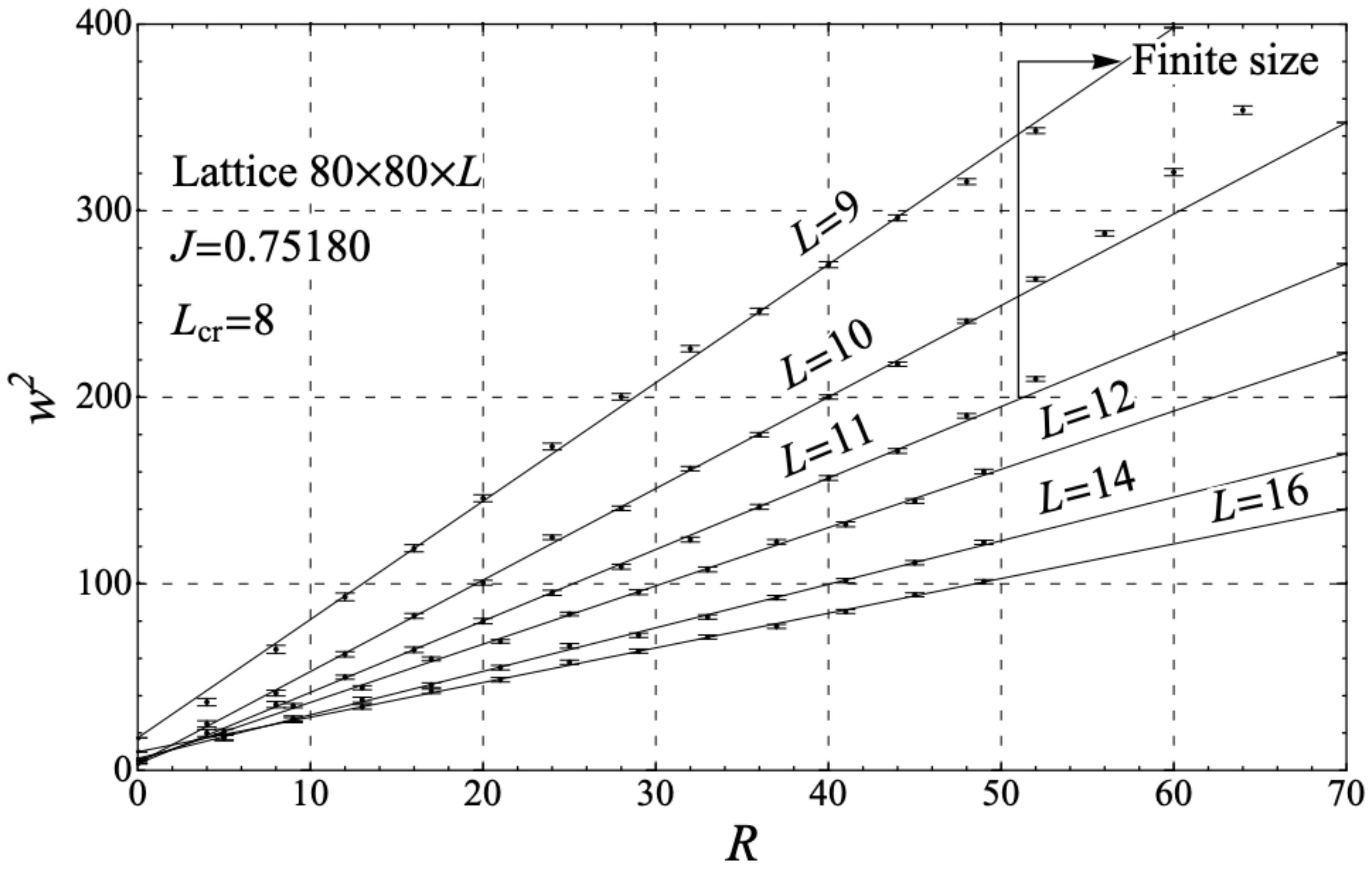


Figure 2: *Flux tube thickness as a function of the interquark distance for various values of the inverse temperature L.*

[Allais and Caselle; 0812.0284]

Lattice simulations of $\mathbb{Z}_2$ gauge model

OBSERVABLES: WIDTH σw^2

Two-loop calculation by **Gliozzi, Pepe and Wiese**

[Gliozzi, Pepe and Wiese; 1006.2252]

$$\sigma w^2 = \left(1 + \frac{\pi}{6\sigma L^2}\right)\frac{R}{4L} + \cdots$$

Never observed in lattice gauge theory simulations (unaccessible regime)

OBSERVABLES: WIDTH σw^2

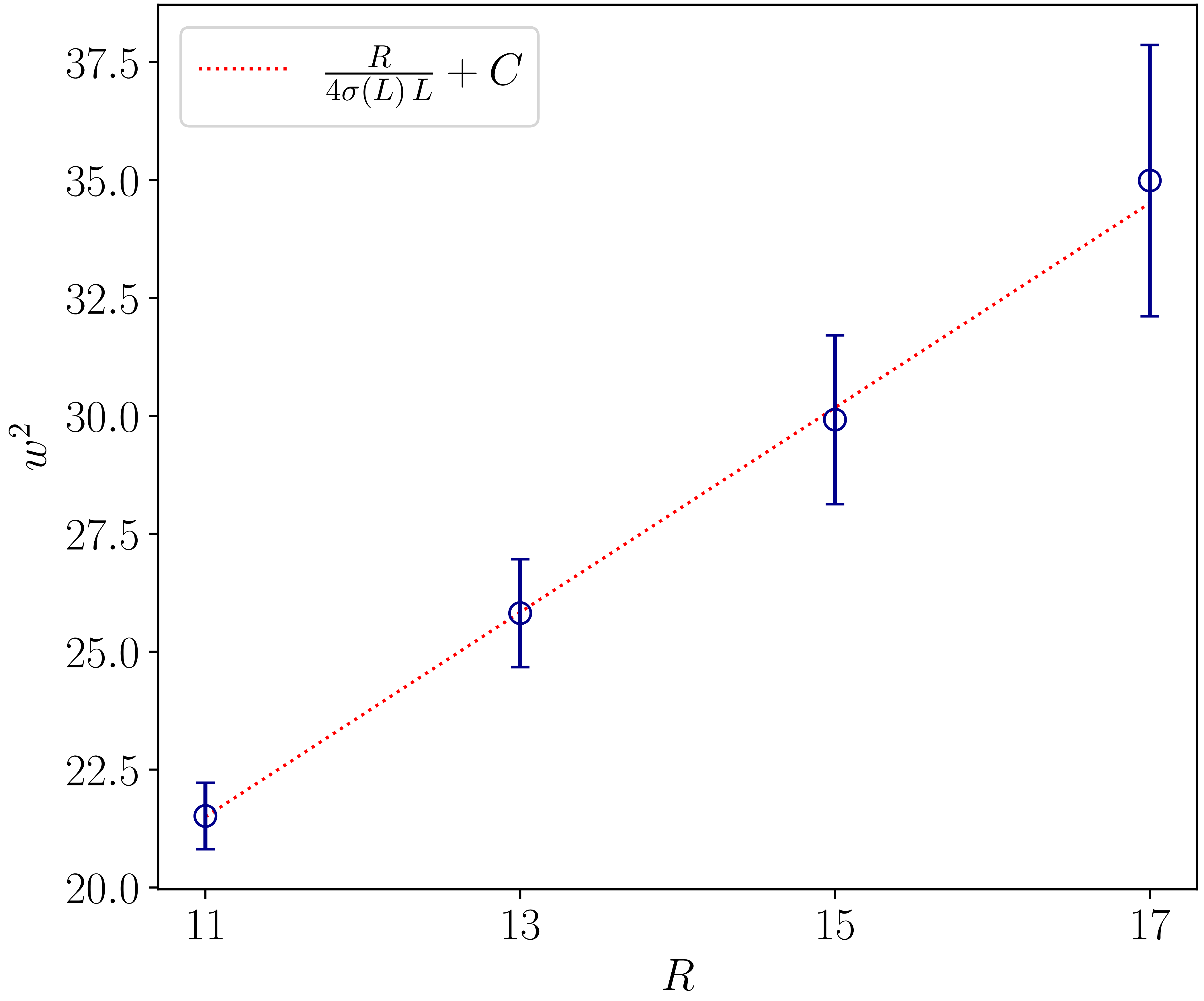
Conjecture on the general behaviour of the width by **Caselle**

