Towards low-dimensionalization of four dimensional QCD

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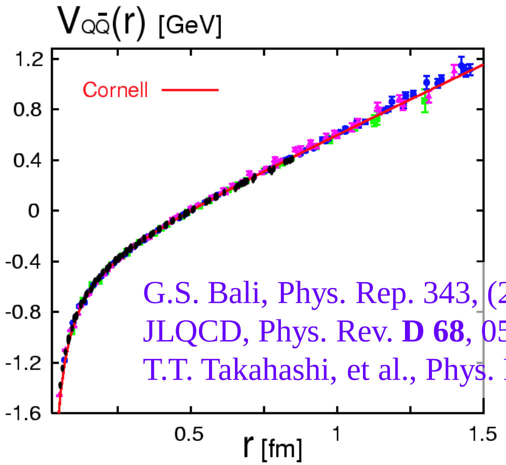
Motivation: Quark Confinement and Dimensional Reduction of 4D QCD

Quark confinement is characterized by a linear interquark potential and one-dimensional squeezing of color-electric fields.



G.S. Bali, Phys. Rep. 343, (2001),
JLQCD, Phys. Rev. **D 68**, 054502
T.T. Takahashi, et al., Phys. Rev. **D 65**, 114509 (2002)

one-dimensional squeezed
color-electric flux

Inspired by low-dimensionalization of color-electric flux, we consider that there might be an aspect of dimensional reduction in non-perturbative 4D QCD.

We try to extract 2D picture from non-perturbative 4D QCD.
To this end, we utilize gauge degrees of freedom.

Dimensional Reduction (DR) Gauge

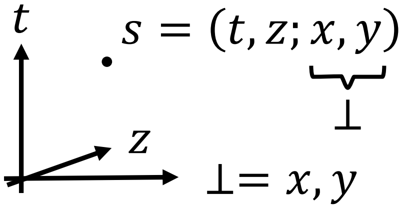
We propose a new gauge fixing, “Dimensional Reduction (DR) gauge”

Dimensional Reduction Gauge

The gauge globally defined so as to ***minimize***

$$R_{\mathrm{DR}} \equiv \int d^4s \sum_{\perp=x,y} \mathrm{Tr}[A_\perp(s)^2] \quad (\perp = x, y)$$

$$= \int d^4s\, \mathrm{Tr}\left[A_x(s)^2 + A_y(s)^2\right]$$



DR gauge can be defined in both Minkowski and Euclidean spacetime.

Local gauge fixing condition is

$$\partial_\perp A_\perp(s) \equiv \partial_x A_x(s) + \partial_y A_y(s) = 0\,.$$

DR-gauged QCD Action and Residual Gauge Symmetry

4D DR-gauged YM action：

$$S_{\mathrm{DR}} = \int d^4 s \left[-\frac{1}{2} \mathrm{Tr} G_{\mu\nu} G^{\mu\nu} + \underbrace{\frac{1}{2\alpha} \sum_{\perp = x,y} \mathrm{Tr}(\partial_\perp A_\perp)^2}_{\text{Gauge-fixing term}} \right]$$

This action has a residual gauge symmetry for $\Omega(t,z)$:

$$A_{t,z}(s) \to \Omega(t,z) \left(A_{t,z}(s) + \frac{1}{ig} \partial_{t,z} \right) \Omega^\dagger(t,z)$$

$$A_\perp(s) \to \Omega(t,z) \left(A_\perp(s) + \frac{1}{ig} \partial_\perp \right) \Omega^\dagger(t,z)$$

$$= \Omega(t,z) A_\perp(s) \Omega^\dagger(t,z)$$

This residual gauge symmetry is the same as 2D QCD on the t-z plane.

tz-projection as removal of the $\perp$-directed gluon fields

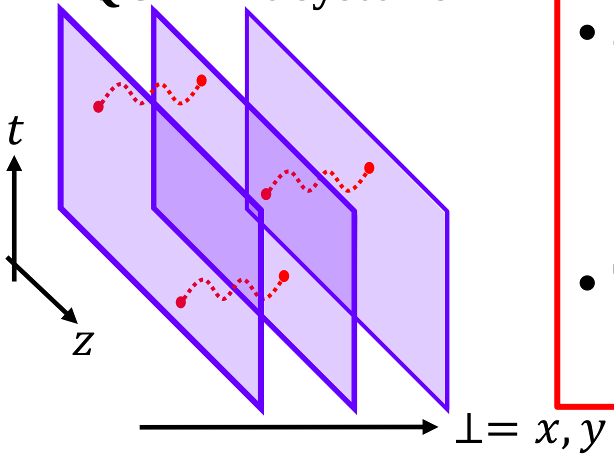
tz-projection : removal of the $\perp$-directed gluon fields, i.e.,

$$A_{x,y}(s) \to 0\,.$$

Applying the tz-projection, 4D DR-gauged tree-level YM action becomes

$$S_{\mathrm{DR}}^{tz} = \int d^4 s \left[\underbrace{\mathrm{Tr} G_{tz}^2}_{\text{2D YM}} + \underbrace{\sum_{\perp = x,y} \mathrm{Tr}\{(\partial_\perp A_t)^2 - (\partial_\perp A_z)^2\}}_{\text{Interaction between neighboring 2D YM systems}} \right] \qquad (\perp = x, y)$$

2D QCD-like systems



- 4D DR-gauged QCD action after tz-projection can be expressed as 2D QCD-like systems on t-z planes.
- These 2D systems are piled in the x, y directions and interact with neighboring planes.

Lattice formalism of DR gauge

On a lattice, gauge fields $A_\mu(s)$ are expressed as link variables $U_\mu(s)$,

$$U_\mu(\mathrm{s}) = \exp\left[iagA_\mu(\mathrm{s})\right].$$

$(a$: lattice spacing, g : gauge coupling$)$

DR gauge on a lattice

The DR gauge on a lattice is defined so as to ***maximize***

$$R_{\mathrm{DR}}^{\mathrm{lat}} \equiv \sum_s \mathrm{Re}\,\mathrm{Tr}\left[U_x(s) + U_y(s)\right].$$

Note: gauge fixing is performed for each gauge configuration generated in LQCD MC.

Lattice formalism of tz-projection

On a lattice, tz-projection is defined as

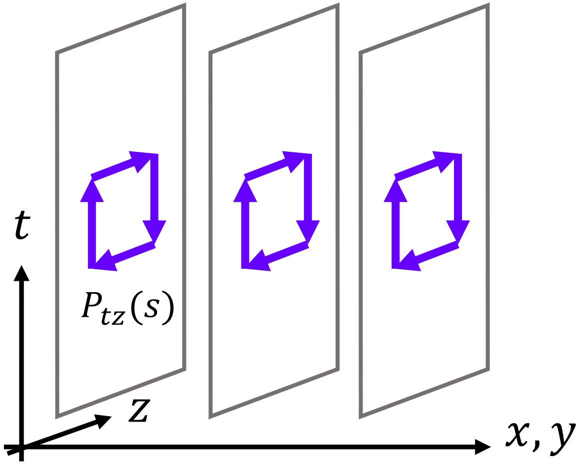
$$U_\perp(\mathrm{s}) \to 1\,. \qquad (U_\perp(\mathrm{s}) = \exp[iagA_\perp(\mathrm{s})])$$

The tz-projection on a lattice changes the standard plaquette action as

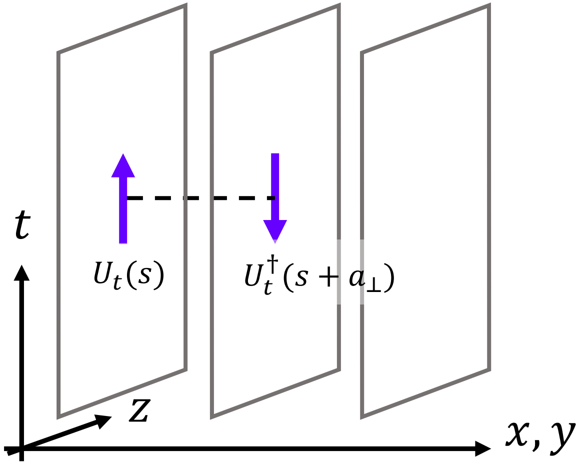
$$S^{\mathrm{lat}}_{tz-\mathrm{DR}} = \beta \sum_s \left[\left\{1-\frac{1}{N_c}\mathrm{ReTr}P_{tz}(s)\right\} + \sum_{\mu=t,z}\left\{1-\frac{1}{N_c}\sum_{\perp=x,y}\mathrm{ReTr}\left[U_\mu(s)U_\mu^\dagger(s+a_\perp)\right]\right\}\right]$$

