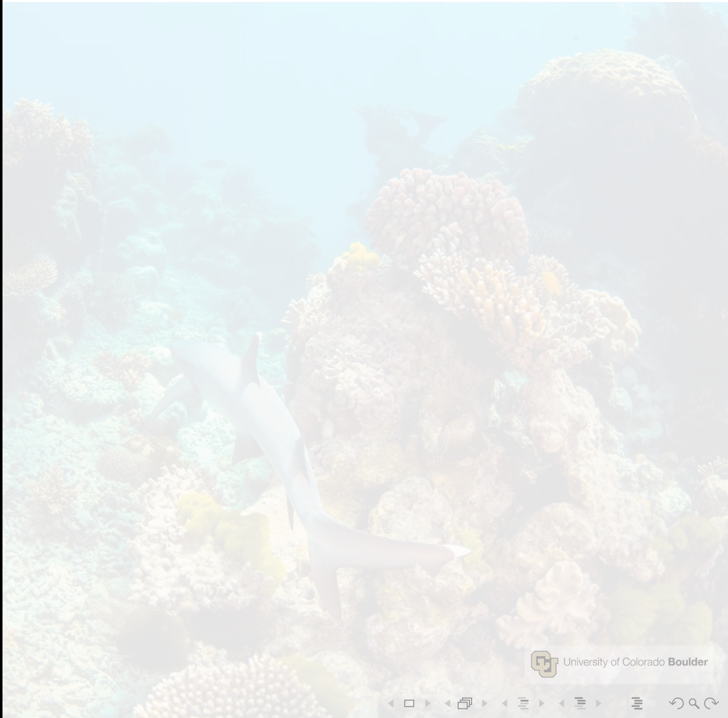
Non-Perturbative Yang-Mills Beyond One-Loop Order

Who? **Seth Grable**

From? Department of Physics
University of Colorado Boulder

When? August 2024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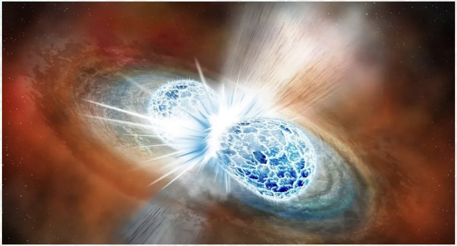

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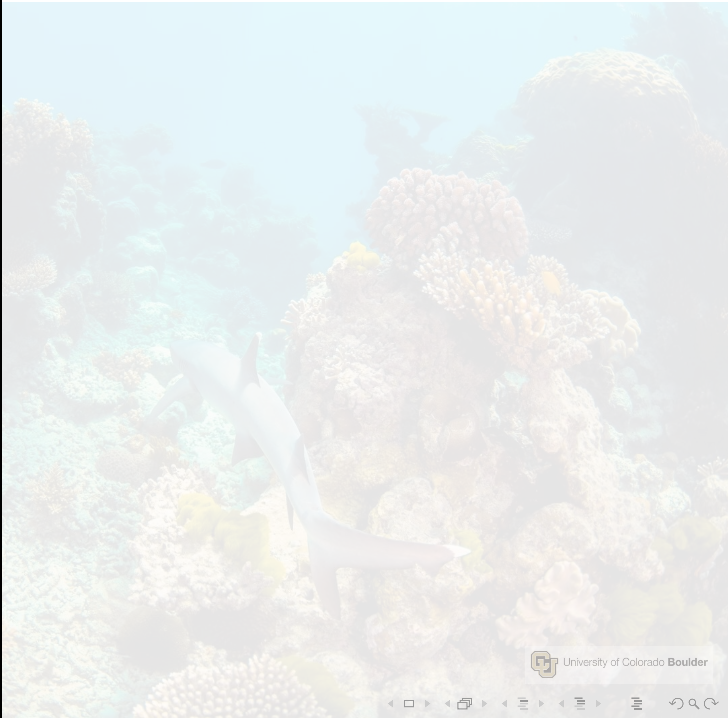
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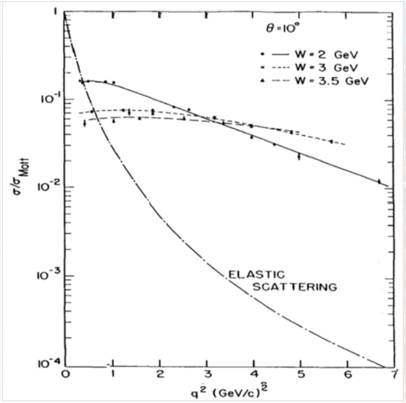

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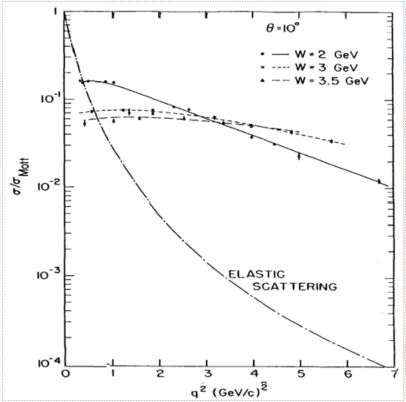

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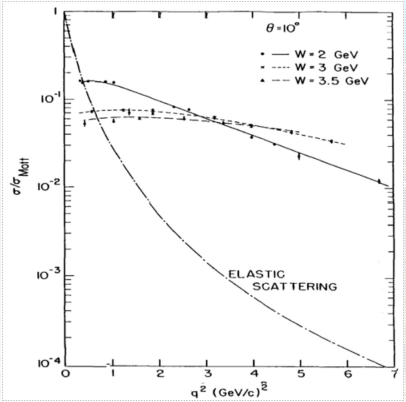

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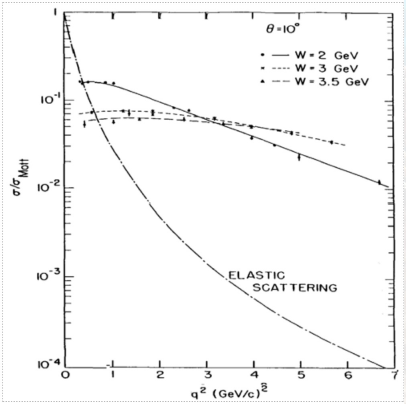

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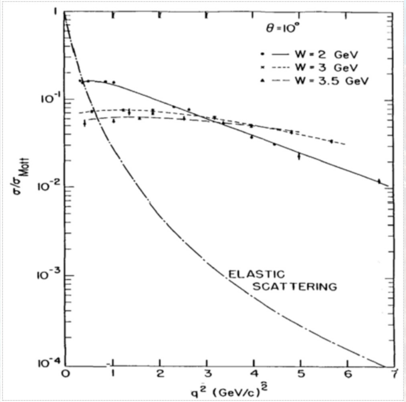

