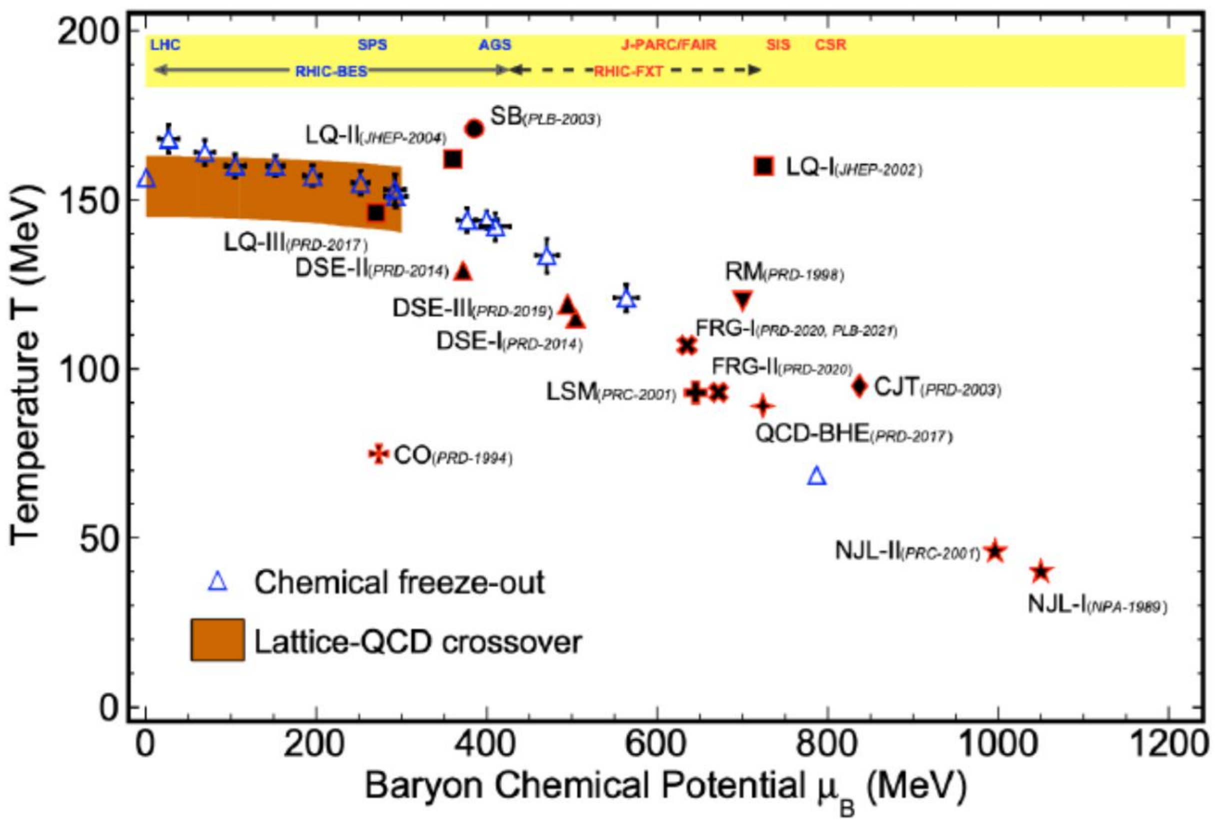
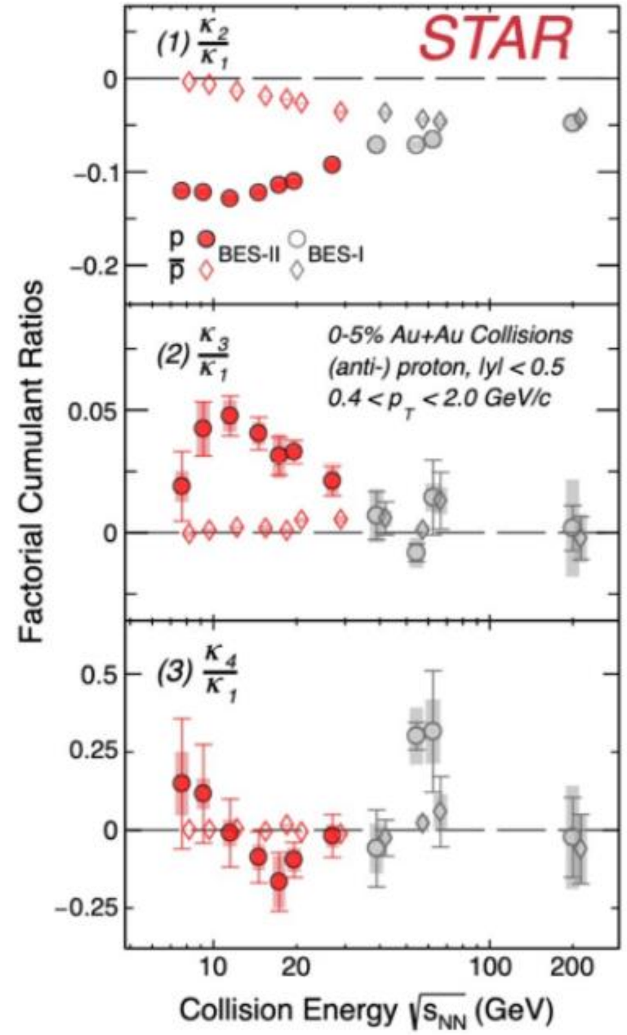
Fluctuations of conserved charges in heavy ion collisions and lattice QCD

Rene Bellwied (University of Houston)





