Further numerical evidences for the gauge-independent separation between Confinement and Higgs phases in lattice SU(2) gauge theory with a scalar field in the fundamental representation

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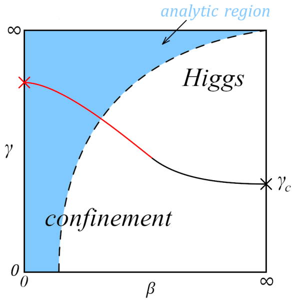
Introduction

- We investigate the gauge-scalar model to clarify the mechanism of confinement in the Yang-Mills theory in the presence of matter fields.
- We also investigate non-perturbative characterization of the Brout-Englert-Higgs (BEH) mechanism providing the gauge field with the mass, in the gauge-independent way (without gauge fixing).
- We reexamine the lattice $SU(2)$ gauge-scalar model with a radially-fixed scalar field (no Higgs mode) which transforms according to the fundamental representation of the gauge group $SU(2)$ without any gauge fixing.
- Note that it was impossible to realize the conventional BEH mechanism on the lattice unless the gauge fixing condition is imposed, since gauge non-invariant operators have vanishing vacuum expectation value on the lattice without gauge fixing due to the Elitzur theorem.
- This difficulty can be avoided by using the *gauge-independent description of the BEH mechanism* proposed recently by one of the authors, *which needs neither the spontaneous breaking of gauge symmetry nor*.
- Therefore, we can study the Higgs phase in the gauge-invariant way on the lattice without gauge fixing based on the lattice construction of gauge-independent description of the BEH mechanism.

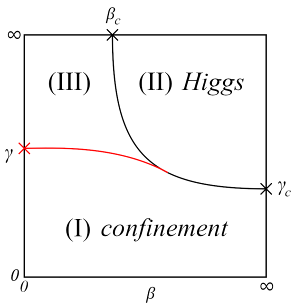
In case of fundamental scalar field

- Confinement and Higgs regions are sub-regions of analytically continued single phase.
  K. Ostewalder and E. Seiler, Annls. Phys. 110, 440 (1978)
  E. Fradkin and S.H. Shenker, PRD 19, 3682 (1979)
- We found a new transition line (red) which separates confinement and Higgs regions completely.
  [Phys.Rev.D 109, 054505 (2024)]



In case of adjoint scalar fields.

- Confinement and Higgs regions are completely separated into the two different phases by a continuous transition line.
  R.C. Brower et. al. PRD 25, 3319 (1982)
- We found a new transition line (red) that divides completely the confinement phase into two parts.
  [Phys.Rev.D 110, 034508 (2024)]



Plan of talk

In this talk, we further investigate the gauge-scalar model with the scalar field in the fundamental representation, so that we give further numerical evidences for the gauge-independent separation between Confinement phase and Higgs phase in the above model to establish its physical origin.

Contents:

- Lattice action
- Gauge-covariant decomposition (CDGSFN decomposition)
- Numerical simulations
  - Lattice result I (Analysis of the action density)
  - Lattice result II (Analysis based on the gauge-covariant decomposition)
    - the scalar-color correlation
    - the magnetic-monopole density
    - the gauge-color correlation (adjoint-scalar-action density)
- Summary

Lattice action

The $SU(2)$ gauge-scalar model with a radially-fixed scalar field in the fundamental representation of the gauge group:

$$
\begin{aligned}
S_{\rm GS} &:= S_g[U] + S_H[U,\Theta]\,, \\
S_g[U] &:= \sum_x \sum_{\mu<\nu} \frac{\beta}{2} \mathbf{tr}\left(\mathbf{1} - U_{x,\mu} U_{x+\mu,\nu} U^\dagger_{x+\nu,\mu} U^\dagger_{x,\nu}\right) + c.c.\ , \\
S_H[U,\Theta] &:= \sum_{x,\mu} \frac{\gamma}{2} \mathrm{tr}\left\{ \left(D_\mu[U]\Theta_x\right)^\dagger \left(D_\mu[U]\Theta_x\right)\right\} \\
&= \sum_{x,\mu} \frac{\gamma}{2} \mathrm{tr}\left\{\mathbf{1} - \Theta^\dagger_x U_{x,\,u} \Theta_{x+\hat{\mu}}\right\} + \text{c.c.}\,,
\end{aligned}
$$

where $U_{x,\mu} \in SU(2)$ represents a (group-valued) gauge variable on a link $< x, \mu >$, $\Theta_x \in SU(2)$ represents a (matrix-valued) scalar field in the fundamental representation of the gauge group on a site x which obeys the unit length (or radial-fixed) condition as $\Theta^\dagger_x \Theta_x = \mathbf{1} = \Theta_x \Theta^\dagger_x$, and $D_\mu[U]\Theta_x$ represents the covariant derivative in defined as

$$
D_\mu[U]\Theta_x := U_{x,\mu}\Theta_{x+\hat{\mu}} - \Theta_x\,.
$$

The action is invariant under the local $SU(2)_{\text{local}}$ gauge transformation and the global $SU(2)_{\text{global}}$ transformation for the link variable $U_{x,\mu}$ and the site variable Θ_x:

$$U_{x,\mu} \longrightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\hat{\mu}}\,, \quad \Omega_x \in SU(2)_{\text{local}}\,,$$
$$\Theta_x \longrightarrow \Theta'_x = \Omega_x \Theta_x \Gamma\,, \quad \Gamma \in SU(2)_{\text{global}}\,.$$

