Perturbatively Confined Phase of QCD under Imaginary Rotation

Based on the collaboration with Shi Chen and Kenji Fukushima:

[Phys. Rev. Lett. 129, 242002 (2022)], [arXiv:2404.00965]

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Overview

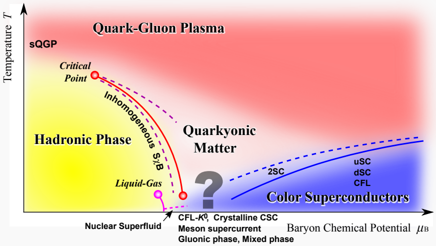
- Introduction: QCD Phase Diagram with Rotation
- Perturbatively Confined Phase under *Imaginary* Rotation
- Characters of the PC Phase
- Toward Real systems (If time remains...)

QCD phase diagram under rotation

QCD mystery:

- **Confinement**
- Spontaneous breaking of **Chiral symmetry**

Phase transitions of QCD matter → phase diagram



[Fukushima, Hatsuda (2011)]

Sign problem for finite μ direction

↓

New parameter axis

QCD phase diagram under rotation

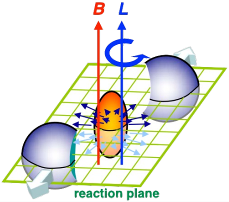
New axes of diagram: **angular velocity** ω

Rotation appears in real world:

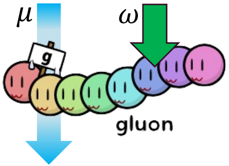
- neutron stars
  $\omega \sim 10^3$ /s
- heavy ion collision
  $\omega \sim 10^{22}$ /s [STAR(2010)]

ω is a good parameter:

- directly interact with gluons
  $\rightarrow$ confinement can be seen



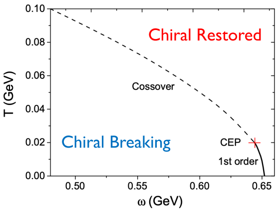
[Niida (2020)]



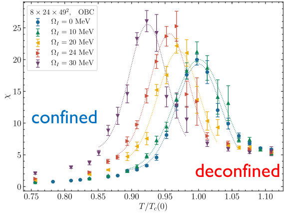
[higgstan.com]

Previous results for rotation

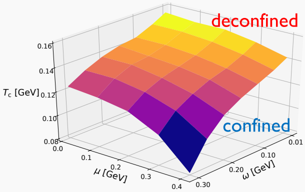
The critical temperature T_c by models and lattice:



[Jiang, Liao (2016)]



[Braguta, Kotov, Kuznedelev, Roenko (2021)]



[Fujimoto, Fukushima, Hidaka (2021)]

Models: $T_c(\omega)$ decreases

Lattice: $T_c(\Omega_{\mathrm{I}})$ decreases

$\rightarrow T_c(\omega)$ increases

Perturbative calculation with Ω_I 1/2

Perturbative calc. under **imaginary rotation** $\omega = i\Omega_I$.

Real ω has causality problems

Taking a background method

$$A_\mu = A_{B\,\mu} + \mathcal{A}_\mu\,, \quad A_{B\,\mu} = \frac{\delta_{\mu,4}}{g\beta}\boldsymbol{\phi}\cdot\boldsymbol{H}\,,$$

$\boldsymbol{H}$: the vector of Cartan sub-algebra of $\mathfrak{su}(N)$

with gauge fixing as

$$D_B^\mu A_\mu = 0, \qquad \text{w/} \quad D_{B\,\mu} := \partial_\mu + igA_{B\,\mu}.$$

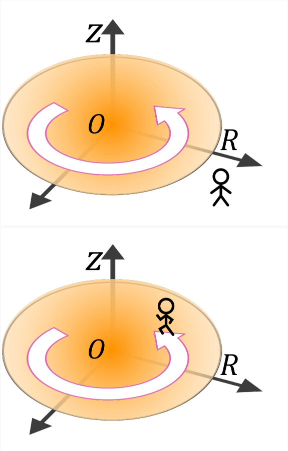
In this gauge condition,

$$\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} \overset{\mathsf{LO}}{=} 2\,\mathrm{tr}\,\mathcal{A}^\mu\left(-D_B^2\right)\mathcal{A}_\mu\,,$$

and FP determinant is given by $\mathrm{Det}[-(D_B^2)]\,.$

Perturbative calculation with Ω_I 2/2

We take cylindrical coordinate rotating along the z axis.
Effect of rotation appears in the metric.