[Caselle;1004.3875]

$$\sigma w^2 = \underbrace{\frac{1}{\sqrt{1-\frac{\pi}{3\sigma L^2}}}}_{\sigma(L)/\sigma} \underbrace{\frac{R}{4L}}_{\text{Gaussian solution}} + \cdots$$

Observed in lattice gauge theory.

However, a rigours prove from the EST side was missing



Lattice simulations of pure $SU(2)$

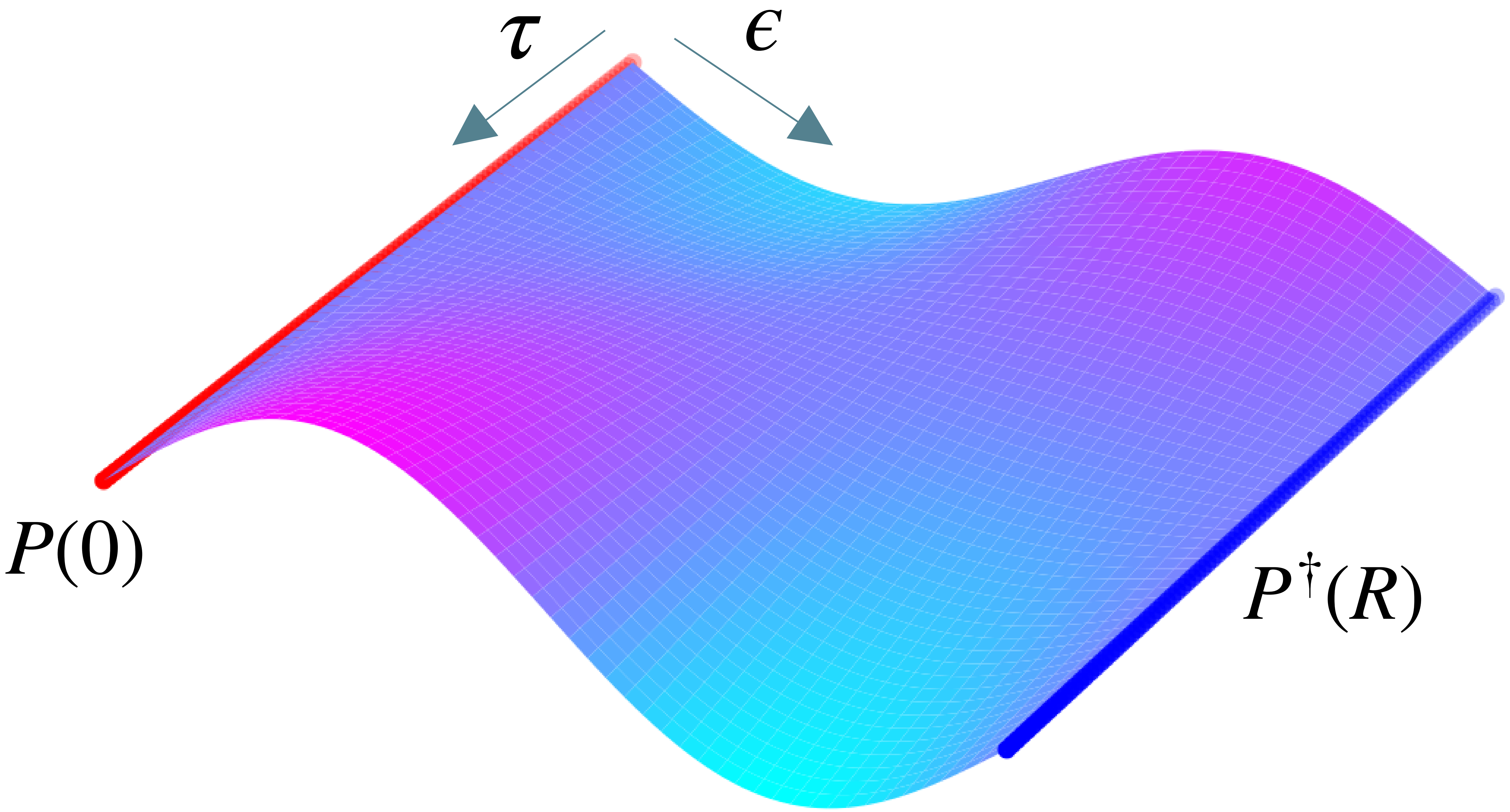
[Caselle et al.; in prep.]

LATTICE REGULARIZATION & DEEP LEARNING

LATTICE NAMBU-GOTO STRING

$$S_{NG}(\phi) = \sigma \sum_{x \in \Lambda} \left[\sqrt{1 + (\partial_\mu \phi(x))^2/\sigma} - 1 \right]$$

- $d = 2 + 1$ target Yang-Mills
- σ string tension
- Λ : square lattice of size $L \times R$, $a = 1$
- $\phi(x) = \phi(\tau, \epsilon) \in \mathbb{R}$
- $\phi(\tau + L, \epsilon) = \phi(\tau, \epsilon)$
- $\phi(\tau, 0) = \phi(\tau, R) = 0$
- $\sigma w^2 = \langle \phi(\tau, R/2)^2 \rangle_\tau$



[Caselle, EC, Nada; 2307.01107]

LACKS OF NUMERICAL METHODS

Numerical problems:

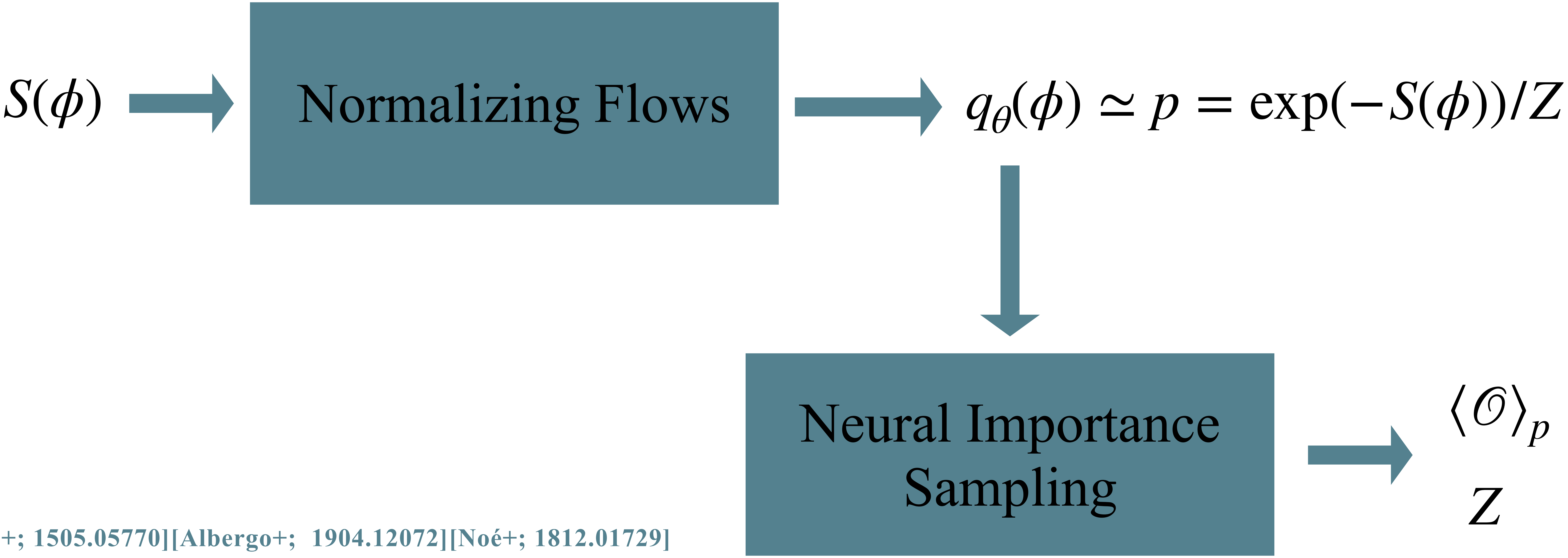
- Strong **non-linearity** → **critical** theory (**Critical Slowing Down**)
- Estimation of **partition functions**