The expectation value of the operator $\mathcal{O}$ in this model is defined by

$$\langle \mathcal{O}[U,\Theta] \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Theta e^{-S_{\text{GS}}} \mathcal{O}[U,\Theta]\,, \quad Z = \int \mathcal{D}U \mathcal{D}\Theta e^{-S_{\text{GS}}}\,,$$

where integration measure $\mathcal{D}U = \prod_{x,\mu} dU_{x,\mu}$ and $\mathcal{D}\Theta = \prod_x d\Theta_x$ are the invariant Haar measure for the $SU(2)$ group.Therefore, this model has $SU(2)_{\text{local}} \times SU(2)_{\text{global}}$ symmetry.
In the naive continuum limit this action reproduces the continuum gauge-scalar model with a scalar field in the fundamental representation with a gauge coupling g and the fixed length v, where $\beta = 4/g^2$ and $\gamma = v^2$.

Gauge-covariant decomposition (CDGSFN decomposition)

We introduce the site variable $\mathbf{n}_x := n_x^A \sigma_A \in su(2) - u(1)$ which is called the color-direction (vector) field, in addition to the original link variable $U_{x,\mu} \in SU(2)$. The link variable $U_{x,\mu}$ and the site variable $\mathbf{n}_x$ transforms under the gauge transformation $\Omega_x \in SU(2)$ as

$$U_{x,\mu} \to \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger = U'_{x,\mu}, \quad \mathbf{n}_x \to \Omega_x \mathbf{n}_x \Omega_x^\dagger = \mathbf{n}'_x.$$

In the decomposition, a link variable $U_{x,\mu}$ is decomposed into two parts:

$$U_{x,\mu} := X_{x,\mu} V_{x,\mu}.$$

$$V_{x,\mu} \to \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger = V'_{x,\mu}. \quad X_{x,\mu} \to \Omega_x X_{x,\mu} \Omega_x^\dagger = X'_{x,\mu},$$

Such decomposition is obtained by solving the defining equations:

$$D_\mu[V]\mathbf{n}_x := V_{x,\mu} \mathbf{n}_{x+\mu} - \mathbf{n}_x V_{x,\mu} = 0, \quad \mathrm{tr}(\mathbf{n}_x X_{x,\mu}) = 0.$$

This defining equation has been solved exactly and the resulting link variable $V_{x,\mu}$ and site variable $X_{x,\mu}$ are of the form

$$V_{x,\mu} := \tilde{V}_{x,\mu} / \sqrt{\mathrm{tr}[\tilde{V}_{x,\mu}^\dagger \tilde{V}_{x,\mu}]/2}, \; \tilde{V}_{x,\mu} := U_{x,\mu} + \mathbf{n}_x U_{x,\mu} \mathbf{n}_{x+\mu},$$

$$X_{x,\mu} := U_{x,\mu} V_{x,\mu}^{-1} \; .$$

Note that this decomposition is obtained uniquely for a given set of link variable $U_{x,\mu}$ once the site variable $\mathbf{n}_x$ is given.

Reduction condition

The configurations of the color-direction field $\{\mathbf{n}_x\}$ are obtained by minimizing the functional:

$$F_{\text{red}}[\{\mathbf{n}_x\}|\{U_{x,\mu}\}] := \sum_{x,\mu} \text{tr}\left\{ \left(D_{x,\mu}[U]\mathbf{n}_x\right)^{\dagger} \left(D_{x,\mu}[U]\mathbf{n}_x\right) \right\},$$

$$D_{x,\mu}[U]\mathbf{n}_x := U_{x,\mu}\mathbf{n}_{x+\hat{\mu}} - \mathbf{n}_x U_{x,\mu}$$

which we call the *reduction condition*.

- Simulation
  - 16^4 lattice with the periodic boundary condition.
  - Updating link variables $\{U_{x,\mu}\}$ and scalar fields $\{\Theta_x\}$ alternately by using the HMC algorithm with integral interval $\Delta\tau = 1$ without the gauge fixing.
  - After 2500 sweep thermalization, we store 1500 configurations every 5 sweeps.
  - The figure below shows the parameters in the β–γ plane where simulations run.

- The search for the phase boundary
  by measuring the expectation value $\langle \mathcal{O} \rangle$ of a chosen operator $\mathcal{O}$ by changing γ (or β) along the $\beta =$const. (or $\gamma =$const.) lines.
- identify the boundary,
  Use of the bent, step, and gap observed in the graph of the $\langle \mathcal{O} \rangle$ plots .
  Use of peaks in the graph of the susceptibility plots.