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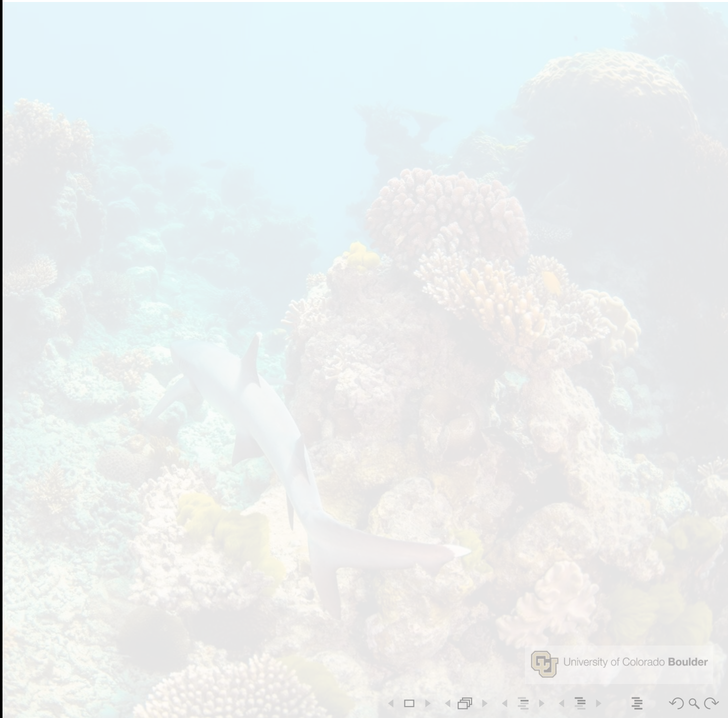
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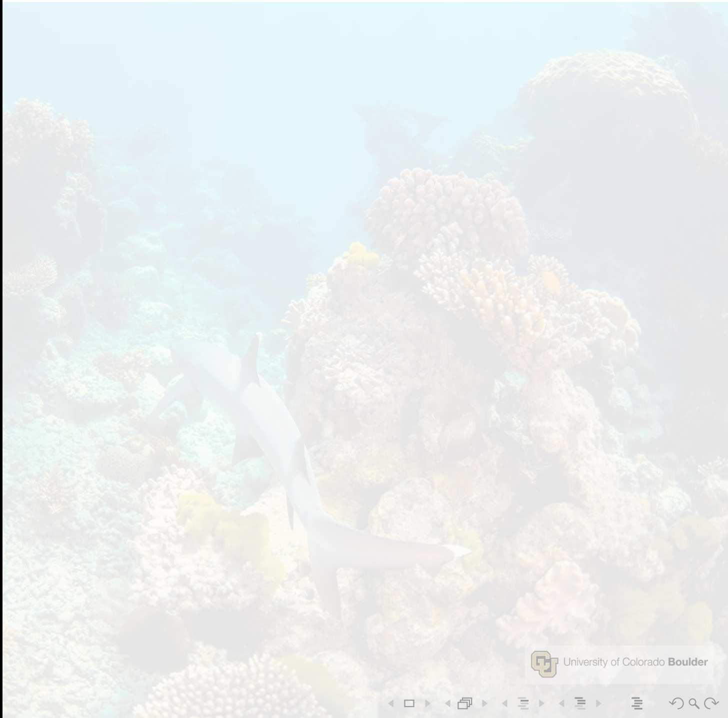
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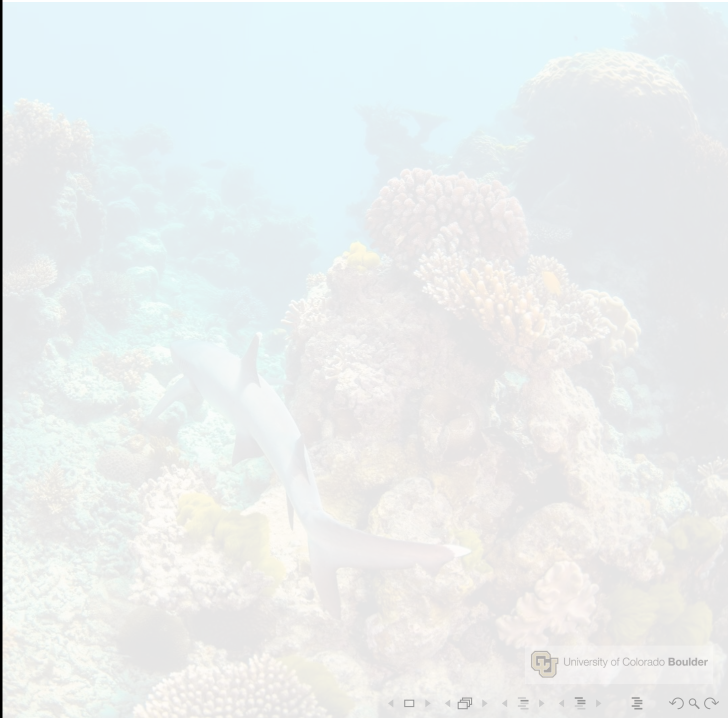
- **Laplace's method.**



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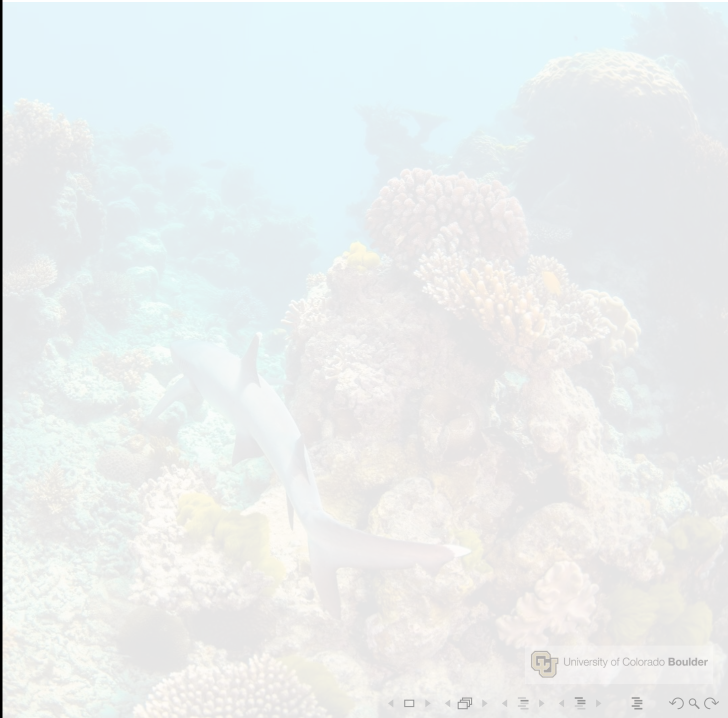
- **This allows for an expansion around B^a_μ in terms of a^a_μ.**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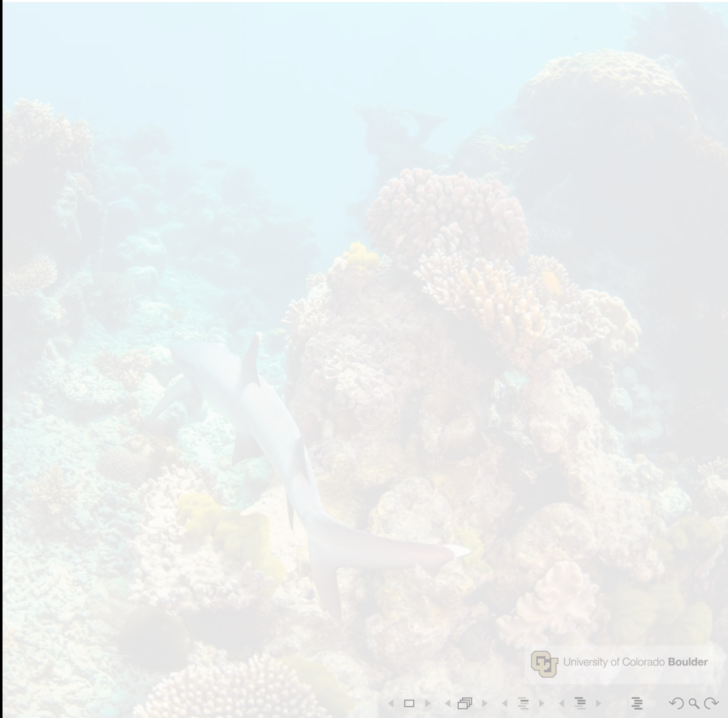
- S_0**, quadratic contributions, are calculable (Gaussian).**



