Possible scenario of dynamical chiral symmetry breaking in the instanton liquid

Reference: YS, Jido, Phys. Rev. D 110, 014037 (2024)

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Introduction

- **Vacuum of strong interaction**
  - non-trivial structure
    - confinement
    - dynamical chiral symmetry breaking (χSB)

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- **non-trivial structure $\approx$ topological structure**
  - EM: Abelian gauge theory
    - no topological gauge configuration
  - Strong: non-Abelian gauge theory
    - topological configurations such as instantons and sphalerons

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- **Vacuum of strong interaction**
  - non-trivial structure
    - confinement
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- **non-trivial structure ≈ topological structure**
  - EM: Abelian gauge theory
    - no topological gauge configuration
  - Strong: non-Abelian gauge theory
    - topological configurations such as instantons and sphalerons

- **difficult to detect the structure experimentally**
  - chiral magnetic effect in QGP by RHIC
  - light scalar and pseudoscalar meson mass spectrum

Introduction

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A. **There might be.** We focus on:

- **topological structure**
- **chiral ($\mathrm{U}(1)_A$) anomaly**
- **chiral symmetry breaking (χSB)**

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Q. Are there any other approaches to reveal the QCD vacuum structure?

A. **There might be.** We focus on:

- **topological structure**
- **chiral ($\mathbf{U(1)}_A$) anomaly**
- **chiral symmetry breaking (χSB)**

- **topological structure relates to chiral ($\mathbf{U(1)}_A$) anomaly**
  - cf. the Atiyah-Singer index theorem

- **chiral ($\mathbf{U(1)}_A$) anomaly** has long been discussed in relation to χSB by chiral effective theories

- from the discussion of χSB by chiral effective theories, we advance our understanding of the QCD vacuum structure

Outline

- **Chiral symmetry breaking (χSB) in the chiral effective theories**
- **Chiral symmetry breaking (χSB) in the instanton liquid [our work]**
- **Summary**

Introduction to χSB in chiral effective theories

[1] Y. Nambu and G. Jona-Lasinio, Phys. Lev. **122**, 345 (1961)

- **NJL** introduces a sufficiently large coupling g_S to dynamically break chiral symmetry

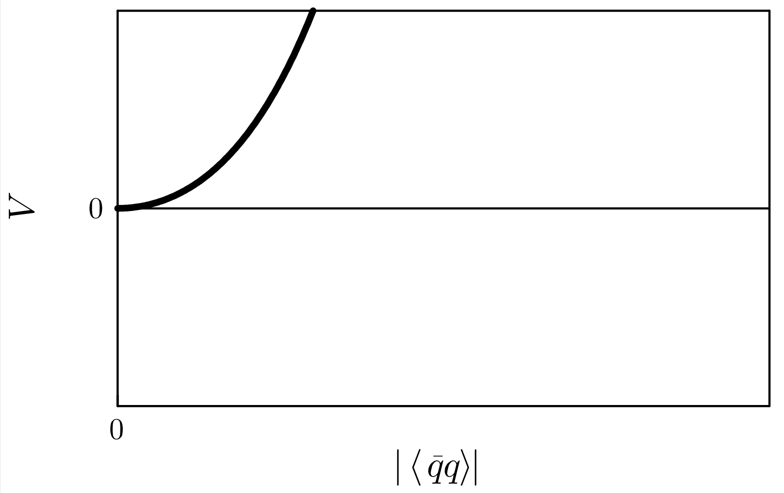
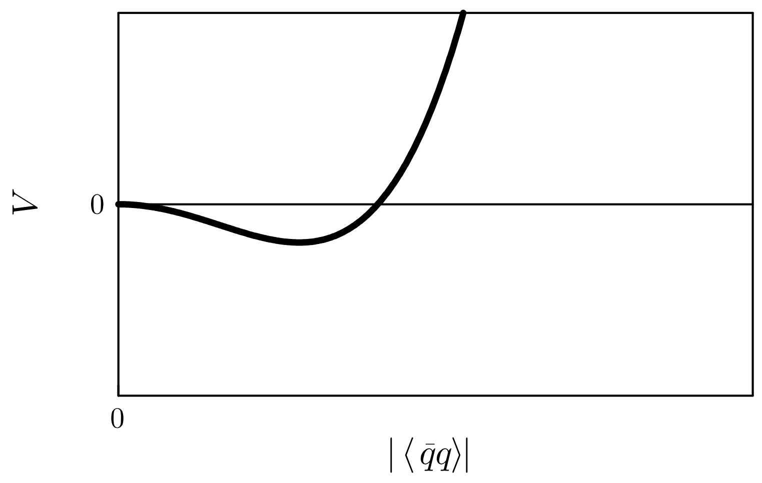
NJL model:

$$\vec{\pi} = \bar{q}\vec{\tau}\gamma_5 q$$

$$\sigma = \bar{q}q$$

$$\mathcal{L}_{\text{int}} = \sum_{a=0}^{8} \frac{g_S}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2\right]$$

Example: NJL model with chiral limit

$G_S = g_S/g_S^{\text{crit.}}$	$G_S < 1$	$G_S > 1$
description	not broken	ordinary breaking
$V(\langle\bar{q}q\rangle)$		

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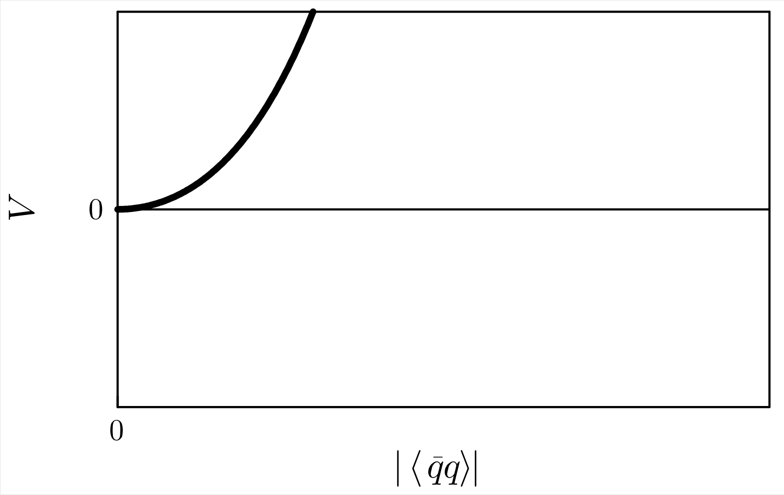
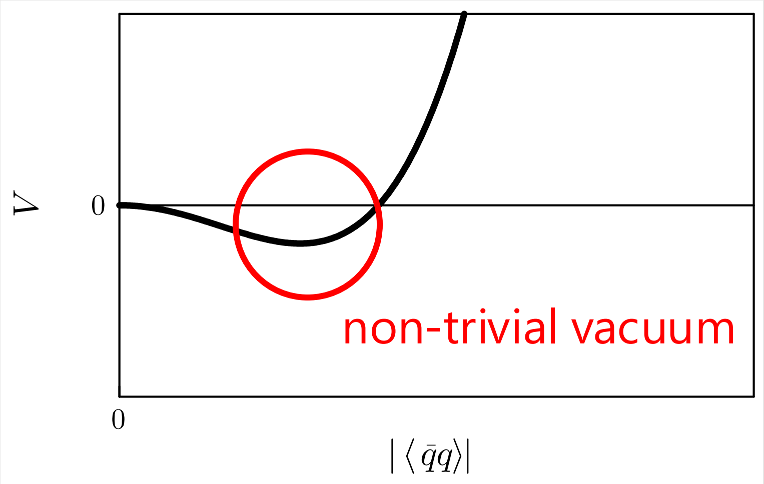
$V(\langle\bar{q}q\rangle)$		

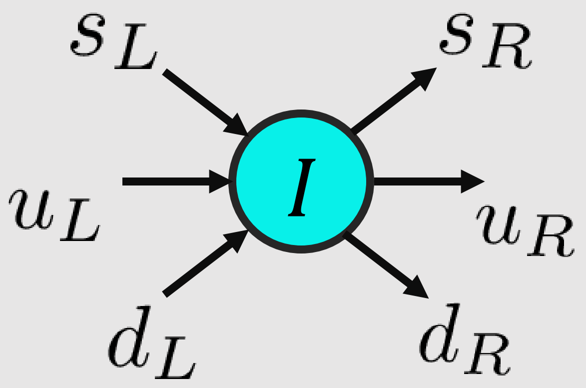
Introduction to χSB in chiral effective theories

- **chiral ($\mathbf{U(1)_A}$) anomaly** has been introduced by interaction to violate the $U(1)_A$ symmetry in chiral effective theories

$$\propto \{\det[\bar{q}_i(q-\gamma_5)q_j] + \text{H.c.}\}$$

Kobayashi-Maskawa-'t Hooft (KMT) term

$\subset$



't Hooft vertex induced by instanton

[2] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976)
[3] M. Kobayashi, H. Kondo and T. Maskawa, Prog. Theor. Phys. 45, 1955 (1971)
[4] E. Shuryak, "Nonperturbative Topological Phenomena in QCD and Related theory" (2021)

Introduction to χSB in chiral effective theories

[5] S. Kono, et al., PTEP **2021**, 093D02 (2021)

- strength of the chiral ($\mathrm{U}(1)_A$) anomaly **changes χSB pattern**

$$\mathcal{L}_{\mathrm{int}} = \sum_{a=0}^{8} \frac{g_S}{2} \left[(\bar{q}\lambda_a q)^2 + (\bar{q} i \gamma_5 \lambda_a q)^2 \right] + \frac{g_D}{2} \left[\det(\bar{q}_i (1-\gamma_5) q_j + \mathrm{H.c.} \right]$$

chiral ($\mathrm{U}(1)_A$) anomaly

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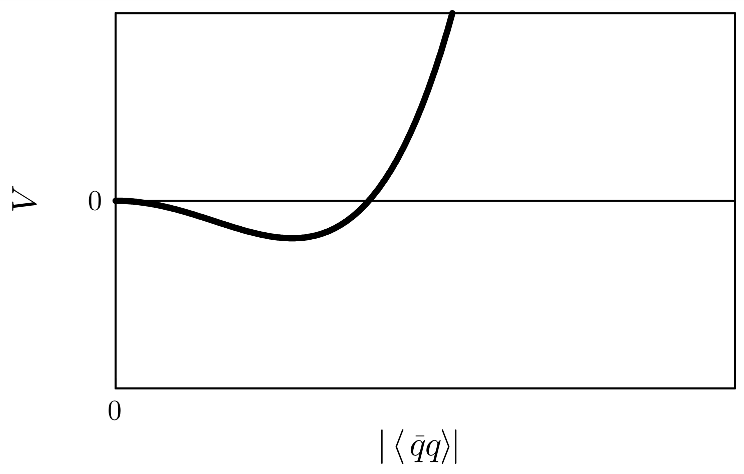
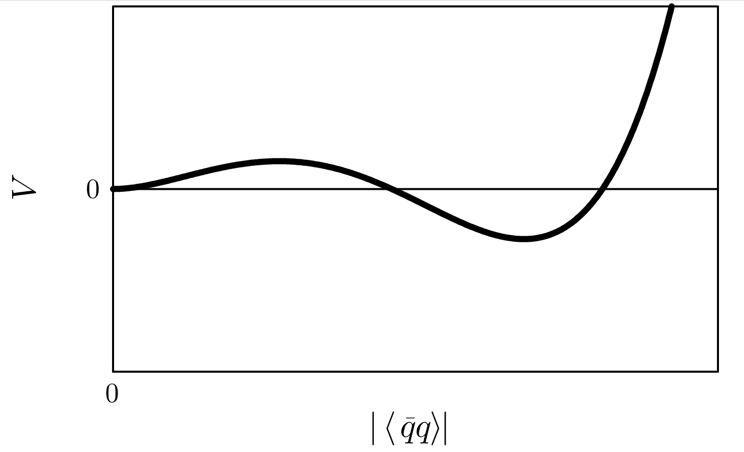
$V(\langle \bar{q}q \rangle)$		