Rotating systems are equivalent to static systems w/ the metric:

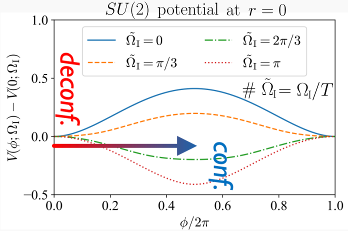
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & r^2\Omega_{\mathrm{I}} \\ 0 & 0 & 1 & 0 \\ 0 & r^2\Omega_{\mathrm{I}} & 0 & 1 + r^2\Omega_{\mathrm{I}}^2 \end{pmatrix}.$$

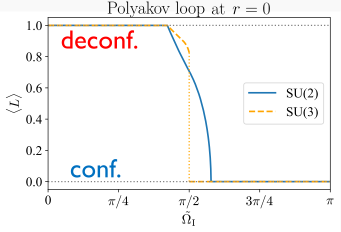
$-D_{B\,\mu}$ for vectors changes into $G_{B\,\mu} A_\nu = D_{B\,\mu} A_\nu - \Gamma^{\lambda}_{\mu\nu} A_\lambda$.

Perturbative Confinement

The Polyakov loop potential under imaginary rotation is

$$V_g = \frac{1}{4\pi^2\beta}\sum_{\alpha}\sum_{l\in\mathbb{Z}}\int_0^\infty k_\perp\,\mathrm{d}k_\perp\int_{-\infty}^{\infty}\mathrm{d}k_z\,\left[J_{l-1}^2(k_\perp r)+J_{l+1}^2(k_\perp r)\right]$$
$$\times\mathrm{Re}\log\left[1-e^{i\phi\cdot\alpha-il\beta\Omega_\mathrm{I}}e^{-\beta\sqrt{k_\perp^2+k_z^2}}\right].$$





With $\tilde\Omega_\mathrm{I} \gtrsim \pi/2$, V_g flips.

$\to$ minimum moves into $\phi = \pi$:

$$L = \frac{1}{N_c}\,\mathrm{tr}\,\mathcal{P}\,e^{ig\int_0^\beta \mathrm{d}\tau A_{B\,4}}$$
$$= \frac{1}{2}\,\mathrm{tr}\,\exp\left[\frac{i}{2}\begin{pmatrix}\phi & \\ & -\phi\end{pmatrix}\right]$$
$$= \cos\left(\frac{\pi}{2}\right) = 0.$$

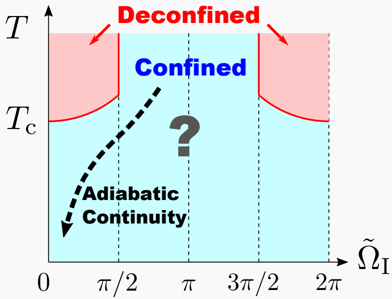
Perturbative Confinement!

Phase diagram of $(T, \tilde{\Omega}_\mathrm{I})$

What occurs at low T?

$\rightarrow$ The ghost contributions are enhanced. (KOGZ mechanism)

$$V_\mathrm{ghost} \sim \sum_\alpha \sum_l \int \mathrm{d}k_\perp^2 \int \mathrm{d}k_z \, J_l^2(0) \times \mathrm{Re}\log\left[1 - e^{i\phi\cdot\alpha - il\tilde{\Omega}_\mathrm{I}} e^{-\beta k}\right]$$



At $r = 0$, only $l = 0$ is taken.

$\rightarrow$ ghost do **NOT** couple with ω.

- $\tilde{\Omega}_\mathrm{I}$ induces confinement.
- At low T, conf. is induced.
- High $\tilde{\Omega}_\mathrm{I}$ phase is always confined! (suggestion)

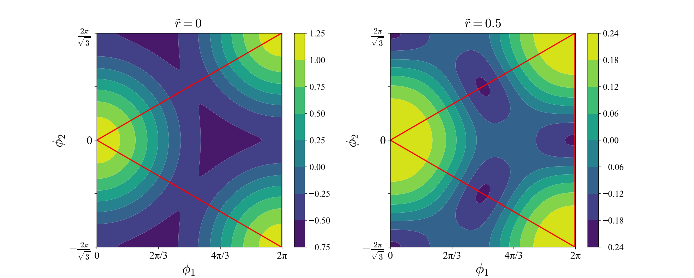
Perturbatively confined phase can be *connected* to the **hadronic phase**.

Inhomogeneity of PC phase

Perturbative calculation is r dependent.

For $SU(3)$, $A_{B\,4} = \frac{1}{g\beta}(\phi_1 T^3 + \phi_2 T^8)$, so $V_g = V_g(\phi_1, \phi_2)$.
$V_g(\phi_1, \phi_2)$ pattern is periodic and minimum patterns show Z_3 center symmetry $\rightarrow$ confinement.