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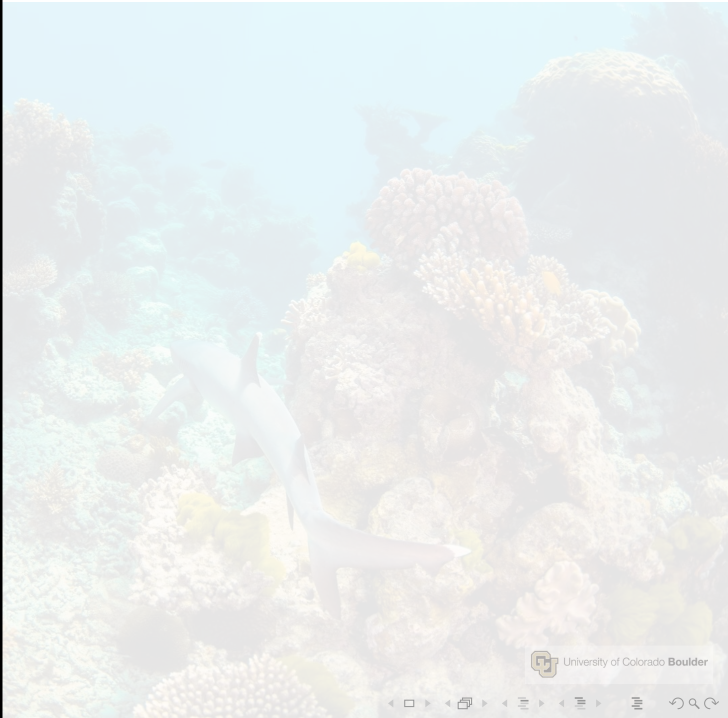
- Elements of Confinement for QCD with Twelve Massless Quarks: arXiv:2310.12203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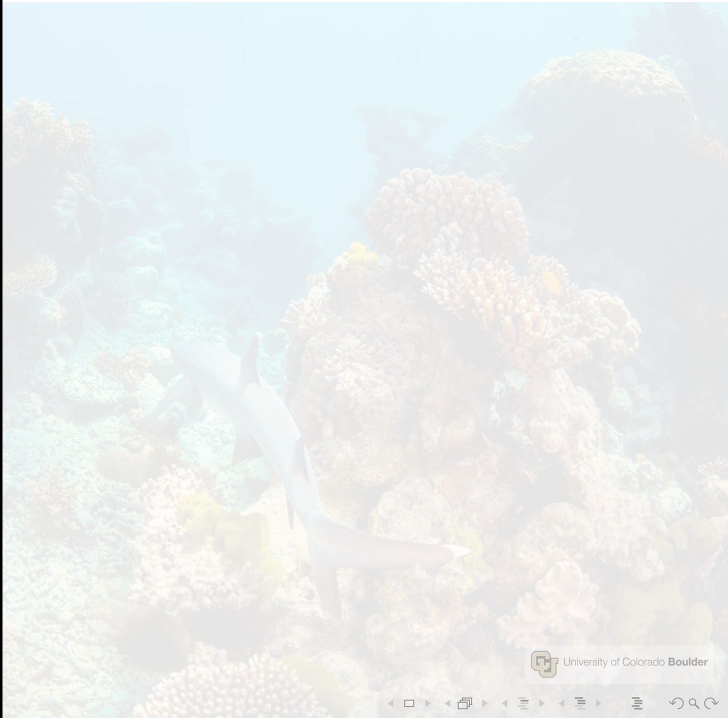
- **We see this as a validation and proof of concept for applying the BGFM to non-abelian theories.**



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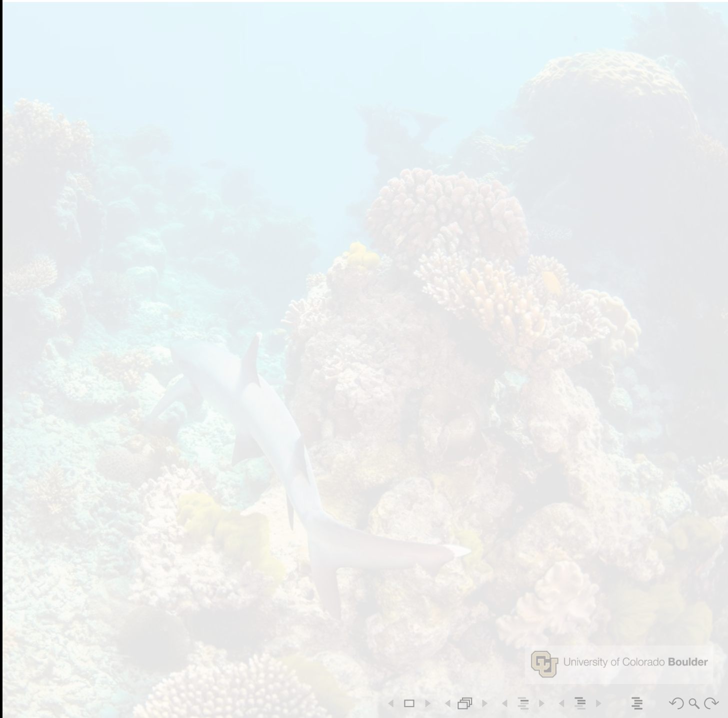
- **Technical method: Hubbard-Stratonovich transformation**



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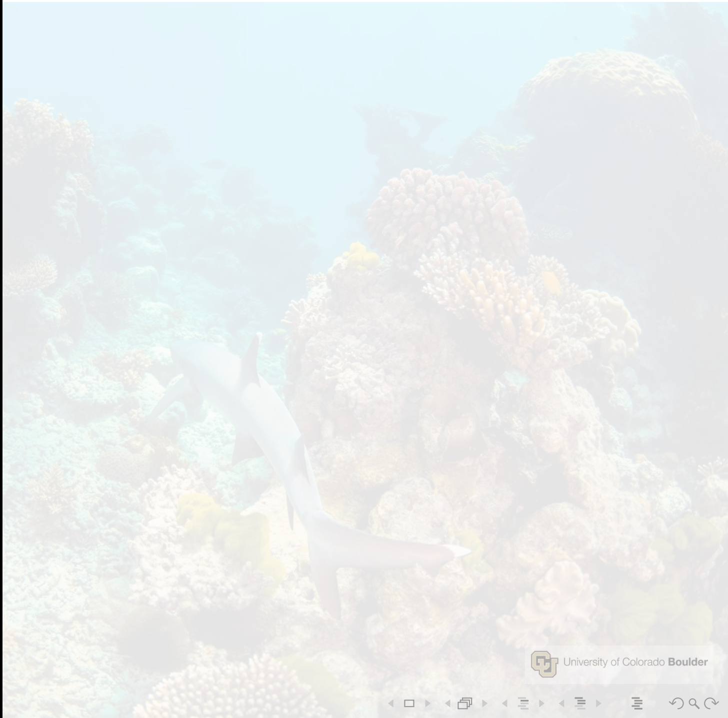
- x **can be integrated out: It's Gaussian!**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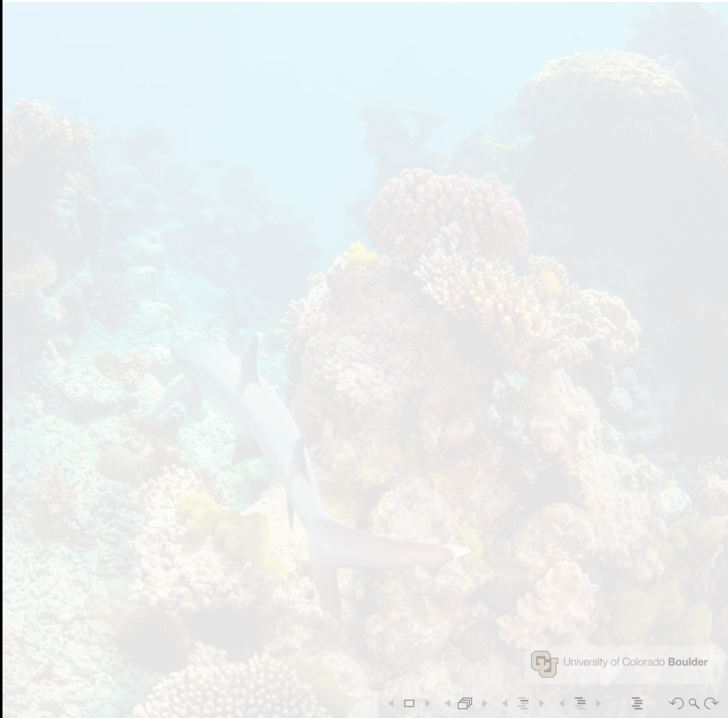
- **Energy gap in the electronic spectrum of a superconductor.**



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$$Z = e^{-\frac{\beta V}{g_0^2}(B\lambda^a)^2 - \frac{1}{2}\ln\det\left[\theta^{R_0}_{\mathbf{Glue}}\right] + \ln\det\left[\theta_{\mathbf{Ghost}}\right]}\Bigg|_{\bar{B}\bar{\Delta}}, \quad (8)$$