↓

Deep Learning

FLOW-BASED SAMPLING

<u>Normalizing Flows</u> are a class of **<u>deep generative models</u>** able to learn an approximation of a **<u>Boltzmann distribution</u>** p using as input only the energy: **<u>no need for training data!</u>**



$S(\phi)$

Normalizing Flows

$q_\theta(\phi) \simeq p = \exp(-S(\phi))/Z$

Neural Importance Sampling

$\langle \mathcal{O} \rangle_p$

Z

[Rezende+; 1505.05770][Albergo+; 1904.12072][Noé+; 1812.01729]
[Nicoli+; 1910.13496, 2007.07115]

ARCHITECTURES

The normalizing flows architectures used to obtain the results of this talk are two:

- **Continuous Normalizing Flows**: the flow is the solution of a Neural ODE, poor scaling in σ

  [Chen et al.;][Gerdes et al.;][Caselle, EC, Nada; 2307.01107]

- **Stochastic Normalizing Flows**: combination of Normalizing Flows and Non-Equilibrium MCMC, fully described by non-equilibrium thermodynamics.

  [Wu et al.; 2002.06707],[Caselle, EC, Nada, Panero; 2201.08862]

RENORMALIZATION

The analysis of the **finite-size numerical simulations** must take into account the **divergent, non-physical, terms**:

$$-\log Z = \text{physical terms} + \text{divergent terms}$$

We fitted every terms, we then isolate the physical (interesting) ones

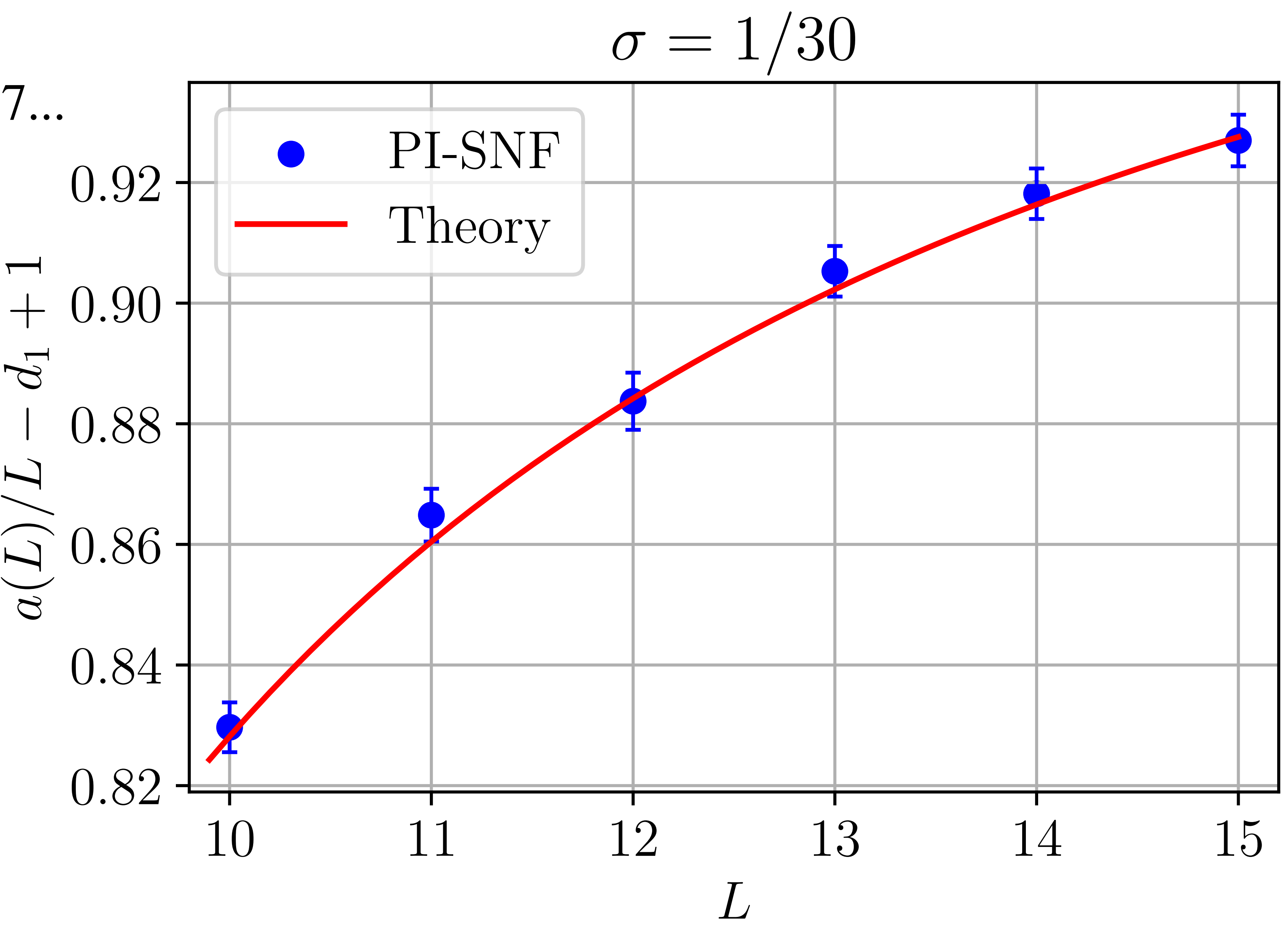
In this talk, I will just show the physical results, see: **[Caselle, EC, Nada; 2307.01107]** for a detailed discussion on this aspect.