(P_{tz}: Plaquette)

Ensemble of 2D YM systems



Interactions between neighboring planes



Short summary of DR gauge

Dimensional Reduction (DR) gauge :

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On a lattice, defined so as to ***maximize***

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- **4D DR-gauged QCD action has a residual gauge symmetry for $\boldsymbol{\Omega(t,z)}$**, and the symmetry is same as the gauge symmetry of 2D QCD on a t-z plane.
- After tz-projection, **4D DR-gauged QCD can be expressed as 2D QCD-like systems on $\boldsymbol{t}$-$\boldsymbol{z}$ planes.**
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Next step: Lattice QCD analysis of DR gauge

Numerical Lattice QCD calculation

To investigate the non-perturbative properties of DR-gauged QCD, we perform SU(3) lattice QCD simulations at the quenched level.

Settings:

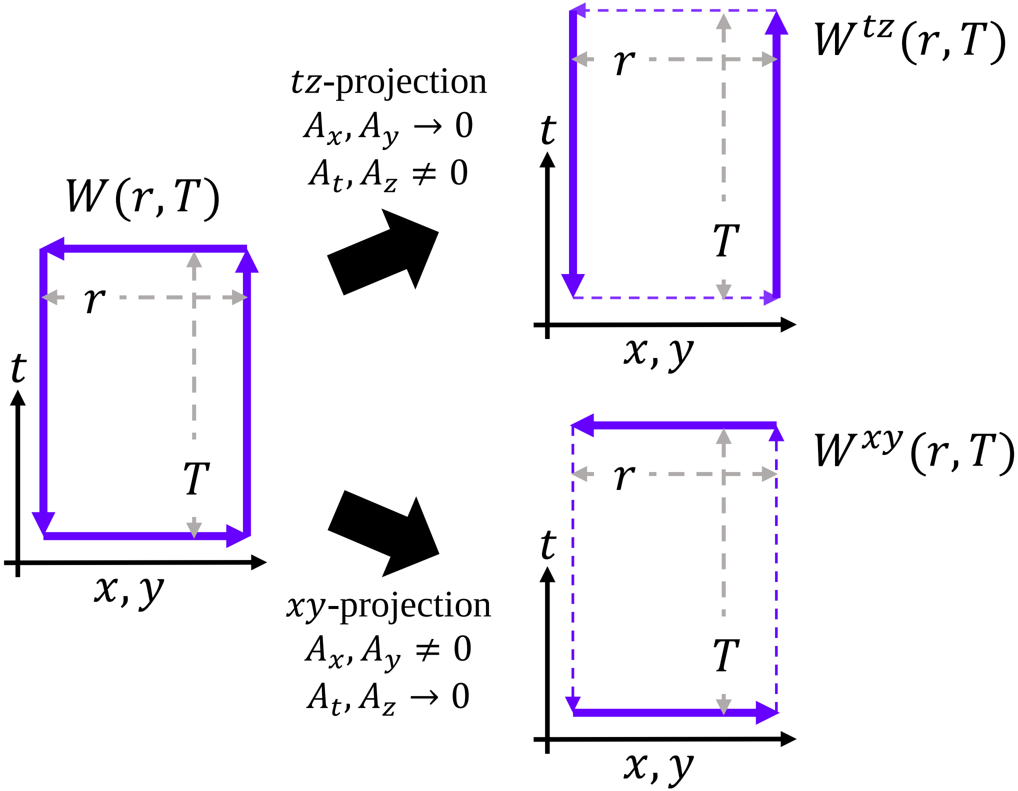
- Generation of gauge configurations
  - Gauge action: Standard plaquette action $(\beta = 6.0 \leftrightarrow a \simeq 0.1 \text{ fm})$
  - Lattice size: 24^4
  - Configurations: 800 configurations

    (picked up every 1,000 sweeps after 20,000 sweeps for thermalization)
- Gauge-fixing (Numerical maximization of $R_{\mathrm{DR}}^{\mathrm{lat}}$)
  - Gauge fixing is performed for each gauge configuration generated in LQCD.
  - Iterative maximization algorithm (similar in Landau or Coulomb gauge).
  - Over-Relaxation method with OR parameter 1.6.

Wilson loop and interquark potential after *tz*-projection in DR gauge

How dominant are A_t and A_z in low-energy phenomena?

To investigate this, we apply the *tz*-projection to the Wilson loop.



$$\langle W^{tz}(r,T)\rangle_{\mathrm{DR}} = Ae^{-V^{tz}(r)T}$$

$$\Downarrow$$

$$V^{tz}(r) = -\frac{1}{T}\ln\langle W^{tz}(r,T)\rangle_{\mathrm{DR}}$$

$$\langle W^{xy}(r,T)\rangle_{\mathrm{DR}} = Ae^{-V^{xy}(r)T}$$

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Note: These projected Wilson loops are residual gauge invariant.

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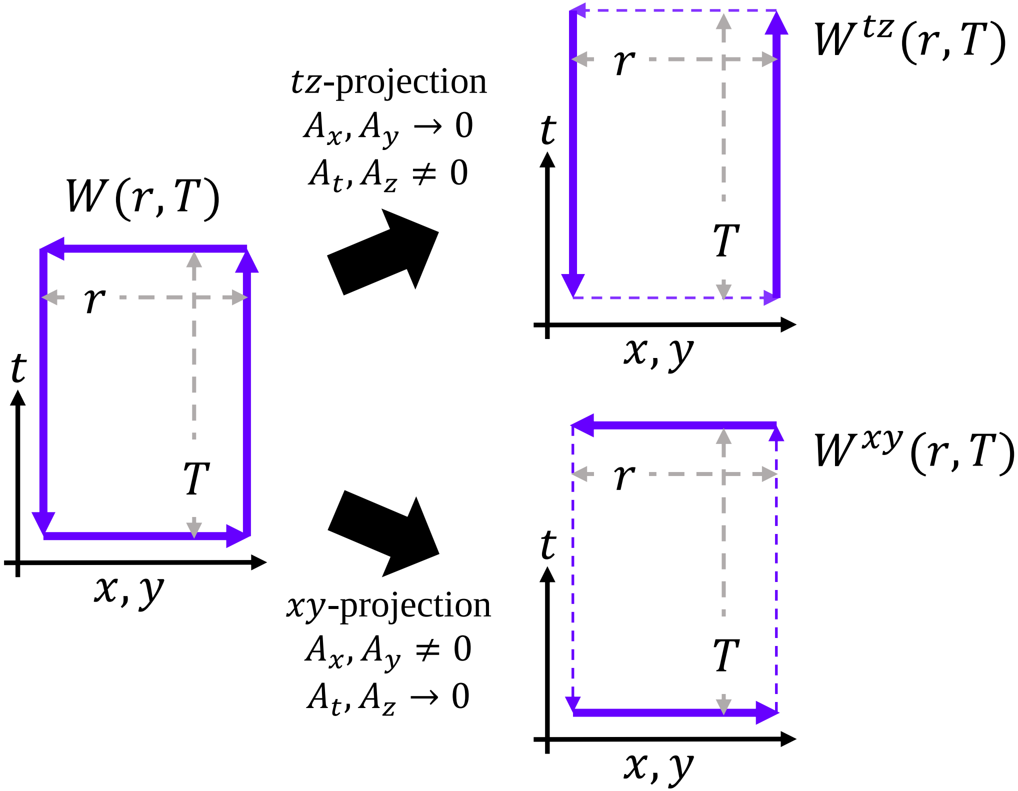