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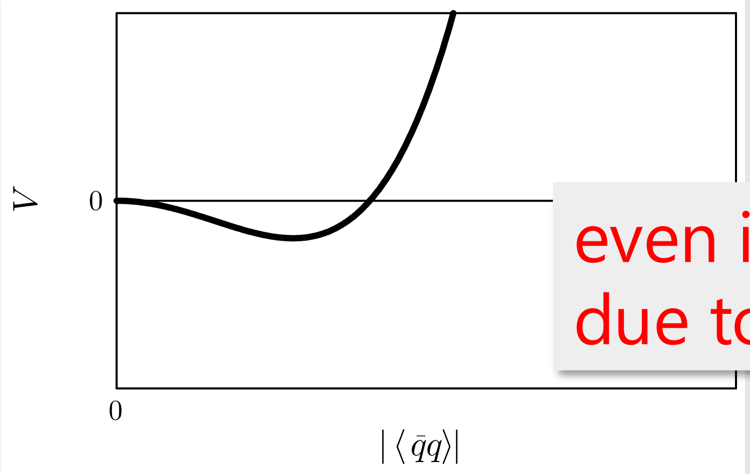
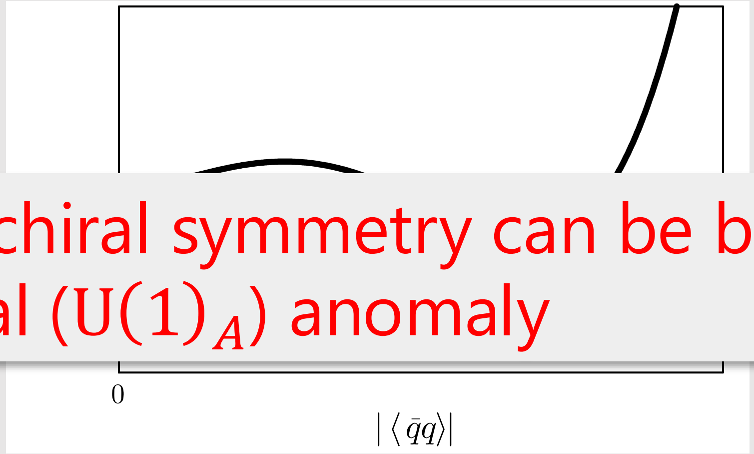
$V(\langle \bar{q}q \rangle)$		

even if $G_S < 1$, chiral symmetry can be broken due to the chiral $(\mathrm{U}(1)_A)$ anomaly

Anomaly driven χSB may link to physical observables

[5] S. Kono, et al., PTEP **2021**, 093D02 (2021)

- show that **size of the sigma mass** depends on the breaking pattern

ordinary breaking $\Rightarrow m_\sigma > 800\ \mathrm{MeV}/c^2$

anomaly driven breaking $\Rightarrow m_\sigma < 800\ \mathrm{MeV}/c^2$

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ordinary breaking ⇒ $m_\sigma > 800\ \mathrm{MeV}/c^2$

anomaly driven breaking ⇒ $m_\sigma < 800\ \mathrm{MeV}/c^2$

- here, the sigma is introduced as chiral partner of pion (chiral sigma)
- to compare the f0(500) resonance, we need to know its composition

Generalization of determination procedure for χSB

[6] YS and D. Jido, PRD **110**, 014037 (2024)

- so far, in chiral effective theories,
  χSB and chiral ($\mathrm{U}(1)_A$) anomaly effect are discussed
  using model specific coupling constants G_S

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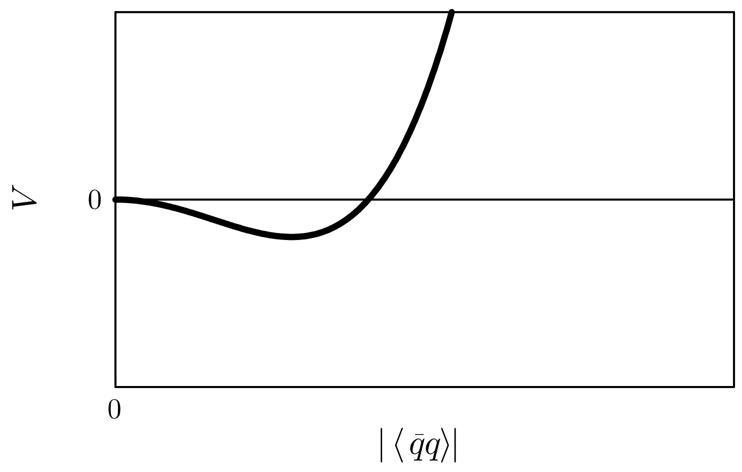
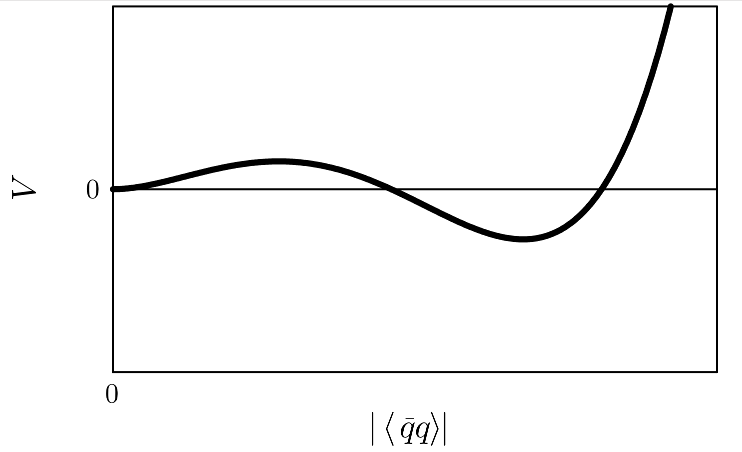
$V(\langle\bar{q}q\rangle)$		

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- so far, in chiral effective theories, **χSB and chiral ($\mathrm{U}(1)_A$) anomaly effect** are discussed using model specific coupling constants G_S
- **use the curvature of the effective potential at the origin** to determine the breaking pattern

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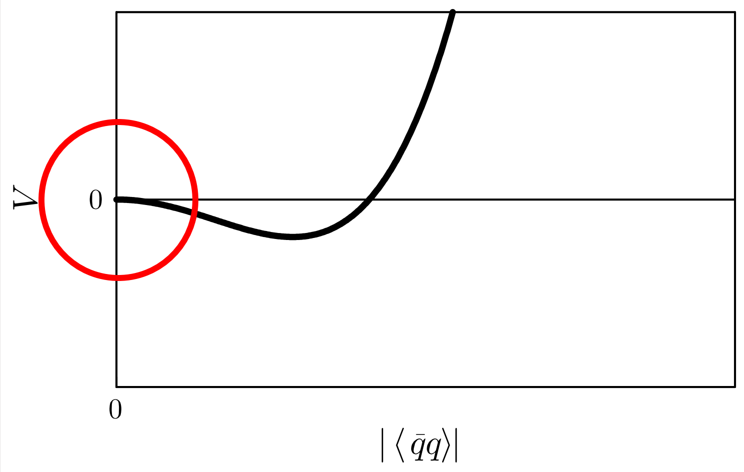
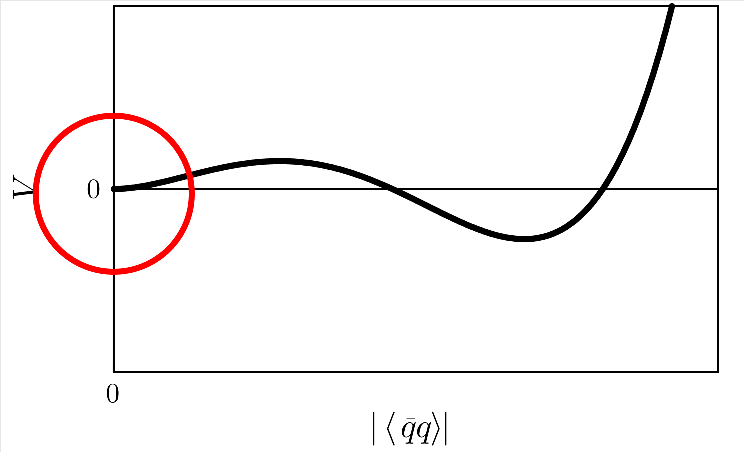
$V(\langle\bar{q}q\rangle)$		

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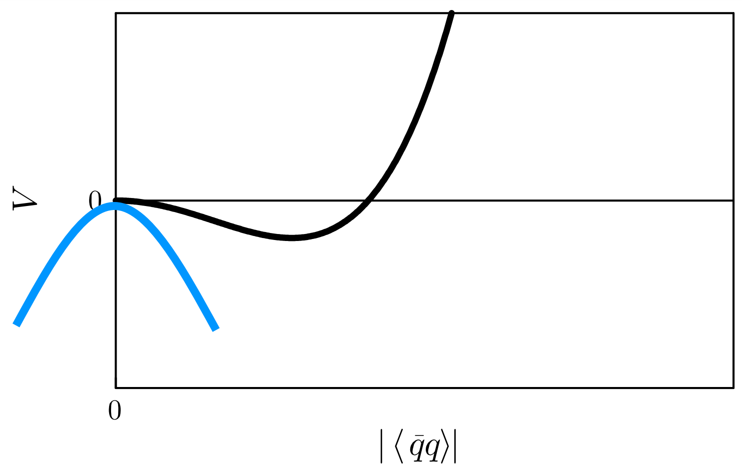
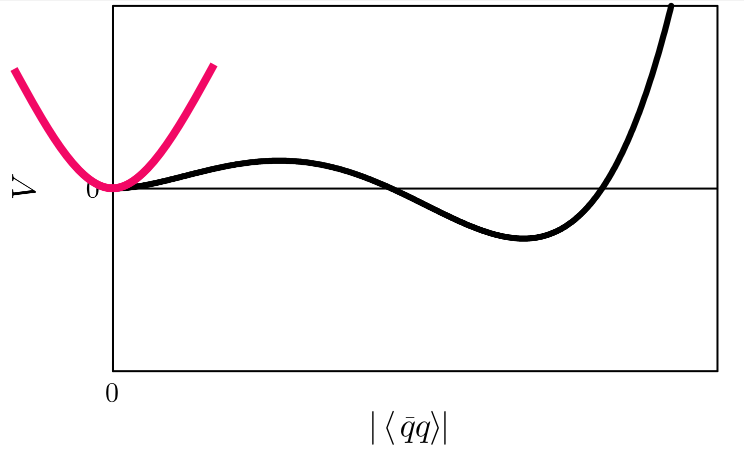
Example: SU(3) NJL model including chiral ($\mathrm{U}(1)_A$) anomaly with chiral limit

$G_S = g_S/g_S^{\text{crit.}}$, $g_D \neq 0$	$G_S > 1$	$G_S < 1$
description	ordinary breaking	anomaly driven breaking
curvature: $C_2 \equiv \left.\dfrac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right\vert_{\langle \bar{q}q \rangle = 0}$	(plot, $G_S > 1$)	(plot, $G_S < 1$)
curvature C_2	$C_2 < 0$	$C_2 > 0$