Special thanks to my UH theory colleagues: C. Ratti, V. Vovchenko, J. Jahan

The QCD Phase Diagram

What we know

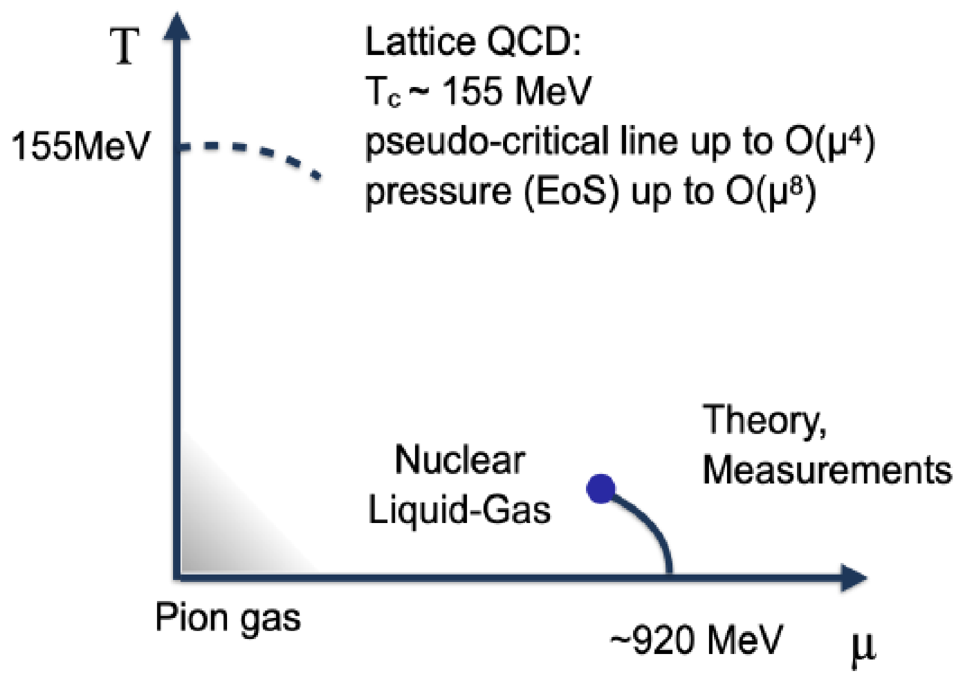


Figure courtesy of V. Koch

What we hope to know

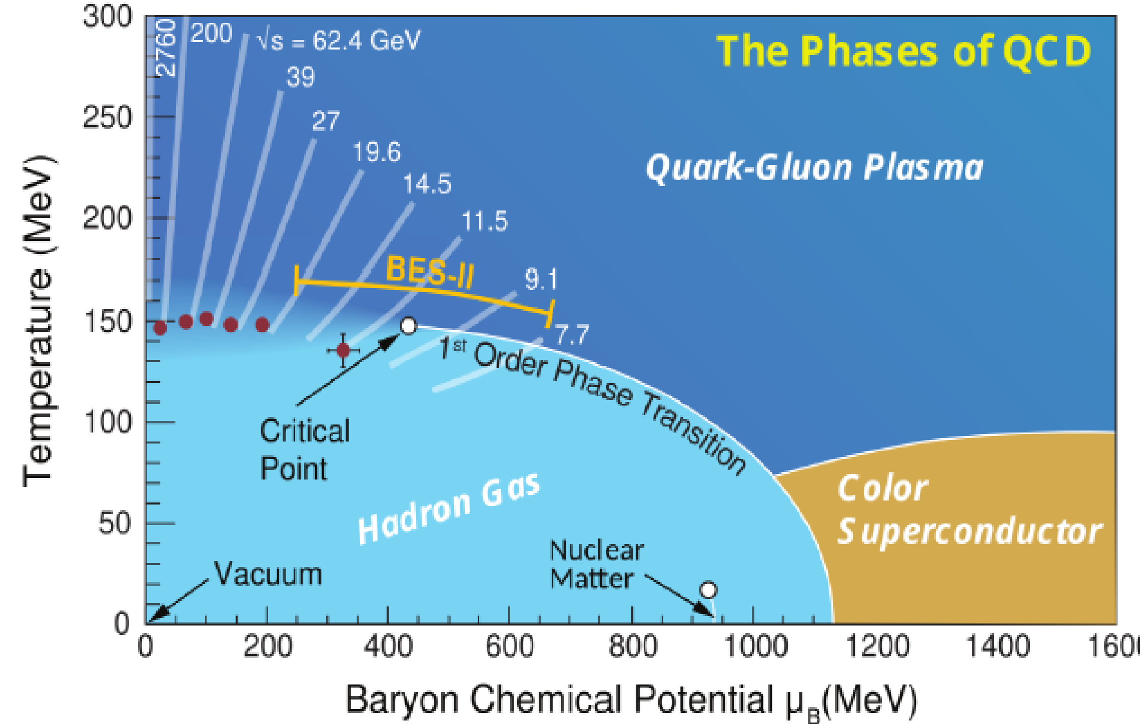
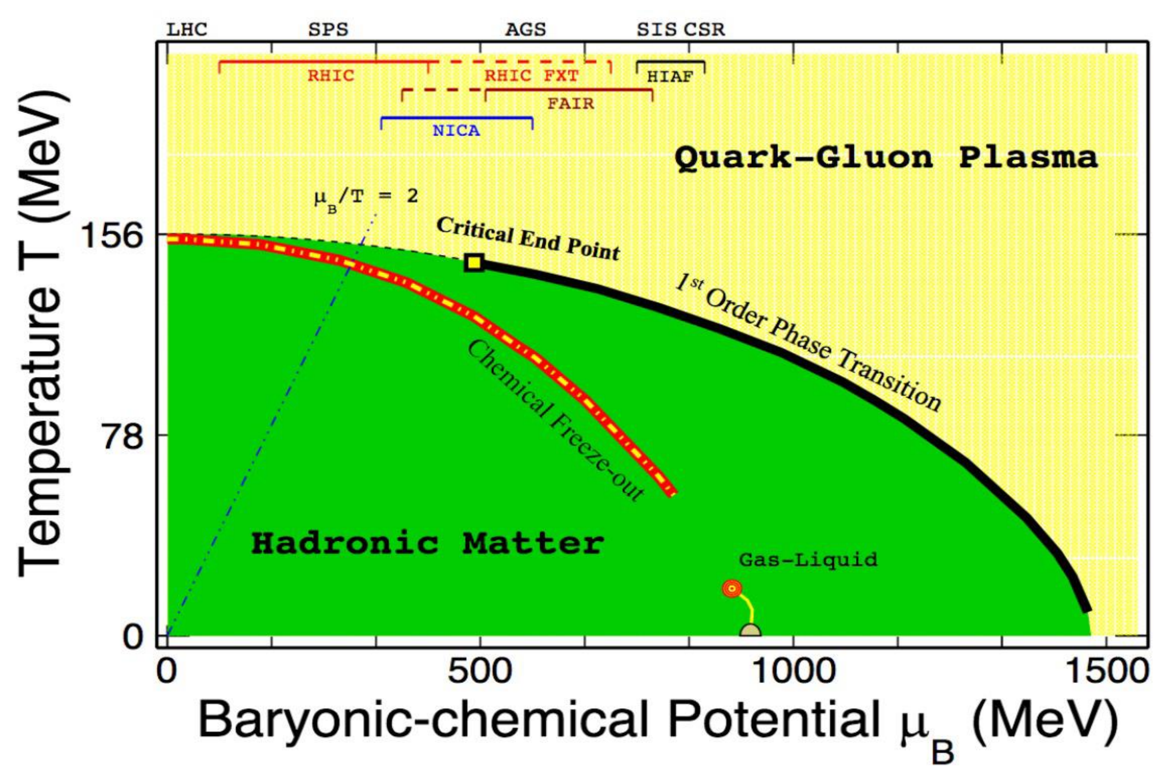


Figure from Bzdak et al., Phys. Rept. '20 & 2015 Long Range Plan



Experimental signatures potentially based on critical multiplicity fluctuations

- Critical point in QCD phase diagram at finite μ_B. Experimental conditions reproduced at RHIC and GSI.

- Chiral transition at $\mu_B = 0$ (which temperature, which order). Experimental conditions reproduced at the LHC.

- Evidence for strong magnetic field at the onset of the deconfined quark gluon phase. LHC measurements.

The Basics: Relation between susceptibilities, event by event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- Cumulants of conserved quantities

Net-baryon (B) (net-proton as proxy)
Net-electric charge (Q)
Net-strangeness (S) (net-kaon as proxy)

$$\delta N = N - \langle N\rangle$$
$$C_1 = \langle N\rangle = M$$
$$C_2 = \langle(\delta N)^2\rangle = \sigma^2$$
$$C_3 = \langle(\delta N)^3\rangle$$
$$C_4 = \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2$$

$$\frac{C_2}{C_1} = \frac{\sigma^2}{M}, \quad \frac{C_3}{C_2} = S\sigma$$
$$\frac{C_4}{C_2} = \kappa\sigma^2$$

1. Sensitive to correlation length

$$C_3 = \langle(\delta N)^3\rangle \sim \xi^{4.5}$$
$$C_4 = \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle \sim \xi^7$$

2. Related to susceptibility

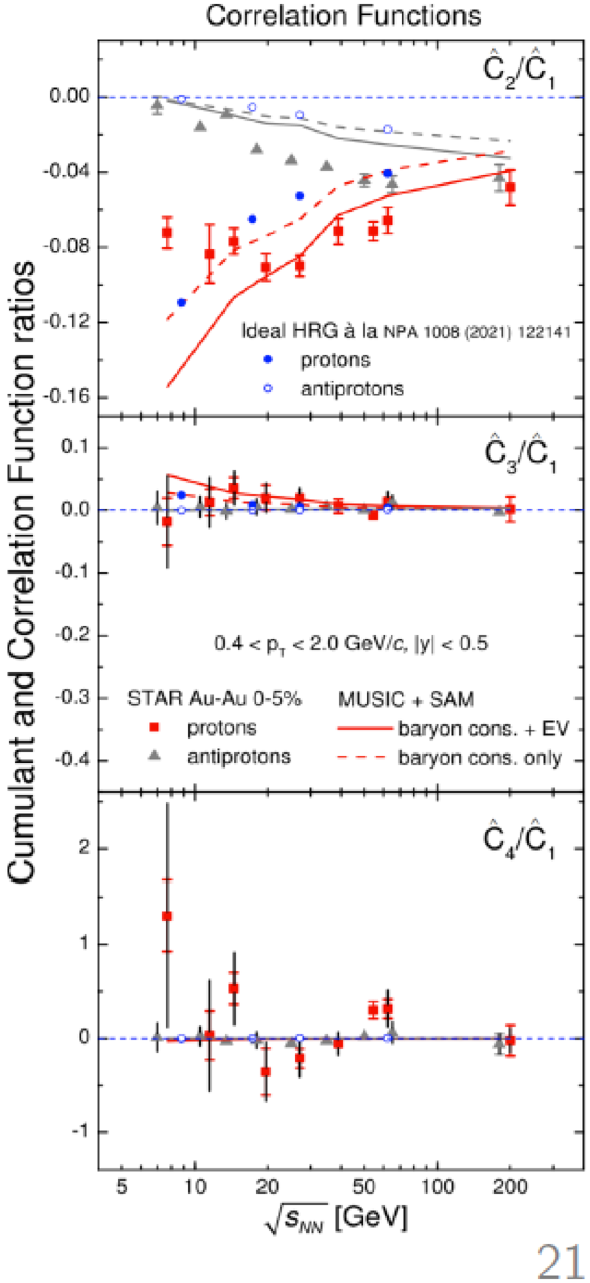
$$\frac{\chi_4^q}{\chi_2^q} = \kappa\sigma^2 = \frac{C_4^q}{C_2^q}, \quad \frac{\chi_3^q}{\chi_2^q} = S\sigma = \frac{C_3^q}{C_2^q}$$
$$\chi_n^q = \frac{1}{VT^3}\cdot C_n^q = \frac{\partial^n(p/T^4)}{\partial(\mu^q)^n}, \quad q = B, Q, S$$

The experimental improvement: factorial moments

- **Factorial cumulants** $\hat{C}_n$ [Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)]
  - Remove the Poisson contribution and probe genuine correlations

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- **Expectation:** High-order ($n > 3$) factorial cumulants
  - have small contributions from non-critical effects [Bzdak, Koch, Skokov, EPJC '17; VV et al, PLB '17]
  - are as singular as ordinary cumulants near the critical point [Ling, Stephanov, PRC '16]
- **Observations from STAR data:**
  - $\hat{C}_3$ & $\hat{C}_4$ are largely consistent with zero within errors
    - Reanalyze (non-)monotonic energy dependence for $\hat{C}_4/\hat{C}_1$ instead of κ_4/κ_2?
  - Statistically significant effects appear to be driven by two-proton correlations in $\hat{C}_2$

