Quark pairing in sQGP

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fQCD collaboration :
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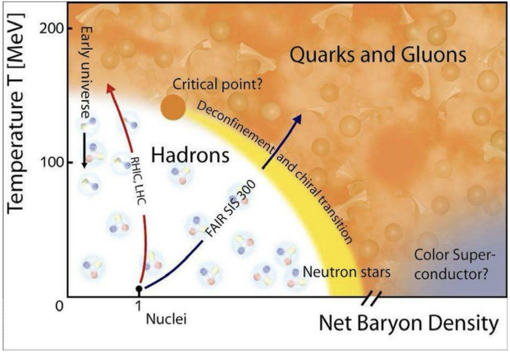
Roughly speaking, a transition between hadrons and asymptotic quarks/gluons.

- Transitions coming from the mass scale change of quark and gluon:
  Quark mass generation=Chiral PT
  Gluon mass generation ≈ Deconfinement
- Crossover at low density connected with a critical end point (CEP)
- Fine structures to classify phases like chiral spin symmetric phase, inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity phase



The principle of fQCD: We don't do models, we do simplification.

QCD in vacuum:

Cyrol, Mitter, Pawlowski, Strodhoff, PRD 97 (2018) 5, 054006.

Binosi, Chang, Papavassiliou, Qin, Roberts, PLB 742, (2015) 183

Williams, Fischer, Heupel, PRD 93, (2016)034026.

Mitter, Pawlowski, Strodthoff, PRD 91, (2015)054035.

Qin, Chang, Liu, Roberts, Schmidt, PLB 722 (2013) 384

Chang, Roberts, PRL 106 (2011) 072001 ...

Yang-Mills sector:

Eichmann, Pawlowski, Silva, PRD 104 (2021) 11, 114016

Aguilar, Ferreira, Papavassiliou, PRD 105 (2022) 1, 014030

Huber, PR 879, 1 (2020)

Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

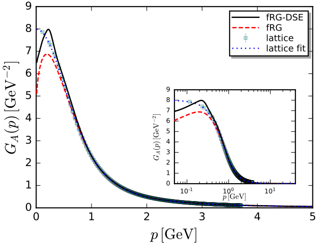
Aguilar, Binosi, Papavassiliou, PRD 86 (2012) 014032 ...

Phase Structure: Fu, Pawlowski, Renneke, PRD 101 (2020) 5, 054032; Gao, Chen, Liu, Roberts, Schmidt, PRD 93 (2016) 9, 094019; Fischer, PPNP 105,(2019)1;Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 Isserstedt, Buballa, Fischer, PRD 100 (2019) 7, 074011; Qin, Chang, Chen, Liu, Roberts, PRL106 (2011) 172301...

The minimal requirements for a truncation scheme that describes QCD:

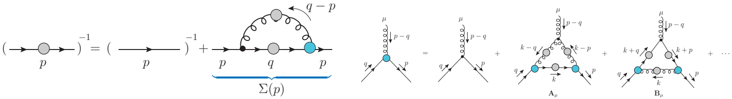
- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*

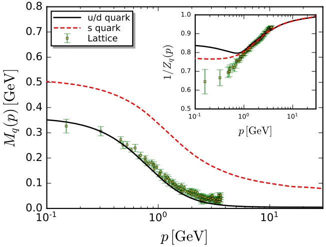
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Solve the DSEs of quark propagator and quark gluon vertex:





lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p) = \sum_{i=1}^{8} t_i(q,p) P^{\mu\nu}(q-p) \mathcal{T}_i^{\nu}(q,p)\,,$$

The dominant structures are Dirac and Pauli term:

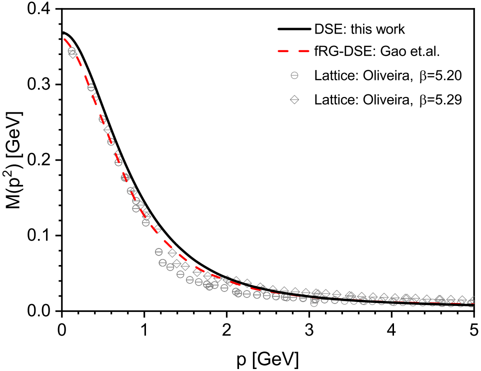
$$\mathcal{T}_1(p,q) = -i\gamma^{\mu}\,, \mathcal{T}_4^{\mu}(p,q) = \sigma_{\mu\nu}(p-q)^{\nu}\,,$$

$$t_1(p,q) = F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

$$t_4(p,q) = \left[Z(k^2)\right]^{-1/2}\frac{B(p^2)-B(q^2)}{p^2-q^2}$$

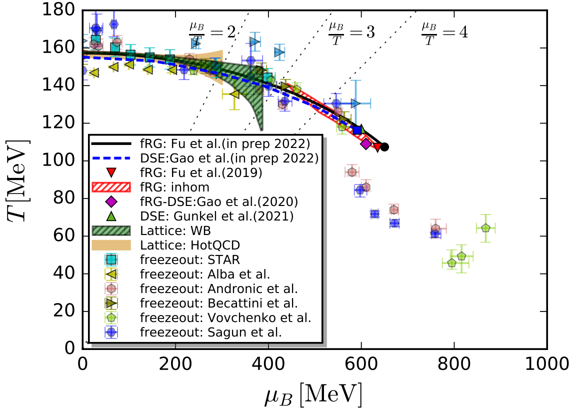
All quantities are expressed by the running of two point functions.