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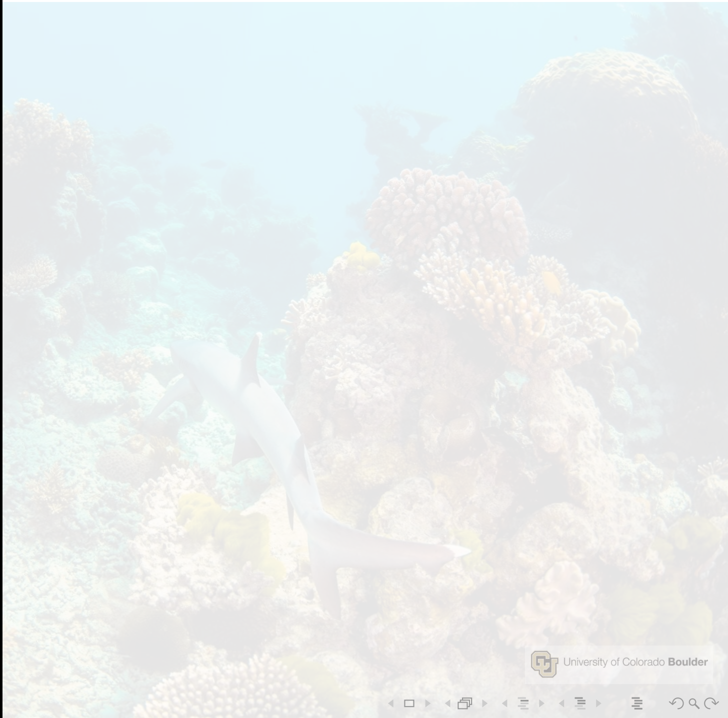
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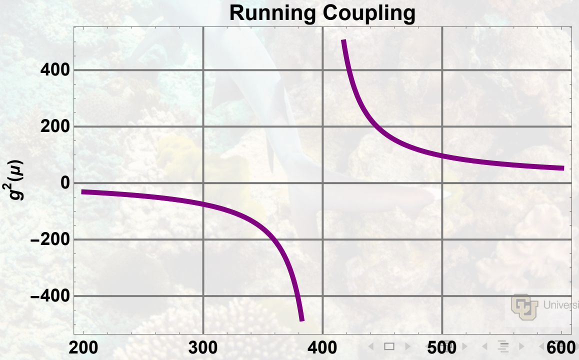

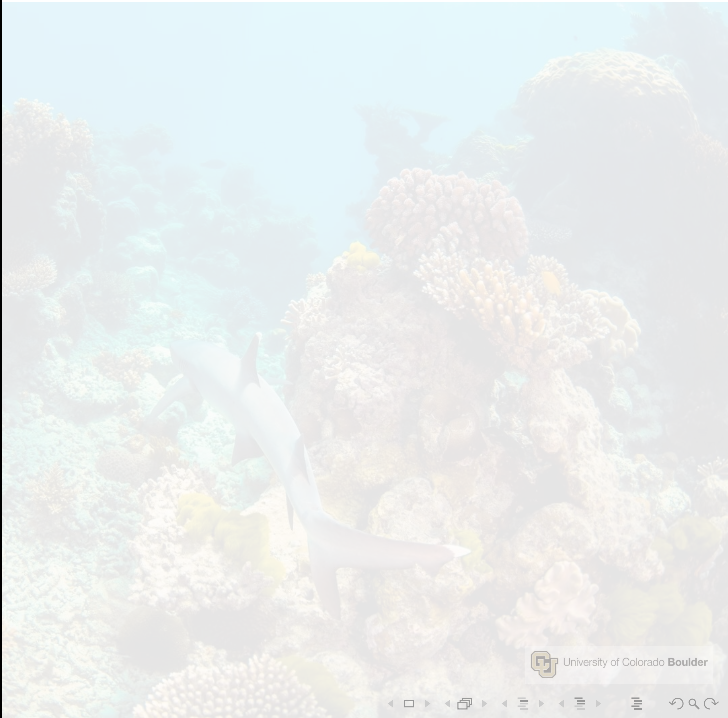

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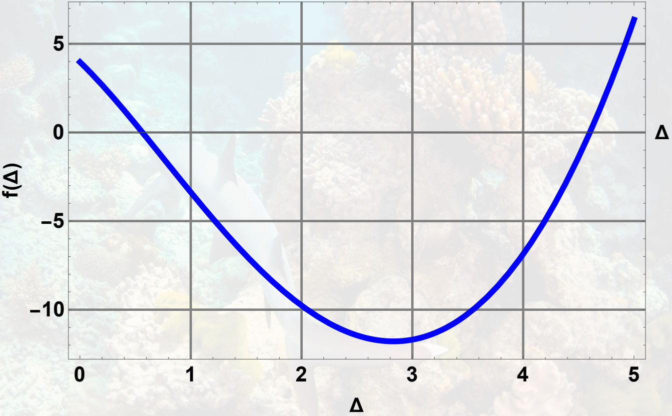
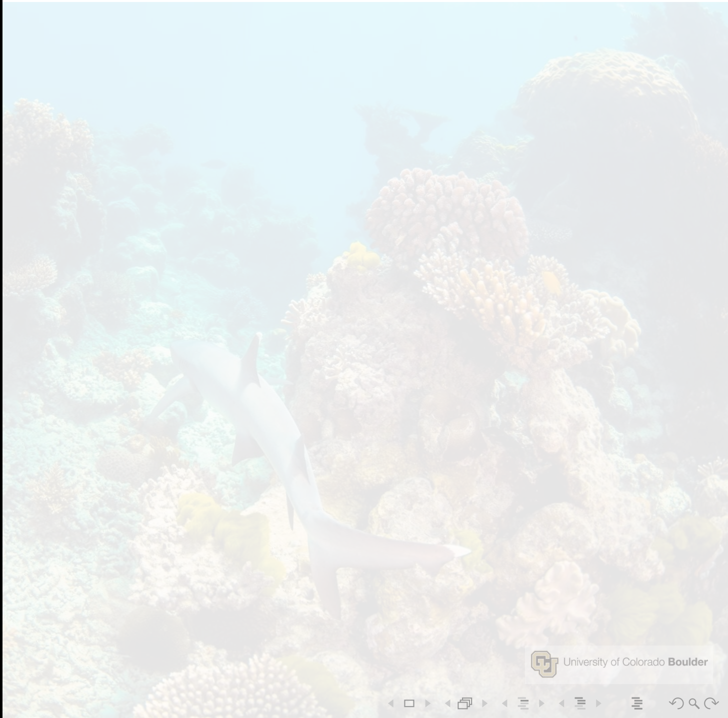
Functional Dependence of the Auxillary field Δ



-
- **Gap:**

$$\bar{\Delta} \rightarrow 2.82898. \quad (10)$$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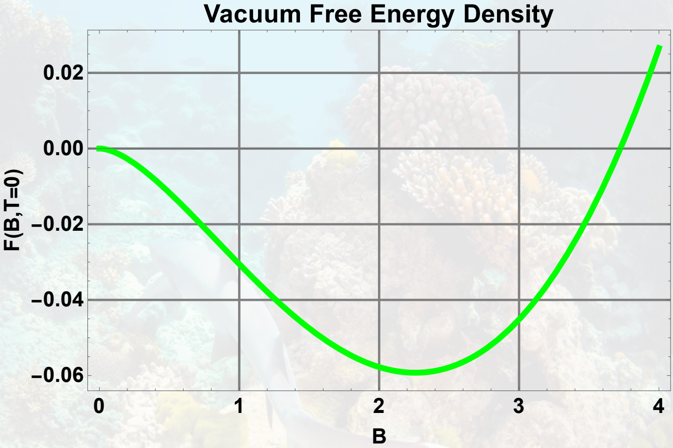