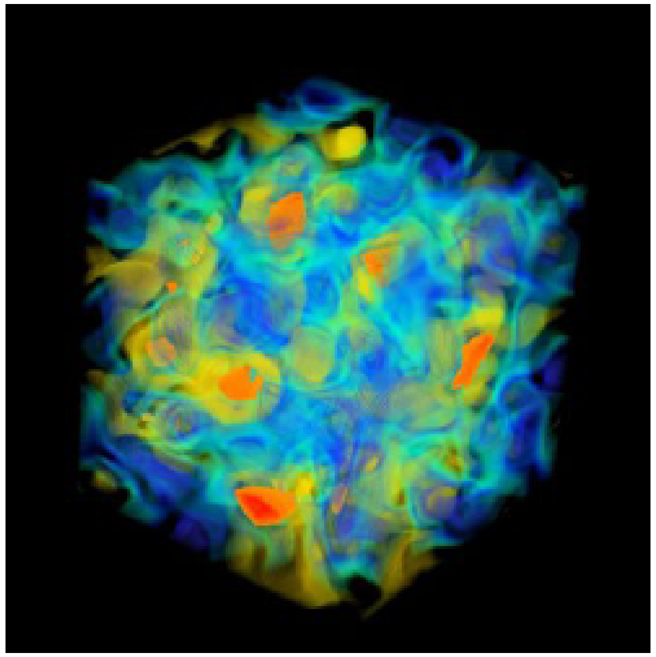


⚠ **Notation:** We use κ_n for cumulants and $\hat{C}_n$ for factorial cumulants, STAR uses the opposite

Theory vs. experiment: the caveats

- Grand-canonical heat bath vs expansion in vacuum
  - Vovchenko et al., PLB 811, 135868 (2020), PRC 105, 014903 (2022)
  - Bzdak et al., PRC 87, 014901 (2013)
- Coordinate vs. momentum space
  - Ling et al., PRC93, 034915 (2016)
- Conserved charges (net-baryon) vs. proxies (net-proton)
  - Kitazawa et al., PRC 85, 021901 (2012)
  - Koch et al., PRC 103, 044903 (2021)
- Volume, initial state, baryon stopping: fixed vs. fluctuating
  - Gorenstein et al., PRC 84, 014904 (2011)
  - Skokov et al., PRC 88, 034911 (2013)
- Hadronic phase
  - Steinheimer et al., PLB 776, 32 (2018)
  - Savchuk et al. PLB 827, 136983 (2022)
- Non-equilibrium (memory) effects
  - Mukherjee et al., PRC92, 034912 (2015)
  - Asakawa et al., PRC 101, 034913 (2020)

Theory



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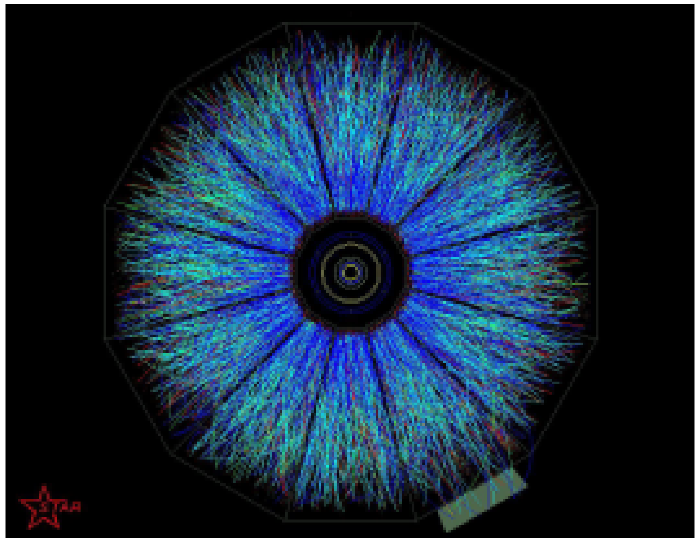
Experiment



STAR event display

Topic 1: Critical Point

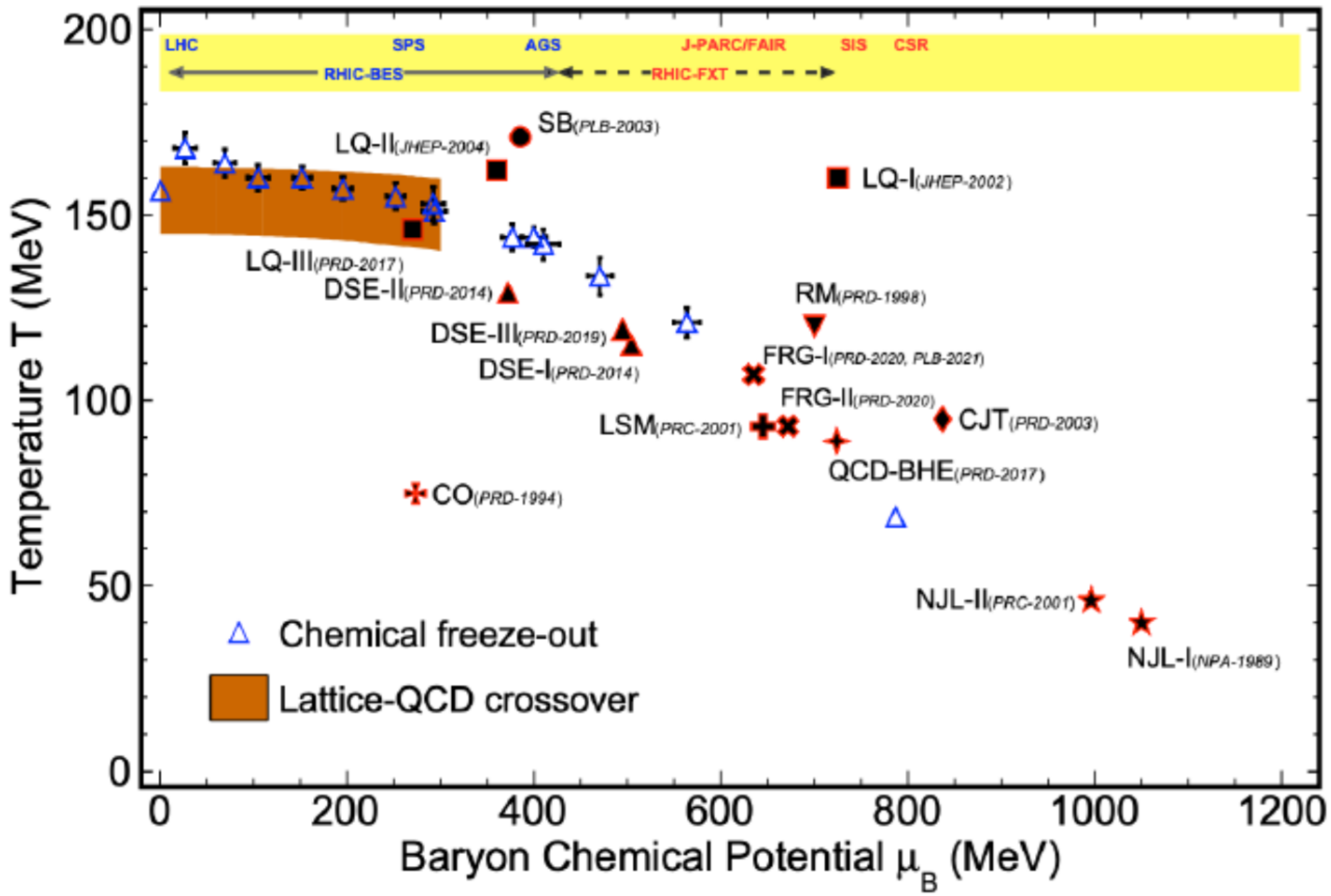


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

Until recently: no viable critical point calculation based on lattice QCD alone

Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)

Sign problem overcome by Standard Taylor Expansion

Taylor Expansion around $\mu_B = 0$

$$\frac{n_B(T,\mu_B)}{T^3} = \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} \chi_{2n}(T,\mu_B=0) \left(\frac{\mu_B}{T}\right)^{2n-1}$$

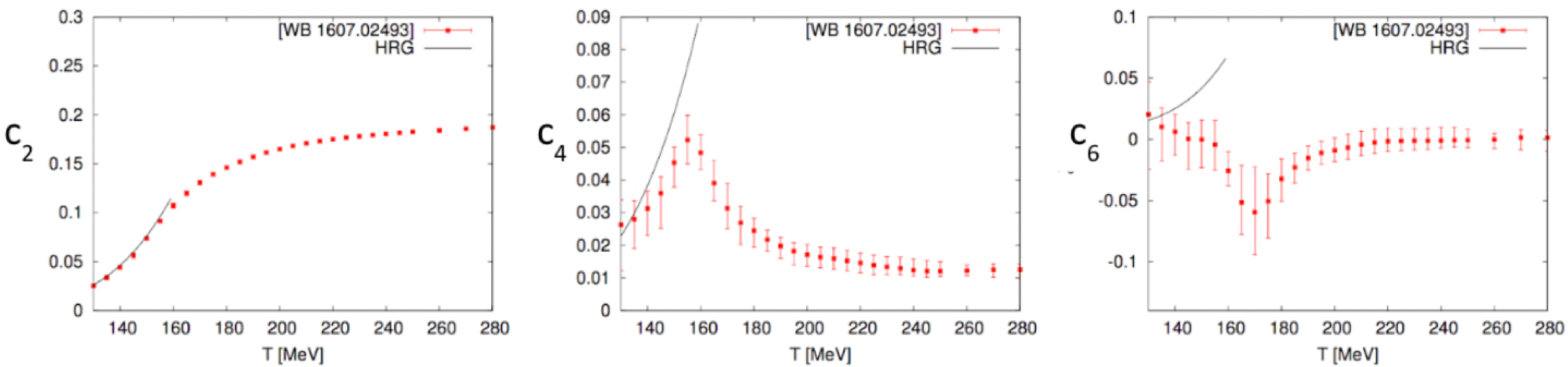
[Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]
[Bazavov, A et al PhysRevD.95, 054504 (2017)]

$$c_n(T) = \frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\Big|_{\mu_B=0}$$

Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational power**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]
[Borsanyi , S et al arXiV:2312.07528v1. (2023)]

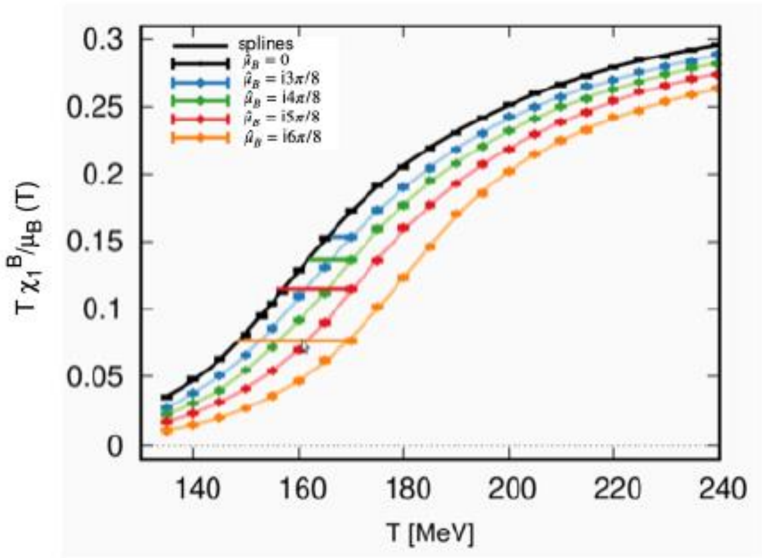


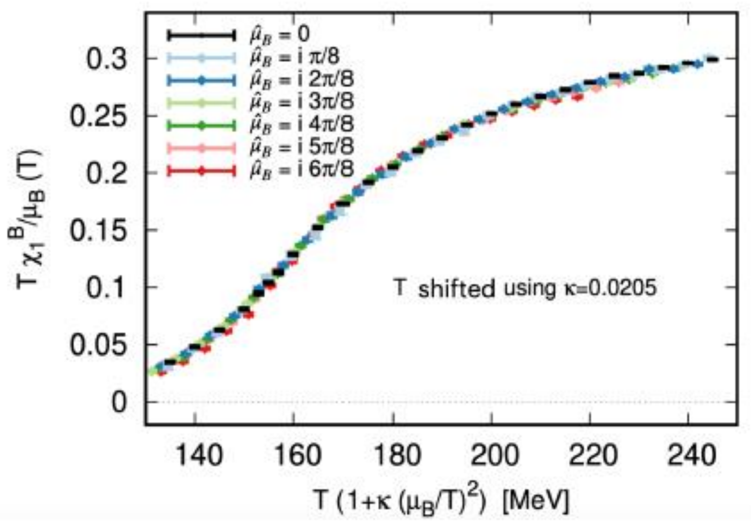
[WB Lattice QCD Collaboration]

Promising new expansion scheme

T′ Expansion scheme (T ExS)

Simulating at Imaginary μ_B





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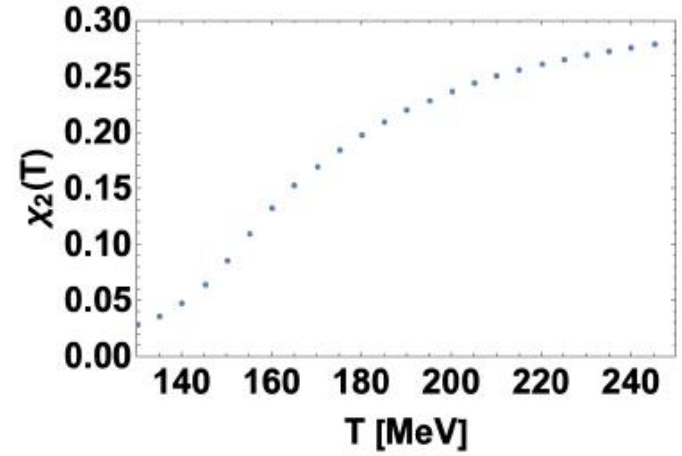


[Borsányi, S et al PRL. 108(1), 101.034901(2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

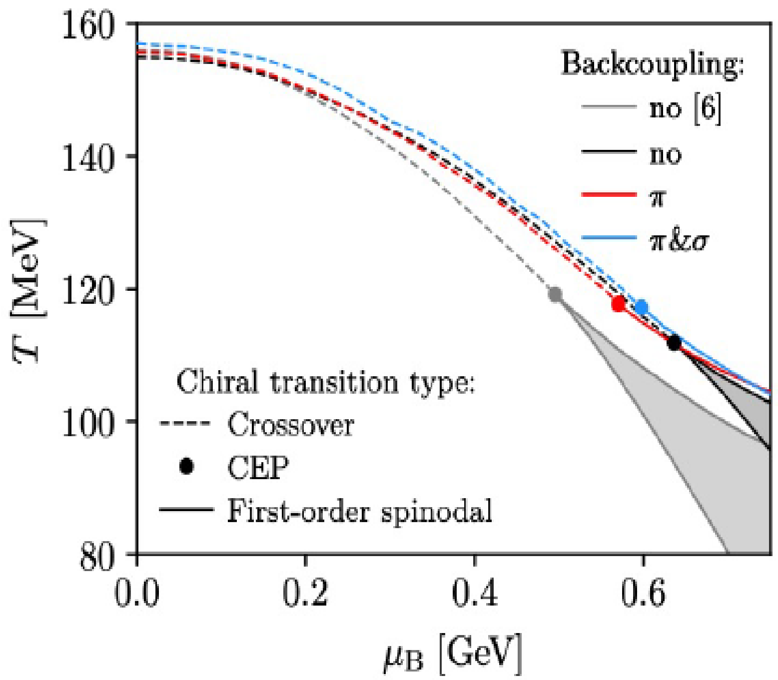
$$\chi_n^B(T,\mu_B) = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

- **μ_B dependence is captured in T-rescaling.**
- **Trusted up to** $\frac{\mu_B}{T} = 3.5$

Updated predictions (anchored to lattice QCD at μ_B=0)

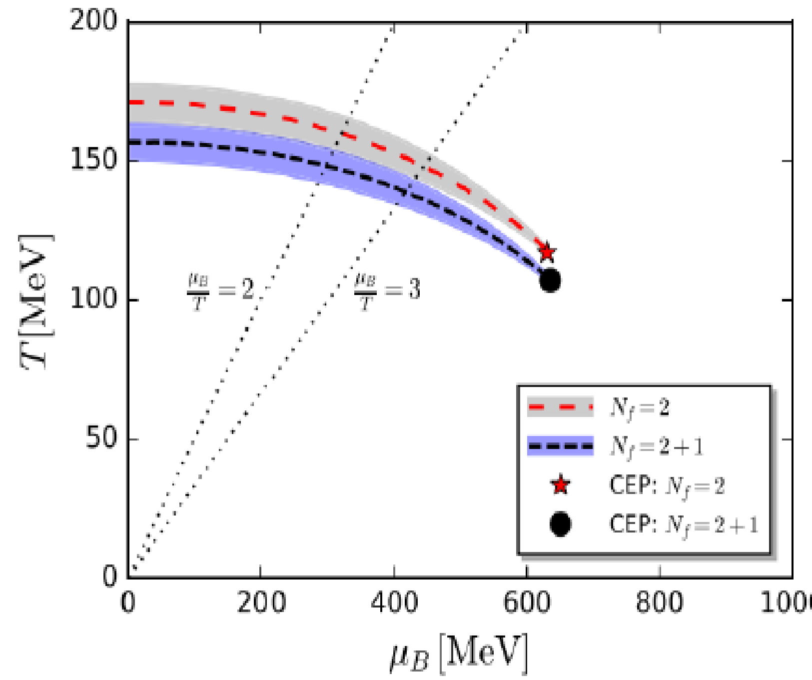
Dyson-Schwinger equations



Gunkel, Fischer, PRD 104, 054202 (2021)

T~120 MeV μ_B~600 MeV

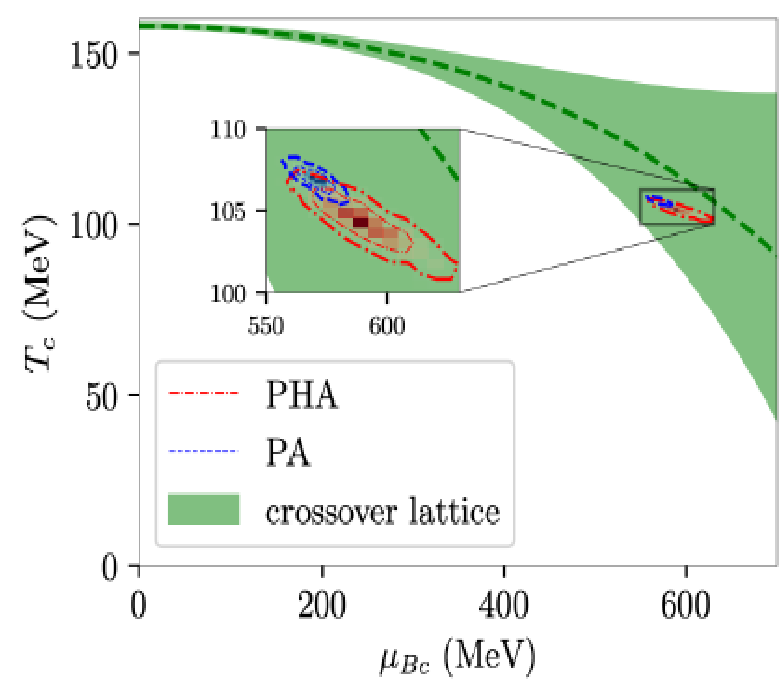
Functional renormalization group



Fu, Pawlowski, Rennecke, PRD 101, 053032 (2020)