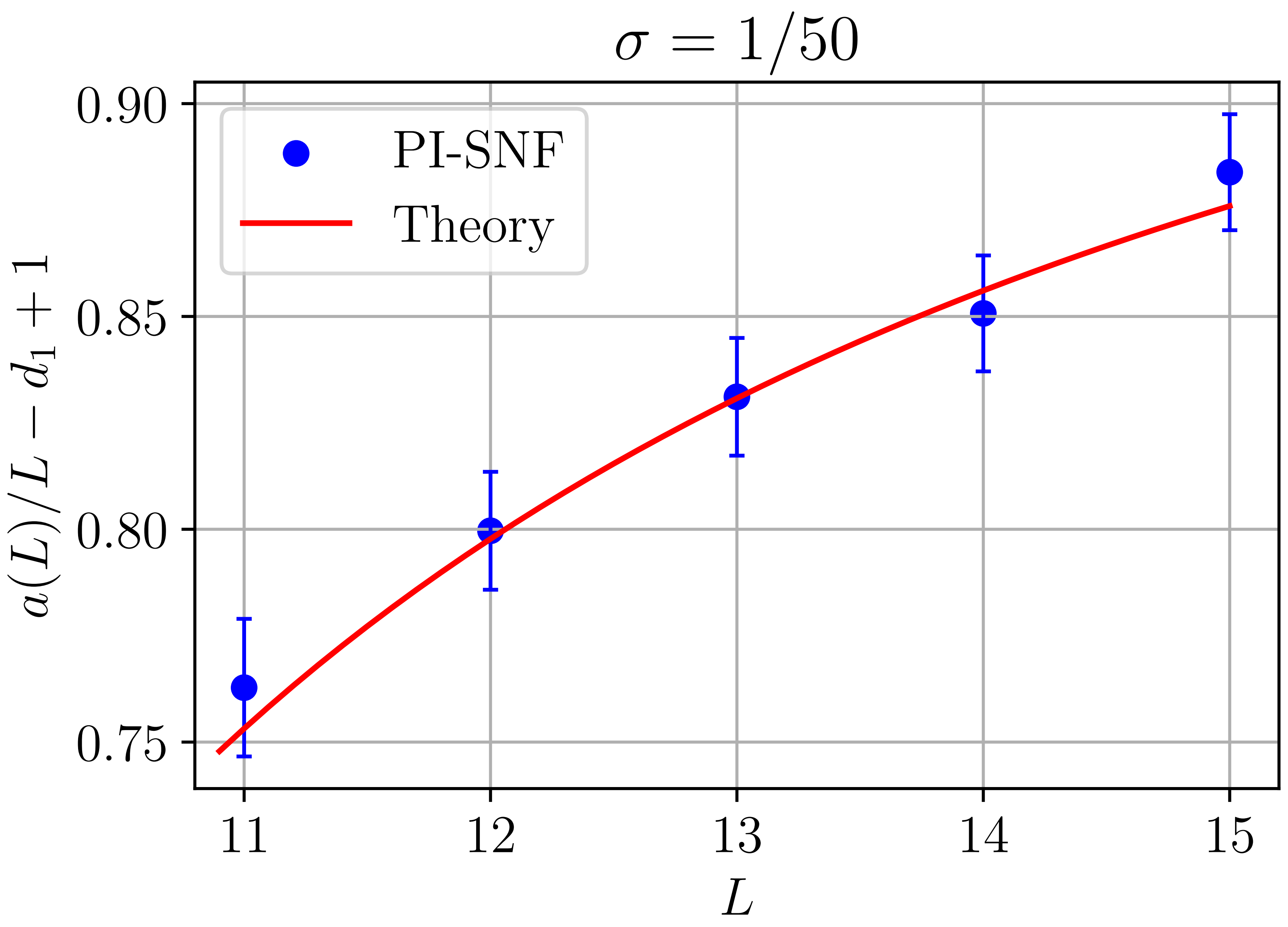
NUMERICAL RESULTS

FREE ENERGY

$$-\log Z = \sigma R L \sqrt{1 - \frac{\pi}{3\sigma L^2}} + \cdots$$

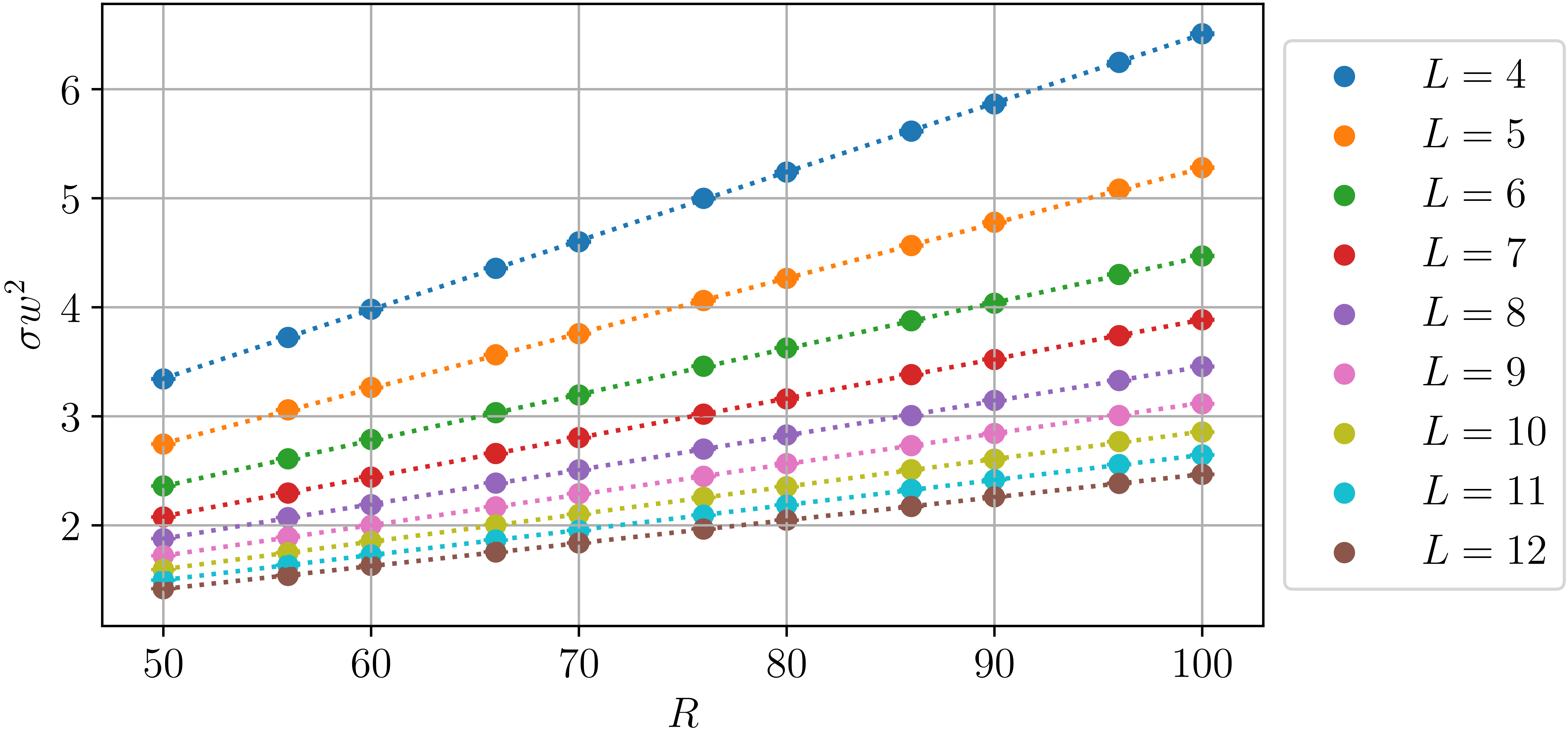
Fitted: $-1.03(2)$,
Target: $-\pi/3 = -1.047...$