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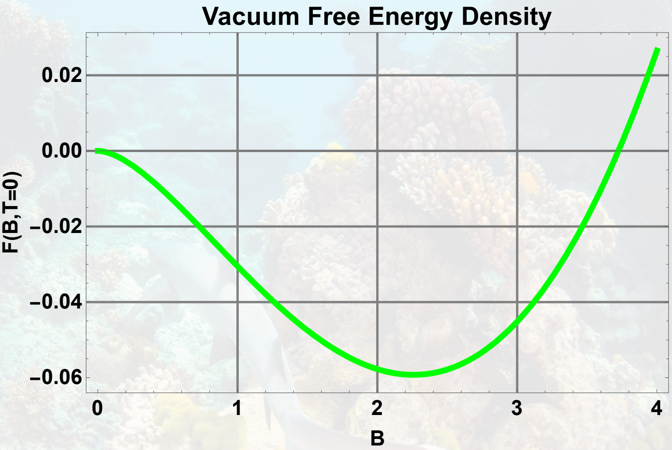
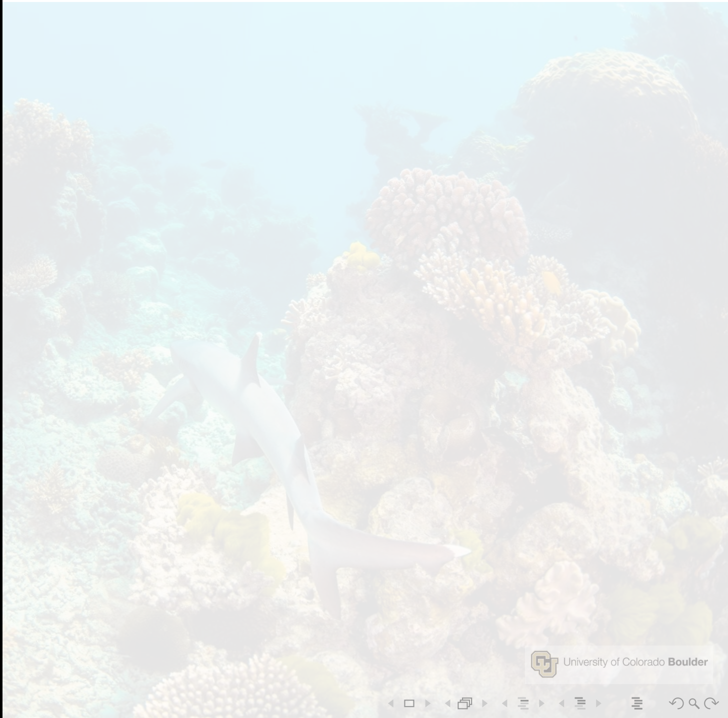


- **Gap:**

$$\bar{B} \to \frac{2.25885\Lambda_{\mathbf{YM}}^2}{\lambda^l}\ ,\ \bar{B} \to 0, \tag{11}$$



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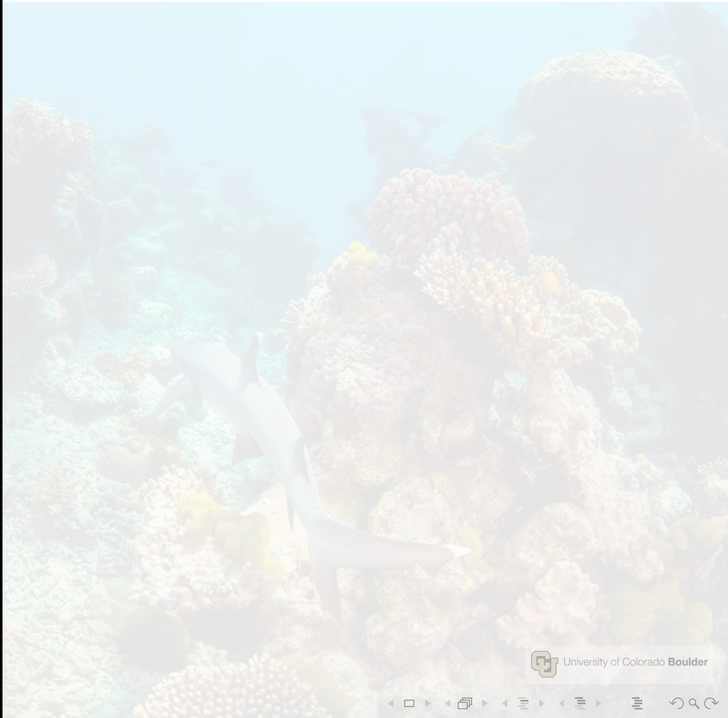

Future Work

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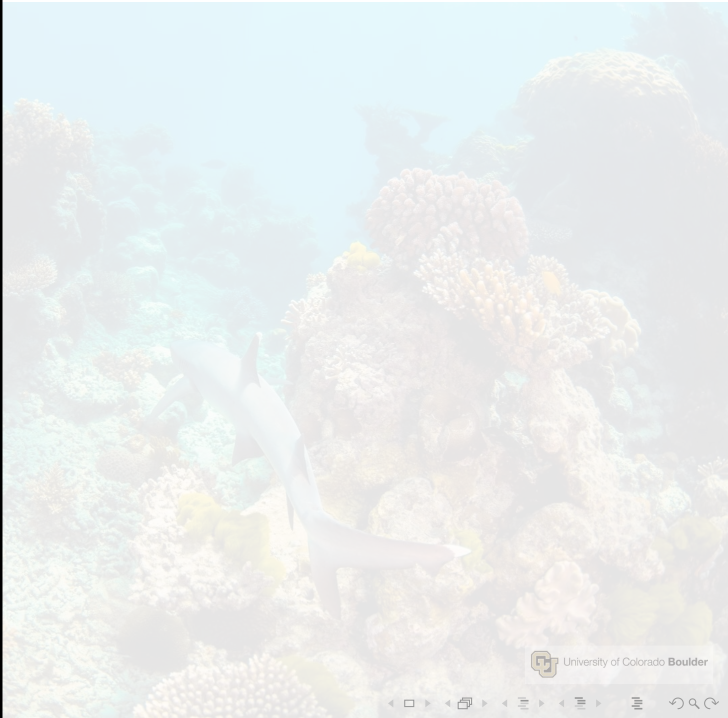

Future Work

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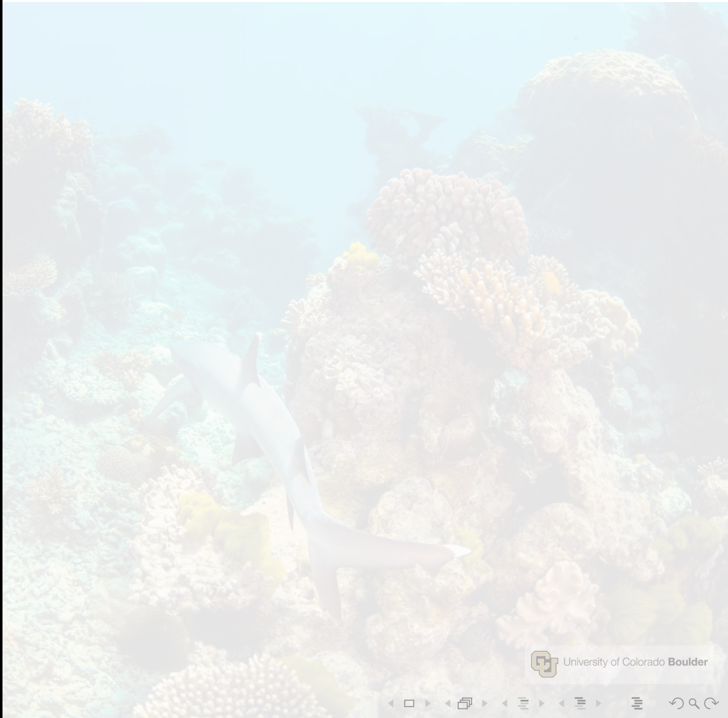
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Fun!

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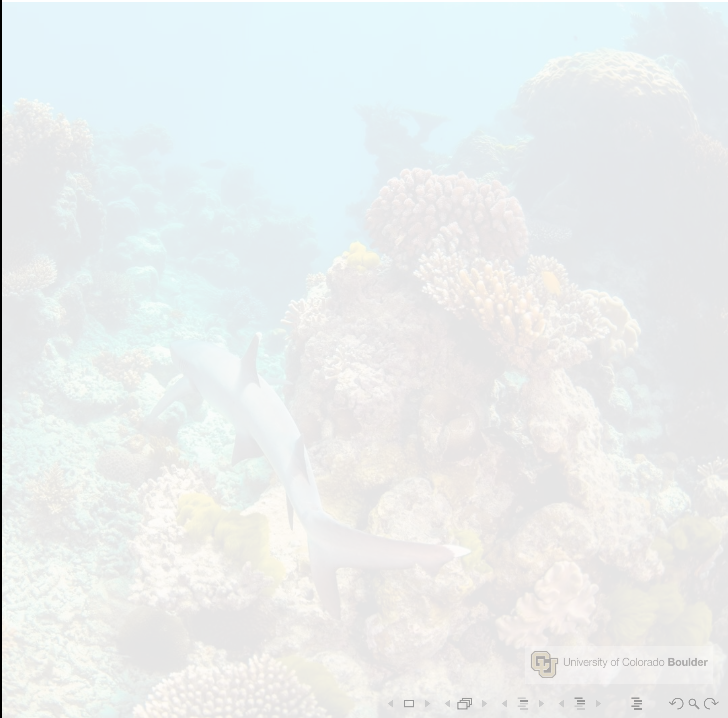