T~100 MeV μ_B~600 – 650 MeV

Black-hole engineering



Hippert et al., arXiv:2309.00579

T~105 MeV μ_B~580 MeV

All in excellent agreement with lattice QCD at $\mu_B = 0$
and predict QCD critical point in a similar ballpark of μ_B/T ~ 5-6

If true, reachable in heavy-ion collisions at $\sqrt{s_{NN}}$~3 – 5 GeV

Interesting new lattice results using Lee-Yang edge singularities

D.A.Clarke et al., 2405.10196

Similar results: G. Basar, 2312.06952

Multi-point Pade approximation

AIC - Akaike informa
criterion

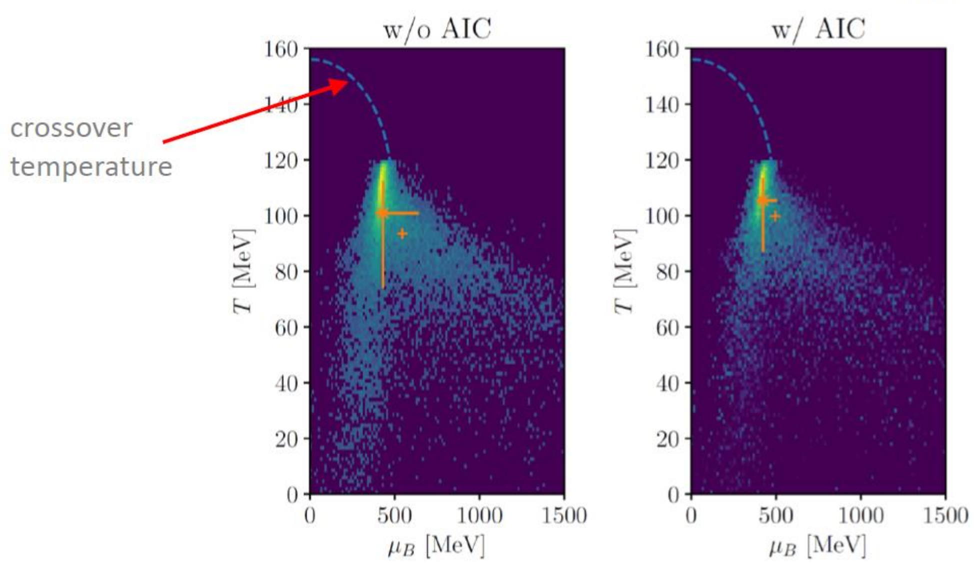
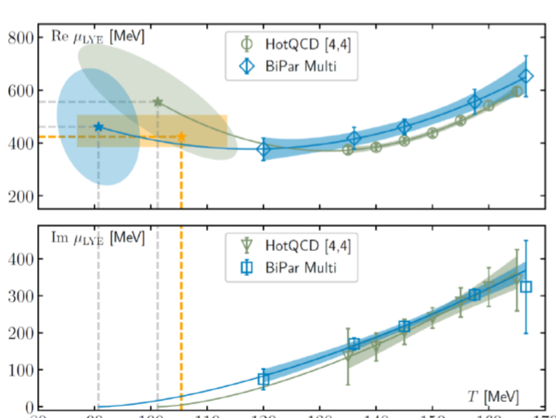


FIG. 1. Probability distribution of the QCD critical point from extrapolating Lee-Yang singularities to the real domain using universal scaling. For a detailed description see the text.

Using the multi-point Pade approach, the Lee-Yang edge singularities are located in the QCD pressure in the complex baryon chemical potential.

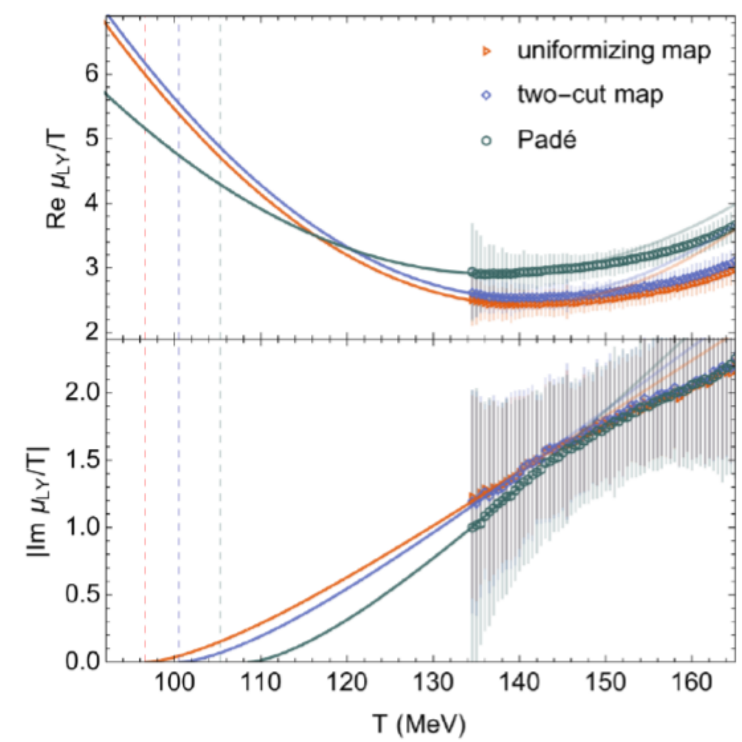
$$(T^{\rm CEP}, \mu_B^{\rm CEP}) = (105^{+8}_{-18}, 422^{+80}_{-35})\ {\rm MeV}$$



"An important limitation of the estimate presented here is that they are not fully extrapolated to the continuum limit"

Can we conclude that there is the QCD critical point?

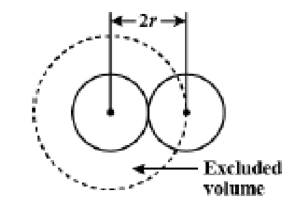
*We analyze the trajectory of the Lee-Yang edge singularities of the QCD equation of state in the complex baryon chemical potential (μ_B) plane... By extrapolating from this information, we estimate for the location of the QCD critical point, **$T_c \approx$ 100 MeV, $\mu_c \approx$ 580 MeV**.*



A more dynamical phenomenological approach

1. **Dynamical model calculations of critical fluctuations** BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]
   - Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
   - Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
   - Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high μ_B: hadronic transport/molecular dynamics with a critical point
[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznietsov et al., PRC 105, 044903 (2022)]



2. **Deviations from precision calculations of non-critical fluctuations**
   - Non-critical baseline is not flat [Braun-Munzinger, Rustamov, Stachel, NPA 1008, 122141 (2021)]
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

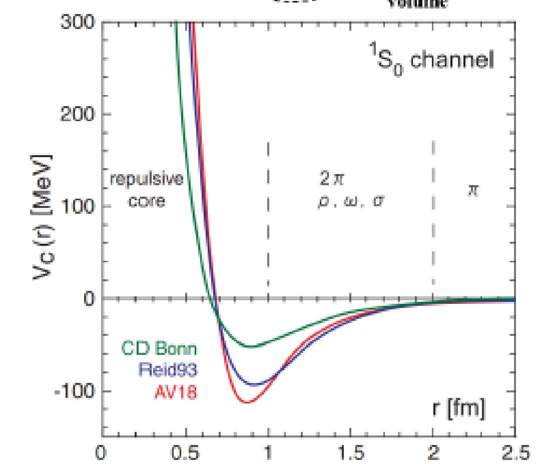
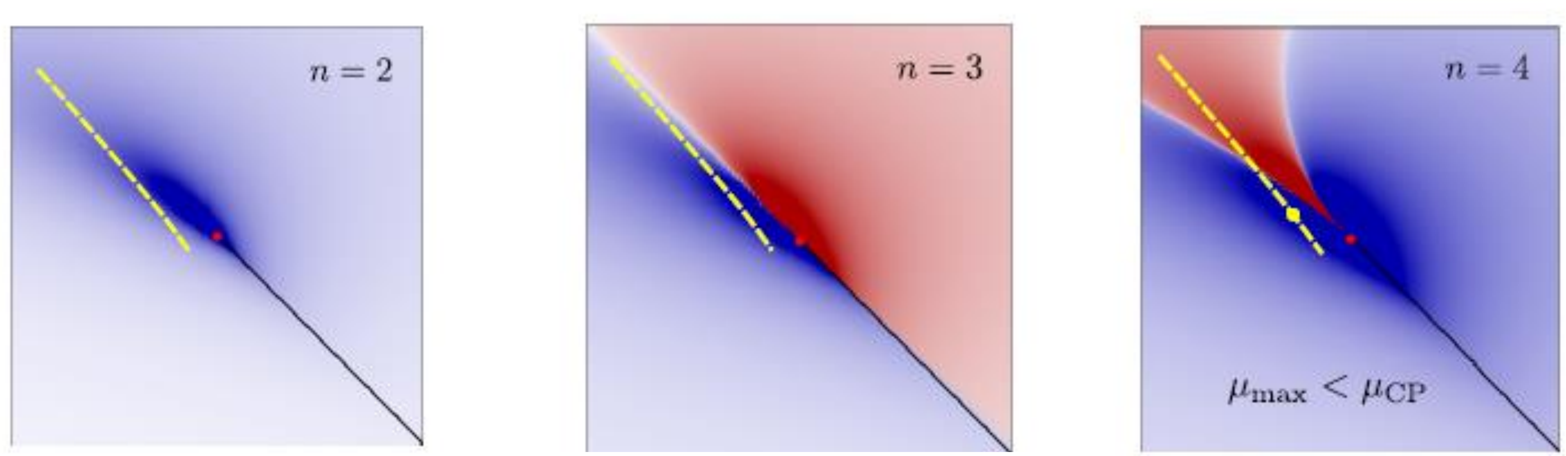