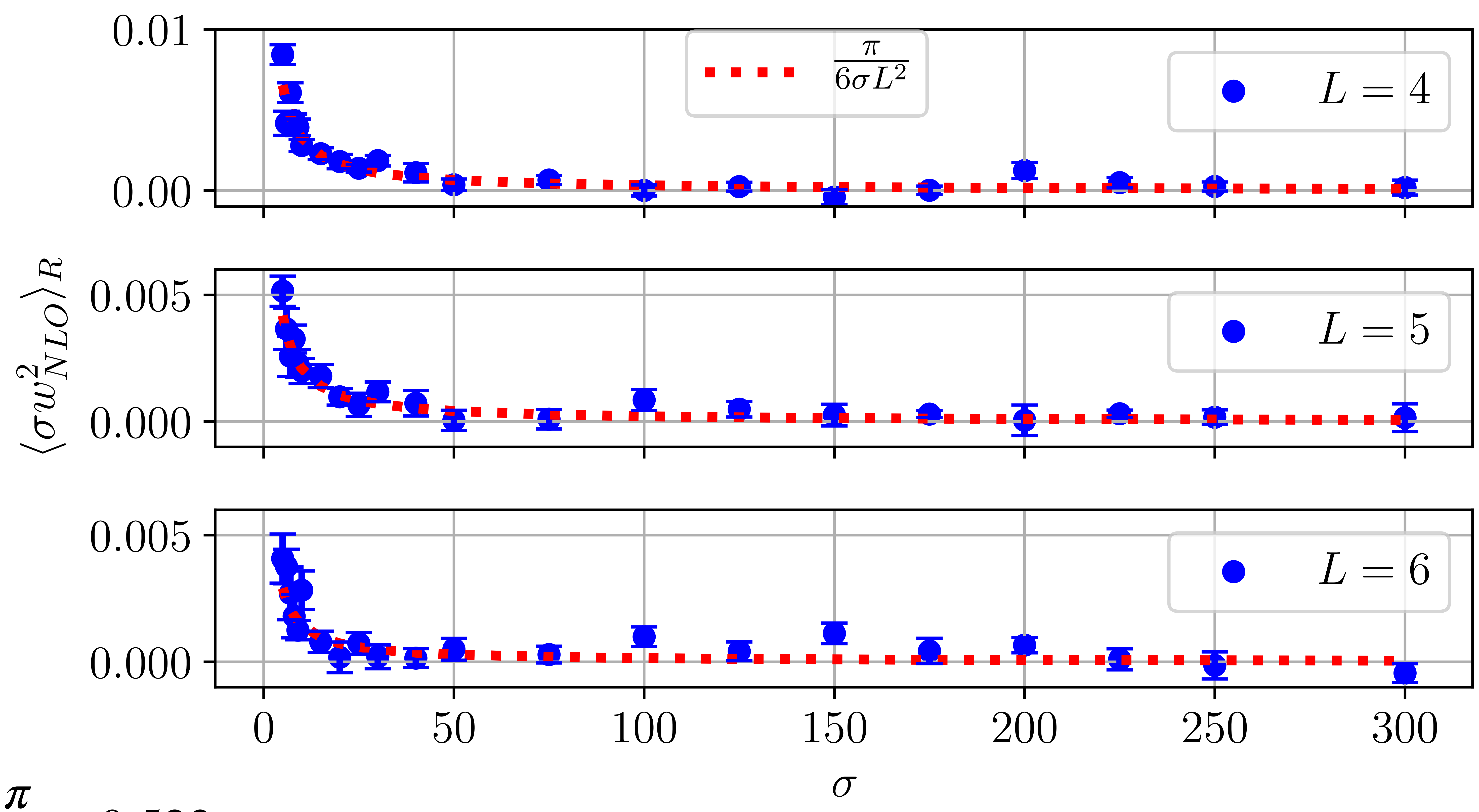
Fitted: $-1.04(7)$,
Target: $-\pi/3 = -1.047...$

WIDTH: LINEAR BROADENING



[Caselle, EC, Nada; 2307.01107]

WIDTH: 2−LOOP LEVEL



Fitted: 0.55(5), target: $\frac{\pi}{6} = 0.523...$

[Caselle, EC, Nada; 2307.01107]

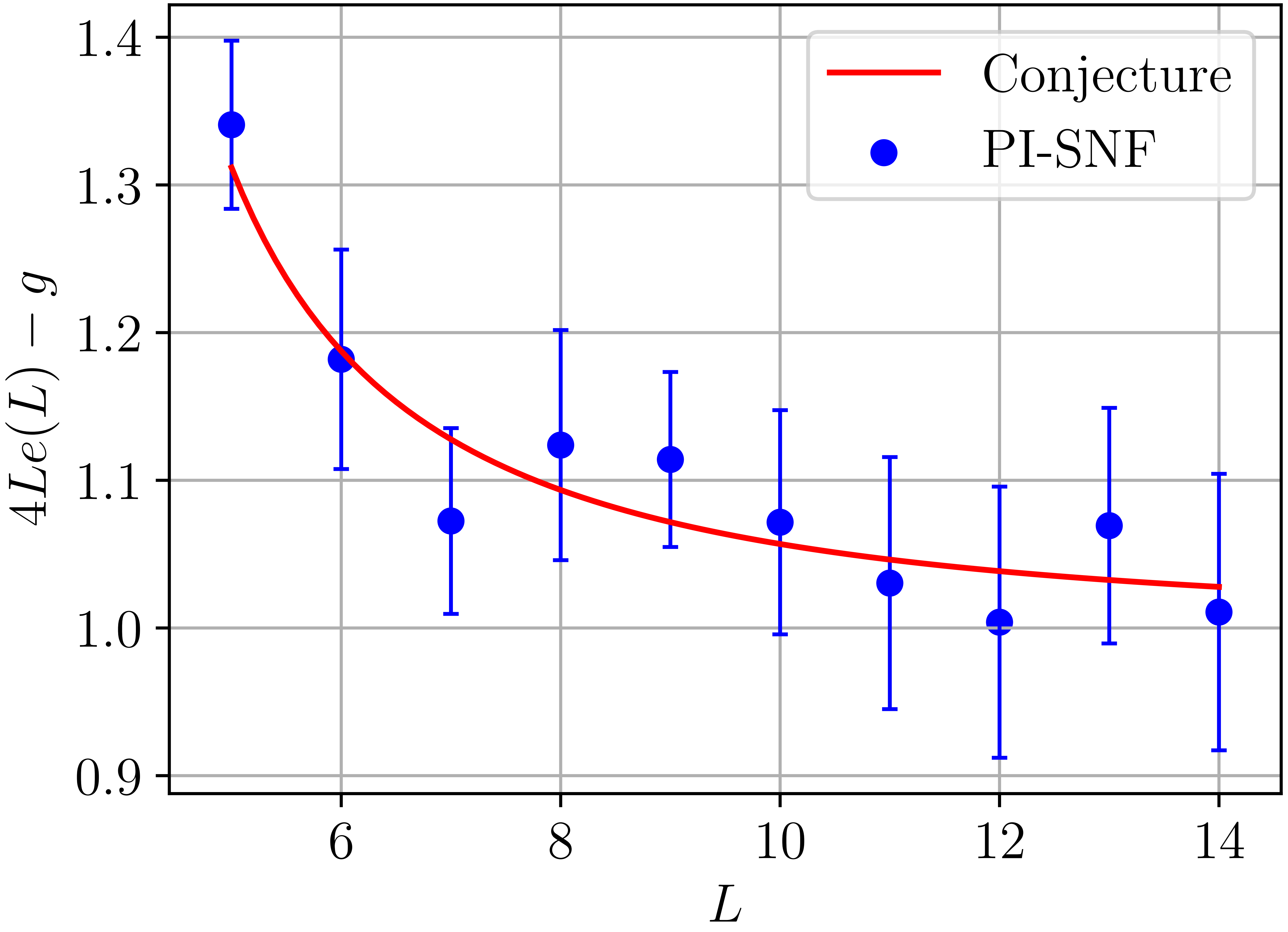
WIDTH: CONJECTURE

$\sigma = 1/10$

Conjecture:

$$\sigma w^2 = \frac{1}{\sqrt{1 - \frac{\pi}{3\sigma L^2}}} \frac{R}{4L} + \cdots$$