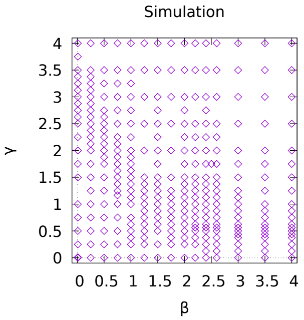


Figure: Simulation points

Numerical Result I [Preliminary]

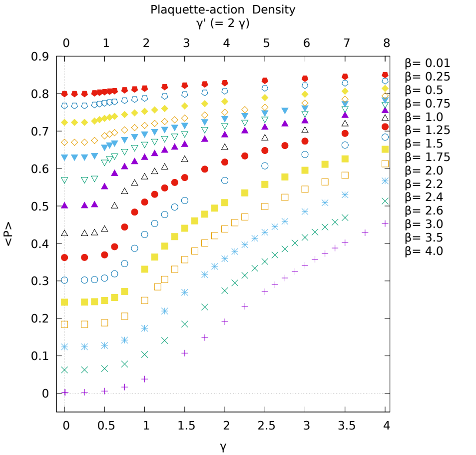
First, we reexamine the phase boundaries for a wider parameter space.

plaquette-action density

$$P = \frac{1}{6N_{\text{site}}} \sum_{x} \sum_{\mu<\nu} \frac{1}{2} \mathrm{tr}(U_{x,\mu\nu}), \qquad U_{x,\mu\nu} = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu},$$

Susceptibility of the plaquette-action density

$$\chi(P) = (6N_{\text{site}}) \left\{ \langle P^2 \rangle - \langle P \rangle^2 \right\}$$



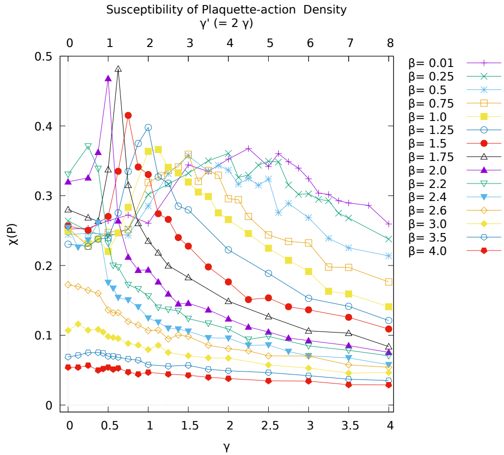


Figure: Left) $\langle P \rangle$ versus γ on various $\beta =$ const. lines. Right: $\chi(P)$ versus γ on various $\beta =$const. lines

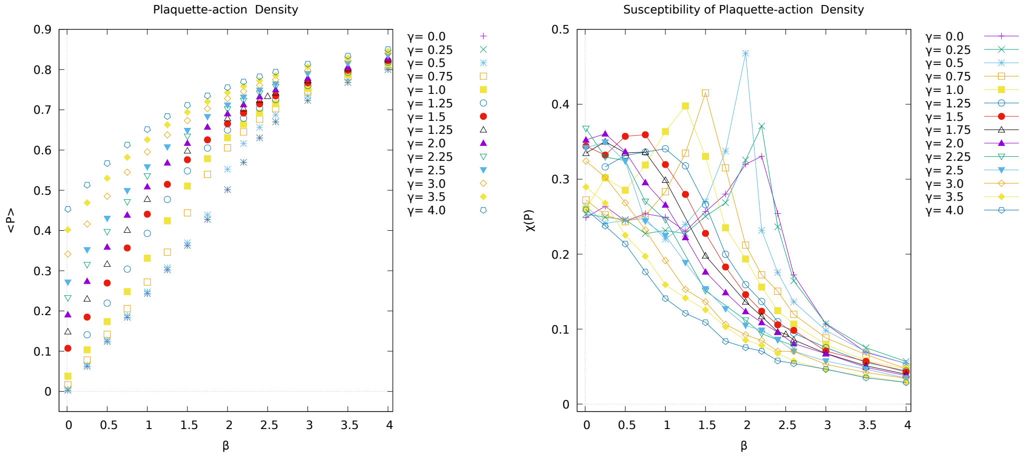
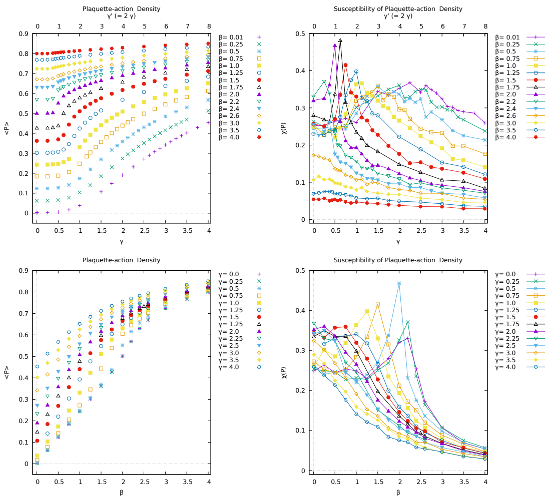
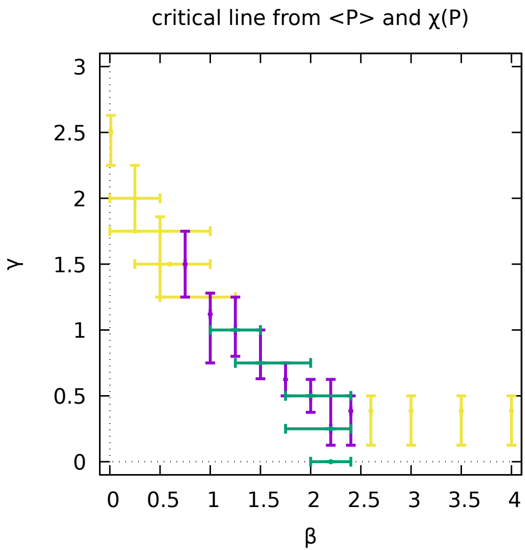