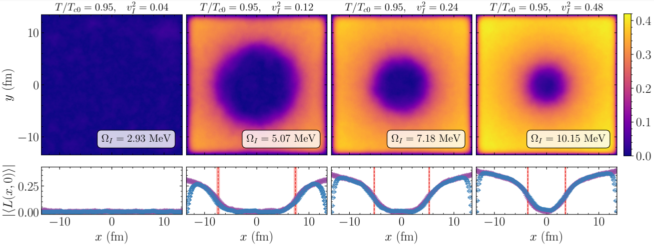
$SU(3)$ Polyakov loop potential at $\tilde{\Omega}_{\rm I} = \pi$



$L = 0$ $\quad\quad$ $|L| = 1/3$

Previous result about inhomogeneity

Lattice QCD calculation also shows the inhomogeneity.



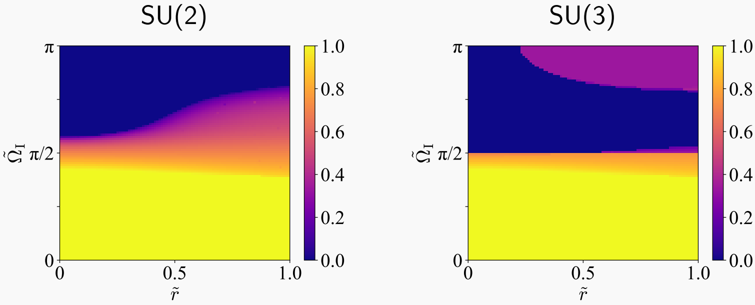
[Braguta, Chernodub, Roenko (2024)]

- $T < T_c$, pure gluonic SU(3)
  w/ periodic & open BC
- center/edge is confined/deconfined
  finite r gives deconfinement

Phase diagram of $(\tilde{\Omega}_I, \tilde{r})$

Polyakov loop values for pure YM.

Blue region w/ $L = 0$ is confined.



At large $\tilde{\Omega}_I$, center is **confined** and edge(large $\tilde{r}$) is **deconfined** as the lattice.
In SU(3), *another deconfined phase* appears.

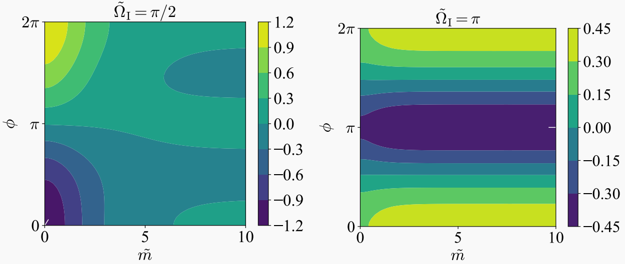
Introduce $N_f = 2$ fermion contribution in the action as

$$\mathcal{L}_{\text{quark}} = \bar{\psi}(\not{G} + m)\psi\,, \quad G_\mu = D_\mu - \Gamma_\mu\,.$$

The dynamical mass m gives the chiral condensate.

Polyakov loop $L = \cos(\phi/2)$ for $SU(2)$.

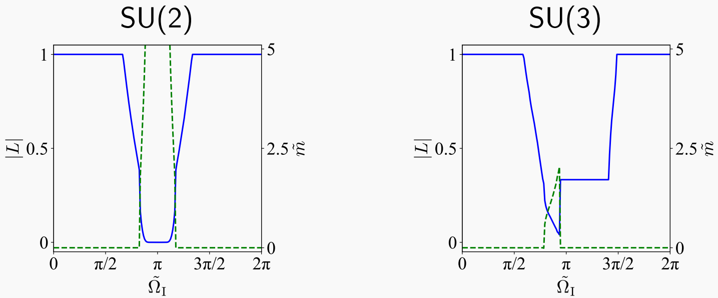
Polyakov loop potential map at $r = 0$



At $\tilde{\Omega}_I = \pi$, **perturbative confined** & **chiral broken**

Chiral symmetry breaking in PC phase 2/3

The Polyakov loop (blue) and the dynamical mass (green) for $N_f = 2$ QCD at $r = 0$.



As the system becomes **confined**,

the dynamical mass m **increases** $\rightarrow$ chiral **broken**.

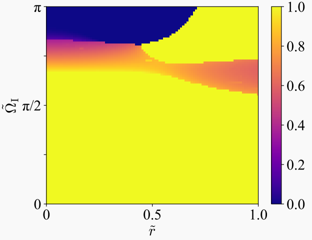
\# Large m weaken the explicit breaking of center symmetry.

\# Result is 4π periodic & $\tilde{\Omega}_I \sim 4\pi - \tilde{\Omega}_I$.