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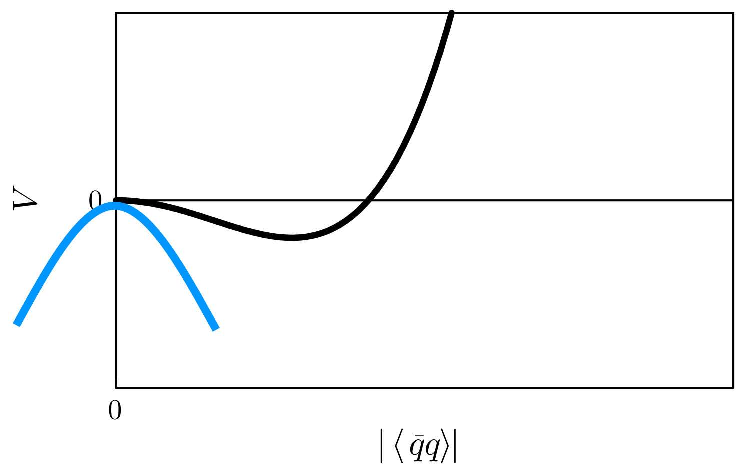
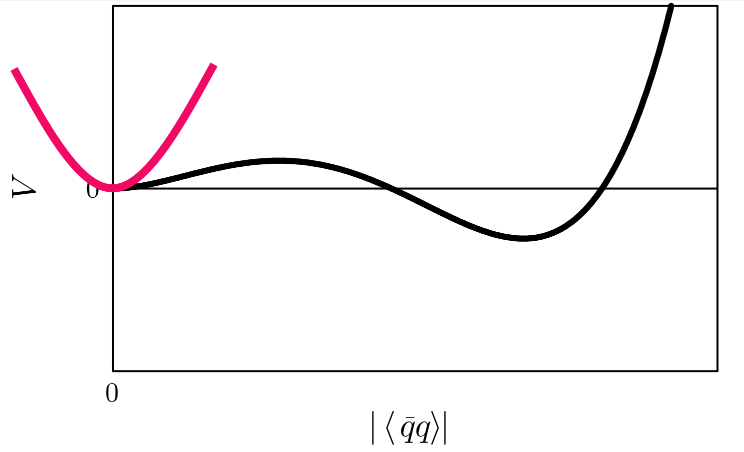
curvature: $C_2 \equiv \left.\dfrac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right\vert_{\langle \bar{q}q \rangle = 0}$		
curvature C_2	$C_2 < 0$	$C_2 > 0$

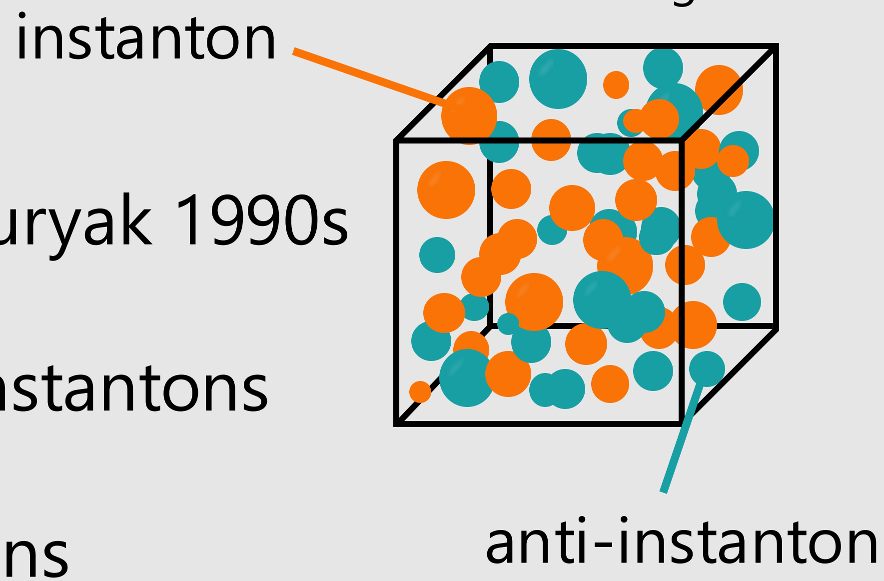
→ take $C_2 < 0$ and $C_2 > 0$ our definitions of the determination procedure for the ordinary and anomaly driven breaking patterns

Model

[7] T. Schafer and E. Shuryak, Rev. Mod. Phys. **70** 323 (1998).

- **I**nteracting **I**nstanton **L**iquid **M**odel (IILM), E. Shuryak 1990s
- allows us to treat the QCD vacuum
  as statistical mechanics of instantons and anti-instantons
- described by Euclidean QCD partition function
  that is saturated by instantons and anti-instantons

*schematic image of IILM



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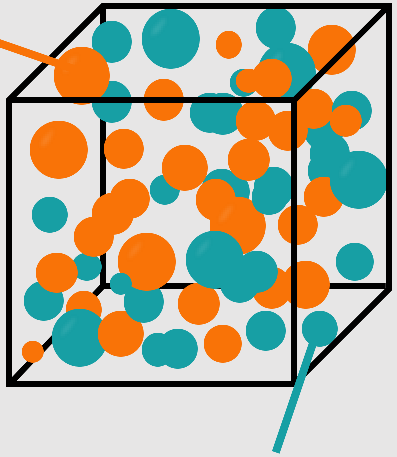


IILM partition function:

$$Z_{\mathrm{IILM}} = \frac{1}{N_+!N_-!}\int\left(\prod_{i=1}^{N_+ + N_-} d\Omega_i\, f(\rho_i)\right)\exp(-S_{\mathrm{int}})\prod_{f=1}^{N_f}\mathrm{Det}\left(\gamma_\mu D_\mu + m_f\right)$$

Instanton-instanton interaction (S_{int})

Instanton-quark interaction ($\mathrm{Det}(\gamma_\mu D_\mu + m_f)$)

Semiclassical Instanton amplitude ($f(\rho_i)$)

Collective coordinates of instantons ($d\Omega_i$)

Simulation detail

[6] YS and D. Jido, PRD **110**, 014037 (2024)

Model action:

full / quench

$$S_{\text{eff}} = -\sum_{i=1}^{N} \log\left[f(\rho_i)\right] + S_{\text{int}} - \sum_{f=1}^{N_f} \log\left[\text{Det}\left(\gamma_\mu D_\mu + m_f\right)\right]$$

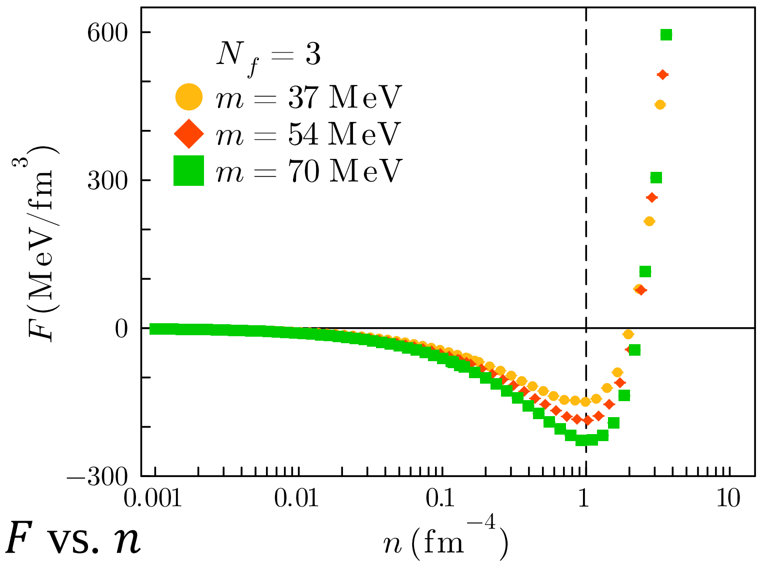
Setup:

	flavor	m_f	# of $I\&\bar{I}$ (fixed)	# of conf.
full	$N_f = 3$	37 MeV~70 MeV	$32 = 16 + 16$	5000
quench	$N_f = 0$	2.8 MeV~28 MeV		

Calculated quantities:

	description
$F = -\ln Z_{\text{IILM}} / V_4$	vacuum energy at zero temp. corresponding to eff. potential
$\langle \bar{q} q \rangle$	quark condensate w/o free contrib. per flavor

Result: F vs. n, $\langle \bar{q}q \rangle$ vs. n and F vs. $\langle \bar{q}q \rangle$ [full]



F vs. n

quantities are obtained as a function of the instanton density n

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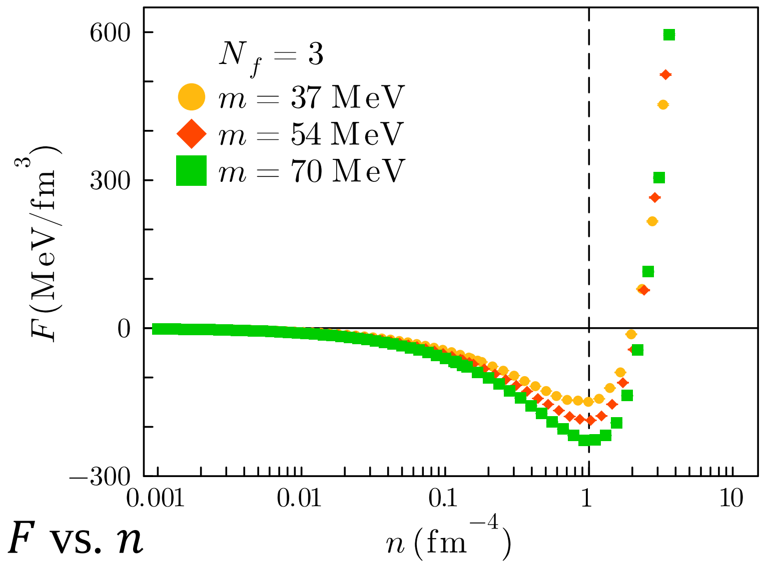

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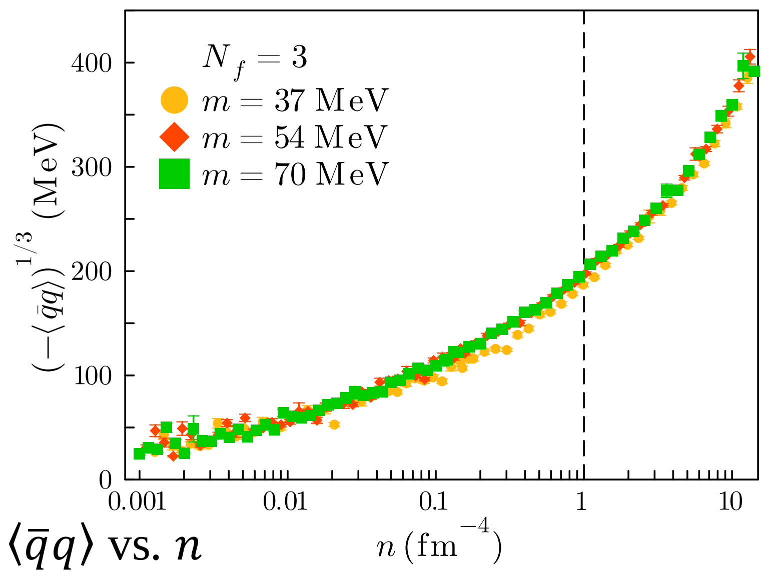