tz-projection
$A_x, A_y \to 0$
$A_t, A_z \neq 0$

xy-projection
$A_x, A_y \neq 0$
$A_t, A_z \to 0$

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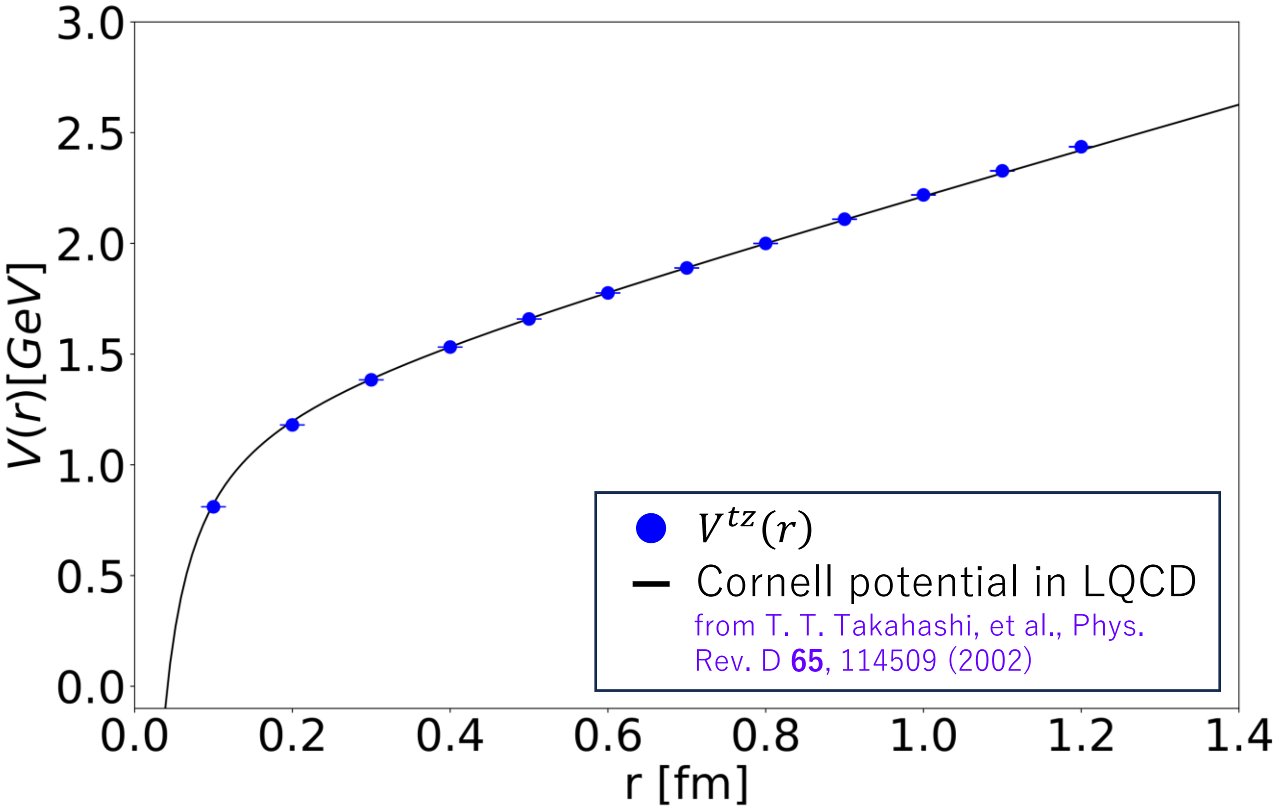
Interquark potential from the tz-projected Wilson loop in DR gauge



The tz-projected interquark potential $V^{tz}(r)$ is in good agreement with the Cornell potential.
→The interquark potential is well reproduced with $A_t(\mathrm{s})$ and $A_z(\mathrm{s})$ in the DR gauge.

How about *xy*-projected case?

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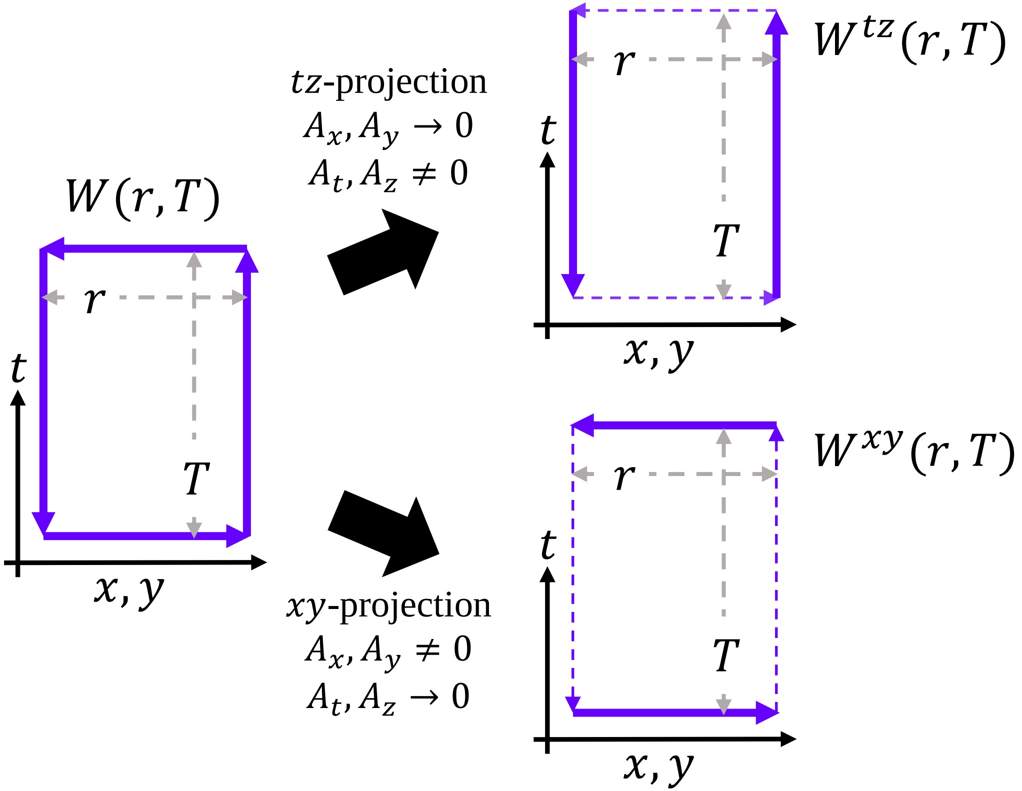

tz-projection
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$$\Downarrow$$

$$V^{tz}(r) = -\frac{1}{T}\ln\langle W^{tz}(r,T)\rangle_{\mathrm{DR}}$$

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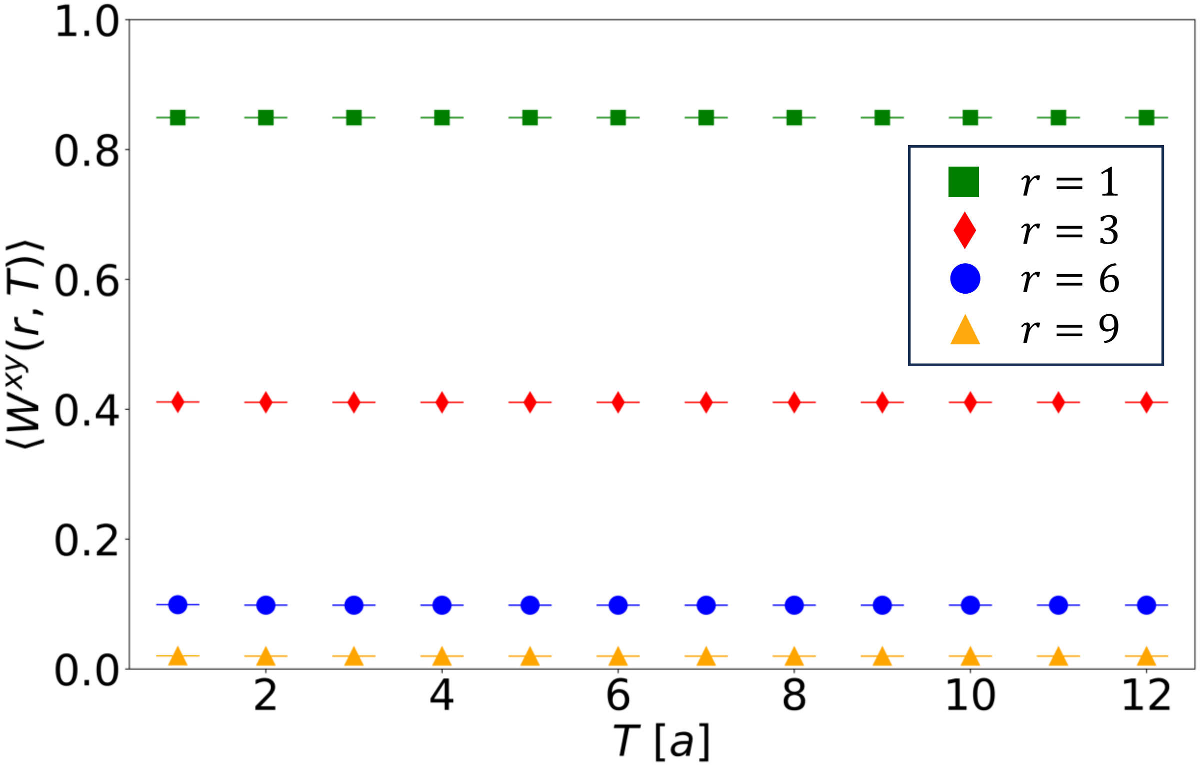
The xy-projected Wilson loop in DR gauge