Figure from Ishii et al., PRL '07

Review by A. Bzdak (1906.00936)

(universal EOS) critical χ_n:



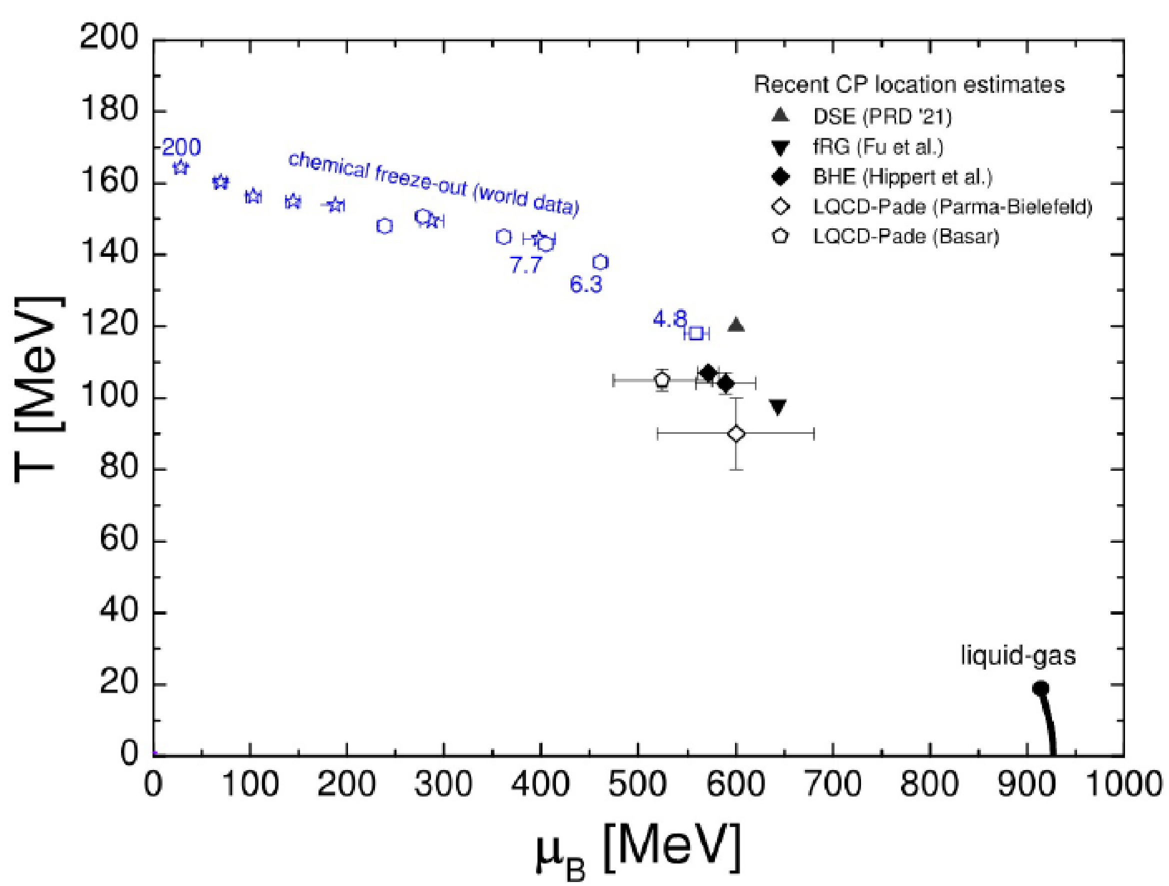
Experiment: Heavy Ion Collisions

Control parameters

- Collision energy $\sqrt{s_{NN}} = 2.4 - 5020$ GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

Measurements

- Final hadron abundances and momentum distributions **event-by-event**



STAR BES-I Program: Au+Au collisions

$\sqrt{s_{NN}}$ (GeV)	Events / 10^6	μ_B (Mev)
200	220	25
62.4	43	75
54.4	550	85
39	92	112
27	31	156
19.6	14	206
14.5	14	264
11.5	7	315
7.7	3	420
3.0	140	750

μ_B from J. Cleymans et al, PRC 73,034905(2006)

STAR BES Program:
Au+Au collisions

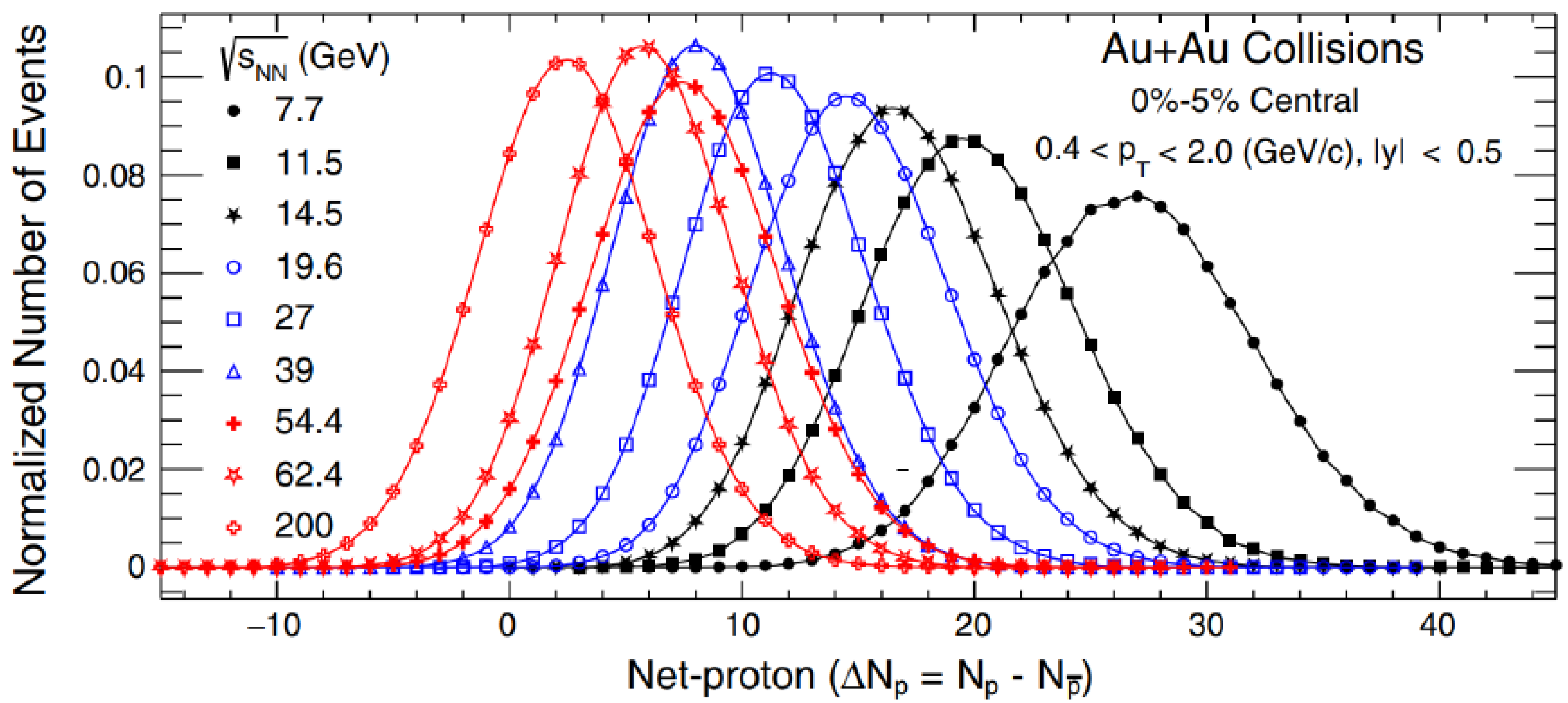
$\sqrt{s_{NN}}$ (GeV)	Events BES-I (10^6)	Events BES-II (10^6)
7.7	3	**45**
9.2	-	**78**
11.5	7	**110**
14.6	20	**178**
17.3	-	**116**
19.6	15	**270**
27	30	**220**

Measuring cumulants in event by event multiplicity distributions
(particles are proxies for conserved quantum numbers)

Count the number of events with given number of e.g. (net) protons

$$P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N^{total}_{events}}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