$\langle \bar{q}q \rangle$ vs. n

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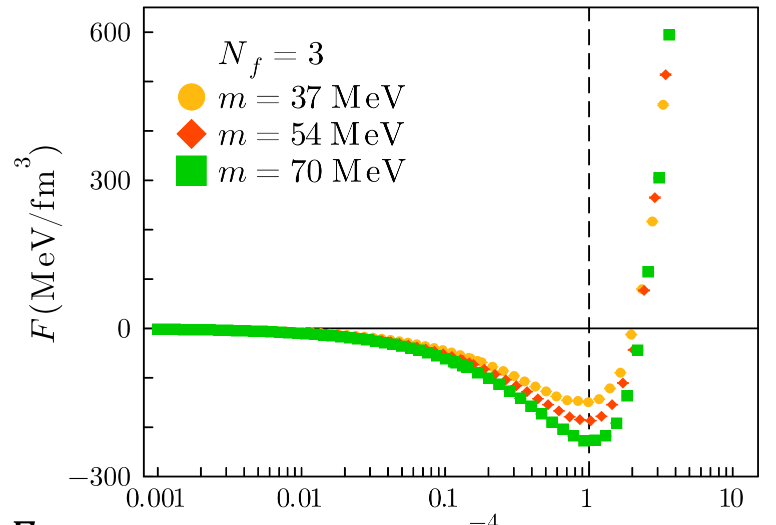

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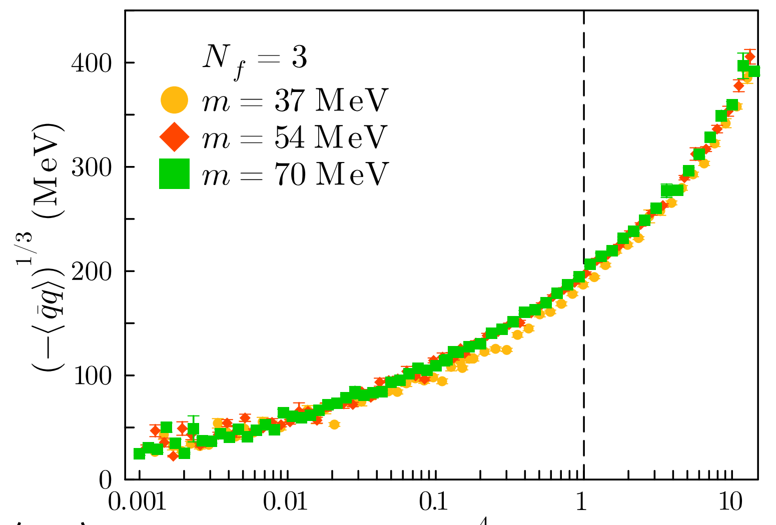
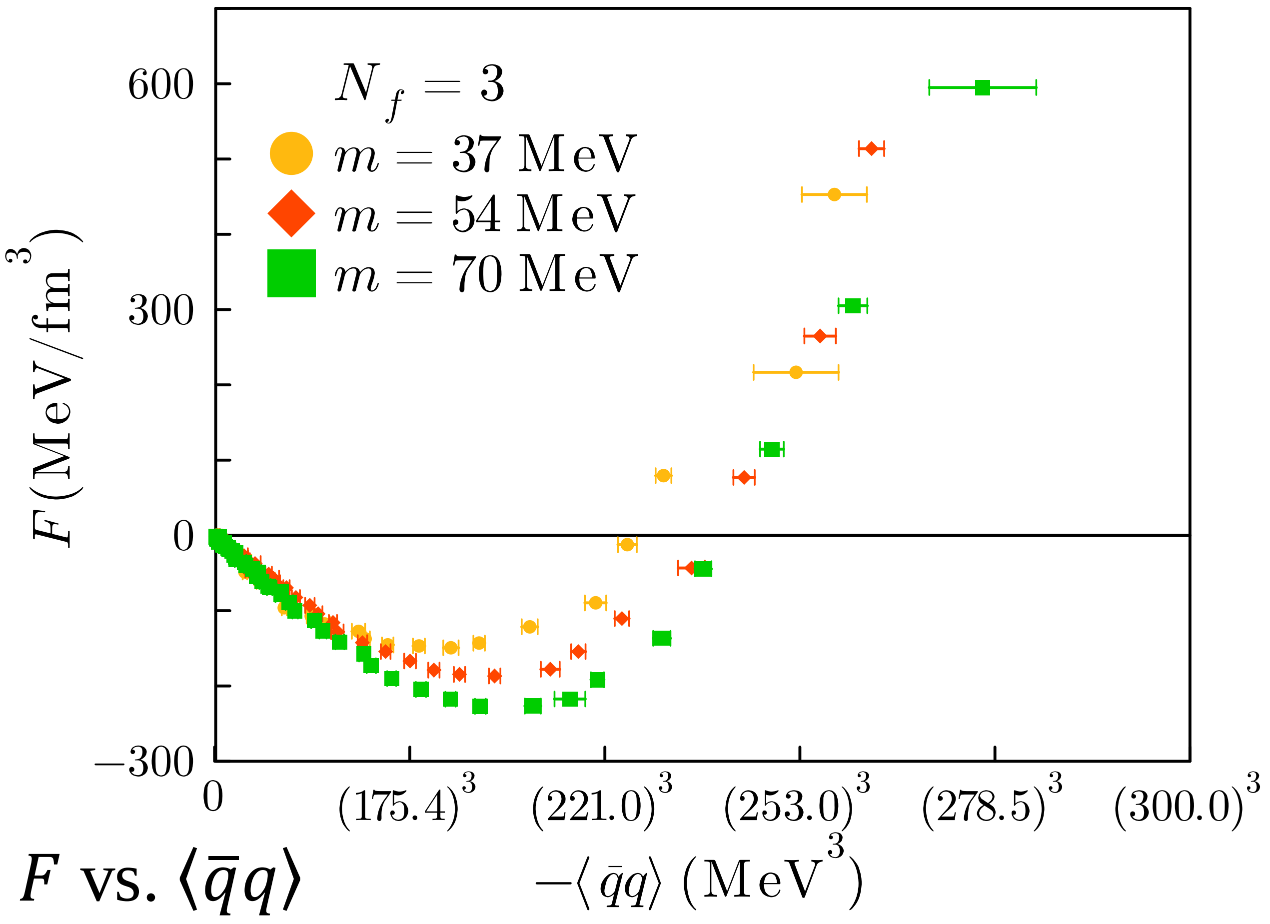


$\langle\bar{q}q\rangle$ vs. n

combine



F vs. $\langle\bar{q}q\rangle$

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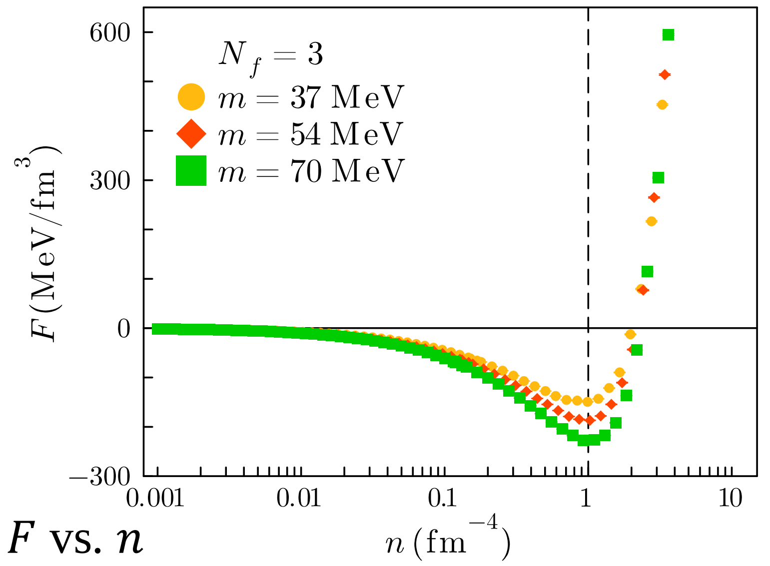

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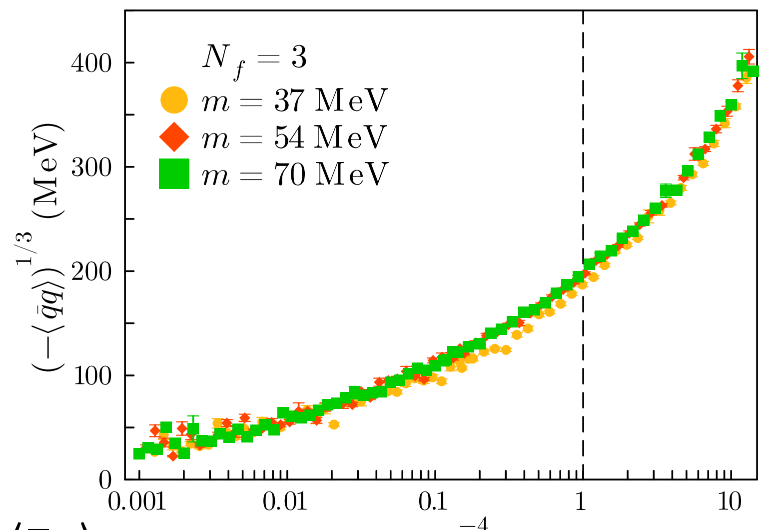
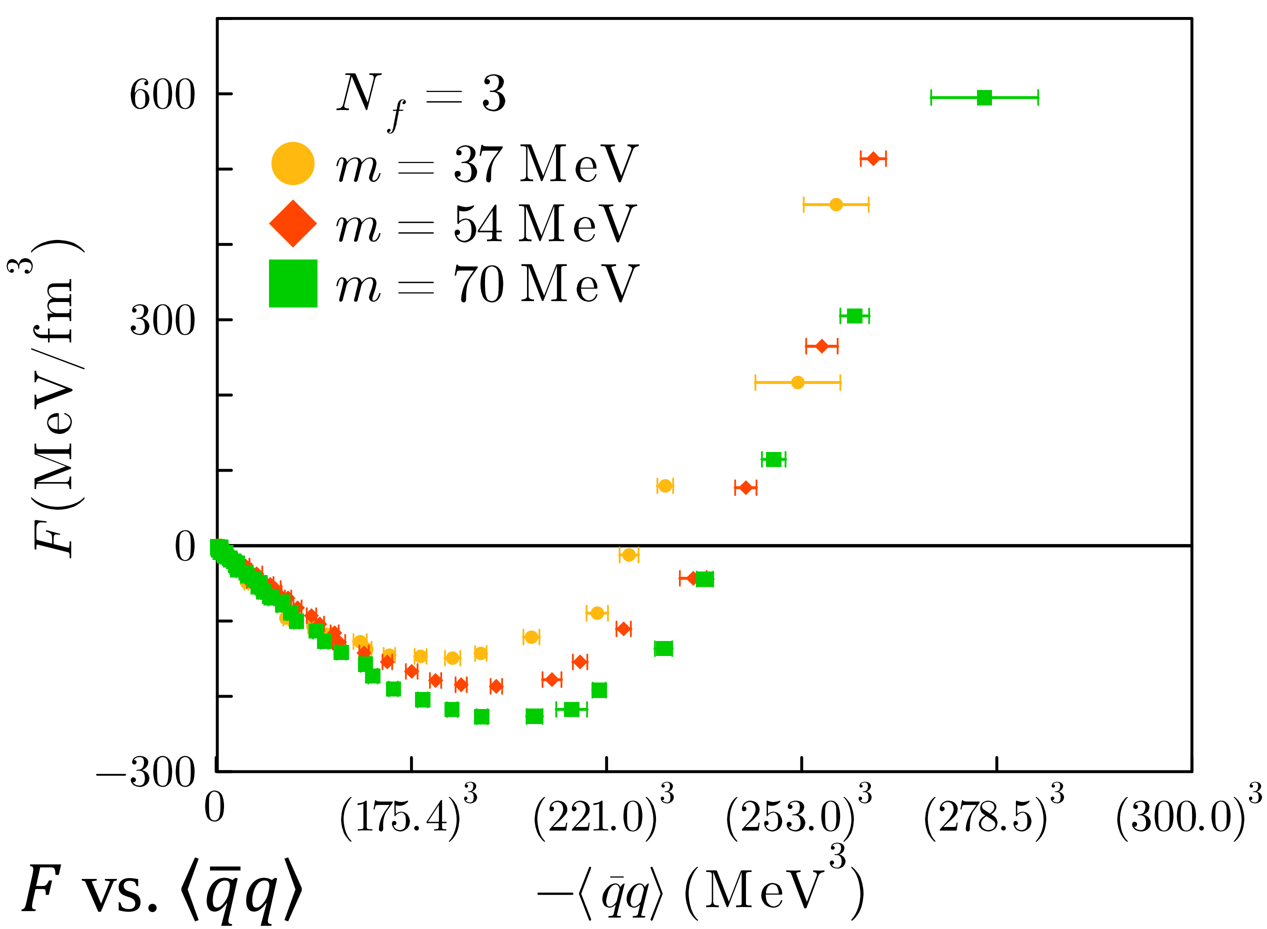