Fitted: $-1.09(8)$, target: $-\pi/3 = -1.047...$

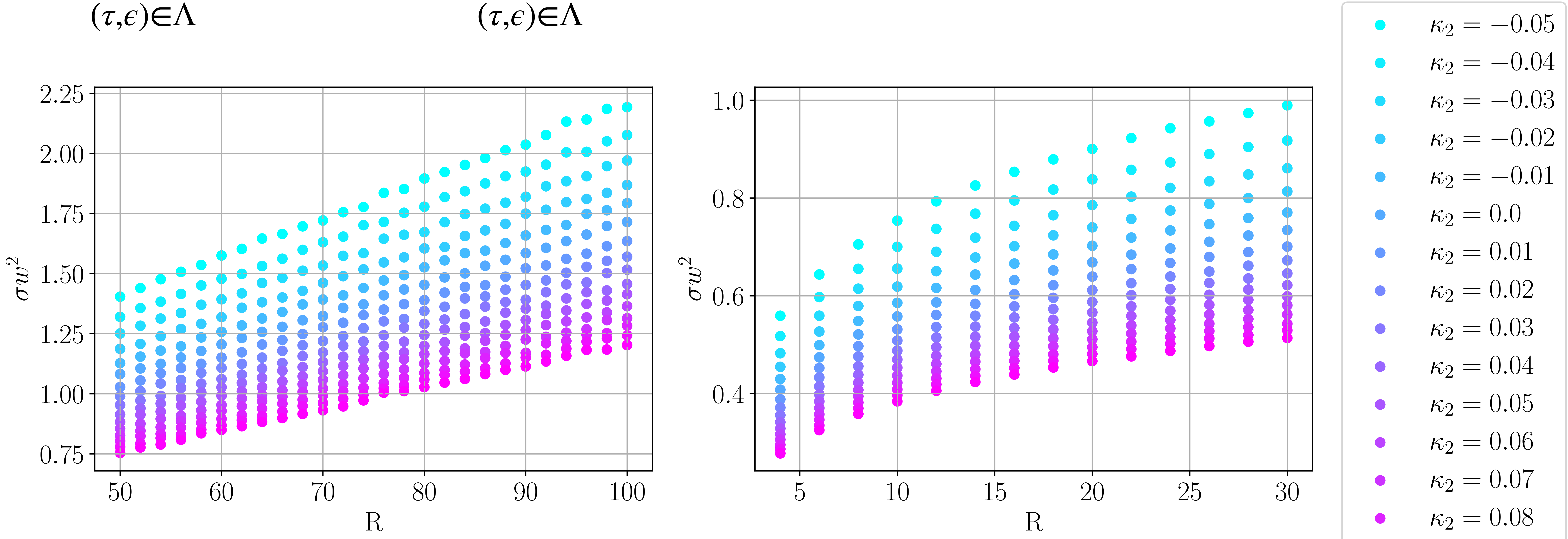


[Caselle, EC, Nada; in prep.]

BEYOND NG: WIDTH

$$S_{EST}(\phi) = S_{NG}(\phi) + \kappa_2 \mathscr{K}^2(\phi)$$

$$\mathscr{K}^2(\phi) = \sum_{(\tau,\epsilon)\in\Lambda} \mathscr{L}^2(\phi(\tau,\epsilon)) = \sum_{(\tau,\epsilon)\in\Lambda} (\partial_\tau\partial_\tau\phi(\tau,\epsilon))^2 + (\partial_\epsilon\partial_\epsilon\phi(\tau,\epsilon))^2 + 2(\partial_\tau\partial_\epsilon\phi(\tau,\epsilon))^2$$

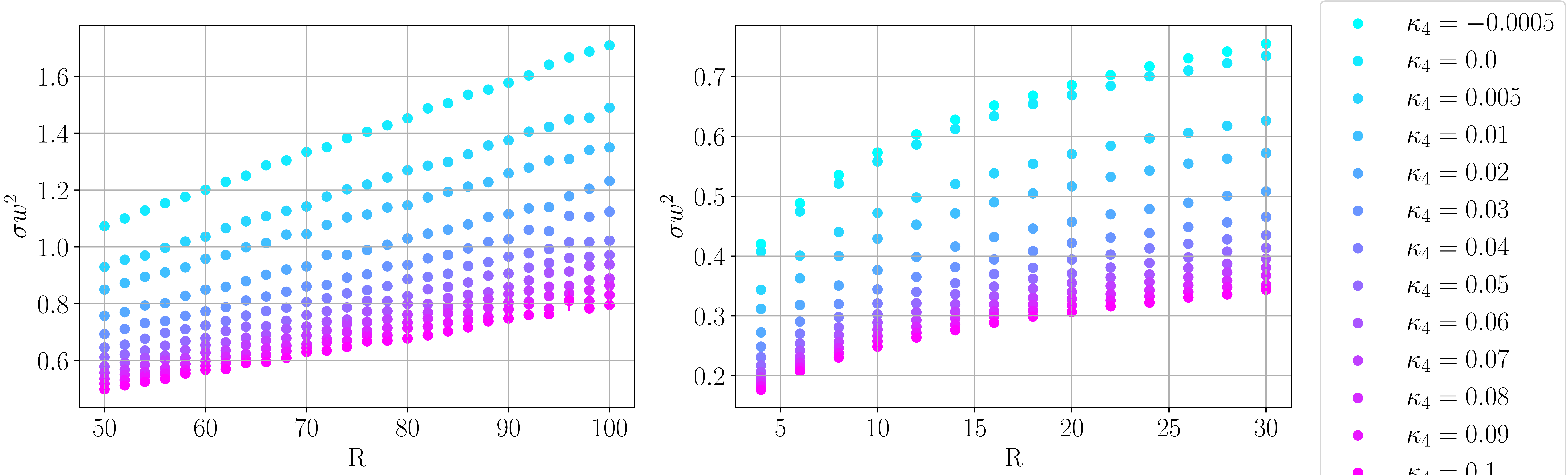


$\sigma = 100$ $R \gg L = 20$ $L = 80 \gg R$

BEYOND NG: WIDTH

$$S_{EST}(\phi) = S_{NG}(\phi) + \kappa_4 \mathcal{K}^4(\phi)$$

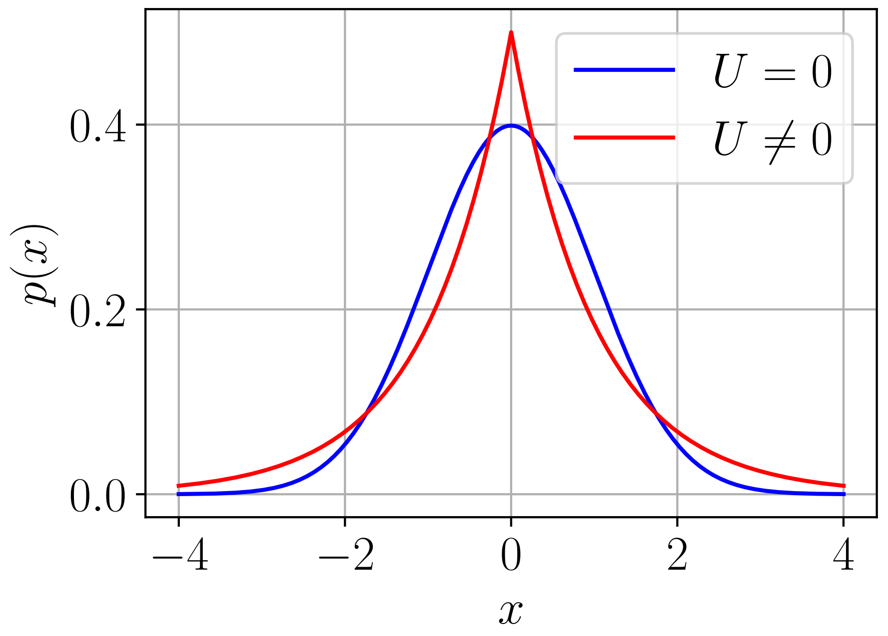
$$\mathcal{K}^4(\phi) = \sum_{x\in\Lambda} \left(\mathcal{L}^2(\phi(x))\right)^2$$