Figure: Left: $\langle P \rangle$ versus β on various $\gamma =$ const. lines. Right: $\chi(P)$ versus β on various $\gamma =$const. lines.

Phase boundary from gauge-action density





scalar-action density

,

$$M = \frac{1}{4N_{\text{site}}} \sum_{x} \sum_{\mu} \frac{1}{2} Re\text{tr}\left(\Theta_x^{\dagger} D_{\mu}[U_{x,\mu}]\Theta_{x+\hat{\mu}}\right),$$

Susceptibility of the scalar-action density

$$\chi(M) = (4N_{\text{site}})\left\{\langle M^2\rangle - \langle M\rangle^2\right\}$$

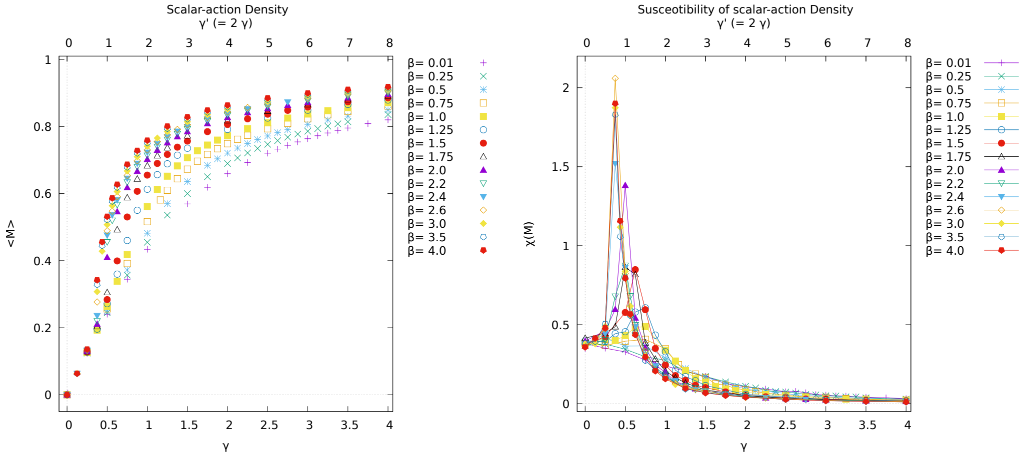


Figure: Left: $\langle M \rangle$ versus γ on various $\beta =$ const. lines. Right:$\chi(M)$ versus γ on various $\beta =$const. lines.

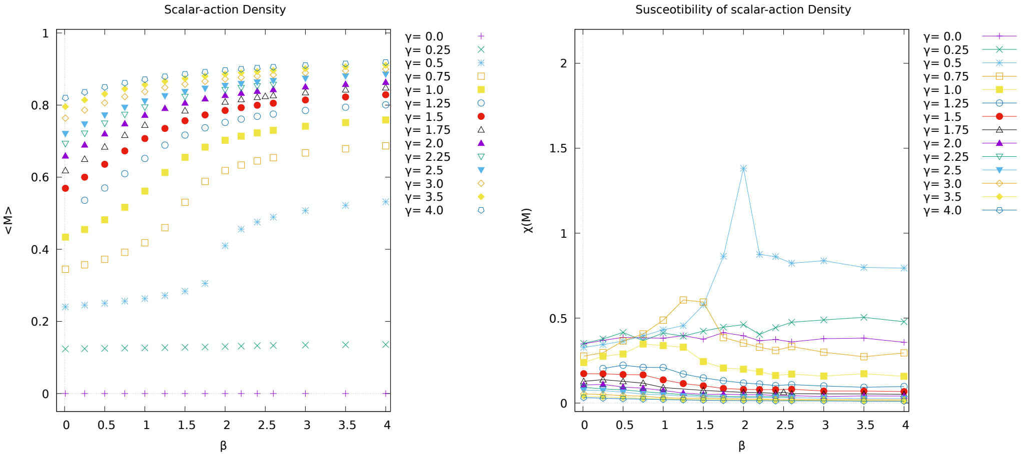
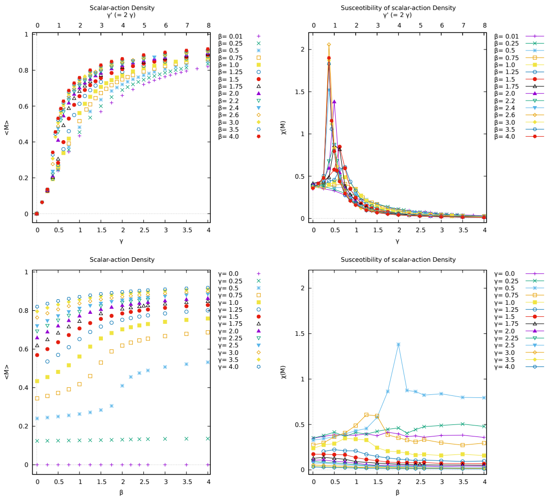
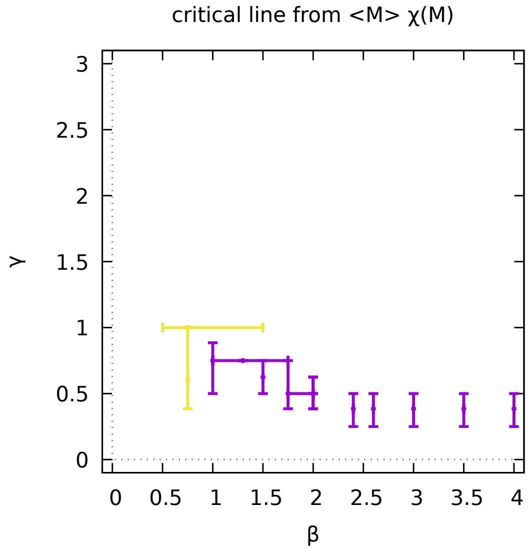


