# The real-time finite-temperature static potential: a higher order calculation

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# Motivation

- Check the robustness of our qualitative understanding of heavy quarkonium dissociation at finite temperature
  - $J/\psi$ suppression (Matsui, Satz, 1986): heavy quarkonium states sequentially melt due to screening
    - ★ The Yukawa potential supports less and less bound states when the screening mass ($\sim T$) increases
  - The potential develops an imaginary part, which is responsible for the melting rather than the screening (M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 0703: 054, 2007)
    - ★ The melting temperature $T_d$ can be parametrically estimated to be $T_d \sim -m_Q \alpha_s^{2/3} / \ln^{1/3} \alpha_s$ (M.A. Escobedo, JS, Phys. Rev. A 78, 032520 (2008))
- Provide additional physically motivated forms of the potential which may help extracting it from lattice data

# The LO potential

- If $p, m_D \ll T$ the momentum space potential reads ($G(p_0, p)$ is the Hard Thermal Loop (HTL) longitudinal gluon propagator, $m_D \sim gT$ the screening mass, $g$ the QCD coupling constant, $C_F$ a color factor)

$$V_{1\mathrm{lo}}(p) = g^2 C_F G(0, p)$$

$$G(0, p) = -\frac{1}{m_D^2 + p^2} + \frac{i\pi T m_D^2}{p\left(m_D^2 + p^2\right)^2}$$

  - When screening is important $p \lesssim m_D \Rightarrow \mathrm{Im}(V_{1\mathrm{lo}}) \gg \mathrm{Re}(V_{1\mathrm{lo}}) \Rightarrow$ No narrow resonance exist
  - $\mathrm{Im}(V_{1\mathrm{lo}}) \sim \mathrm{Re}(V_{1\mathrm{lo}}) \Rightarrow p \equiv p_d \sim (m_D^2 T)^{\frac{1}{3}} \sim g^{\frac{2}{3}} T \Rightarrow m_D \ll p_d \ll T$
  - The typical $p$ must fulfill $p \gg p_d$ for a narrow resonance to exist

# The potential beyond LO

- The LO calculation corresponds to the one-gluon exchange in HTL

$$\delta\mathcal{L}_{HTL} = \frac{1}{2} m_D^2 F^{\mu\alpha} \int \frac{d\Omega}{4\pi} \frac{k_\alpha k_\beta}{-(k.\partial)^2} F^{\mu\beta} + m_f^2 \bar{\psi}\gamma^\mu \int \frac{d\Omega}{4\pi} \frac{k_\mu}{k.\partial}\psi$$

$$k = (1, \hat{\mathbf{k}}), \qquad m_D^2 = g^2 T^2 (N_c + N_f/2)/3, \qquad m_f^2 = g^2 T^2/16$$

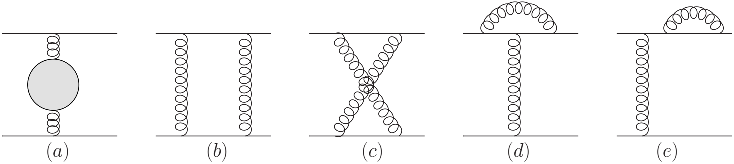
(Braaten, Pisarsky, 92)

  - HTL $\sim$ Integrating out the scale $T$ $\sim$ One-loop selfenergies at LO in the $p/T$ expansion

- Power corrections to HTL $\sim$ One-loop selfenergies at NLO in the $p/T$ expansion (Manuel, JS, Stetina, 16; Carignano, Manuel, JS, 17; Carrington, Carignano, JS, 20; Ekstedt, 23; Gorda, Paatelainen, Säppi, Seppänen, 23)

# The potential beyond LO

- We work in the Coulomb gauge. The relevant diagrams are




  - Gluon lines in (b)-(e) = HTL longitudinal time-ordered gluon propagators
  - Self-energy blob (a):
    - ★ Longitudinal gluon self-energy calculated in HTL
    - ★ Power correction calculated in QCD
- The hierarchy $m_D \ll p \ll T$ produces enormous simplifications $k \sim$ loop momentum
  - (a), the longitudinal gluon self-energy calculated in HTL is dominated by $k \sim p \Rightarrow$ HTL propagators and vertexes reduce to QCD ones up to $m_D^2/p^2$ corrections
  - (b)-(e) are dominated by $k \sim m_D \Rightarrow$ can be expanded in $k/p$, $m_D/p$