The xy-projected Wilson loop $\langle W^{xy}(r,T)\rangle_{\mathrm{DR}}$ is independent of T.
Thus, the xy-projected interquark potential becomes zero,

$$V^{xy}(r) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W^{xy}(r,T)\rangle_{\mathrm{DR}} = 0\,.$$

Short summary of projected Wilson loops in DR gauge

In DR gauge,

- ***tz*-projection**

  The tz-projected interquark potential $V^{tz}(r)$ is in good agreement with the Cornell potential.

  → The interquark potential is well reproduced with $A_t(\mathrm{s})$ and $A_z(\mathrm{s})$.

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  → The xy-projected interquark potential becomes zero, $V^{xy}(r) = 0$.

In DR gauge,

- $A_t(\mathrm{s})$ and $A_z(\mathrm{s})$ play a dominant role in quark confinement.
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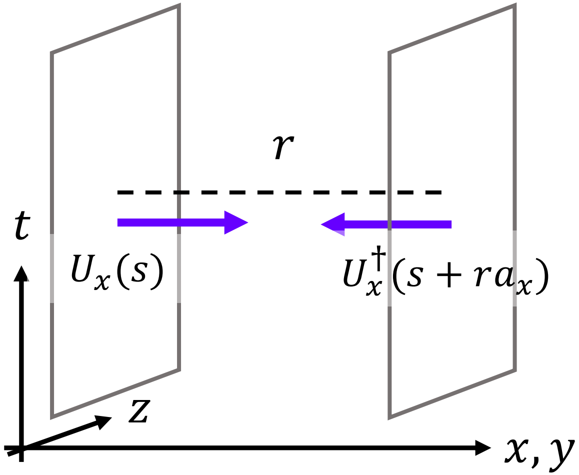
Why are $A_x(s)$ and $A_y(s)$ inactive in the infrared region in the DR gauge?

Spatial correlation and spatial mass of $A_x(s)$ and $A_y(s)$

Why are $A_x(s)$ and $A_y(s)$ inactive in the infrared region in the DR gauge?

The spatial correlation of two link-variables:

$$F(r) \equiv \frac{1}{N_c}\left\langle \mathrm{Tr}\, U_x(0) U_x^\dagger(ra_x) \right\rangle_{\mathrm{DR}}$$
$$= \frac{a^2}{\beta}\langle A_x^a(0) A_x^a(ra_x)\rangle_{\mathrm{DR}} + \left\{1 - \frac{a^2}{\beta}\langle A_x^a(0)^2\rangle_{\mathrm{DR}}\right\} + O(a^3)$$



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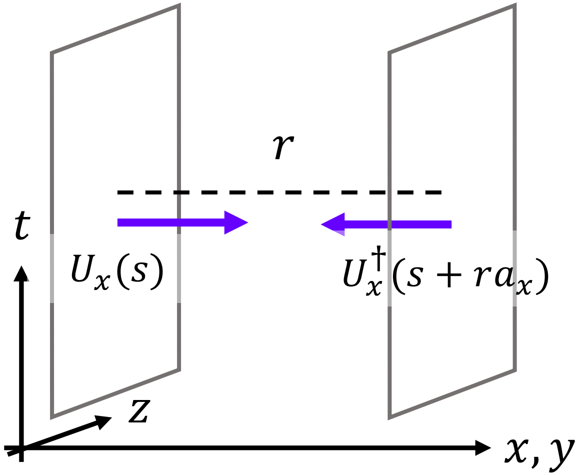
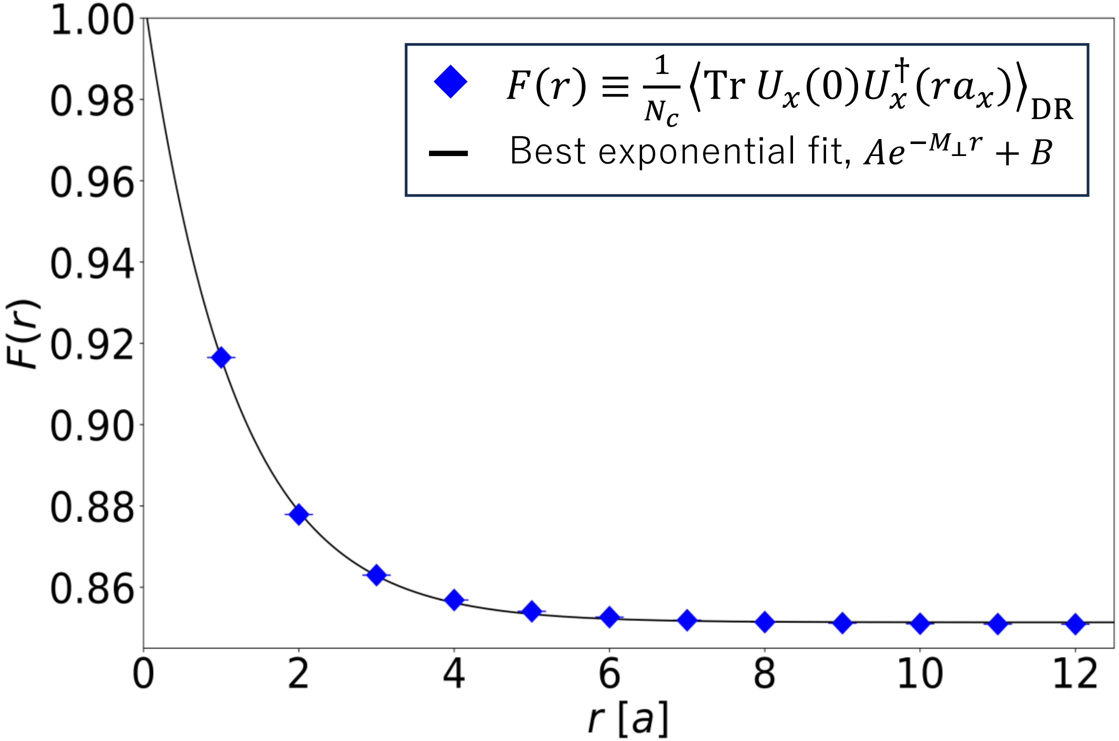
$$= \underbrace{\frac{a^2}{\beta}\langle A_x^a(0) A_x^a(ra_x)\rangle_{\mathrm{DR}}}_{\textbf{Gluon propagator in DR gauge}} + \underbrace{\left\{1 - \frac{a^2}{\beta}\langle A_x^a(0)^2\rangle_{\mathrm{DR}}\right\}}_{\text{constant}} + O(a^3)$$



Estimate the spatial "mass" of $A_x(s)$ and $A_y(s)$ from the infrared behavior of $F(r)$.

Spatial correlation and spatial mass of $A_x(s)$ and $A_y(s)$



The spatial link correlation is well reproduced with $F(r) \simeq Ae^{-M_\perp r} + B$ and the fit parameters are

$A \simeq 0.155$,
$M_\perp \simeq 0.87a^{-1} \simeq \mathbf{1.71}$ GeV,
$B \simeq 0.851$.