$\langle\bar{q}q\rangle$ vs. n

combine

F has min. at $\langle\bar{q}q\rangle \neq 0 \rightarrow \chi$SB



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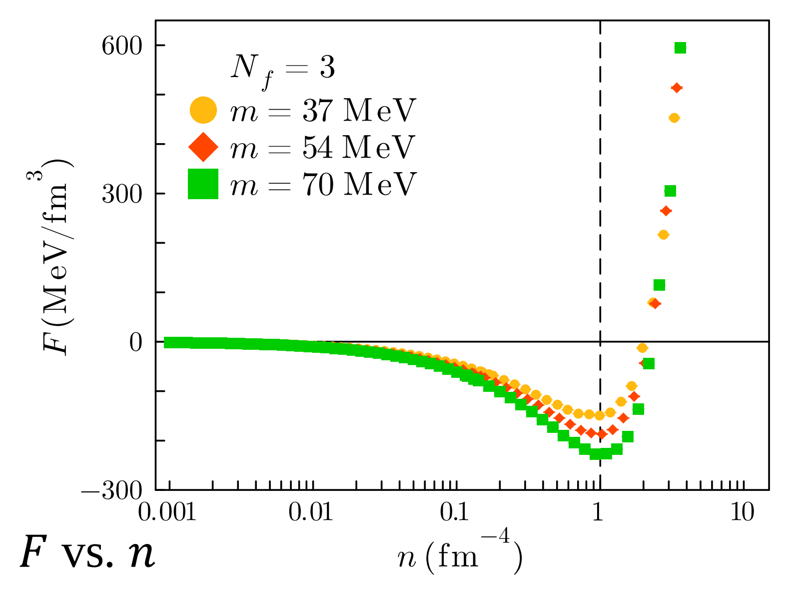

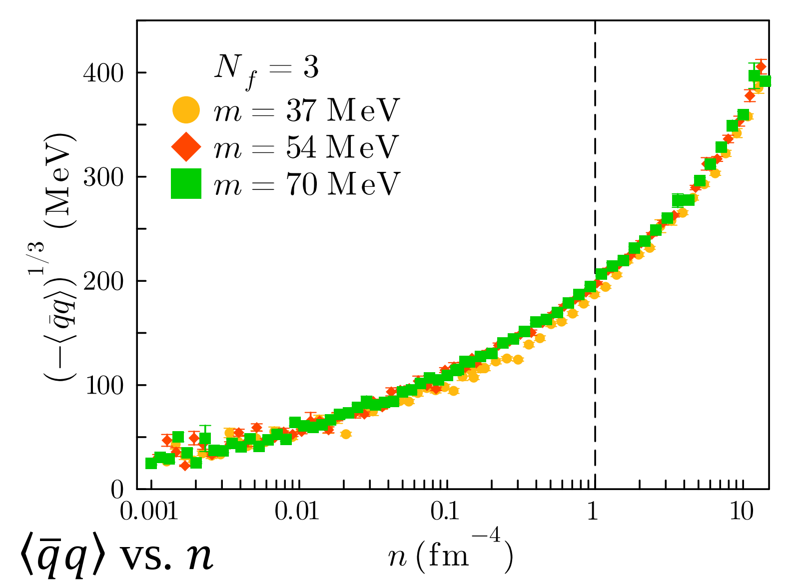

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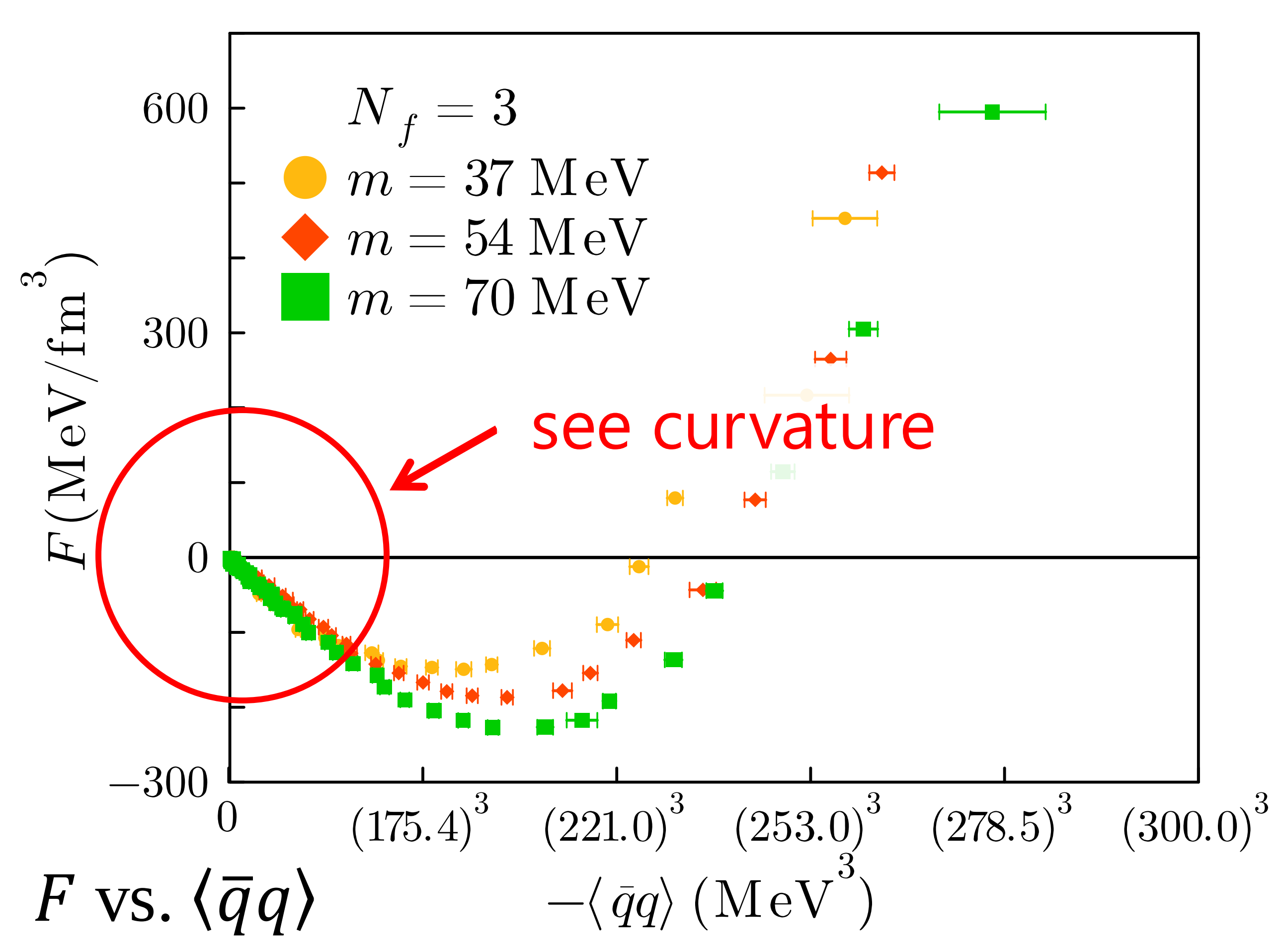
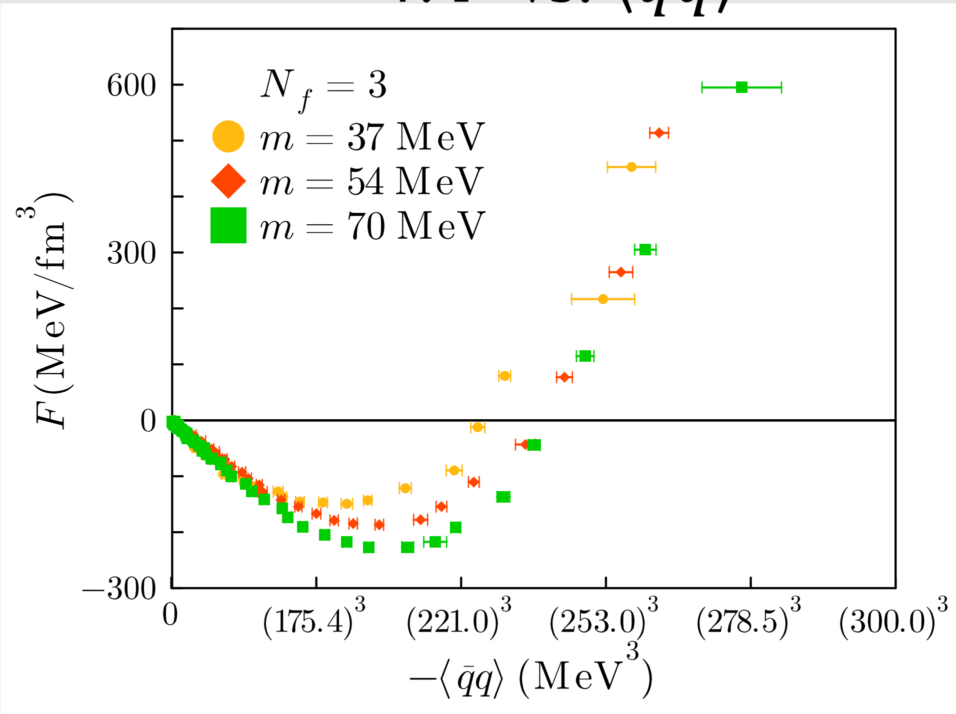


[7] YS and D. Jido, PRD **110**, 014037 (2024)

Result: Curvature [full]