# The potential beyond LO

- The fully expanded result reads

$$V_{2,\mathrm{exp}}(p) = -\frac{g^4 C_F N_c T}{16\pi m_D p^2}\left(1-\frac{3\pi^2}{16}+\frac{4\pi m_D}{p}+\frac{m_D^2}{p^2}\left(\frac{5\pi^2}{24}-\frac{4}{3}\right)\right)$$
$$-i\frac{g^4 C_F T^2}{16p^4}\left(N_c\left(\frac{56}{3\pi}-\left(1-\frac{3\pi^2}{16}\right)\frac{m_D}{p}\right)-\frac{4}{\pi}\left(N_c-\frac{N_f}{2}\right)\frac{p}{T}\right)$$

  - Let $p \sim g^a T$, then the formula above holds up to corrections $g^2$ ($g^{3a}$,$g^{2-a}$) for the real (imaginary) part ($1/3 < a < 2/3$)
- We define the damped approximation by keeping factors $1/(p^2 + m_D^2)$ in the gluon propagators unexpanded
  - Expected to be more realistic for $m_D \lesssim p$
  - Reproduces the expanded formula above

# The potential beyond LO

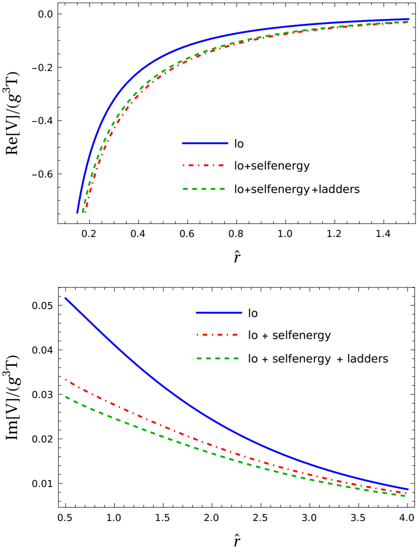
- Upon Fourier transforming, the coordinate space potential in the damped approximation reads $(\hat{r} = r m_D)$

$$\mathrm{Re}[V_2] = \frac{g^4 N_c C_F T}{64\pi^2 \hat{r}} \left\{ 8\left(I_2(\hat{r}) - I_1(\hat{r})\right) + \frac{e^{-\hat{r}}}{16}\left(3\pi^2 - 16 + \frac{\hat{r}}{6}\left(16 - \pi^2\right)\right) \right\}$$

$$i\mathrm{Im}[V_2] = -i \frac{g^3 C_F\, T}{16\pi^2 \hat{m}_D} \left\{ \frac{3\pi^2 - 16}{32\,\hat{r}} I_2(\hat{r}) + \frac{7}{3}\, N_c e^{-\hat{r}} - \frac{2g\hat{m}_D}{\pi\hat{r}} \left(N_c - \frac{N_f}{2}\right) \left(I_1(\hat{r}) - I_2(\hat{r})\right) \right\}$$

$$I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \frac{\sin(\hat{p}\hat{r})}{(\hat{p}^2 + 1)^j} \quad , \quad m_D = gT\hat{m}_D$$

# The potential beyond LO ($g = 1.8$)



# The potential beyond LO

- The coordinate space potential for $r \ll 1/m_D$ gets contributions from $p \sim 1/r$ and $p \lesssim m_D$
  - The contribution from $p \sim 1/r$ can be obtained by Fourier transforming $V_{2,exp}$
  - The contribution from $p \lesssim m_D$ is a polynomial in $r^2$, since the $\exp(i\vec{p}\vec{r})$ in the Fourier transform can be expanded
  - The damped approximation reshuffles part of the $p \lesssim m_D$ contribution out of the polynomial

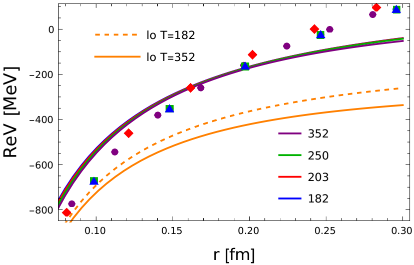
# Comparison with lattice data

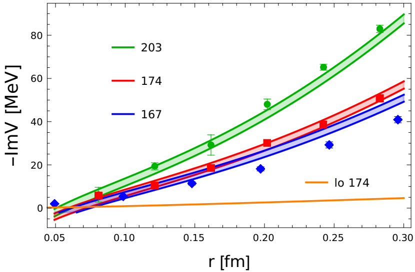
- $g$ small $\Rightarrow$ short distance observables
  - Data for the potential up to 0.3 fm (Bazavov, Hoying, Kaczmarek, Larsen, Mukherjee, Petreczky, Rothkopf, Weber, 23)$\equiv$ [1]
  - Mass shift and decay width of $\Upsilon(1s)$ (Larsen, Meinel, Mukherjee, Petreczky, 20)$\equiv$ [2]
- We fix $g$ from the best fit to $T = 0$ data of the first reference ($g = 1.8$)
- The soft contribution to the coordinate space potential is accounted for by including