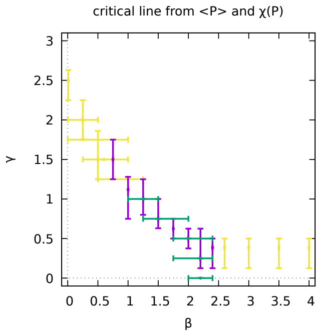
Figure: Left: $\langle M \rangle$ versus β on various $\gamma =$ const. lines. Right: $\chi(M)$ versus β on various $\gamma =$const. lines.

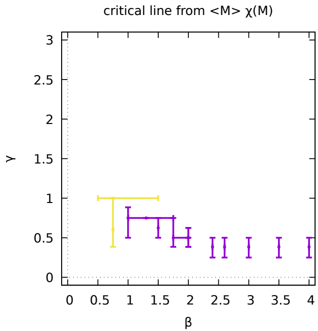
Phase boundary from the scalar-action density

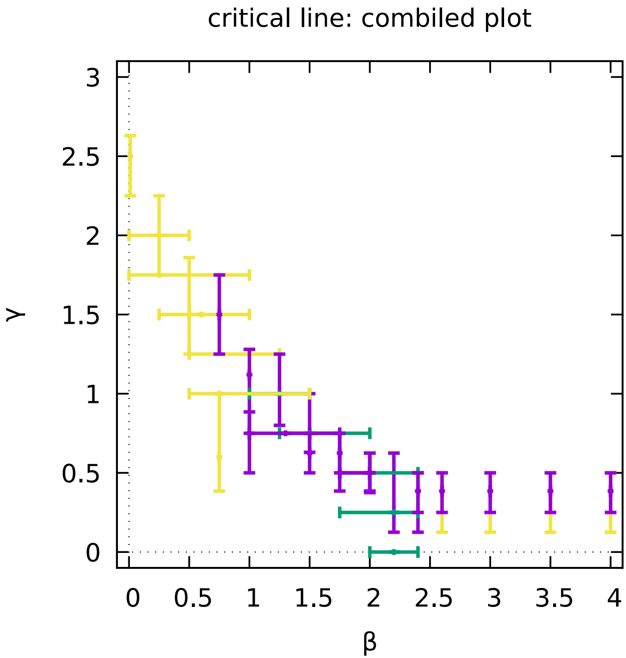




Phase boundary from action density (combination plot)







Numerical Result II [Preliminary]

scalar-color correlation

$$\boldsymbol{R} = \frac{1}{N_{\text{site}}} \sum_x \Theta_x^\dagger \mathbf{n}_x \Theta_x \,, \quad R_A = \text{tr}\left(\boldsymbol{R}\sigma^A\right)$$

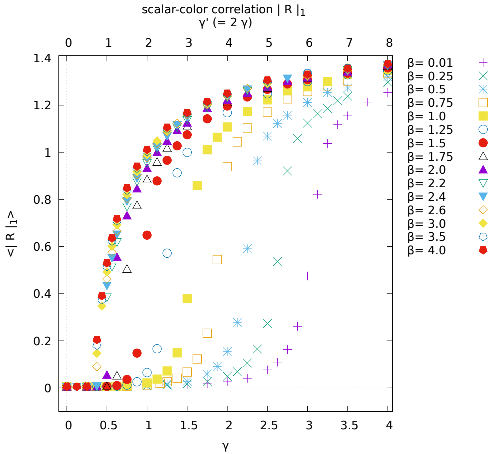
We investigate the correlations between the scalar field and the color-direction field through the gauge covariant decomposition.

We need to solve the reduction condition to obtain the color-direction field $\mathbf{n}_x$, which however has two kinds of ambiguity.