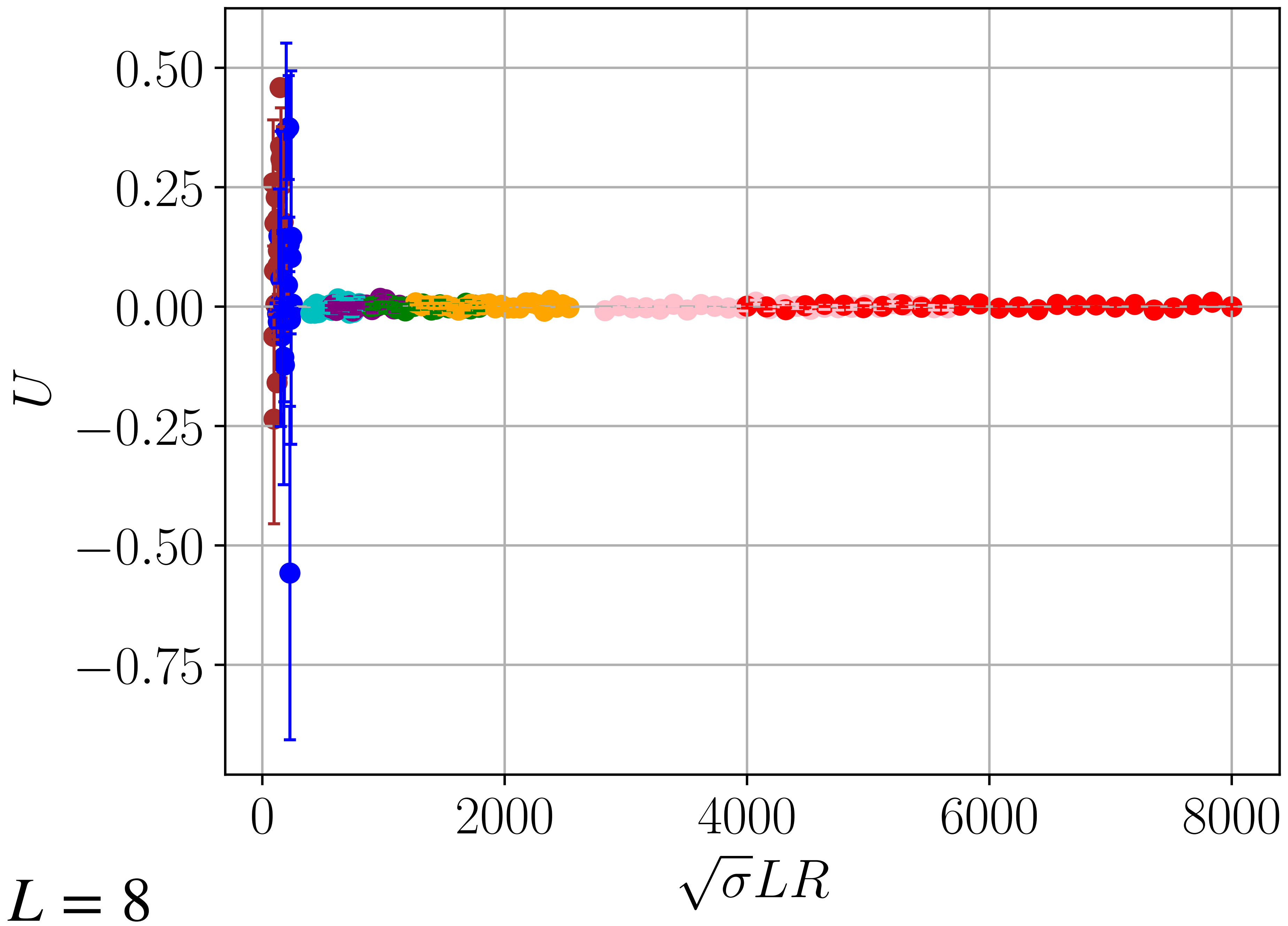


$\sigma = 100$ $R \gg L = 20$ $L = 80 \gg R$

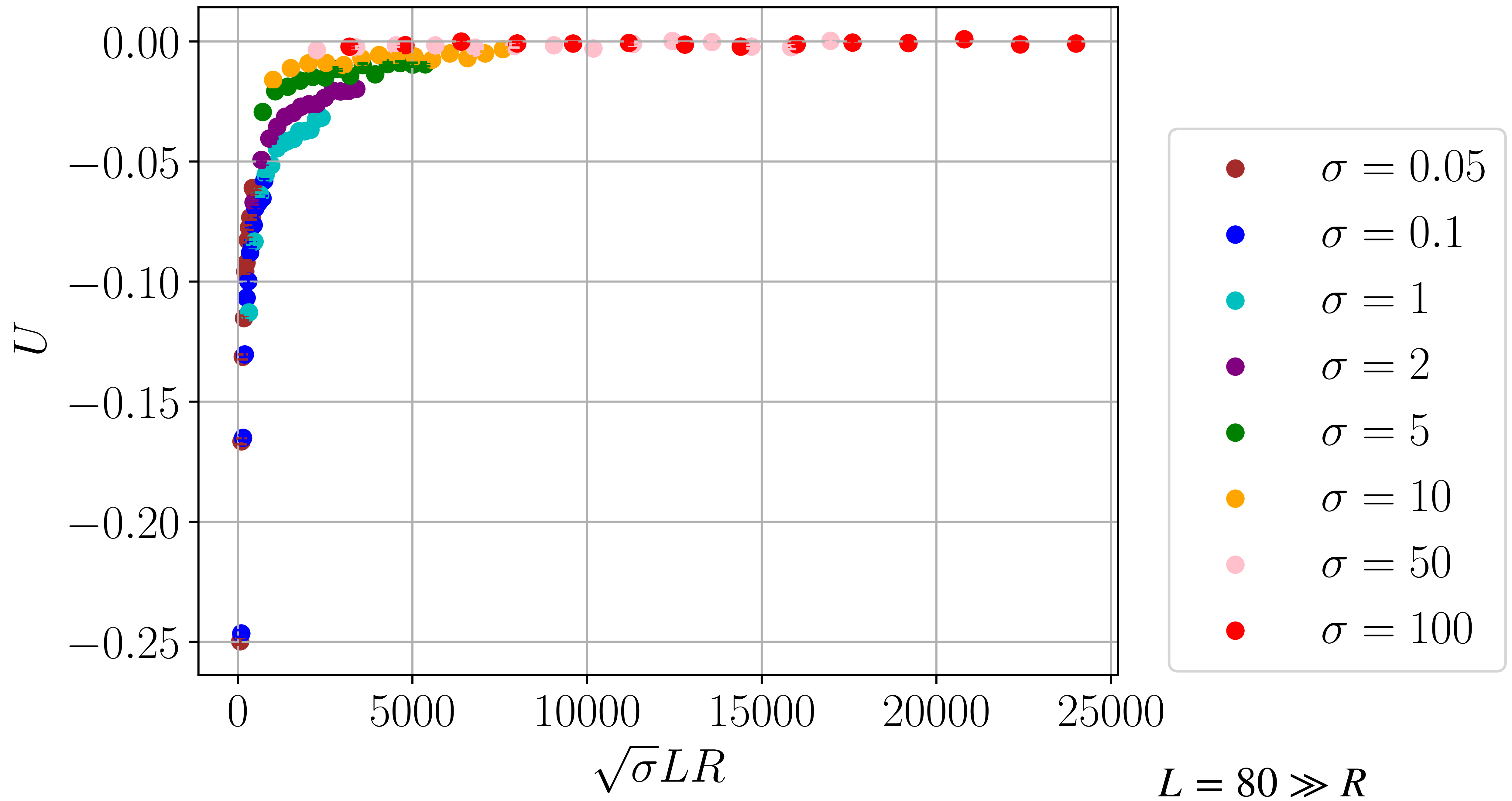
IS THE NG FLUX TUBE SHAPE GAUSSIAN?