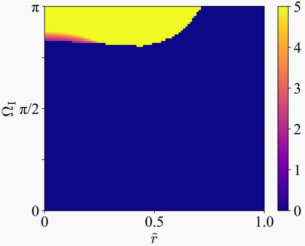
Chiral symmetry breaking in PC phase 3/3

Inhomogeneous behavior of Polyakov loop and dynamical mass for $N_f = N_c = 2$ QCD.

Polyakov loop



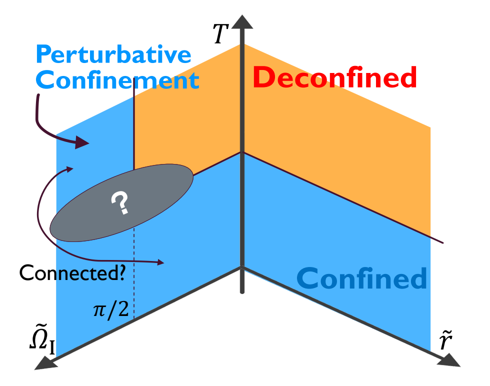
Dynamical Mass



- The chiral broken phase and the perturbatively confined phase are related.
- PT behavior has changed.
- Spatial PT also appears.

Phase diagram of $(T, \tilde{\Omega}_{\rm I}, \tilde{r})$ 1/3

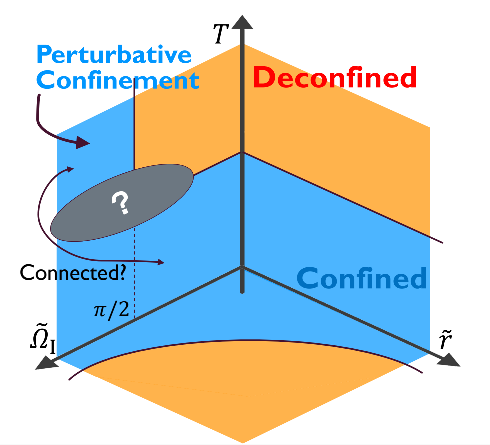
Summarize the phase diagram along T, $\tilde{\Omega}_{\rm I}$ and $\tilde{r}$.



At large $\tilde{\Omega}_{\rm I}$, the perturbatively confined phase exists.
$T_c(\tilde{\Omega}_{\rm I})$ behavior is NOT clear.

Phase diagram of $(T, \tilde{\Omega}_{\mathrm{I}}, \tilde{r})$ 2/3

Summarize the phase diagram along T, $\tilde{\Omega}_{\mathrm{I}}$ and $\tilde{r}$.

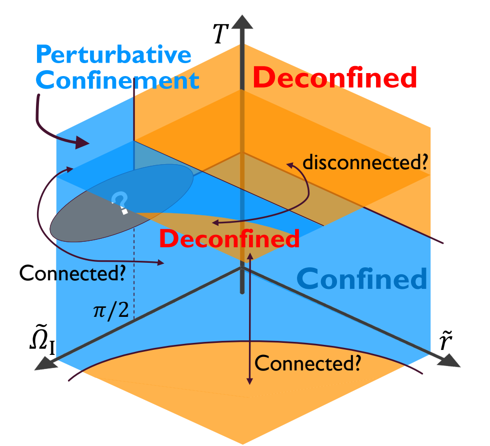


$T_c(\tilde{\Omega}_{\mathrm{I}}, \tilde{r})$ should be considered.

Lattice shows that inhomogeneous phase appears.

Phase diagram of $(T, \tilde{\Omega}_{\mathrm{I}}, \tilde{r})$ 3/3

Summarize the phase diagram along T, $\tilde{\Omega}_{\mathrm{I}}$ and $\tilde{r}$.



At high T, another deconfined phase appears.

How these phases connect?

Summary and Future prospects

Our research discovered

- **Perturbative confinement** with **imaginary** rotaiton

  Confined phase at high T

  $\rightarrow$ connected to the hadronic phase?

- **Spatial phase transition**

  Mixture of confined/deconfined matter

- **Chiral symmetry breaking**

  Dynamical mass increases as Polyakov loop disappears

Further researches are needed to

- solve the discrepancy in $T_c(\omega)$
- reveal what the *imaginary* rotation is
- achieve the prediction of real phenomena

Thank you!

Appendix

Periodicity of imaginary rotation

When we are in the *rest* frame and seeing rotating matter.