1. F vs. $\langle \bar{q}q \rangle$



2. fit to polynomial

$$F(\langle \bar{q}q \rangle) = C_0 + C_1 \langle \bar{q}q \rangle + C_2 \langle \bar{q}q \rangle^2 + \cdots + C_K \langle \bar{q}q \rangle^K$$

slope $\propto m_q$
∵ finite quark mass

curvature
= what we want

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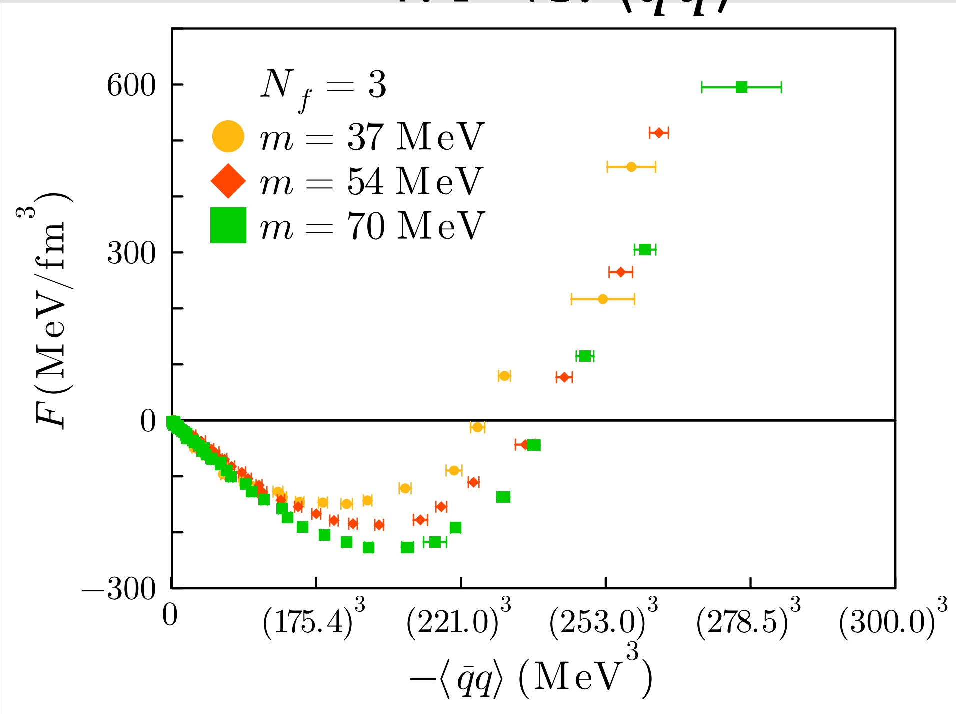

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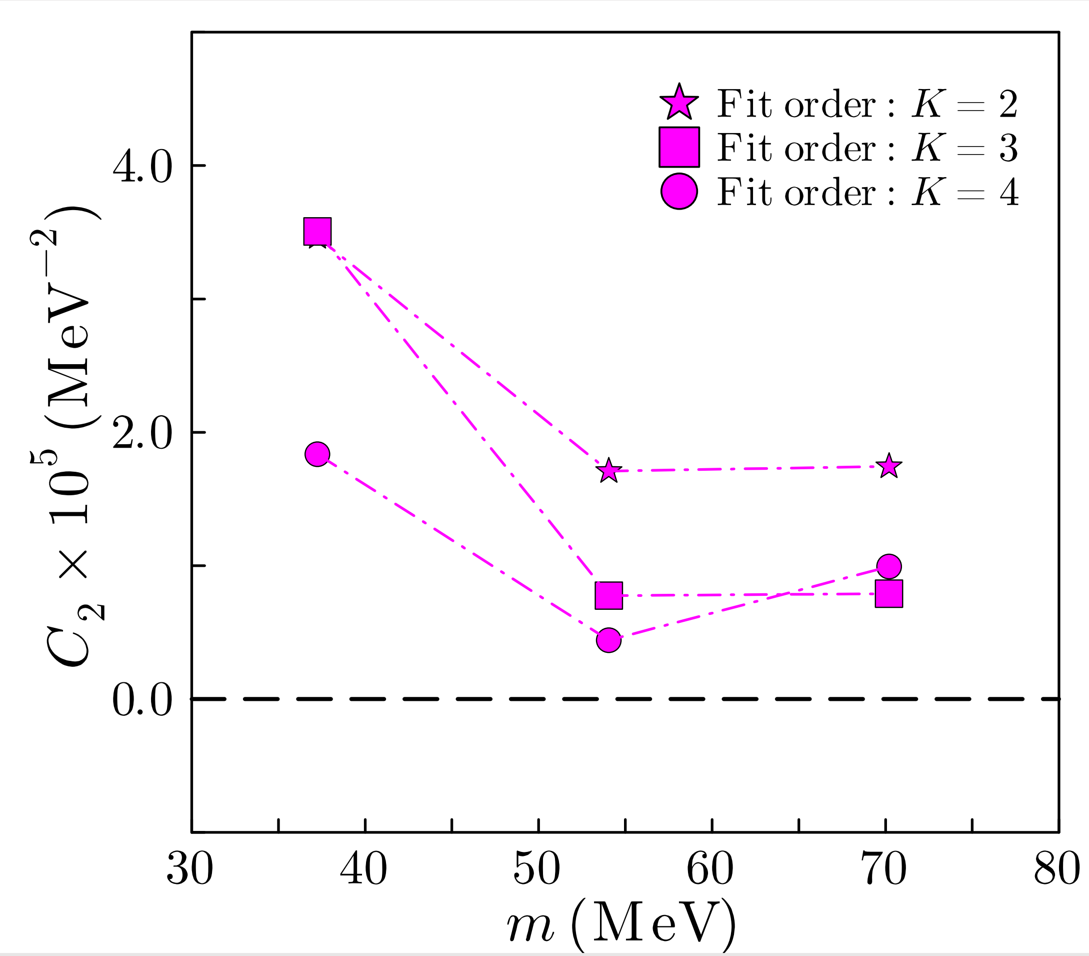

positive curvature ($C_2 > 0$)
in wide quark mass ranges

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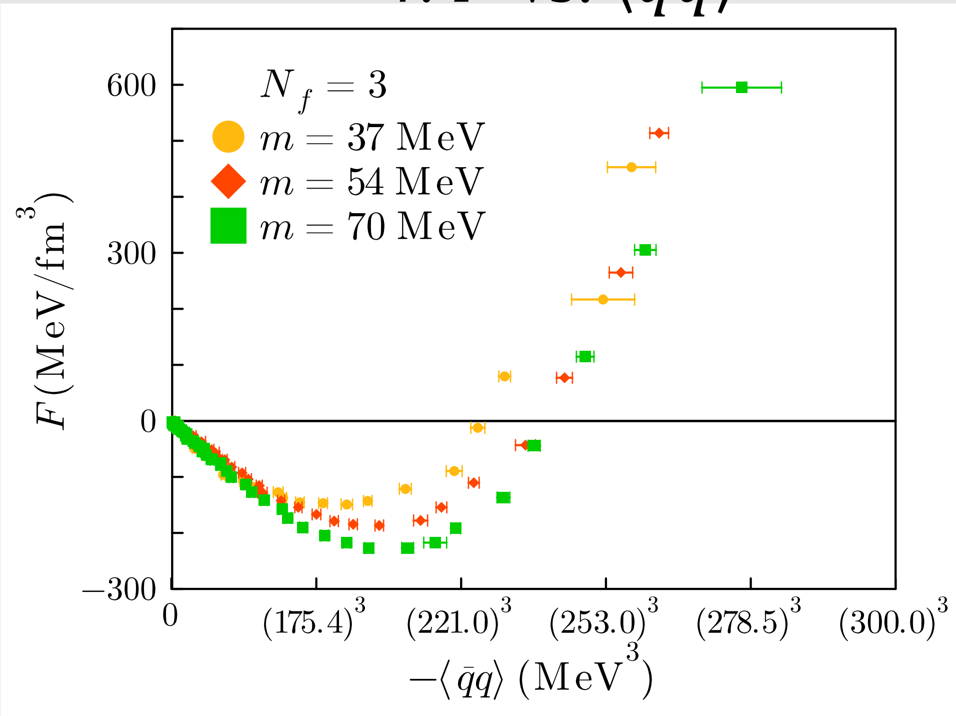

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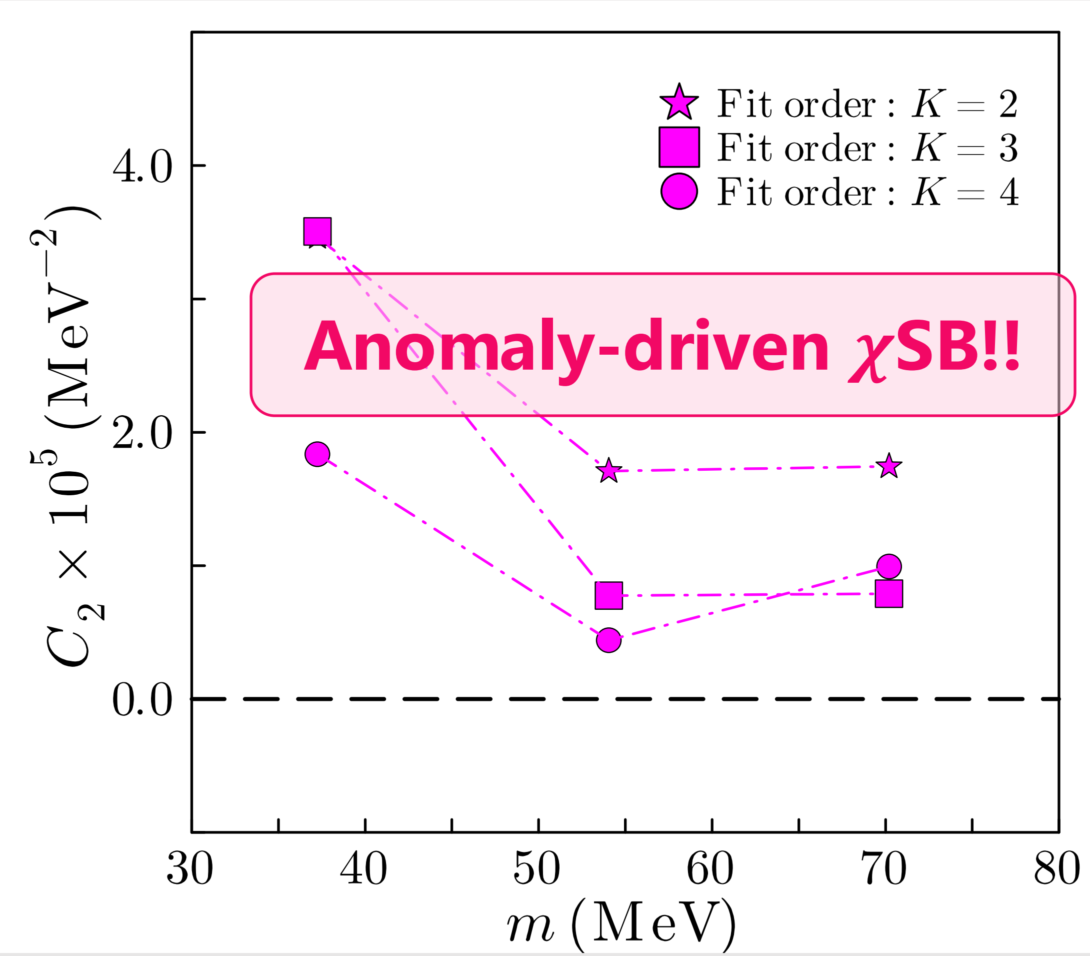

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[7] YS and D. Jido, PRD **110**, 014037 (2024)