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Defining the Background Operators

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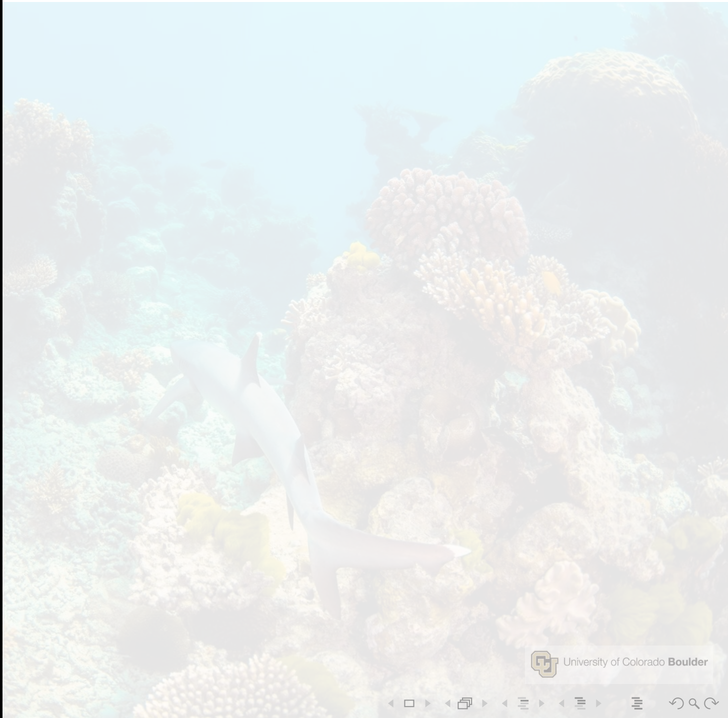

Gauge Fixing and Local Gauge Invariance

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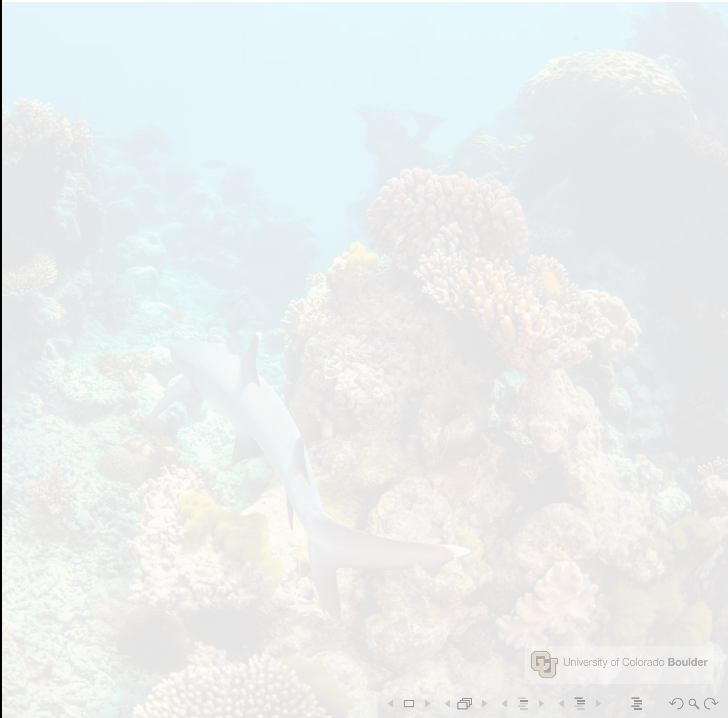
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The Eigenspectrum of S_0

$$D_{\mu}^{ac} = \partial_{\mu}\delta^{ac} + \frac{i}{2}A^{ac}F_{\mu\nu}x_{\nu} \tag{20}$$

where A is the hermitian matrix consisting of the sum generators in the adjoint representation.

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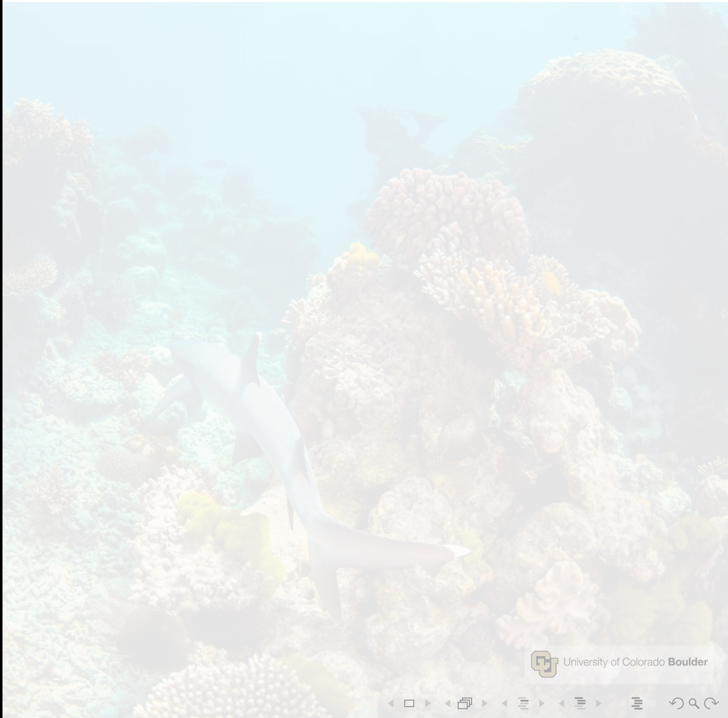
- **Then:**

$$\begin{aligned} -(D^2)^{ac} = &-(\partial_0^2)\delta^{ac} - (\partial_1^2)\delta^{ac} + i(AB)^{ac}\big(x_1\partial_0 - x_0\partial_1\big) \\ &+ \frac{1}{4}(A^2B^2)^{ac}\big(x_0^2 + x_1^2\big) \\ &- (\partial_2^2)\delta^{ac} - (\partial_3^2)\delta^{ac} + i(AB)^{ac}\big(x_3\partial_2 - x_2\partial_3\big) \\ &+ \frac{1}{4}(A^2B^2)^{ac}\big(x_2^2 + x_3^2\big). \end{aligned} \tag{21}$$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$$\begin{aligned} c_\mu &= \left[\frac{\partial_\mu \ \mathbb{1}}{(B\mathbf{A}')^{\frac{1}{2}}} + \frac{1}{2}(B\mathbf{A}')^{\frac{1}{2}} \hat{x}_\mu \right] \\ c_\mu^\dagger &= \left[-\frac{\partial_\mu \ \mathbb{1}}{(B\mathbf{A}')^{\frac{1}{2}}} + \frac{1}{2}(B\mathbf{A}')^{\frac{1}{2}} \hat{x}_\mu \right]. \end{aligned} \tag{22}$$

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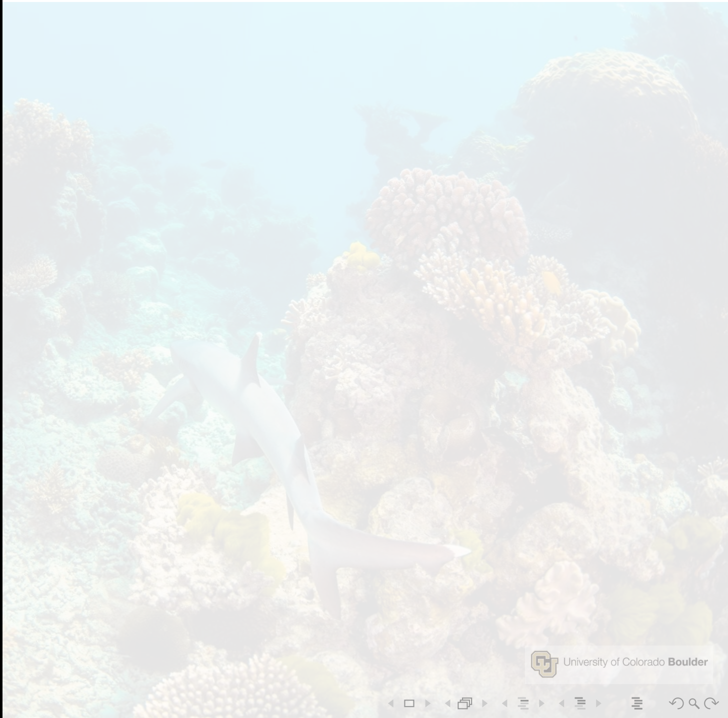
-