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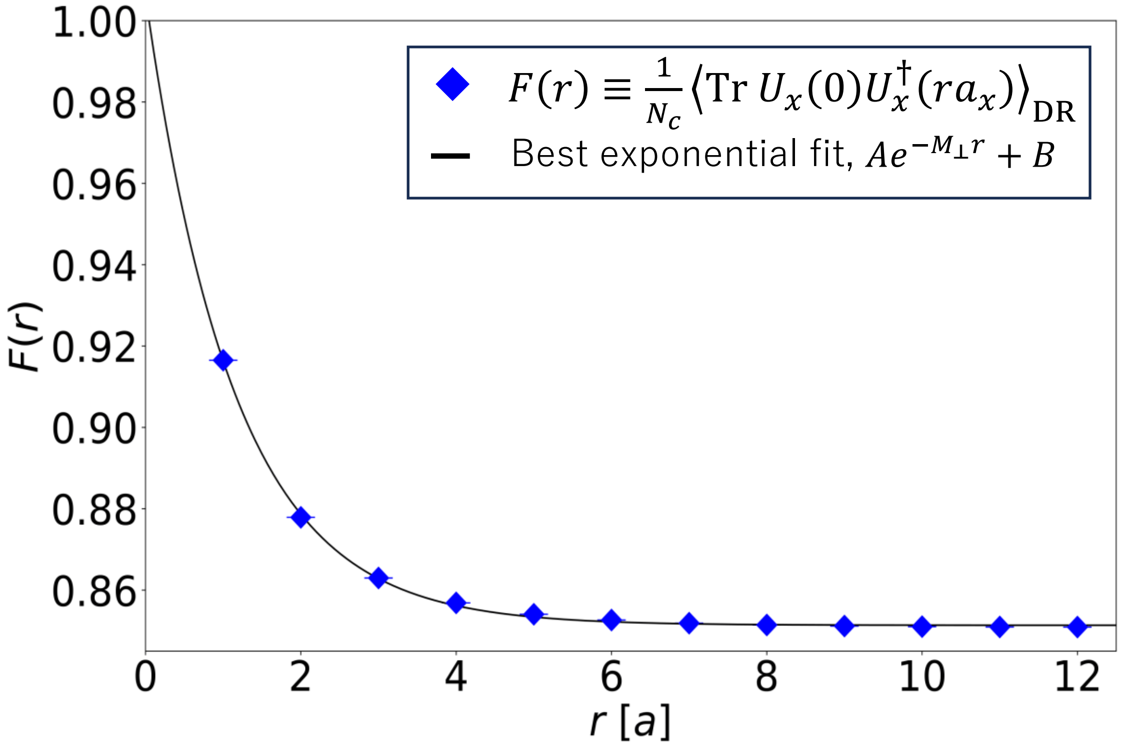

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This results implies that the large mass of $A_x(s)$ and $A_y(s)$ becomes them inactive in the infrared region.

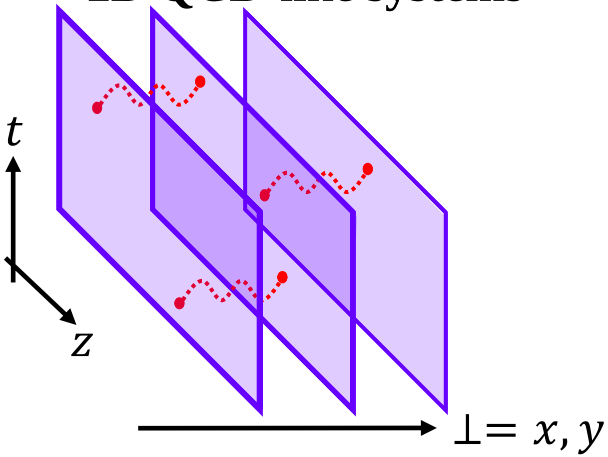
Signs of low-dimensionalization of 4D QCD in DR gauge

In DR gauge,

- $A_t(\mathrm{s})$ and $A_z(\mathrm{s})$ play a dominant role in quark confinement.
- $A_x(s)$ and $A_y(s)$ are inactive in the infrared region.
- The large mass of $A_x(s)$ and $A_y(s)$ seems to make them infrared inactive.

We try to consider a possibility that low-energy phenomena are described in 2D degrees of freedom, $A_t(\mathrm{s})$ and $A_z(\mathrm{s})$, in DR-gauged QCD.

2D QCD-like systems



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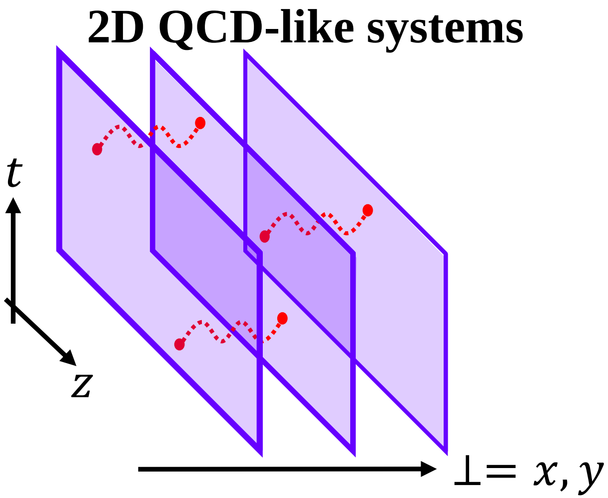


DR-gauged YM action has a neighboring interaction between 2D systems.

⬇

We investigate a correlations produced by this interaction in LQCD.

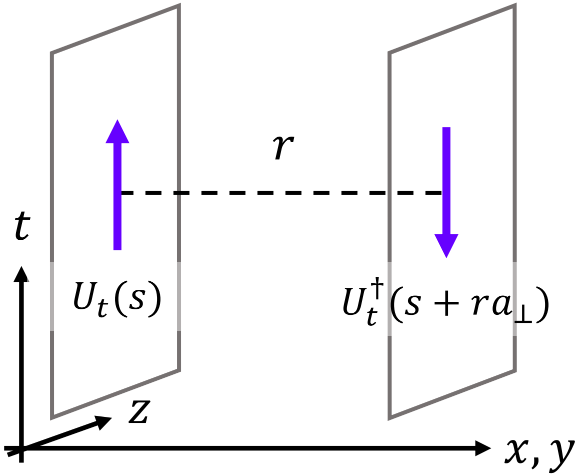
Neighboring interaction and spatial correlation of temporal-links

The tz-projected lattice action has a local interaction

$$\beta \sum_{s} \sum_{\mu=t,z} \left\{ 1 - \frac{1}{N_c} \sum_{\perp=x,y} \mathrm{ReTr}\left[U_\mu(s) U_\mu^\dagger(s+a_\perp) \right] \right\}.$$