At finite T, $\tau \sim \tau + \beta$.
$+$ After τ, polar coordinate moves θ to $\theta + \Omega_I \tau$.

$$\rightarrow \ (r, \theta, z, \tau) \sim (r, \theta + \beta\Omega_{\mathrm{I}}, z, \tau + \beta)$$

$\#$ imaginary rotating $\sim$ spatial periodicity

$\#$ In fact, $Z(\Omega_{\mathrm{I}})$ is given by the flat space-time w/ above new periodicity: if the eigenfunctions are periodic as

$$\Psi \propto E_{\boldsymbol{\alpha}} \ e^{i\left[(2\pi n\beta^{-1} - m\Omega_{\mathrm{I}})\tau + m\theta + k_z z\right]} \ J_m(k_\perp r) \, ,$$

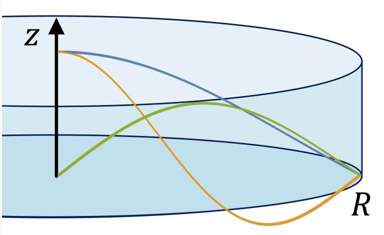
the same spectra with the rotating frame are given by the flat covariant derivative square $-D_{B\,s}^2 = -D_{B\,\tau}^2 - r^{-1}\partial_r(r\partial_r) - r^{-2}\partial_\theta^2 - \partial_z^2$.

Real rotation 1/2

Can we do analytical continuation toward real ω ?

$$V_g = \frac{1}{4\pi^2\beta}\sum_{\alpha}\sum_{l\in\mathbb{Z}}\int_0^{\infty} k_\perp\,\mathrm{d}k_\perp \int_{-\infty}^{\infty}\mathrm{d}k_z\left[J_{l-1}^2(k_\perp r) + J_{l+1}^2(k_\perp r)\right] \times \mathrm{Re}\log\left[1 - e^{\pm i\phi\alpha - il\beta\Omega_{\mathrm{I}}} e^{-\beta\sqrt{k_\perp^2 + k_z^2}}\right].$$

$\rightarrow$ Integration of $\log\left[1 - e^{l\beta\omega} e^{-\beta\sqrt{k_\perp^2+k_z^2}}\right]$ diverge.



To avoid the problem, BC is needed.

e.g.)

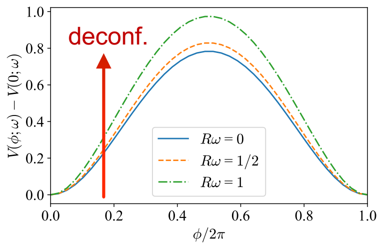
$$A_\mu(r = R) = 0$$

by discretized momentum

$$J_v(k_v^i R) = 0$$

Real rotation 2/2

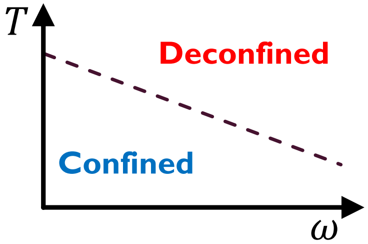
Larger real ω sharpen the potential.



[Chen, Fukushima, Shimada (2022)]

$\rightarrow$ more deconfined

T_c decreases by ω.



This agrees with models, but disagrees with lattice.

Perturbative calculation 1/3

Our new attempt is **perturbative calculation**.

As Weiss (1981), Weiss (1982), Gross, Pisarski, Yaffe (1981)

To achieve the results, separate the gauge configuration into diagonal background and dynamical:

$$A_\mu = A_{B\,\mu} + \mathcal{A}_\mu\,, \quad A_{B\,\mu} = \frac{\delta_{\mu,4}}{g\beta}\boldsymbol{\phi}\cdot\boldsymbol{H}\,,$$

where $\boldsymbol{H}$ is a vector of Cartan sub-algebra of $\mathfrak{su}(N)$.

Gauge fixing is

$$D_B^\mu A_\mu = 0$$
$$\text{w/} \quad D_{B\,\mu} := \partial_\mu + igA_{B\,\mu}$$

Perturbative calculation 2/3

In this gauge condition,

$$\mathrm{tr}\, F_{\mu\nu} F^{\mu\nu} \overset{\text{LO}}{=} -2\, \mathrm{tr}\, \mathcal{A}^{\mu} \big(D_B^2 \big) \mathcal{A}_{\mu}\,,$$

and FP determinant is given by $\mathrm{Det}[-(D_B^2)]$.

So the partition function is

$$Z = \underbrace{\mathrm{Det}\big(-D_B^2\big)}_{\text{ghost part}} \underbrace{\mathrm{Det}^{-1/2}\ \big(-D_B^2\big)}_{\text{gluon part}}$$

The ghost part cancels non-physical modes of the gluon part

This gives the energy $F = -\frac{1}{\beta}\ln Z = \int \mathrm{d}v\, V(r, \phi)$.
$\rightarrow$ The minimal energy states of rotating systems.