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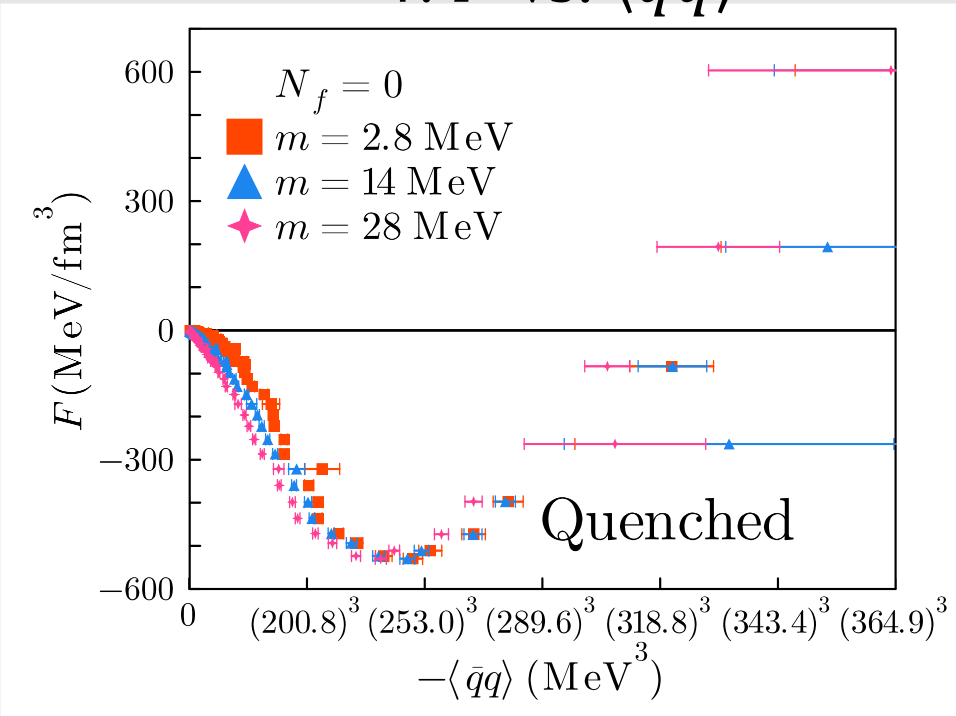

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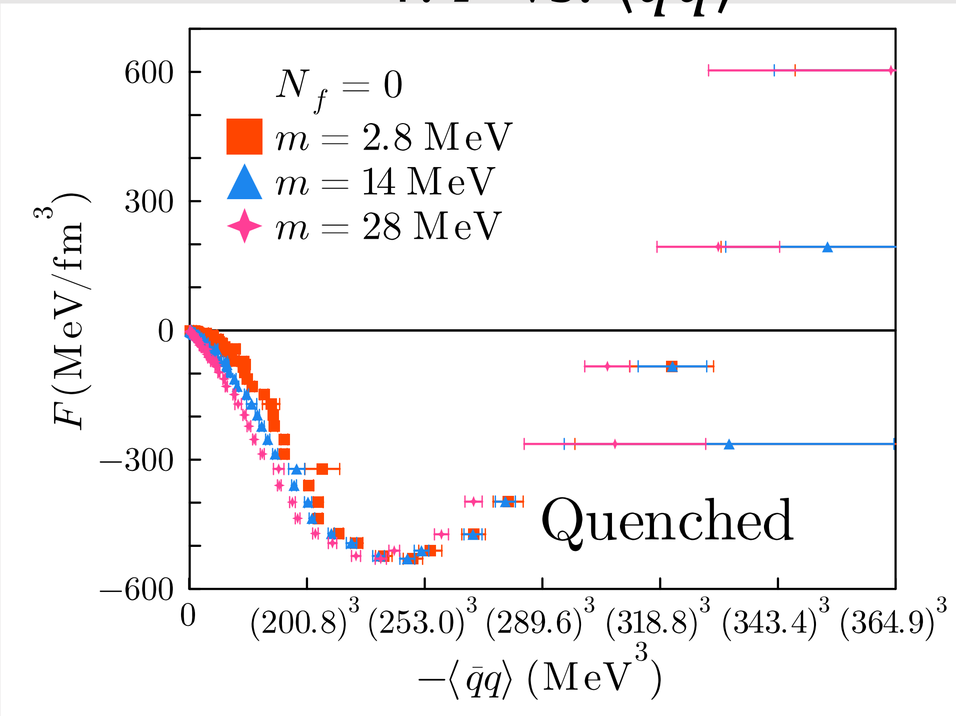

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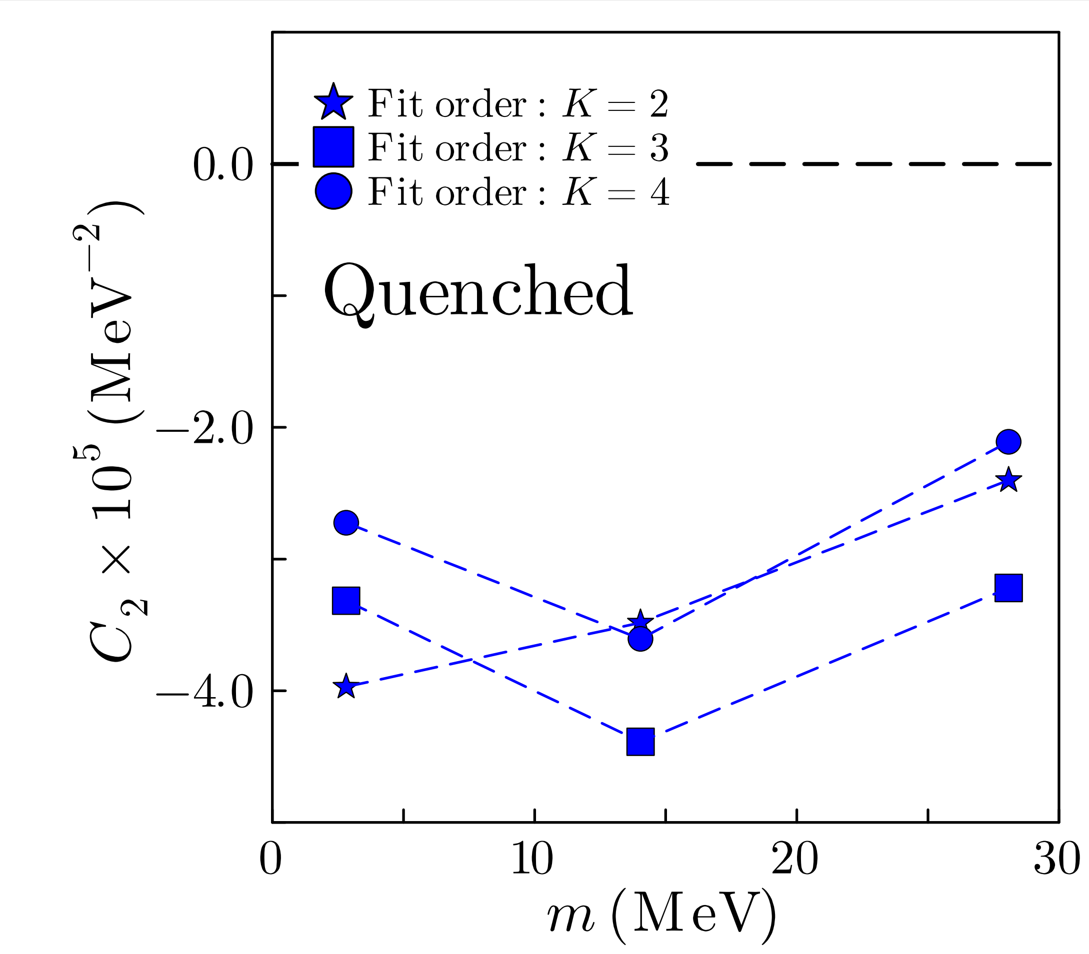

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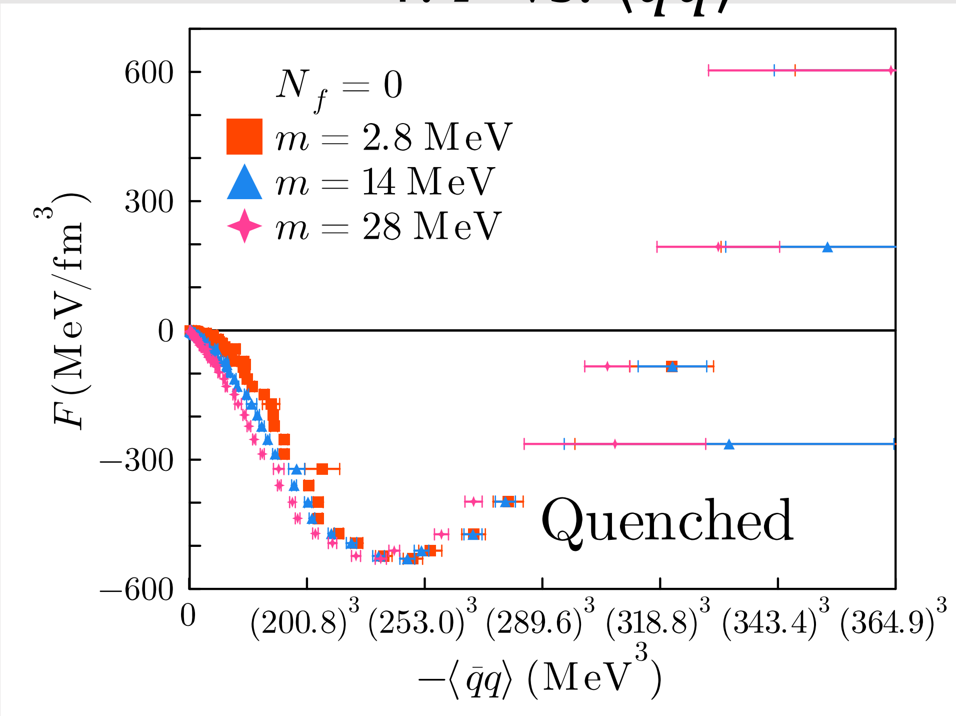

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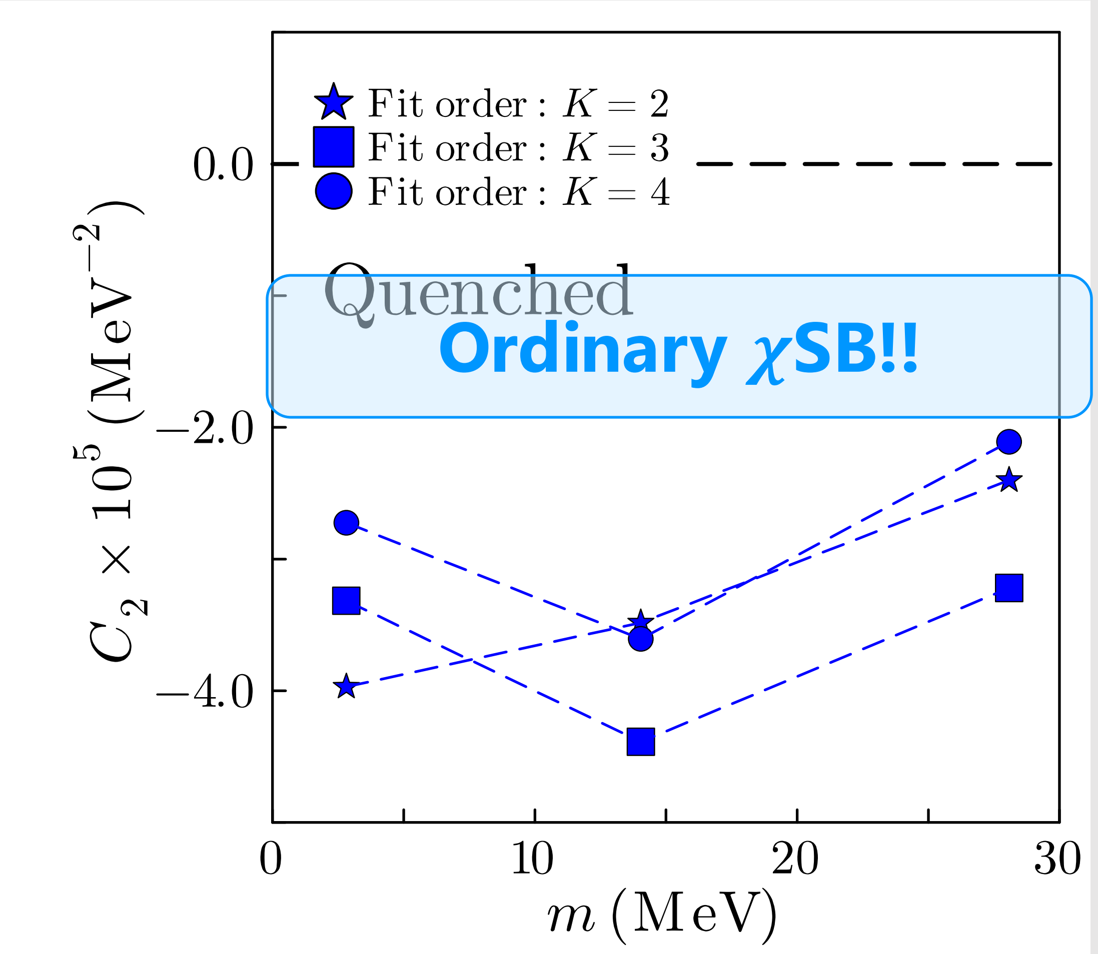