$$\begin{aligned} -(D^2)' =& B\mathbf{A}' \Big[\big(c_0^\dagger + i c_1^\dagger\big)\big(c_0 - i c_1\big) + 1 \Big] + \\ & B\mathbf{A}' \Big[\big(c_2^\dagger + i c_3^\dagger\big)\big(c_2 - i c_3\big) + 1 \Big], \end{aligned} \tag{23}$$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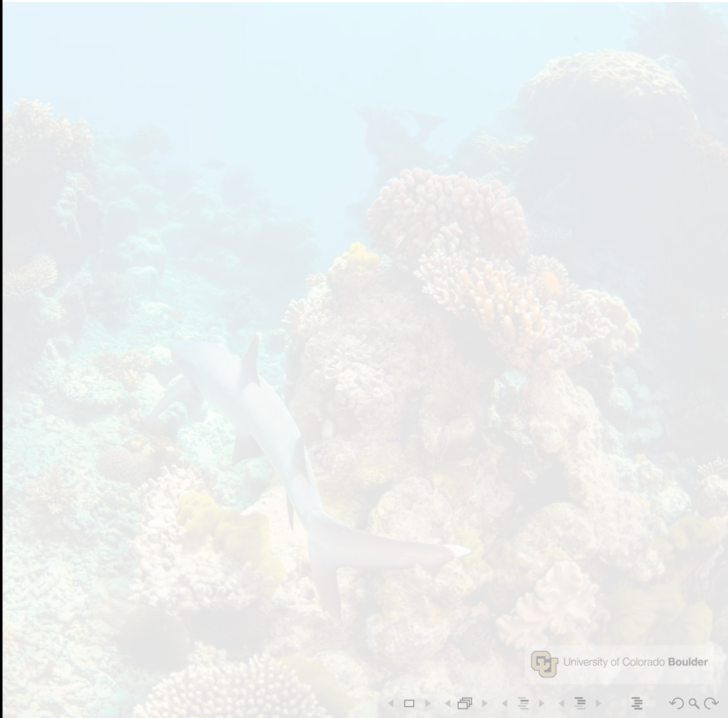
$$\begin{aligned}(\Lambda_a^+)_{m,n} &= (2n+1)B\lambda_a + (2m+1)B\lambda_a + 2B\lambda_a \\ (\Lambda_a^-)_{m,n} &= (2n+1)B\lambda_a + (2m+1)B\lambda_a - 2B\lambda_a,\end{aligned} \tag{24}$$



Evaluating the Path Integral

- **HS transformation is applied to Z of the form**

$$\int_{-\infty}^{\infty} \mathcal{D}\sigma \int_{0}^{\infty} \mathcal{D}\xi \mathbf{Re}[e^{i\int_x \xi^a_{\mu\nu}(\sigma^a_{\mu\nu} - f^{abc} a^b_\mu a^c_\nu)}] = 1, \quad (25)$$

under the ansatz that $\xi^a_{\mu\nu}$ is diagonal in Lorentz space.

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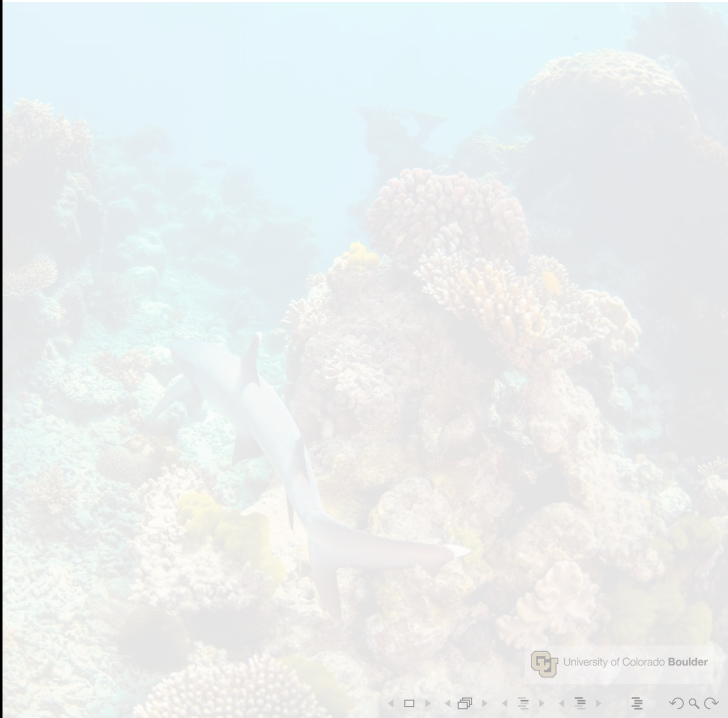
- **Some elbow grease gives:**

$$Z=\int\mathcal{D}a\mathcal{D}c\mathcal{D}\bar{c}\;\mathbf{Re}\Big[\exp[-\int_x\frac{1}{4g_0^2}(\bar{F}^{ab}_{\mu\nu})^2+\frac{(B\Delta^a_{\mu\nu})^2}{g_0^2}$$
$$+a^a_\mu\big[-(D^2)^{ac}\delta_{\mu\nu}+2A^{ac}F_{\mu\nu}$$
$$+\Delta BA^{ac}\delta_{\mu\nu}\big]a^c_\nu+\bar{c}^a\big[(-(D_\mu)^2)^{ac}\big]c^c]\Big]_{\bar{B},\bar{\Delta}}. \quad (26)$$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Heat Kernels

$$\ln \det \theta = -\Big[\frac{d}{ds}\frac{1}{\Gamma[s]}\int_0^\infty d\tau \tau^{s-1} K_\theta\Big]_{s=0}, \qquad (27)$$

where

$$K_\theta = \mathbf{Tr}^{ab}_{\mu\nu} \sum_{n,m} e^{-\tau\theta/\mu^2} \qquad (28)$$

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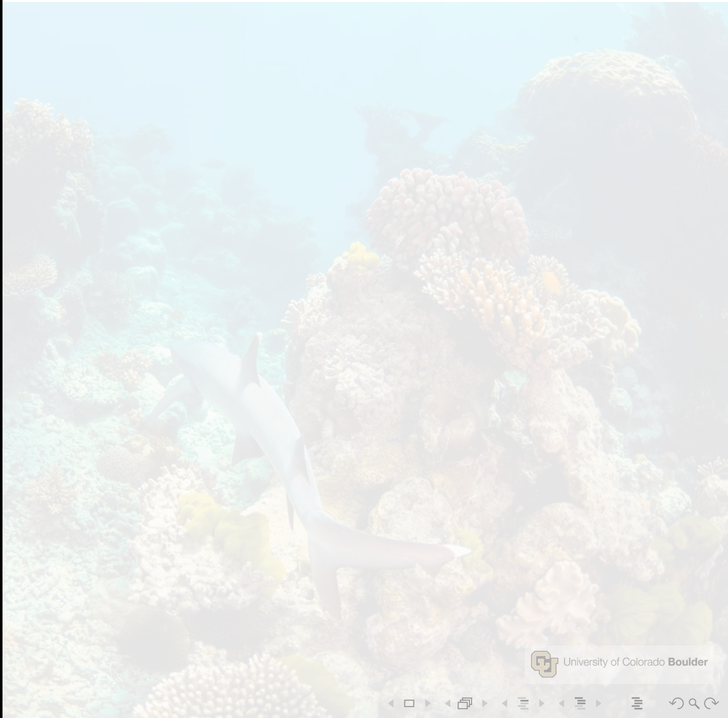
$$K_{\mathbf{Ghost}} = \sum_l \beta V\frac{(B\lambda^l)^2}{16\pi^2}\left[\frac{1}{\sinh^2(\frac{B\lambda^l\tau}{\mu^2})}\right]$$

&

$$K^{R_0}_{\mathbf{Glue}} = \sum_l \beta V\frac{(B\lambda^l)^2}{4\pi^2}\left(e^{-\frac{\tau B\lambda^l\Delta}{\mu^2}}\right)\left[2+\frac{1}{\sinh^2(\frac{B\lambda^l\tau}{\mu^2})}\right]. \tag{29}$$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ln det θ's