Perturbative calculation 3/3

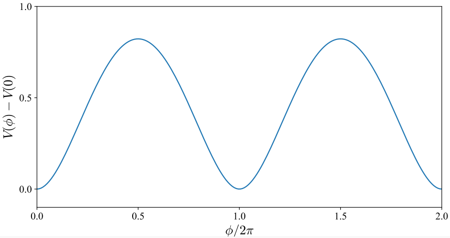
The system is confined if the Polyakov loop

$$L = \frac{1}{N_c} \operatorname{tr} \mathcal{P} \exp\left(ig \int_0^\beta \mathrm{d}\tau \, A_{B\,4} \right)$$

is zero. For $SU(2)$, $L = \cos(\phi/2)$; for $SU(3)$,

$$L = \frac{1}{3}\left(2\cos(\phi_1/2) e^{i\sqrt{3}\phi_2/6} + e^{-i\sqrt{3}\phi_2/3} \right).$$

SU(2) free energy density



re-plotting of [Weiss (1981)]

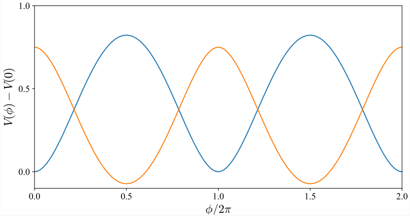
$\phi = 0, 2\pi, 4\pi, \cdots$.
gives deconfinement;

$\phi = \pi, 3\pi, \cdots$.
gives confinement;

Approach to low T

What would occur to the perturbative matter at low T?

KOGZ mechanism[*1]: ghost contribution is enhanced.



Since $V_{\text{gluon}} = -V_{\text{ghost}}$, the potential flips.

Minima are $\pi, 3\pi, \cdots$,
$\rightarrow$ **confined!**

[*1] [Gribov (1978)], [Kugo, Ojima (1979)], [Zwanziger (1994)], [Kugo (1995)]

Definition of G_μ

Covariant derivative in the curved space G_μ is

For scalars $G_\mu\theta = D_\mu\theta\,,$

For vectors $G_\mu A_\nu = D_\mu A_\nu - \Gamma^\lambda_{\mu\nu} A_\lambda\,,$

and $F_{\mu\nu} = [D_\mu, D_\nu] = [G_\mu, G_\nu]\,,$ so

$$Z = \mathrm{Det}\big(-G^2_{B\ \ s}\big)\mathrm{Det}^{-1/2}\big(-G^2_{B\ \ v}\big)\,.$$

Z is obtained by calculating eigenfunctions and eigenspectra of $-G_B^2$

$-G_B^2$ is different for the scholars and the vectors.

$-G_B^2$ spectrum of ghosts

$-G_B^2$ for the scalars (ghosts) is

$$
\begin{aligned}
-G_{B\,s}^2 &:= -\,G_B^{\mu} D_{B\,\mu} = -g^{\mu\nu}\left[D_{B\,\mu} D_{B\,\nu} - \Gamma^{\lambda}_{\mu\nu} D_{B\,\lambda}\right] \\
&= -\left(D_{B\,\tau} - \Omega_{\mathrm{I}}\partial_\theta\right)^2 - \frac{1}{r}\partial_r(r\partial_r) - \frac{1}{r^2}\partial_\theta^2 - \partial_z^2\,.
\end{aligned}
$$

Eigenfunctions and eigenspecta are (in rotating frame)

$$
\begin{aligned}
\Psi &\propto E_{\boldsymbol{\alpha}}\ \exp\left\{i(2\pi n\beta^{-1}\tau + m\theta + k_z z)\right\}\, J_m(k_\perp r)\,, \\
\lambda &= \left(2\pi n\beta^{-1} + \beta^{-1}\boldsymbol{\phi}\cdot\boldsymbol{\alpha} - m\Omega_{\mathrm{I}}\right)^2 + k_\perp^2 + k_z^2\,.
\end{aligned}
$$

So the ghost contribution is

$$
\mathrm{Tr}\ln\left(-G_{B\,s}^2\right) \propto \int k_\perp\,\mathrm{d}k_\perp \int \mathrm{d}k_z\, J_m^2 \ln\left[1 + e^{i\phi\alpha - im\tilde{\Omega}_{\mathrm{I}} - \beta k}\right].
$$

$-G_B^2$ spectrum of gluons

For gluons, $-G_{B\,v}^2$ is

$$\left(-G_{B\,v}^2\right)_\mu{}^\nu = \begin{pmatrix} -G_{B\,s}^2 + r^{-2} & 2r^{-3}\,\partial_\theta & 0 & 0 \\ -2r^{-1}\,\partial_\theta & -r\,G_{B\,s}^2\,r^{-1} + r^{-2} & 0 & 0 \\ 0 & 0 & -G_{B\,s}^2 & 0 \\ -2\Omega_{\mathrm{I}} r^{-1}\,\partial_\theta & 2\Omega_{\mathrm{I}} r^{-1}\,\partial_r & 0 & -G_{B\,s}^2 \end{pmatrix}.$$

Corresponding eigenspectra are the same.