$$\delta\mathrm{Re}(V) = q_0\, g^3 T \quad , \quad \delta\mathrm{Im}(V) = i_0\, g^3 T + i_2\, g^5 r^2 T^3$$

and fitting $q_0$, $i_0$ and $i_2$ to the lattice data

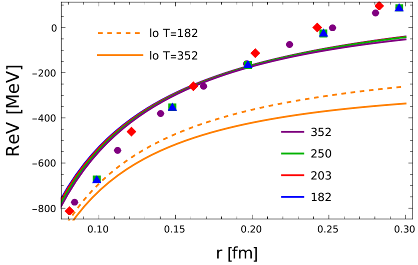
# Comparison with [1]

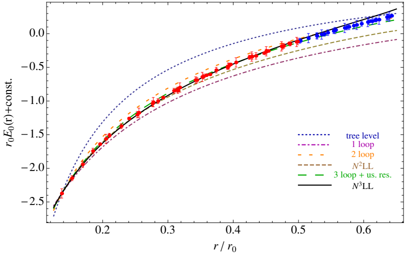



- Re($V$) depends very little on T like the lattice data
  - The slight difference in the shape may be accounted for by higher order T-independent corrections
- The dramatic improvement in Im($V$) is due to the inclusion of the soft contribution $(q_0, i_0, i_2) = (0.049, -0.021 \pm 0.002, 0.205 \pm 0.001)$

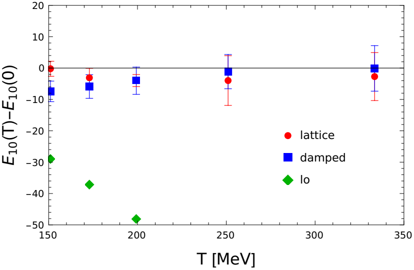
# Comparison with [1]

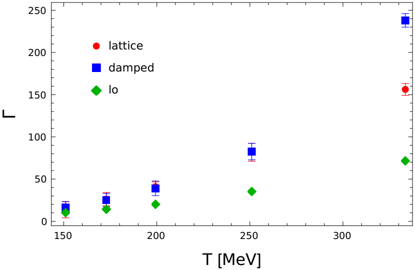



- The rhs plot shows the $T = 0$ potential at different orders of perturbation theory, $r_0 = 0.468$ fm (Bazavov, Brambilla, Garcia i Tormo, Petreczky, Soto, Vairo, 12)
- The slightly different shape between our results and data in the lhs plot is similar to the one between tree level and data in the rhs plot $\Rightarrow$ It may be fixed by a higher order calculation

# Comparison with [2]




- Dramatic (considerable) improvement in the binding energy (decay width)
- The inclusion of soft contributions is crucial $(q_0, i_0, i_2) = (0.078 \pm 0.004, -0.026 \pm 0.009, 0.053 \pm 0.002)$
- The value of $i_2$ is not consistent with the one obtained from [1]
- The same size for $(q_0, i_0, i_2)$ is expected. This is fulfilled except for $i_2$ from [1]

# Dissociation temperature

- We define the dissociation temperature ($T_d$) as the temperature for which the binding energy equals the decay width
  - At LO we obtain: $T_d = 193.2$ MeV
  - Damped approximation, fit to [1]: $T_d = 151.8 \pm 1.2$ MeV
  - Damped approximation, fit to [2]: $T_d = 225 \pm 10$ MeV
- Fit to [1] produces a very low $T_d$ and an unnatural value of $i_2$ $\Rightarrow$ there might be a problem with the data

# Conclusions

- We have calculated higher order corrections to the momentum space potential, both to the real and to the imaginary parts, when $m_D \ll p \ll T$
- We have proposed an approximation (damped) that partially includes soft contributions
- We have pointed out that the soft contributions $p \lesssim m_D$ to the coordinate space potential are universal for $r \ll 1/m_D$, and can be described by a polynomial in $r^2$ (up to logs)
- We get a reasonable description of lattice data at short distances
  - We have been able to identify an inconsistency between two sets of lattice data
- Our results provide additional inputs for the Bayesian methods to obtain the potential from Euclidean lattice data.