$$U = 1 - \frac{\langle \phi^4(\tau, R/2)\rangle_\tau}{3\langle \phi^2(\tau, R/2)\rangle_\tau^2}$$





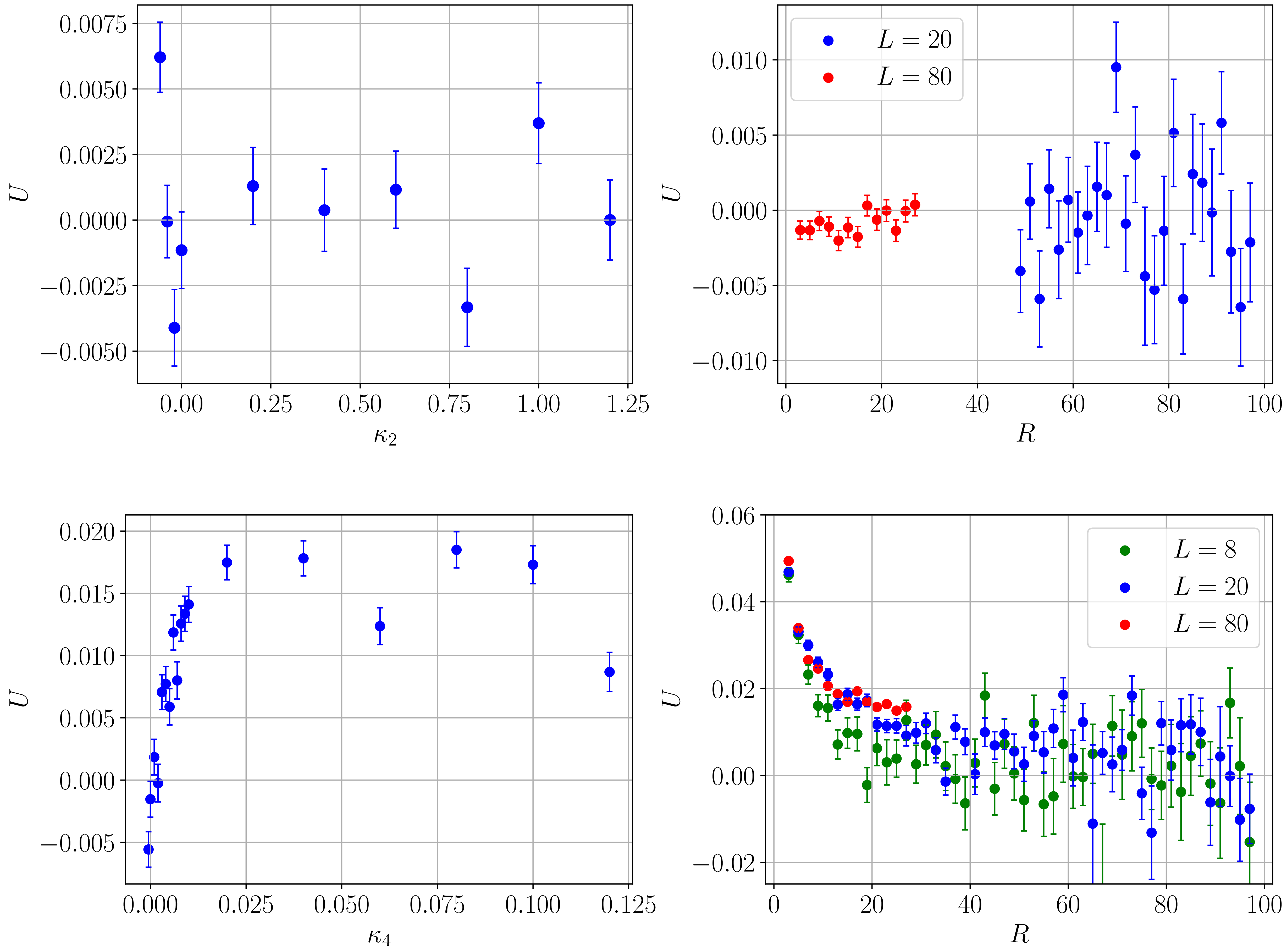
$R \gg L = 8$



$L = 80 \gg R$

BEYOND NG: GAUSSIANITY

$\sigma = 100$



$$S_{NG} + \kappa_2 \mathscr{K}^2$$

$$S_{NG} + \kappa_4 \mathscr{K}^4$$

OUTLOOKS

- Flow-based sampler can be successful applied to sample Lattice EST:
  1. Numerical solution of the high temperature NG width
  2. Numerical studies of the Beyond NG EST
  3. Numerical studies of the Gaussianity of the flux tube shape

OUTLOOKS

- Flow-based sampler can be successful applied to sample Lattice EST:
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THANK YOU FOR YOUR ATTENTION!

JHEP 02 (2024) 048

CNF PyTorch Notebook

BACKUP SLIDE

NORMALIZING FLOWS

A Normalizing Flow (**NF**) g_θ is a **parametric**, **invertible** and **differentiable** function:

[Rezende+; 1505.05770]

$$g_\theta : \; q_0 \to q_\theta \simeq p \qquad \phi = g_\theta(z) \qquad q_\theta(\phi) = q_0(g^{-1}(\phi)) \, | \det J_g |^{-1}$$

NFS: LEARNING BOLTZMANN DISTRIBUTIONS

NFs can be trained to $q_\theta \simeq p(\phi)$ with $p(\phi) = \exp(-S[\phi])/Z$ by minimizing the reverse **Kullback-Leibler divergence**:

[Albergo+; 1904.12072][Noé+; 1812.01729]

$$D_{KL}(q_\theta || p) = \int d\phi q_\theta(\phi) \log \frac{q_\theta(\phi)}{p(\phi)} \geq 0.$$

NFS: SAMPLING BOLTZMANN DISTRIBUTIONS

Partition functions and **observables** can be computed using a re-weighting procedure also called **Importance Sampling**:

[Nicoli+; 1910.13496, 2007.07115]

$$\langle \mathcal{O} \rangle_{\phi \sim p} = \frac{1}{Z} \langle \mathcal{O} \tilde{w} \rangle_{\phi \sim q_\theta} \qquad Z = \langle \tilde{w} \rangle_{\phi \sim q_\theta} \qquad \tilde{w} = \frac{e^{-S[\phi]}}{q_\theta(\phi)}$$

σw^2 IN LGT

$$s(L,R,h) = \frac{\langle P(0)P^{\dagger}(R)U(h)\rangle}{\langle P(0)P^{\dagger}(R)\rangle} - \langle U(h)\rangle$$

$$w^2(L,R) = \frac{\sum_h h^2 s(L,R,h)}{\sum_h s(L,R,h)}$$