Eigenfunctions are 4-vectors, and two are proportional to that of ghosts, remaining two are

$$\Psi \propto E_{\boldsymbol{\alpha}} \exp\left\{i(2\pi n\beta^{-1}\tau + m\theta + k_z z)\right\} J_{m\pm1}(k_\perp r)\,.$$

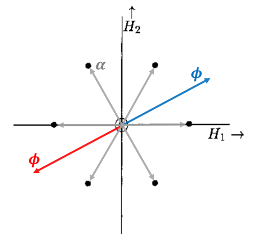
So the gluon contribution is as the ghost one but with $J_{m\pm1}(k_\perp r)$.

Weyl symmetry

Eigenvalues of $D_{B\,\mu} = \partial_\mu + i\beta^{-1}\boldsymbol{\phi}\cdot\boldsymbol{H}$ are that of $\boldsymbol{H}$: roots $\boldsymbol{\alpha}$ or weights $\boldsymbol{\mu}$ of the $\mathfrak{su}(N)$ Lie algebra.
& The potential contains the coupling $\boldsymbol{\phi}\cdot\boldsymbol{\alpha}$ or $\boldsymbol{\phi}\cdot\boldsymbol{\mu}$.

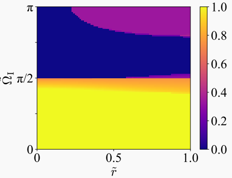
For $\mathfrak{su}(2)$, $H = \sigma_3/2 = \hat{S}_z$; the weights are spin singlet eigenvalue $\pm 1/2$ and the roots are triplet eigenvalue $\pm 1, 0$.

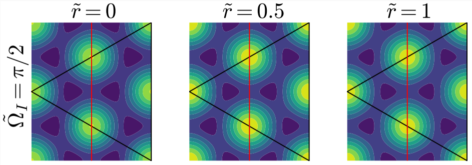
$\mathfrak{su}(3)$ root diagram



The sum of $\sum_{\boldsymbol{\alpha}} \boldsymbol{\phi}\cdot\boldsymbol{\alpha}$ is taken.
From the Weyl symmetry of $\{\boldsymbol{\alpha}\}$s, some $\boldsymbol{\phi}$ gives the same sum.
e.g.)
Red & blue $\boldsymbol{\phi}$ gives the same sum.

Emergent symmetry





At $\tilde{\Omega}_{\rm I} = \pi/2$, the system is always confined & an emergent Z_2 symmetry $\phi_1 \leftrightarrow 2\pi - \phi_1$ (reflection by the red line) appears.

This comes from the vanishing of odd-n terms in the analytic form of the one-loop potential,

$$V_g(\boldsymbol{\phi}; \tilde{\Omega}_{\rm I}) = -\frac{2T^4}{\pi^2} \sum_{\boldsymbol{\alpha}\in\Phi} \sum_{n=1}^{\infty} \frac{\cos(n\boldsymbol{\phi}\cdot\boldsymbol{\alpha}) \cos\left(n\tilde{\Omega}_{\rm I}\right)}{\left\{n^2 + 2\tilde{r}^2\left[1 - \cos\left(n\tilde{\Omega}_{\rm I}\right)\right]\right\}^2} .$$

It could be either a one-loop artifact or a genuine symmetry.

Quark action for imaginary rotating systems

Let us consider $N_f = 2$ QCD.

Introduce fermion contribution by the action

$$\mathcal{L}_{\text{quark}} = \bar{\psi}(\not{G} + m)\psi \,, \quad G_\mu = D_\mu - \Gamma_\mu \,,$$

where $\Gamma_\mu = -\frac{i}{4}\sigma^{ij}\,\omega_{\mu ij}$ is the effect of the curved space-time:

$$\sigma^{ij} = \frac{i}{2}[\hat{\gamma}^i, \hat{\gamma}^j] \,, \quad \omega_{\mu ij} = g_{\rho\sigma}\, e_i{}^\rho \left(\partial_\mu e_j{}^\sigma + \Gamma^\sigma_{\mu\nu}\, e_j{}^\nu\right) ,$$

and m is quark dynamical mass.

In our case, m is equal to the chiral condensate value.