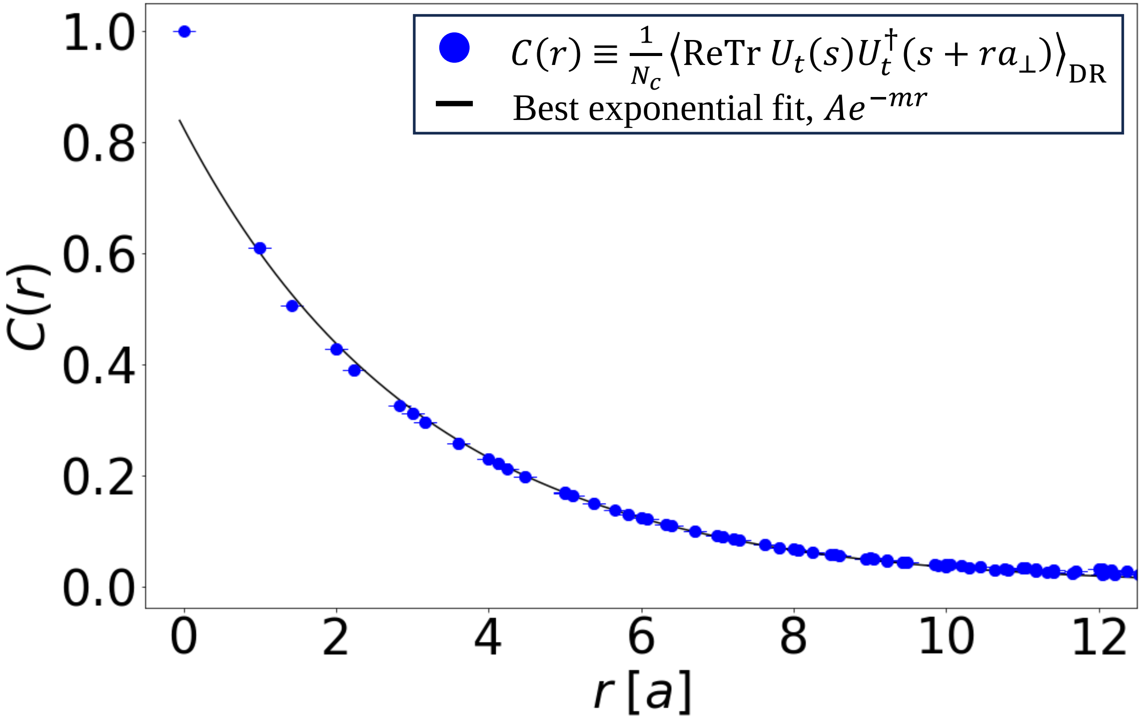
This interaction provides a distant correlation between t-z planes in the x and y directions.

We calculate the spatial correlation between two temporal-links.



$$C(r) \equiv \frac{1}{N_c} \left\langle \mathrm{ReTr}\, U_t(s) U_t^\dagger(s+ra_\perp) \right\rangle_{\mathrm{DR}}$$

Neighboring interaction and spatial correlation of temporal-links



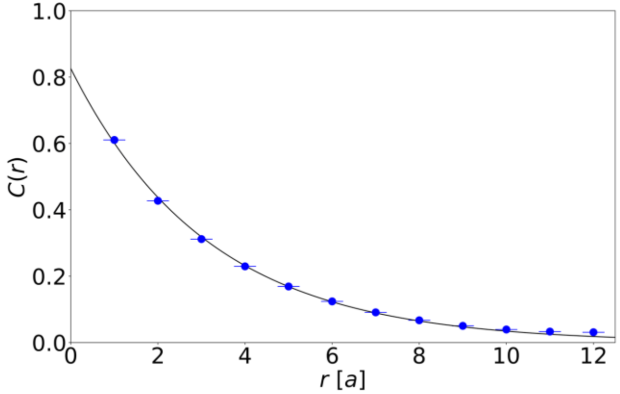
This correlation is well reproduced with **exponential function** $\boldsymbol{C(r) \simeq Ae^{-mr}}$ and the exponent m has a value of

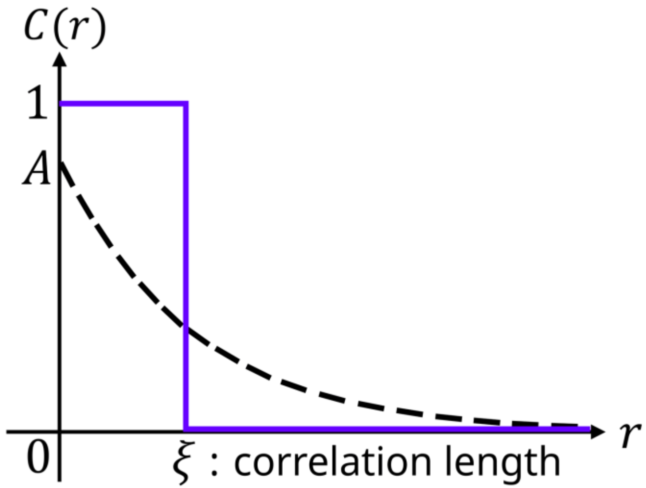
$$\boldsymbol{m \simeq 0.32a^{-1} \simeq 0.64\,\mathrm{GeV}}\,. \qquad (A \simeq 0.83)$$

Thus, **the correlation length** $\boldsymbol{\xi \equiv 1/m \simeq 0.3\,\mathrm{fm}}$.

To get a rough picture of low-dimensionalization of 4D QCD

For an analytical modeling of the tz-projected 4D YM theory,
we make a crude approximation of the exponential correlation $C(r)$.





That is, **we make a replacement of**

$$\boldsymbol{C(r) = \frac{1}{N_c}\left\langle \mathrm{ReTr}\, U_t(s) U_t^\dagger(s + r a_\perp) \right\rangle_{DR} \rightarrow \theta(\xi - r) = \begin{cases} 1 & (r < \xi) \\ 0 & (r > \xi) \end{cases}}$$

Under this approximation,

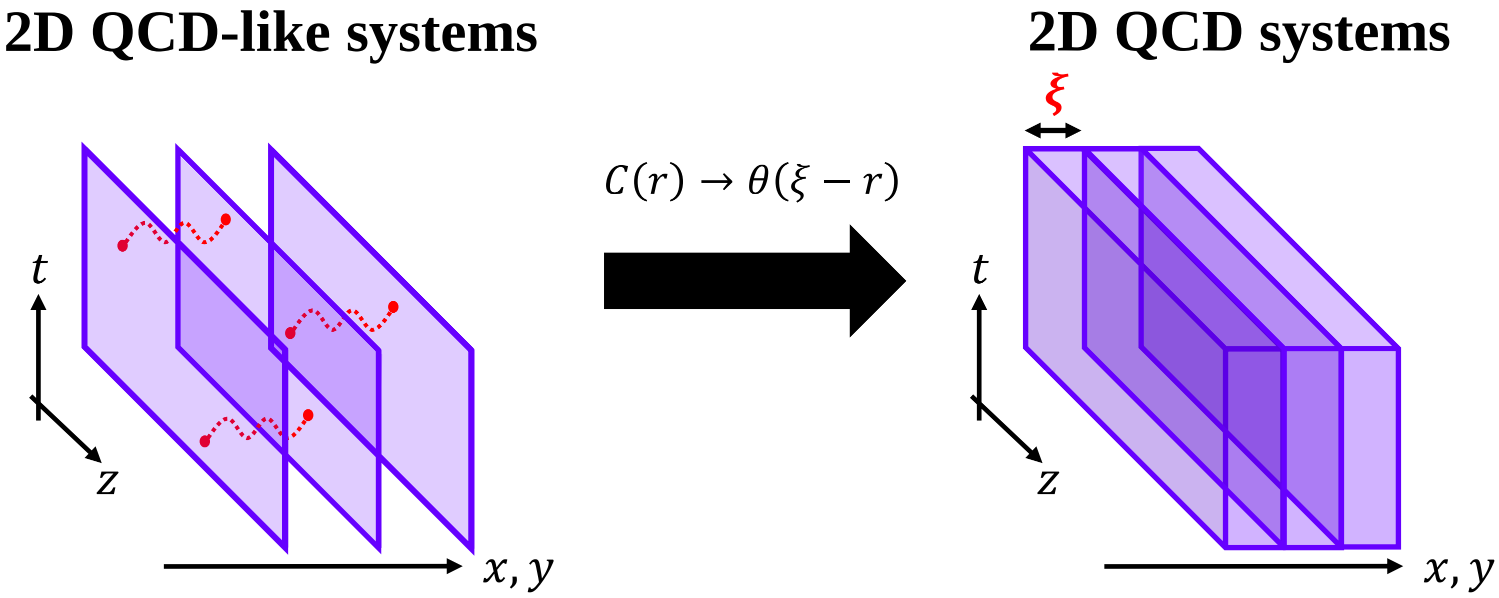
$r < \xi$: $U_t(s)$ and $U_t(s + r a_\perp)$ are same.

$r > \xi$: $U_t(s)$ and $U_t(s + r a_\perp)$ have no correlation in the x and y directions.

Crude approximation of temporal-link-correlation

Under the crude approximation, $C(r) \to \theta(\xi - r)$, DR-gauged 4D QCD can be regarded as **an ensemble of 2D QCD systems on $\boldsymbol{t}$-$\boldsymbol{z}$ layers, which have the width $\boldsymbol{\xi}$ and are piled in the $\boldsymbol{x}$ and $\boldsymbol{y}$ directions**.