- **Ghosts:**

$$\ln \det \theta_{\mathbf{Ghost}} = -\sum_l \beta V \frac{(B\lambda^l)^2}{48\pi^2} \left[\ln\left(\frac{B\lambda^l}{\mu^2}\right) + \ln\left(\frac{2e}{A^{12}}\right) \right] \tag{30}$$

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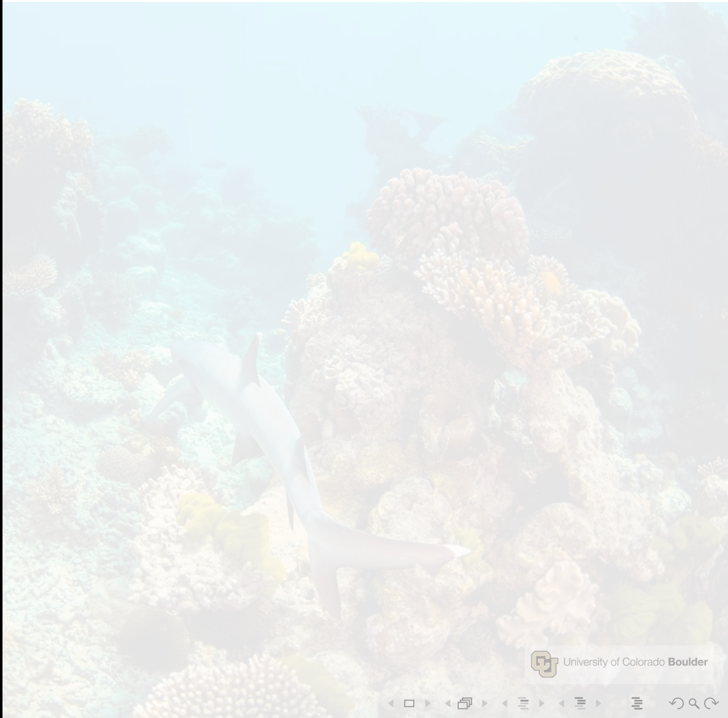
- **Glues:**

$$-\frac{1}{2}\ln\det[\theta^{R_0}_{\textbf{Glue}}] = \sum_l \frac{\beta V (B\lambda^l)^2}{8\pi^2}\times$$
$$\frac{d}{ds}\left[\frac{1}{\Gamma(s)}\left(2\int_0^\infty d\tau \tau^{s-1} e^{-\tau\frac{B\lambda^l\Delta}{\mu^2}}\right.\right.$$
$$+\int_0^\infty d\tau \sum_{n=0}^{2} \frac{\tau^{n-1}(-\frac{B\lambda^l\Delta}{\mu^2})^n}{n!}\frac{1}{\sinh^2\left(\frac{B\lambda^l\tau}{\mu^2}\right)}$$
$$\left.\left.+\int_0^\infty d\tau \sum_{n=3}^{\infty} \frac{\tau^{n-1}(-\frac{B\lambda^l\Delta}{\mu^2})^n}{n!}\frac{1}{\sinh^2\left(\frac{B\lambda^l\tau}{\mu^2}\right)}\right)\right]_{s=0}. \tag{31}$$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Evaluating the Path Integral

- **The equation:**

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-

$$
\frac{\ln Z}{\beta V} = -\frac{(B\lambda^l)^2}{g_0^2} - \frac{(B\lambda^l)^2}{48\pi^2}\left[11\ln\left(\frac{B\lambda^l}{\mu^2}\right) + C + f(\Delta) + g(\Delta, B, \mu)\right]\Bigg|_{\bar{B}\bar{\Delta}} \tag{32}
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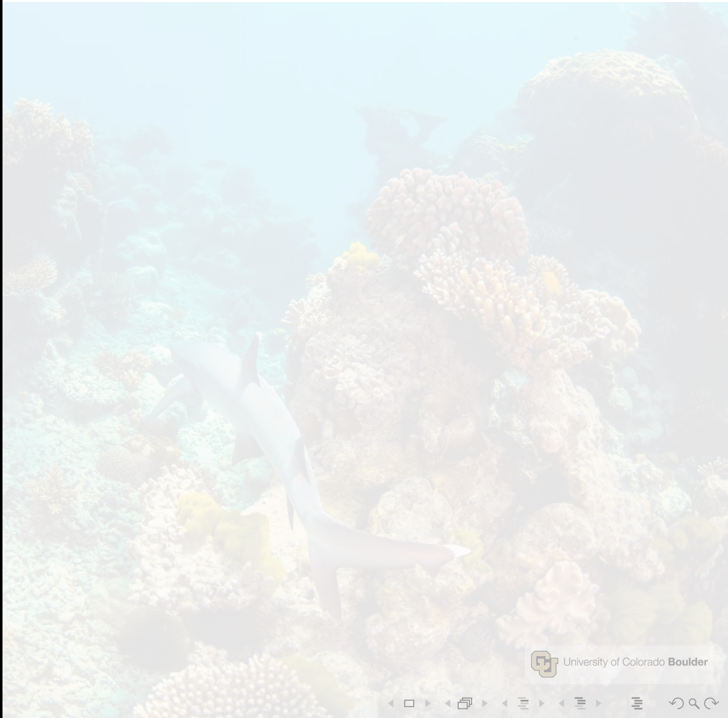
- **where**

$$C = \ln\left(\frac{e}{2A^{12}}\right)$$

$$g(\Delta, B, \mu) = 3\Delta^2 \ln\left(\frac{B\lambda^l}{\mu^2}\right) + 12\ln(\Delta)$$

$$f(\Delta) = -\Delta\left(\ln\left(64\pi^6\right) - 3\Delta\ln(2)\right) + 12\Delta\ln\left(\Gamma\left[\frac{\Delta+2}{2}\right]\right) - 24\zeta^{(1,0)}\left(-1, \frac{\Delta+2}{2}\right). \tag{33}$$



Renormalization

- **Use:**

$$\frac{1}{g^2(\mu)} = \frac{1}{g_0^2} + \frac{1}{48\pi^2} g(\Delta, B, \mu) \tag{34}$$

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$$\frac{\ln Z}{\beta V} = -\sum_a \frac{(B\lambda^l)^2}{48\pi^2} \left[11 \ln \left(\frac{B\lambda^l}{\Lambda^2_{\mathbf{YM}}} \right) + C + f(\Delta) \right]_{\bar{B}, \bar{\Delta}} . \qquad (35)$$

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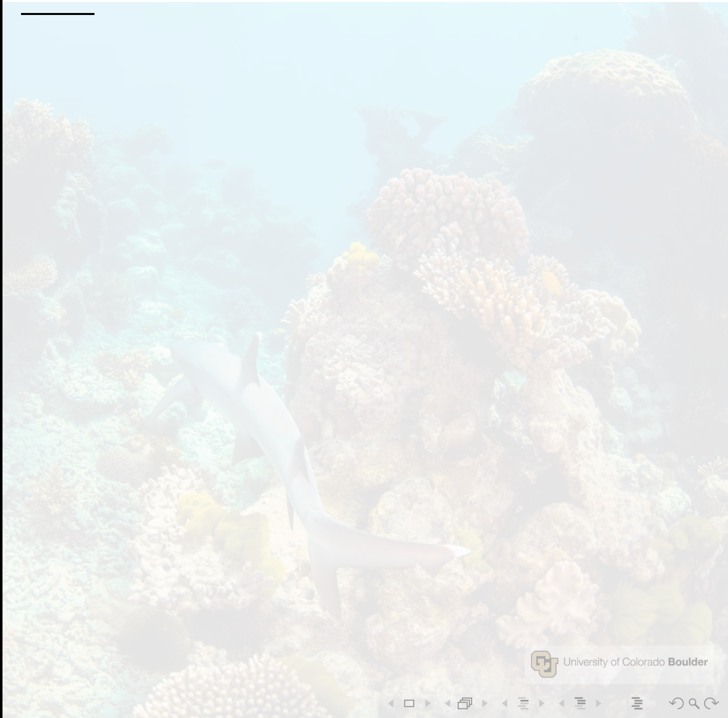
- **Solve the gap equations and get results!**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