The Quark Mass function:



FG, J. Papavassiliou, J. Pawlowski, PRD 103.094013 (2021).
Y, Lu, **FG**, YX Liu, J. Pawlowski, arXiv:2310.16345.

Chiral Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\text{MeV}$ and $\kappa \sim 0.016$
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (600, 700)$ MeV
- $\sqrt{s_{\rm NN}} \approx 3 - 5\text{GeV}$

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and J. Pawlowski, PRD 102, 034027 (2020)
FG and J. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

Deconfinement phase transition

A direct measurement of deconfinement is Polyakov loop:

- Reflects the $Z(N_c)$ center symmetry
- Stands for a nontrivial stationary point in gauge field potential.
- Related to the gluon mass scale (L Fister, J. Pawlowski, PRD 88, 045010 (2013)).

However, there might be some different scenarios for deconfined phase:

- Quasi quarks that breaks center symmetry
- The quark is confined into colored bound states (Diquark) , with no asymptotic quarks but still breaks the symmetry(Partial deconfinement).

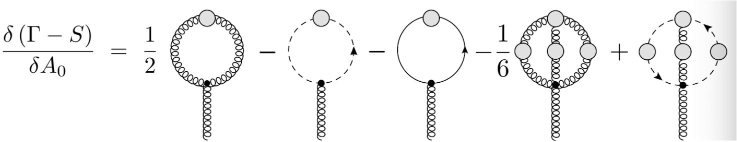
The diquark/quark pairing in the deconfined phase is an additional characteristic property.

Polyakov loop in background field approach is related to A_0^a condensate as:

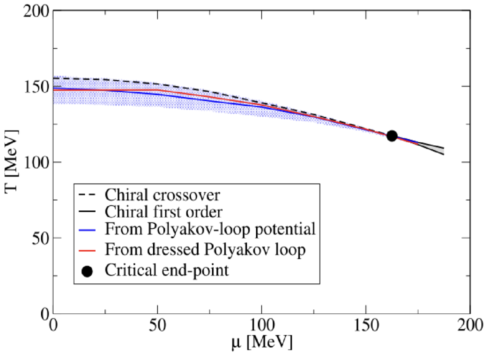
$$\mathcal{L}(A_0) = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{ig\int dx_0 A_0} = \frac{1}{3}\left[1 + 2\cos\left(g\beta A_0/2\right)\right]$$

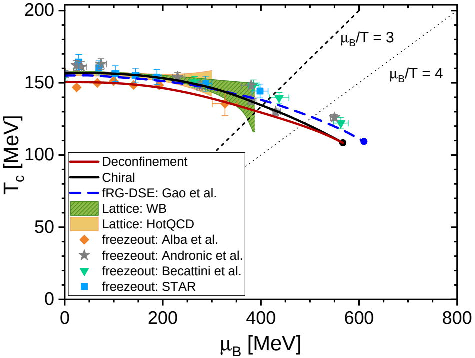
A_0^a condensate is equivalent to the colored imaginary chemical potential: Center symmetry is a symmetry for the shift in the imaginary time axis which is thus related to the pole structure of propagators.

One can obtain A_0 condensate by solving the DSE of A_0^a as $\frac{\delta(\Gamma - S_A)}{\delta A_0} = 0$.
The diagrammatic representation is:



The deconfinement characterized by Polyakov loop is in agreement with Chiral PT with the same CEP location.





C. S. Fischer et al, PLB 732, 273(2014); Yi Lu, FG, J. Pawlowski, Yuxin Liu, in preparation

The quark pairing in gap equation

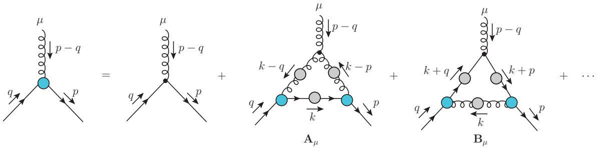
To study the quark pairing in QCD, one needs to compute the gap equation i.e. the quark propagator Schwinger-Dyson equation, in the Nambu-Gorkov basis. It is to extend the fermion field as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C),$$

$$\mathbf{S}^{-1}(p) = \mathbf{S}_0^{-1}(p) + \Sigma(p), \qquad \Sigma(p) = \int_q g^2 G^{aa'}_{\mu\nu}(q-p)[\Gamma^{(0)}_\mu]^a \mathbf{S}(q)\, \Gamma^{a'}_\nu(q,p)$$

The key point: the quark gluon vertex
Essential element for solving quark pairing gap equation.

In vertex DSE, diagram A is non Abelian diagram and diagram B is the Abelian diagram similar to QED.