These layers are independent and **do not interact each other**.



Demonstration: interquark potential in a t-z layer

These 2D layers can be labeled by two integers (m, n) instead of (x, y).

Tree-level action of 4D DR-gauged QCD:

$$S_{\mathrm{DR}}^{tz} \simeq \sum_{M=(m,n)} \xi^2 \int dtdz\, \mathrm{Tr}\{G_{tz}(t,z;m\xi,n\xi)^2\} = \sum_{M} \int dtdz \frac{1}{2} \mathrm{Tr}\left\{\mathcal{G}_{\mu\nu}^{M}(t,z)^2\right\}$$

Rescaling with the correlation length ξ :

$$A_\mu(t,z;m\xi,n\xi) \to \mathcal{A}_\mu^M(t,z) \equiv \xi A_\mu(t,z;m\xi,n\xi)$$

$$G_{tz}(t,z;m\xi,n\xi) \to \mathcal{G}_{\mu\nu}^M(t,z) \equiv \partial_\mu \mathcal{A}_\nu^M - \partial_\nu \mathcal{A}_\mu^M + i\mathrm{g}_{\mathrm{2D}}\left[\mathcal{A}_\mu^M, \mathcal{A}_\nu^M\right]$$

Relation between 4D and 2D QCD coupling: $\boldsymbol{\mathrm{g}_{\mathrm{2D}} \equiv g/\xi}$

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Relation between 4D and 2D QCD coupling: $\boldsymbol{\mathfrak{g}_{\mathrm{2D}} \equiv g/\xi}$

The tree-level interquark potential for quarks in a t-z layer is calculated as

$$\boldsymbol{V_{\mathrm{tree}}(r) = \frac{\mathfrak{g}_{\mathrm{2D}}^2}{2} \cdot \frac{4}{3} r = \frac{g^2}{\xi^2} \cdot \frac{2}{3} r = \sigma_{\mathrm{2D}} r} \, .$$

For $\beta = 2N_c/g^2 = 6.0$ $(g = 1.0)$, because of $\xi \simeq 0.3$ fm, we find $\sigma_{\mathrm{2D}} \simeq 1.37$ GeV/fm

(c.f. $\sigma_{\mathrm{4D}} \simeq 0.89$ GeV/fm)

The scale of “reduced” 2D QCD is determined by the correlation length ξ .

Summary

We proposed a new gauge fixing of “Dimensional Reduction (DR)” gauge and investigated the properties of DR gauge.
As a result, we have found that, **in DR gauge**,

1. **Interquark potential is reproduced only with $A_t(s)$ and $A_z(s)$**, and they play a dominant role in quark confinement.
2. Two-gauge components, $A_x(s)$ **and** $A_y(s)$ **are infrared inactive**, which seems to be caused by **their large spatial “mass ”**.
3. **The spatial correlation of $U_t(s)$ and $U_t(s + ra_\perp)$ decreases exponentially as $C(r) \propto e^{-mr}$ with $m \simeq 0.64$ GeV**.
4. Using a crude approximation of $C(r) \rightarrow \theta(\xi - r)$, 4D DR-gauged QCD theory can be regarded as an ensemble of 2D QCD systems.

For details, please see our PRD paper, Phys. Rev. **D 110**, 034505 (2024).

Future works

1. We are performing this subject with multi beta to investigate scaling properties.

2. Improvement of modeling to include the exponential correlation $C(r)$ properly.

3. Including quark degrees of freedom.

   - Chiral symmetry breaking

     Note: quarks are not bounded in each 2D layer
     but spread over 4D spacetime.

   - Hadron spectroscopy (e.g., N-Δ splitting)

Back Up

Local property of link-variables in DR gauge

Distance between a link variables $U_\mu(s) = e^{iagA_\mu(s)}$ and a unit matrix I :

$$d(U_\mu, I)^2 = \frac{1}{2N_c}\mathrm{Tr}\left[(U_\mu - I)^\dagger (U_\mu - I)\right]$$

gauge	$\langle d(U_\mu, I)^2 \rangle$
No fixing	1.000
DR ($\mu = t, z$)	1.000
DR ($\perp = x, y$)	0.076

$(\beta = 6.0, 24^4)$

The amplitudes of two components A_x and A_y are strongly suppressed by the DR gauge fixing.

The reason of unity values of $\langle d(U_\mu, I)^2 \rangle$

Distance between a link variables $U_\mu(s) = e^{iagA_\mu(s)}$ and a unit matrix I ：

$$
\begin{aligned}
d(U_\mu, I)^2 &= \frac{1}{2N_c}\mathrm{Tr}\left[(U_\mu - I)^\dagger (U_\mu - I)\right] \\
&= \frac{1}{N_c}\mathrm{Tr}\left[I - \mathrm{Re}U_\mu\right]
\end{aligned}
$$

Then, the VEV of $d(U_\mu, I)^2$ becomes

$$
\begin{aligned}
\left\langle d(U_\mu, I)^2 \right\rangle &= \int \mathcal{D}U_\mu \frac{1}{N_c}\mathrm{Tr}\left[I - \mathrm{Re}U_\mu\right] \\
&= \int \mathcal{D}U_\mu \left\{1 - \underline{\frac{1}{N_c}\mathrm{ReTr}U_\mu}\right\} \\
&= 1
\end{aligned}
$$

This term is not gauge invariant and disappears because of Elitzur’s theorem.

Comparison of DR gauge and Maximally Abelian gauge

DR gauge

Definition:

$$R_{\mathrm{DR}} \equiv \int d^4s \sum_{\perp=x,y} \mathrm{Tr}[A_\perp(s)^2]$$

- Residual gauge symmetry of 2D QCD.
- The amplitudes of $A_\perp(s)$ strongly suppressed.
- t and z components $A_{t,z}(s)$ play a dominant role for confinement.
- x and y components $A_{x,y}(s)$ are inactive in the low-energy region? This is because of the large mass of $A_{x,y}(s)$.

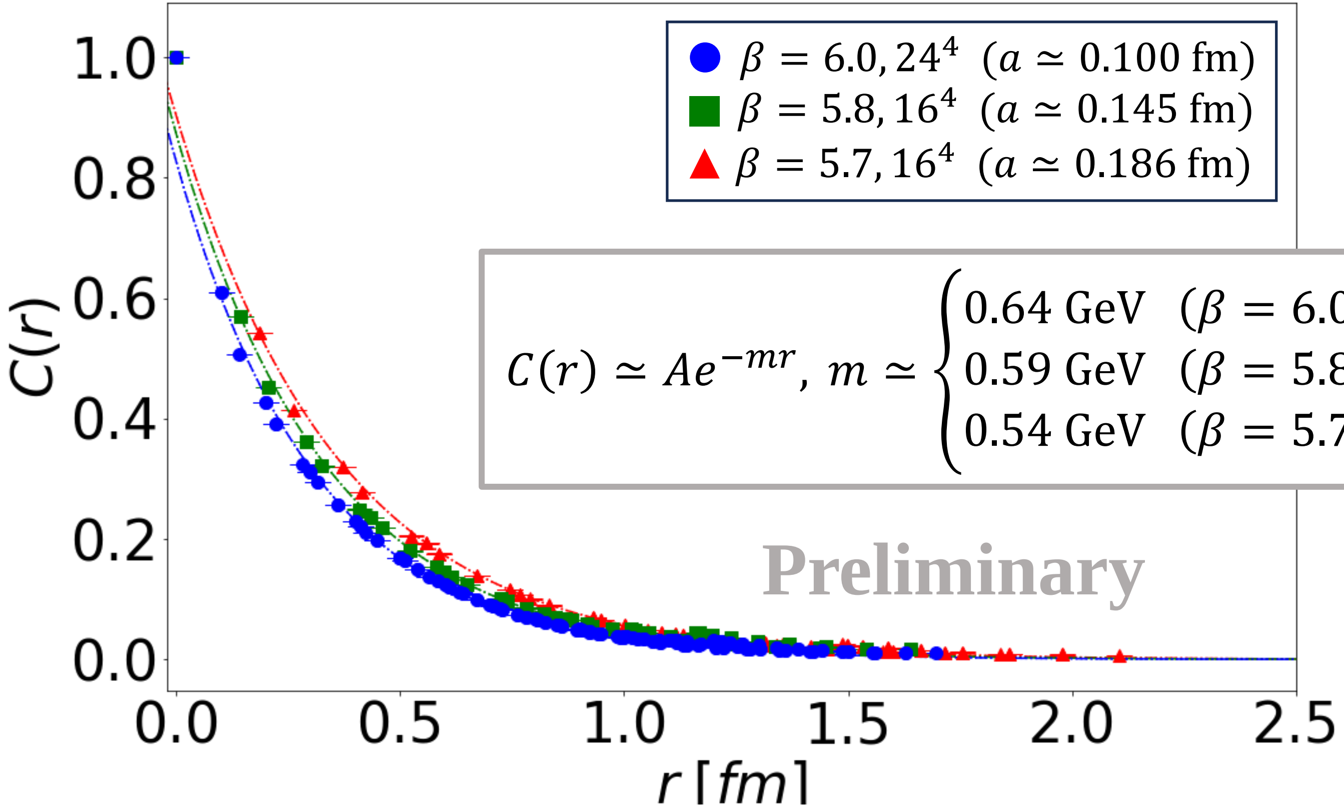
Maximally Abelian gauge

Definition:

$$R_{\mathrm{MA}} \equiv \int d^4s \sum_{\mu} \sum_{\alpha} A_\mu^\alpha(s) A_\mu^{-\alpha}(s)$$

- Residual gauge symmetry of $\mathrm{U(1)}^{\mathrm{N_c}-1}$.
- The amplitudes of $A_\mu^\alpha(s)$ is strongly suppressed.
- Abelian components $A_\mu^i(s)$ play a dominant role for confinement.
- Off-diagonal components $A_\mu^\alpha(s)$ are inactive in the low-energy region. This is because of the large mass of $A_\mu^\alpha(s)$.

Multi beta calculation of correlation of two temporal-links

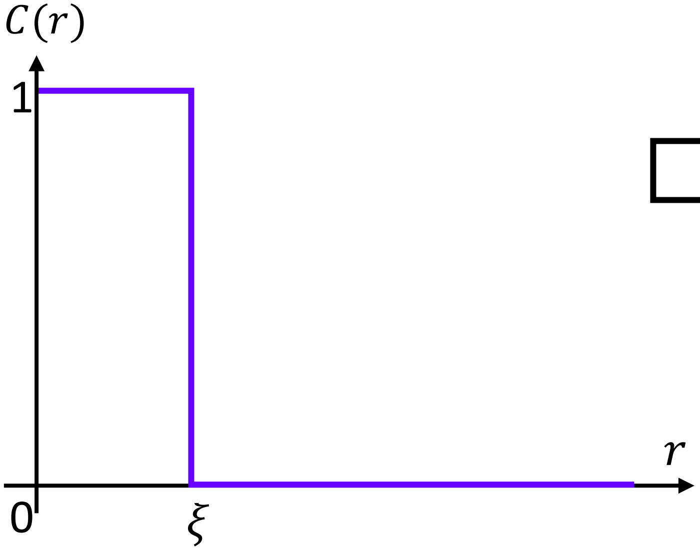


$$C(r) \simeq Ae^{-mr},\ m \simeq \begin{cases} 0.64\ \mathrm{GeV} & (\beta = 6.0, ma \simeq 3.2) \\ 0.59\ \mathrm{GeV} & (\beta = 5.8, ma \simeq 3.0) \\ 0.54\ \mathrm{GeV} & (\beta = 5.7, ma \simeq 2.8) \end{cases}$$

The exponent m may increase as β increase.
Can we understand this behavior in physical viewpoint?

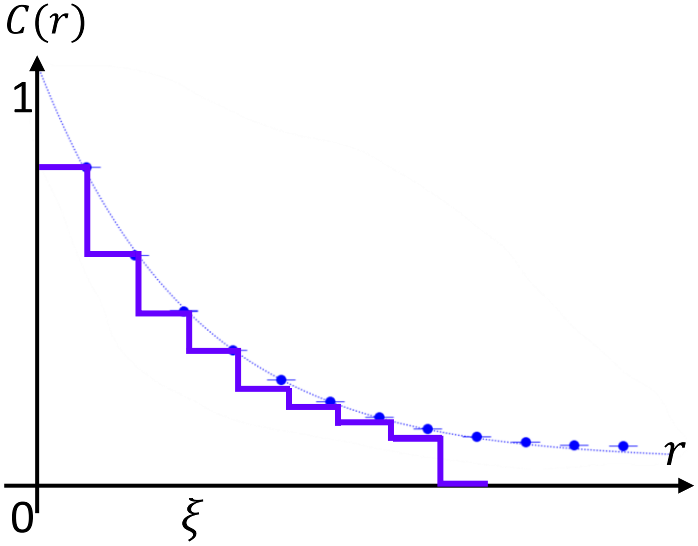
A way to improve the crude approximation

Crude approximation



$$C(r) = \theta(\xi - r)$$

Improved approximation



$$C(r) = \sum_n A_n \, \theta(\xi_n - r) \text{ ?}$$

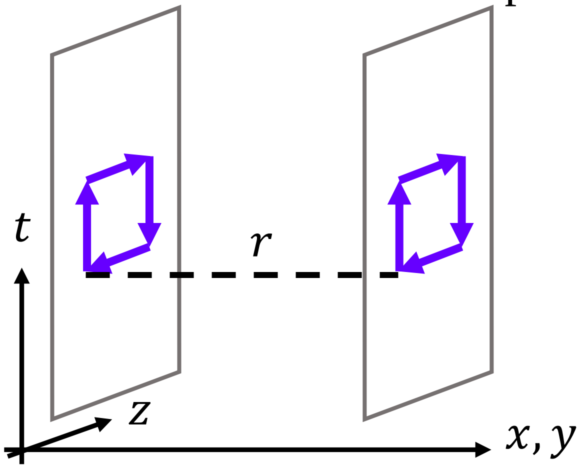
Will improved approximations allow for a more realistic consideration of the impact of correlations?

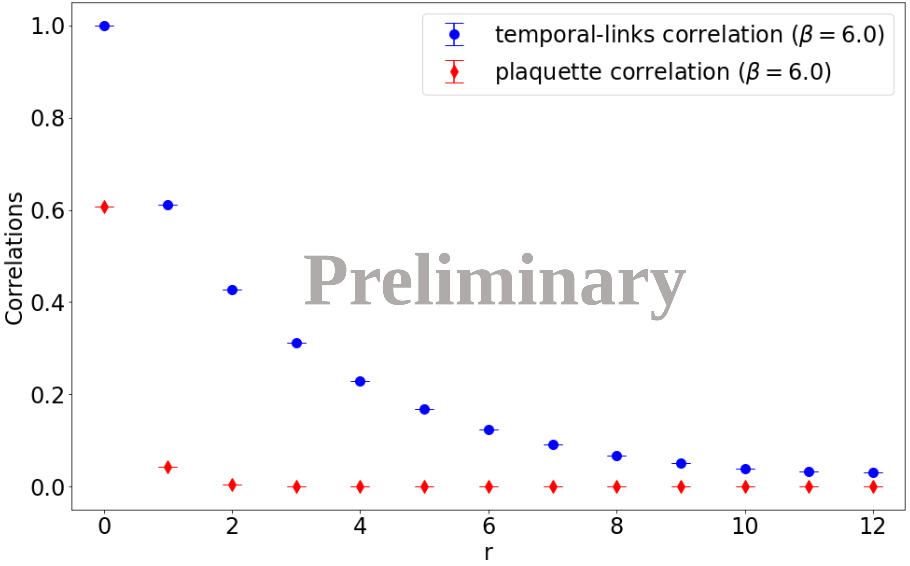
Correlation between two plaquettes

We consider the spatial (x, y-directed) correlation of two temporal-links.

How about the spatial correlation of two plaquettes (which are on t-z planes)?

with subtraction of trace part





The correlation decrease more rapidly.
The data is good agreement with two functions,

- Yukawa function Ae^{-mr}/r , ($m \simeq 3.0$ GeV)
- modified Bessel function of 2nd kind $AK_{3/2}(mr)$. ($m \simeq 3.3$ GeV)