Cumulants are extensive, $\kappa_n \sim V$, use ratios to cancel out the volume

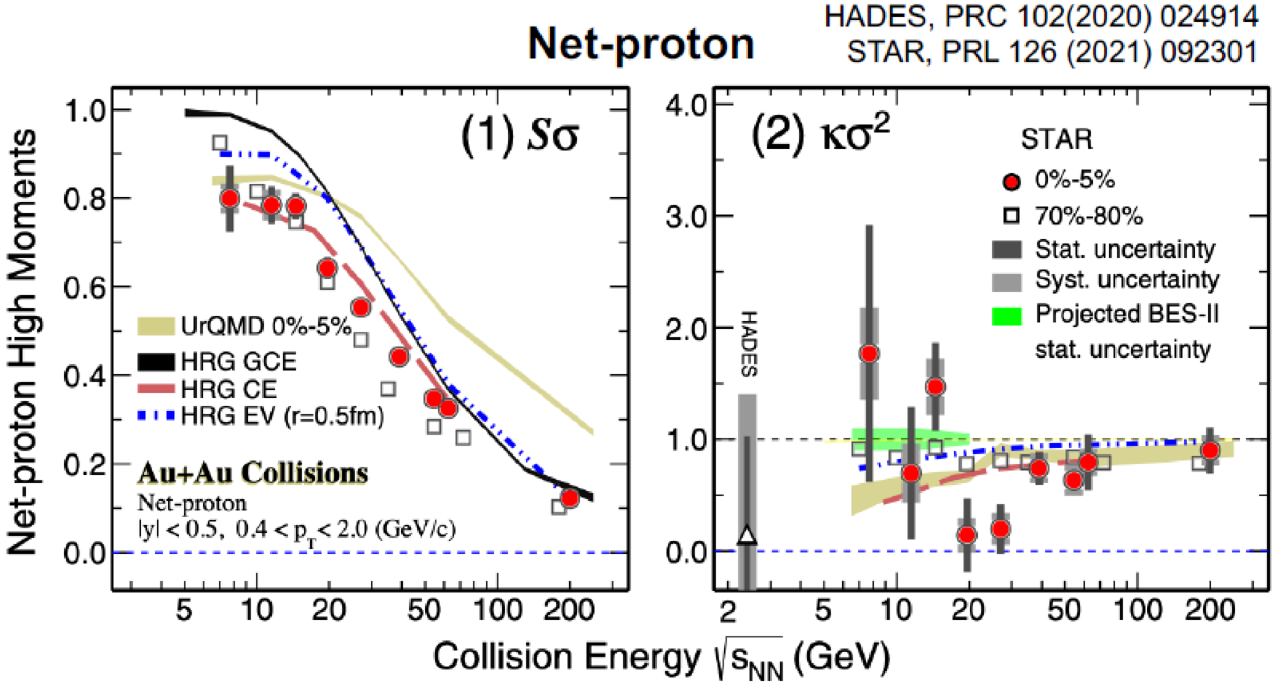
$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals (tails of the distribution)

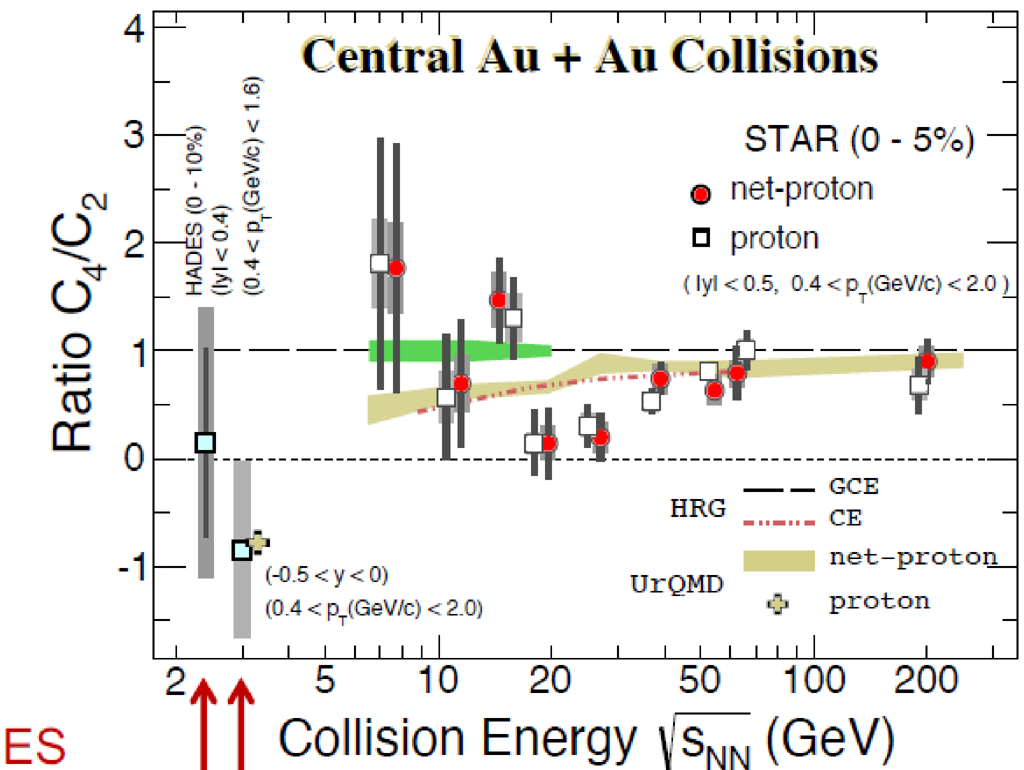
Experimental results: cumulant ratios

HADES, STAR BES-I (2020, 2021)

HADES, STAR BES-II FXT (2020,2022)



STAR, PRL 128 (2022) 202303
HADES, PRC 102(2020) 024914



HADES
$\sqrt{s_{NN}}$ = 2.4 GeV
Au+Au collisions

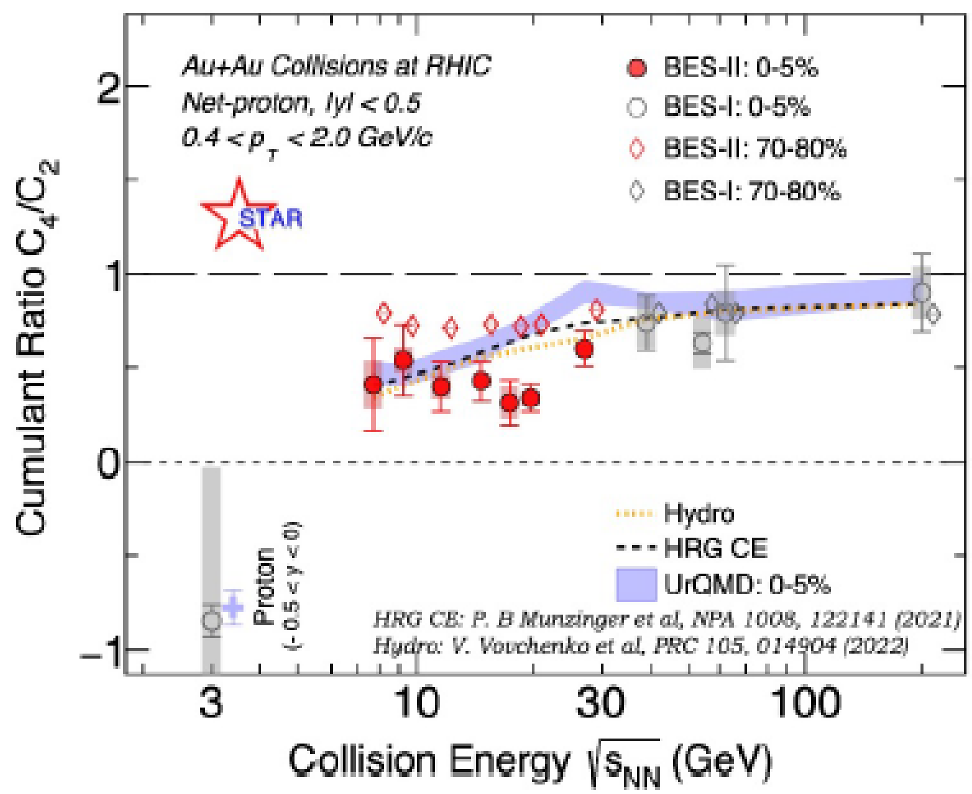
STAR
$\sqrt{s_{NN}}$ = 3.0 GeV
Au+Au collisions

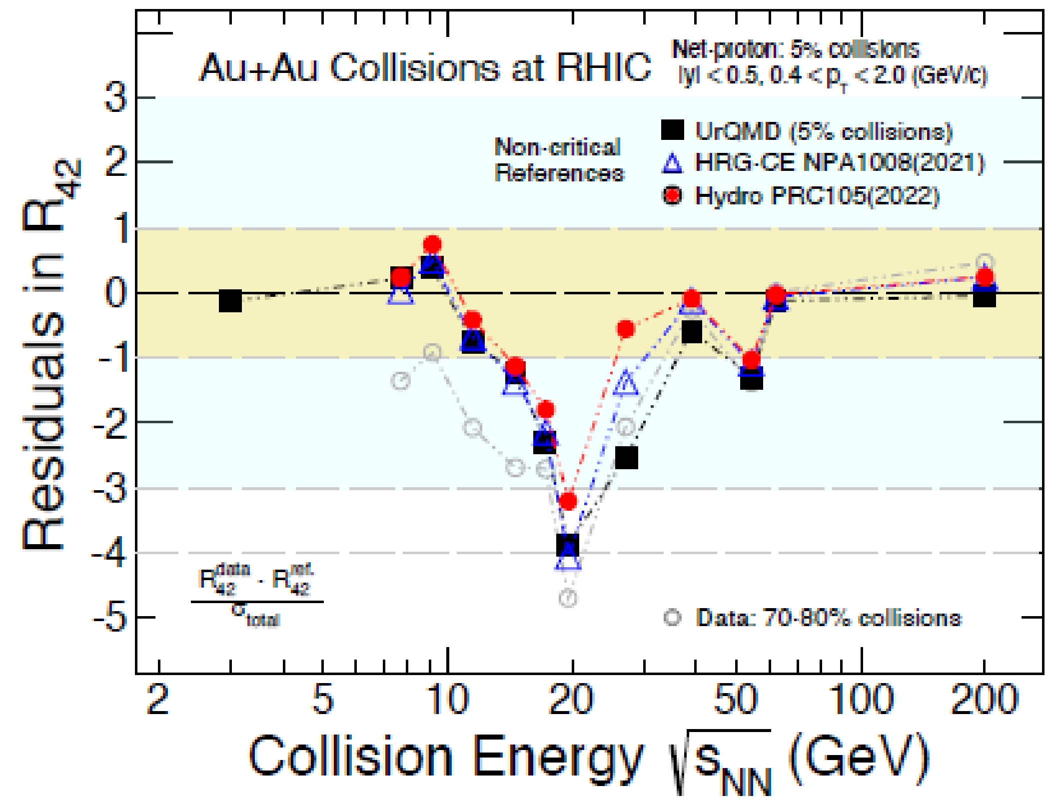
1. Full measurement on BES-I datasets
2. With TOF detector, p_T coverage is extended to 2.0 GeV/c
3. **Non-monotonic energy dependence trend is observed with 3.1σ significance**