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$\mathbf{SU}(3)_f$ NJL model including chiral $(\mathrm{U}(1)_A)$ anomaly

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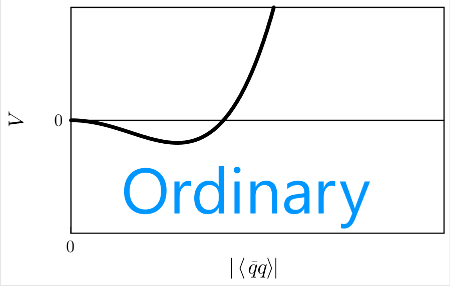

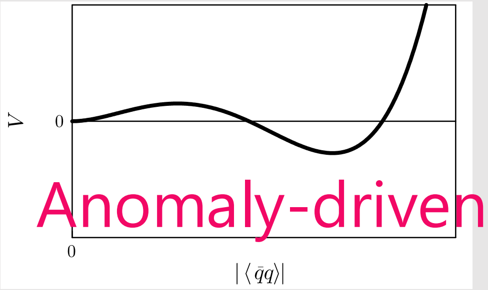

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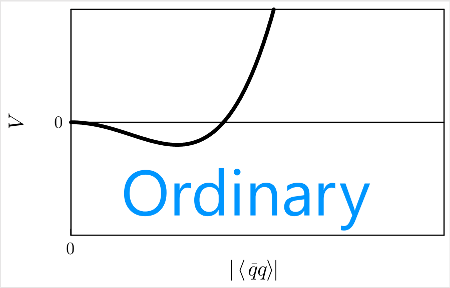

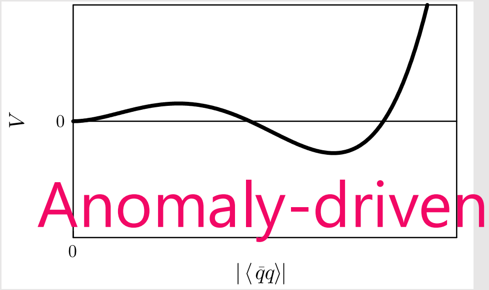

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sums all orders of 't Hooft vertex in the quark det. part:

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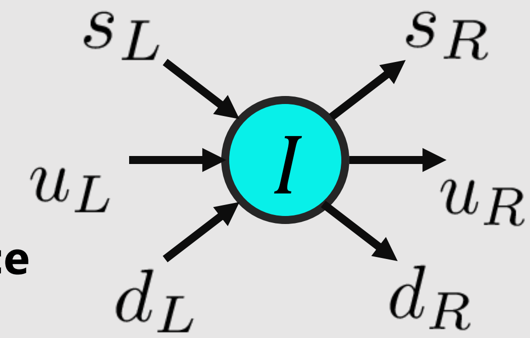
implicitly includes
6-quark interaction
without any parameters →



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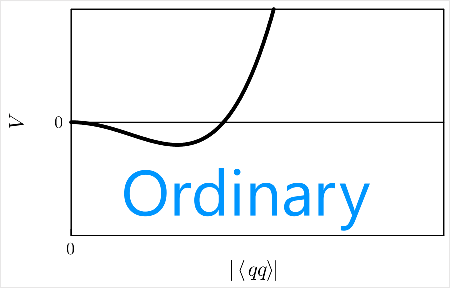

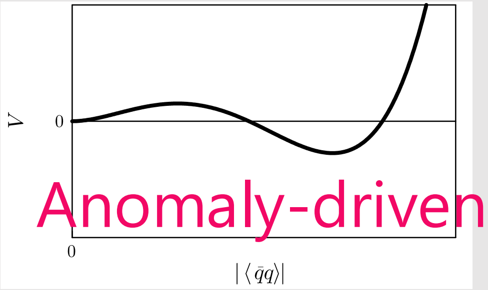

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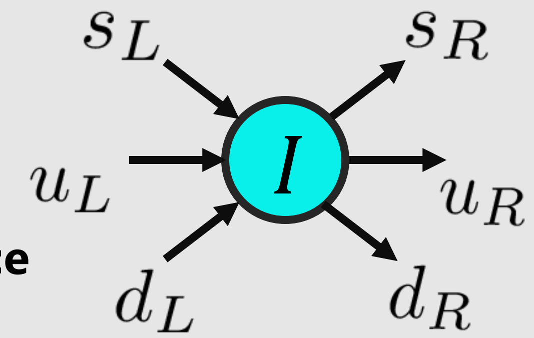


thus, it is **natural to reproduce anomaly driven χSB in IILM** as in the NJL model with the KMT term

suggestion: to explore the phenomena linked to the anomaly driven χSB topological configuration would be important

Summary

- **We studied the χSB pattern driven by chiral ($\mathrm{U}(1)_A$) anomaly in IILM**
  - ✓ PRD110(2024)014037

- To determine the χSB pattern we used the curvature of the energy density w.r.t. the quark condensate at the origin:

  curvature: $C_2 \equiv \frac{\partial^2 V}{\partial \langle \bar{q} q \rangle^2}\Big|_{\langle \bar{q} q \rangle = 0}$
  - ordinary breaking: $C_2 < 0$
  - anomaly driven breaking: $C_2 > 0$
- IILM calculations show that the curvature is positive $C_2 > 0$
- **implies the anomaly driven breaking may be taken place in IILM**

- Further calculations are running with the aim of the systematic study
  - $N_f = 2 + 1$ situation: SU(3) breaking effect
  - $N_f = 2$ situation: interpolation between SU(3) and quench calculations

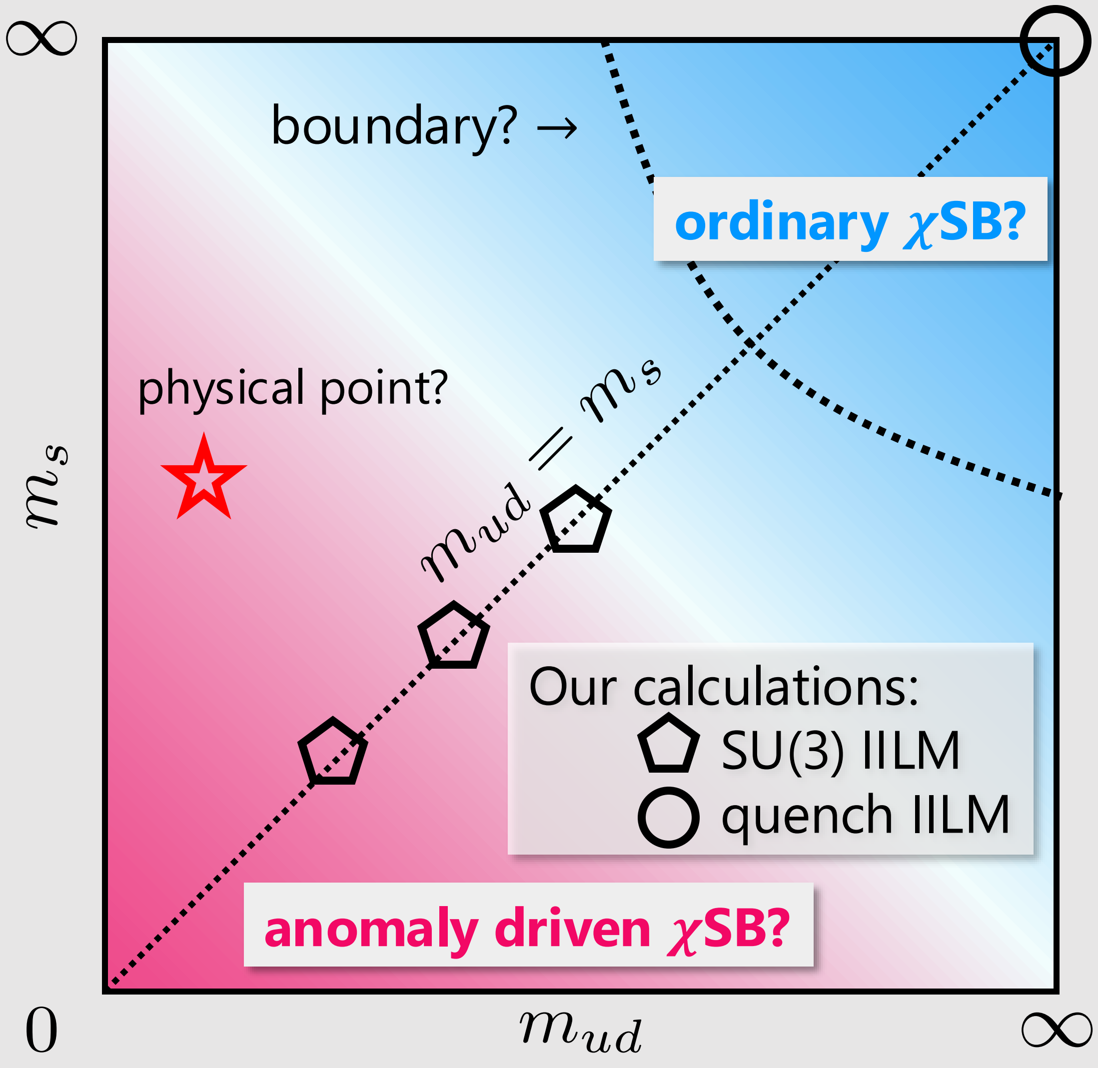
Supplementary: Nf-dependence

- if dominant chiral $(\mathrm{U}(1)_A)$ anomaly exists, ordinary/anomaly driven χSB pattern would depend on quark mass
- impact should manifest itself in some form for the cases $N_f = 3, N_f = 2 + 1$ and $N_f = 2$
- expect that these impacts also realizes in QCD

[8] K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74** (2011) 014011

NOTE: This is NOT the Columbia phase diagram

schematic figure of the phase diagram
for the ordinary/anomaly driven χSB pattern in IILM



Supplementary: Why the instanton liquid model?

- if the anomaly-driven χSB is a universal feature of strong interaction
  ⇒ could provide some way to detect
  the topological structure of the QCD vacuum

- verify whether the anomaly driven breaking takes place or not
  in other systems rather than chiral effective theories

- **instanton liquid model model** is a suitable model to check it:
  - can describe the QCD vacuum in terms of topological configuration
  - can reproduce the dynamical χSB
  - can compute effective potential (vacuum energy)
  - can compute quark condensate

Thank you very much