FG, Y Lu, YX Liu, arXiv:2403.16816

The quark gluon vertex in NG basis

The dynamics related to diagram A is very different from that from diagram B.

With diagram A:

- In ultraviolet region with for instance $p \to 0$, the term $Z_1 \Delta$ is dominant as Z_1 is proportional to $1/p^2$ and leads to the coefficient of quark gluon vertex $t_4 \sim \Delta/p^2$.
- In the infrared limit, Z_1 and Z are finite constants. Considering Δ to be small, the two solutions become $t_4 = Z_1 \Delta$ and $t_4 = \frac{1}{Z\Delta}$.

With diagram B:

- one only gets the first solution $t_4 \propto \Delta$;

Putting the coefficient of vertex t_4 into off diagonal part of gap equation:

- With $t_4 \propto \frac{1}{\Delta}$, the gap equation becomes: $\Delta \propto \int \frac{Z}{k^2 \Delta} G(\bar{k}^2)$

For $Z > 0$, a finite solution for Δ; For $Z < 0$, the trivial solution as $\lambda_4 = \Delta = 0$.

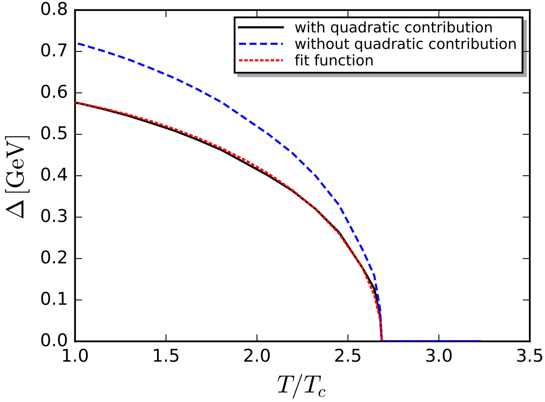
- With $t_4 \propto \Delta$, the gap equation becomes: $\Delta \propto \int_k \frac{\Delta}{k^2} G(\bar{k}^2)$

which gives the conventional CSC gap and proportional to chemical potential μ.

A new type of pairing at zero chemical potential

The pairing can be expanded as:

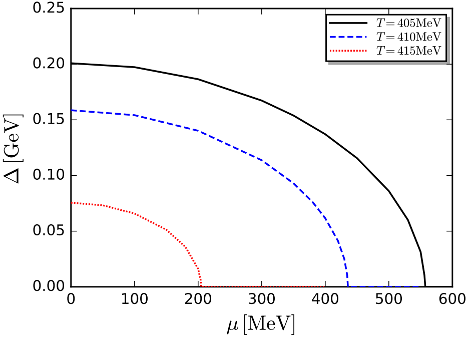
$$\Delta \propto \frac{3}{2}\langle g^2 A^2\rangle - \frac{3}{2}\langle g^2 \frac{k_4^+ p_4^+}{k_+^2}(G_L(\bar{k}^2) + 2G_T(\bar{k}^2))\rangle,$$

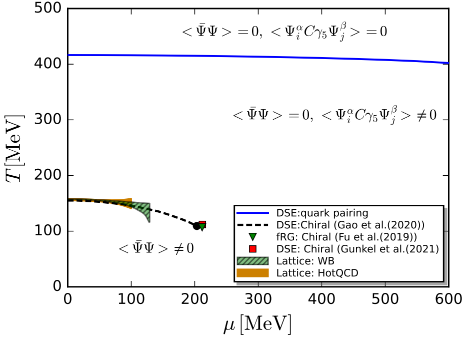


- The quark pairing gap is related to the dimensional 2 gluon condensate and thus dominant by the glue dynamics.
- A second order phase transition at temperature T_Δ, as one has $\Delta = 0$ above T_Δ, and near below T_Δ:

$$\Delta^2 \propto 1 - (T/T_\Delta)^a,$$

  with the best fit as $a = 2.16$.
- The relation then yields a mean field critical exponent as $\beta = 1/2$.





The pairing phase in $T - \mu$ plane:

- Represents a color deconfined phase above the chiral phase transition;
- Possibly a partial deconfined phase;
- Temperature range $T \in [T_c, T_\Delta \approx 2 - 3\,T_c]$, overlaping with Chiral Spin Symmetric phase and the other conjectured strongly coupled states in sQGP.

Discussions and Outlook:

Conclusions:

- The deconfinement characterized with the Polyakov loop is consistent with the Chiral phase transition of QCD.
- A quark pairing appears in the temperature region at $T \in [T_c, 3T_c]$, which defines a partial deconfined phase.
- The quark pairing is induced by the non-Abelian feature in the quark gluon vertex, and is related to the dimensional two gluon condensate.

Momentum dependence of the gap and the vertex coefficients:

- The gluon condensate contains a quadratic divergence that is artificial due to the neglect of the momentum dependence of t_4.
- After incorporating the momentum dependence as $\Delta(p^2)$ and $t_4(p^2, q^2)$, a finite gap can be generated without the bothering of the divergence.
- Further investigations in T-μ_B plane will be done in our future work.

Thank you!