$\not{G} + m$ spectrum of quarks

After a little calculation, we obtain

$$\gamma^\mu G_{B\,\mu} = \hat{\gamma}^1\left(\partial_r + \frac{1}{2r}\right) + \hat{\gamma}^2\frac{\partial_\theta}{r} + \hat{\gamma}^3\partial_z + \hat{\gamma}^4(\partial_\tau + igA_{B4} - \Omega_{\rm I}\partial_\theta)\,.$$

Then eigenfunctions and eigenspectra of $\mathrm{Det}\big[\not{G} + m\big]$ are

$$\xi_{n,l,s,k_\perp,k_z,\boldsymbol{\mu}}(x) \propto u_s\,|\mu\rangle\, e^{i\left[\frac{(2n+1)\pi}{\beta}\tau + \left(l+\frac{1}{2}\right)\theta + k_z z\right]}\, J_{l+1/2-s}(k_\perp r)\,,$$

$$\lambda_{n,l,s,k_\perp,k_z,\boldsymbol{\mu}} = \left[\frac{(2n+1)\pi + \boldsymbol{\phi}\cdot\boldsymbol{\mu}}{\beta} - \left(l + \frac{1}{2}\right)\Omega_{\rm I}\right]^2 + k_\perp^2 + k_z^2 + m^2\,.$$

Perturbative quark potential

Z_{quark} is obtained by $\operatorname{Det}\left(\not{G}_{\text{B}} + m\right)$.

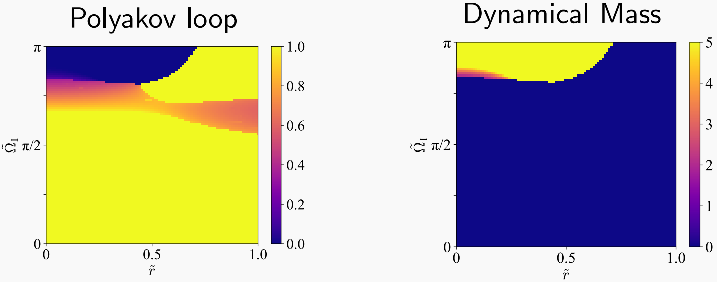
For $N_c = N_f = 2$,

$$V_f = -\frac{1}{2\pi^2\beta} \sum_{\mu=\pm 1/2} \sum_{l\in\mathbb{Z}} \int_0^\infty k_\perp \, \mathrm{d}k_\perp \int_{-\infty}^{\infty} \mathrm{d}k_z \left[J_l^2(k_\perp r) + J_{l+1}^2(k_\perp r)\right] \times \operatorname{Re}\log\left[1 + e^{i\boldsymbol{\phi}\cdot\boldsymbol{\mu} - i(l+1/2)\beta\Omega_{\text{I}}} e^{-\beta\sqrt{k_\perp^2 + k_z^2 + m^2}}\right].$$

Since the quark is a spin $1/2$ spinor, V_f is 4π periodic for θ or $\tilde{\Omega}_{\text{I}} = \beta\Omega_{\text{I}}$.

Chiral symmetry phase diagram (copy)

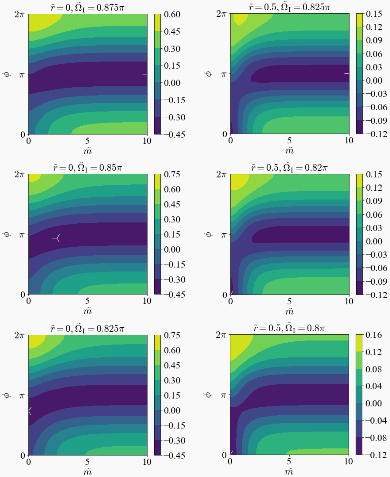
Inhomogeneous behavior of Polyakov loop and dynamical mass for $N_f = N_c = 2$ QCD.

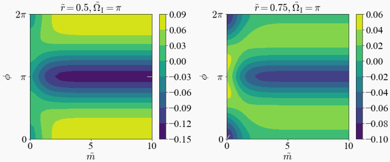


- The chiral broken phase and the perturbatively confined phase are related.
- PT behavior has changed.
- Spatial PT also appears.

PT order of $N_f = N_c = 2$ QCD

About the phase transition of $SU(2)_f$ QCD





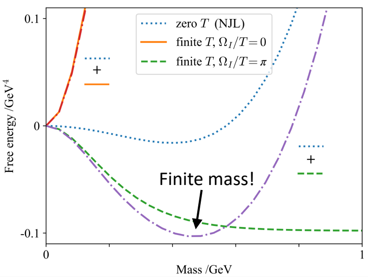
The order of PT to the PC phase changes at $r \sim 0.5$.

At large r, $\phi = m = 0$ point becomes the minimum again by first order PT.

Polyakov loop potential against m

Perturbative calculation gives the decreasing behavior of the potential.

With NJL quark-quark interaction model, the potential increases at large m.



Above the critical m, the sign of $\partial^2 V/\partial m^2$ changes.

$\rightarrow$ The minimum m becomes infinite.

With adding the NJL potential, finite minimum m is obtained.