- One comes form so-called the Gribov copies that are the local minimal solutions of the reduction condition.
- Another comes from the choice of a global sign factor, which originates from the fact that whenever a configuration $\{\mathbf{n}_x\}$ is a solution, the flipped one $\{-\mathbf{n}_x\}$ is also a solution, since the reduction functional is quadratic in the color fields.

To avoid these issues, we propose to use $|\boldsymbol{R}|_1$ and $|\boldsymbol{R}|_2$, where $|\boldsymbol{R}|_n$ represents the n-norm defined by $|\boldsymbol{R}|_n = \sqrt[n]{|R_1|^n + |R_2|^n + |R_3|^n}$

scalar-color correlation $\langle |\boldsymbol{R}|_1 \rangle$



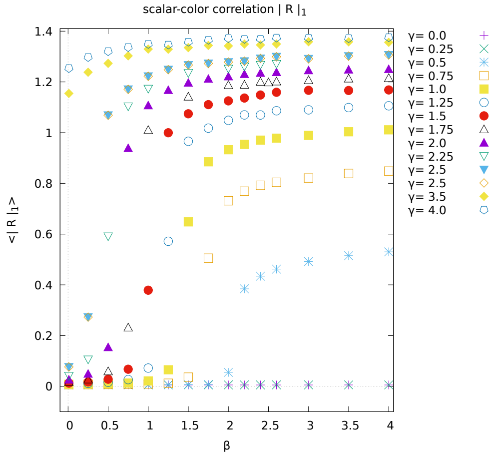


Figure: Average of the scalar-color composite field $\langle |Q| \rangle$: Left: $\langle |\boldsymbol{R}|_1 \rangle$ versus γ on various $\beta =$const. lines. Right: $\langle |\boldsymbol{R}|_1 \rangle$ versus β on various $\gamma =$const. lines.

scalar-color correlation: $|\boldsymbol{R}|_2$

$$\langle |\boldsymbol{R}|_2 \rangle \ , \quad \chi(|\boldsymbol{R}|_2) = (4N_{\rm site}) \left\{ \left\langle \ |\boldsymbol{R}|_2^2 \right\rangle - \langle \ |\boldsymbol{R}|_2 \rangle^2 \right\}$$

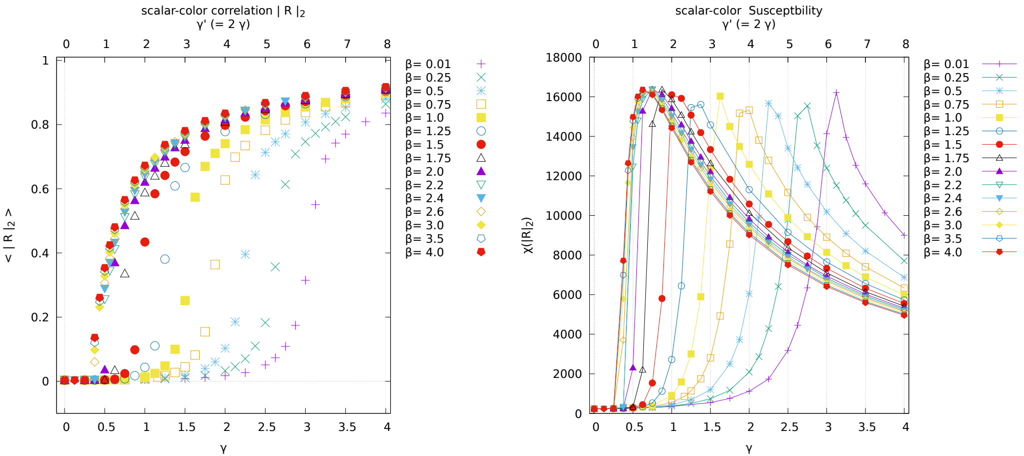


Figure: Left:$\langle |\boldsymbol{R}|_2 \rangle$ versus γ on various $\beta =$const. lines. Right:$\chi(|\boldsymbol{R}|_2)$ versus γ on various $\beta =$const. lines.

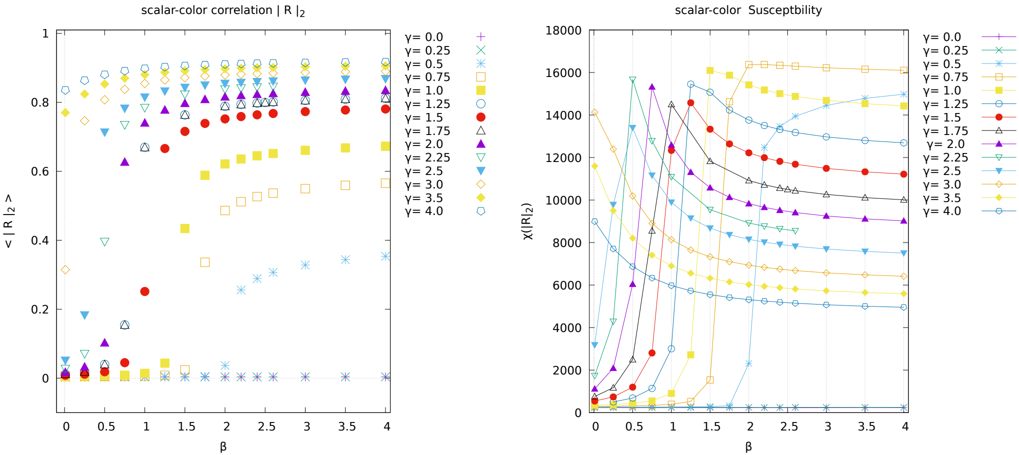
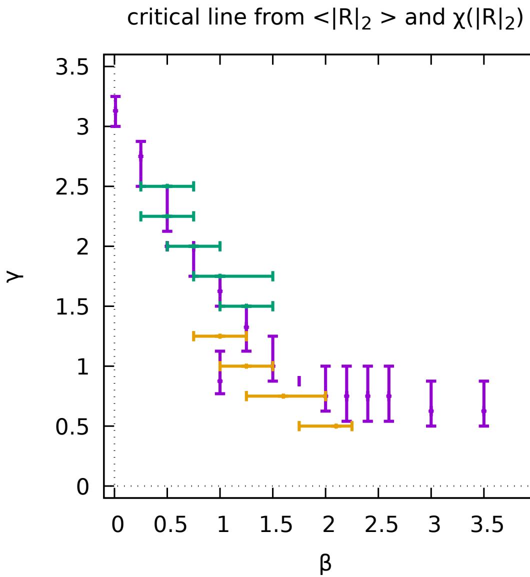


Figure: Left:$\langle|\boldsymbol{R}|_2\rangle$ versus β on various $\gamma=$const. lines. Right:$\chi(|\boldsymbol{R}|_2)$ versus β on various $\gamma=$const. lines.

Phase boundary (critical line) from scalar-color correlation: $|R|_2$



Contribution from the magnetic monopole

Next we investigate the contributions from magnetic monopoles to determine their role in confinement and mass generation (mass gap) from the viewpoint of the electric-magnetic duality.
Through the gauge-covariant decomposition (CDGSFN decomposition), we can define the magnetic monopole in the gauge-independent way:

$$V_{x,\mu,\nu} := V_{x,\mu} V_{x+\hat{\mu},\nu} V^{\dagger}_{x+\hat{\nu},\mu} V^{\dagger}_{x,\nu} = exp(-iF(x)_{\mu,\nu} \boldsymbol{n}_x) ,$$
$$F(x)_{\mu,\nu} := \arg_F \mathrm{tr}\left\{ (\mathbf{1} + \boldsymbol{n}_x) V_{x,\mu,\nu} \right\} ,$$
$$k_{x,\mu} := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} \left(F(x+\hat{\nu})_{\alpha,\beta} - F(x)_{\alpha,\beta} \right\} =: 2\pi m_{x,\mu} , \quad m_{x,\mu} = 0, \pm 1, \pm 2, \cdots$$

where $V_{x,\mu}$ represents the restricted field obtained from CDGSFN decomposition, $\boldsymbol{n}_x$ represents the color-direction field, and $k_{x,\mu}$ represents the magnetic monopole which satisfies the current conservation law, i.e., $\partial_\mu k^{x,\mu} = \sum_\mu (k_{x+\hat{\mu},\mu} - k_{x,\mu}) = 0$.
Therefore, we can define the magnetic-monopole-charge density as

$$\rho_k := \frac{1}{4N_{\mathrm{site}}} \sum_{x,\mu} |m_{x,\mu}| .$$

Magnetic-monopole density: ρ_k

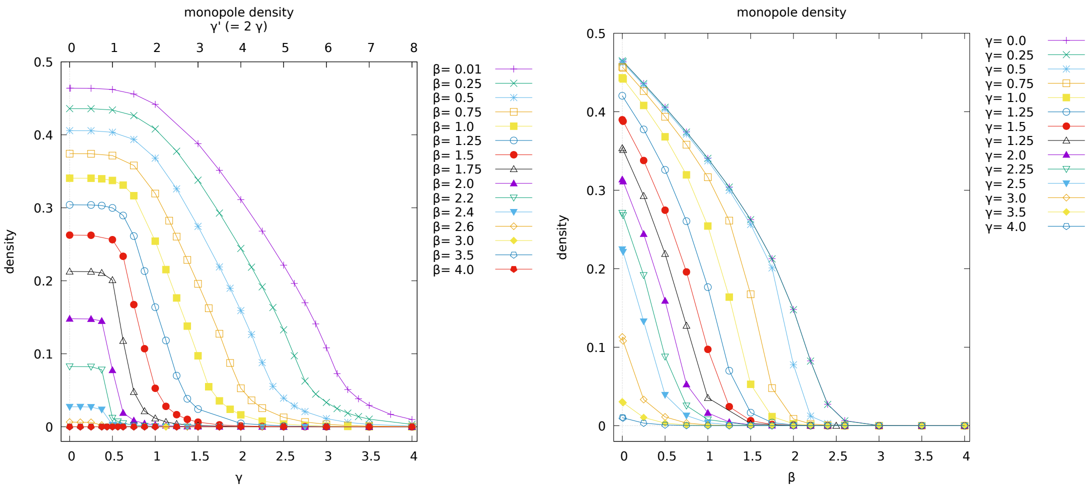


Figure: Left: $\langle \rho_k \rangle$ versus γ on various $\beta =$const. lines. Right: $\langle \rho_k \rangle$ versus β on various $\gamma =$const. lines.

Gauge-scalar correlation

Let us remind the gauge-scalar model with the scalar field in the adjoint representation:

$$
\begin{aligned}
&S_{GS}^{Ad} := S_g[U] + S_H^{Ad}[U, \boldsymbol{\phi}] \\
&S_H^{Ad}[U, \boldsymbol{\phi}] := \frac{\gamma}{2} \sum_{x,\mu} \mathrm{tr} \left\{ \left(D_{x,\mu}[U] \boldsymbol{\phi}_x \right)^\dagger \left(D_{x,\mu}[U] \boldsymbol{\phi}_x \right) \right\} , \\
&D_{x,\mu}[U] \boldsymbol{\phi}_x := U_{x,\mu} \boldsymbol{\phi}_{x+\hat{\mu}} - \boldsymbol{\phi}_x U_{x,\mu} ,
\end{aligned}
$$

where $S_g[U]$ represents the gauge action, $\boldsymbol{\phi}_x := \phi_x^A \sigma_A \in su(2) - u(1)$ represents the scalar field in the adjoint representation.
Note that the functional for the reduction has the same form as the action for the scalar field in the adjoint representation:

$$
S_H^{Ad}[U, \boldsymbol{\phi}] = \frac{\gamma}{2} F_{\mathrm{red}}[\{\boldsymbol{\phi}_x\} | \{U_{x,\mu}\}] ,
$$

and the color-field configuration $\{\boldsymbol{n}_x\}$ for the CDGSFN decomposition is obtained as the solution of motion of the equation for the scalar filed.
Therefore, we investigate a "color-action density", where a scalar field is replaced by a color field in the adjoint-scalar-action density.

Gauge-color correlation (color-action density)

$$S_{\boldsymbol{n}}^{Ad} = \sum_{x,\mu} \mathrm{tr}\left\{ \left(D_{x,\mu}[U]\mathbf{n}_x\right)^{\dagger} \left(D_{x,\mu}[U]\mathbf{n}_x\right) \right\}$$

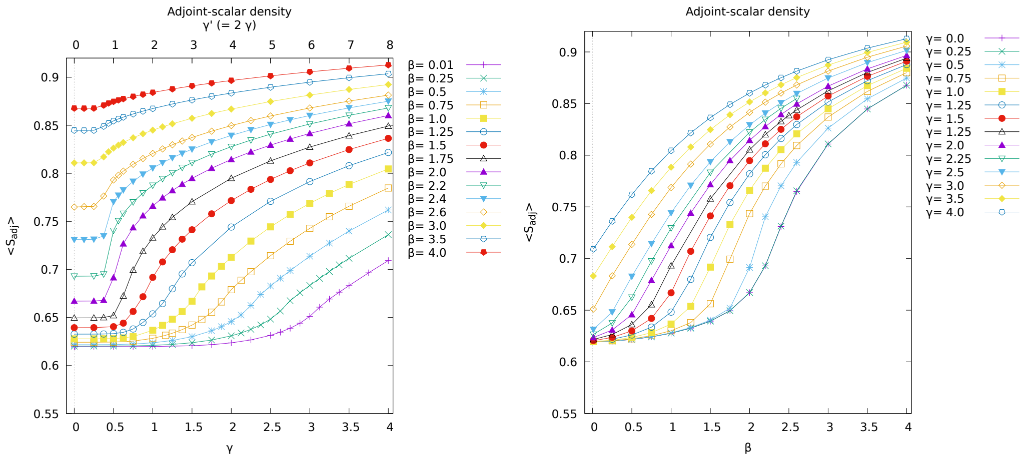


Figure: Left: $\langle S_{\boldsymbol{n}}^{Ad} \rangle$ versus γ on various $\beta =$const. lines. Right: $\langle S_{\boldsymbol{n}}^{Ad} \rangle$ versus β on various $\gamma =$const. lines.

Summary

- We investigate the gauge-scalar model with the scalar field in the fundamental representation to obtain further numerical evidence for the gauge-independent separation between Confinement phase and Higgs phase.
- For this purpose, we reexamine the phase structure without gauge fixing based on the lattice construction of gauge-independent description of the BEH mechanism.for a wider parameter space.
  - In addition to the operators used in the previous paper, we focus on the susceptibility to determine the phase boundary. ¡
  - We confirm the phase diagram in view of the thermodynamic phase transition.
  - We confirm the gauge-independent separation between Confinement phase and Higgs phase.
- Moreover, we investigate the contributions from magnetic monopoles to determine their role in confinement and the mass generation (mass gap) from the viewpoint of the electric-magnetic duality.
- We further investigate the gauge-color correlation ("color-action density").
- Note that these results are obtained by investigating the correlation functions between the gauge-invariant composite operators and the the color-direction field obtained through the gauge-covariant decomposition.