STAR 3 GeV and HADES data indicate a purely hadronic system

New STAR results (CPOD 2024, SQM2024)

Precision Measurement on BES-II

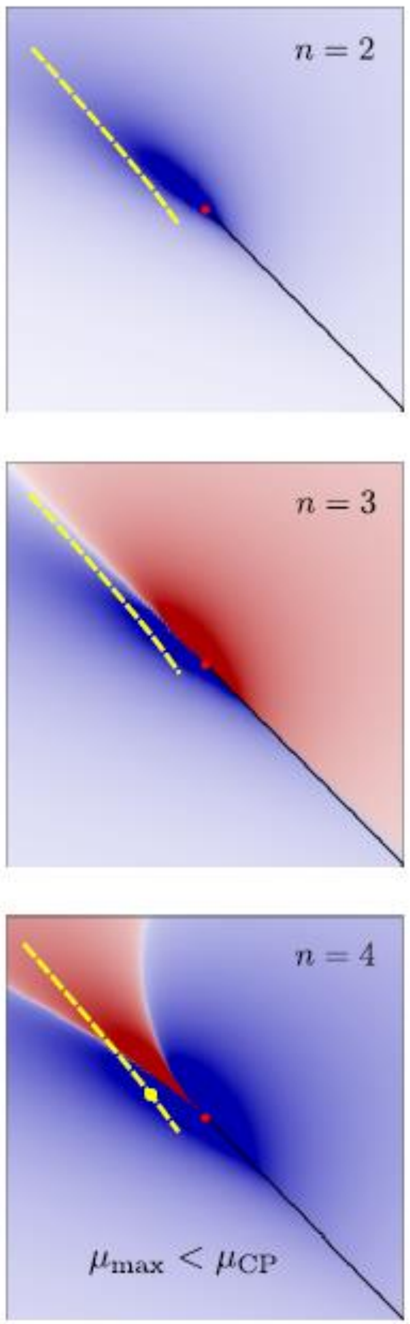




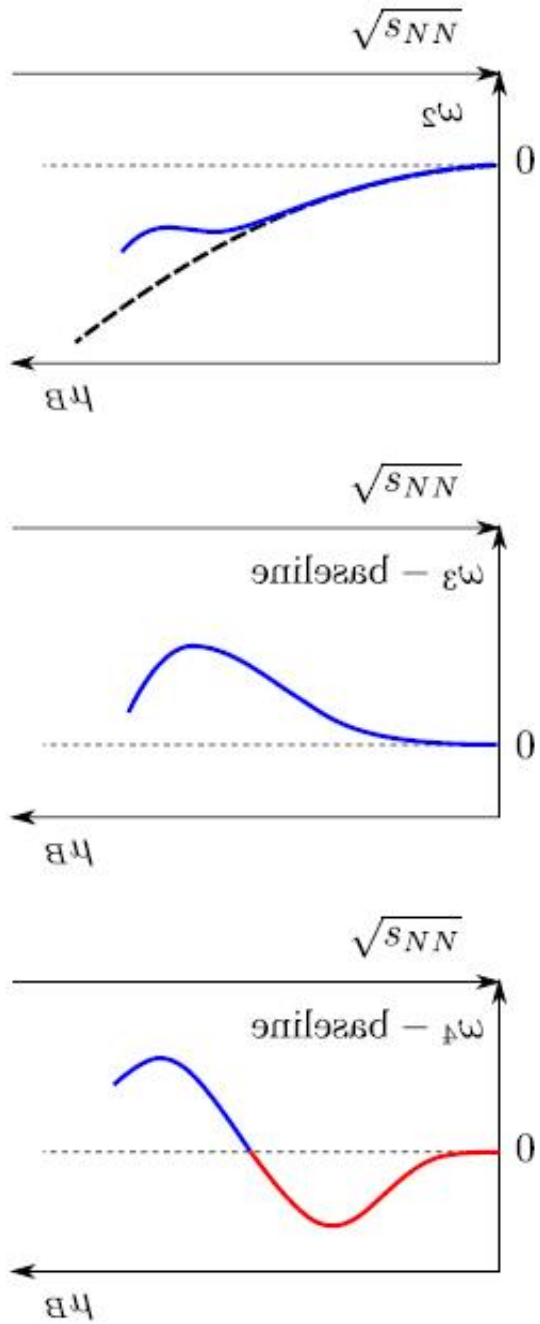
1. **Net-proton C_4/C_2 in 0-5% shows a minimum around ~20 GeV comparing to non-CP models (Hydro, HRG, UrQMD) and 70-80% data.**
2. Maximum deviation: 3.2-4.7σ at 20 GeV (1.3-2σ at BES-I)
3. Overall deviation from 7.7-27 GeV: 1.9–5.4σ (1.4–2.2σ at BES-I)

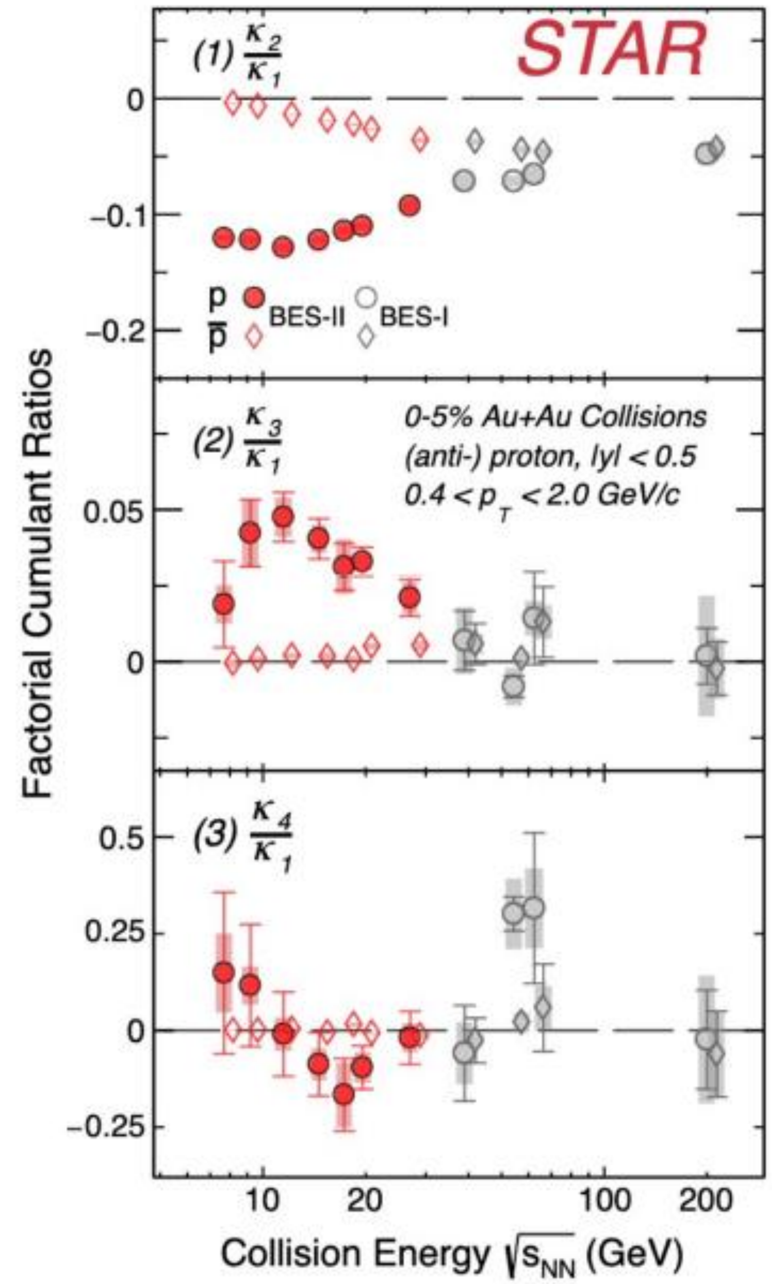
Comparison to theory: no smoking gun but still intriguing structures in all measured cumulants

(universal EOS) critical χ_n:



(irreducible correlations) $\text{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \text{FC}_n/\text{FC}_1$



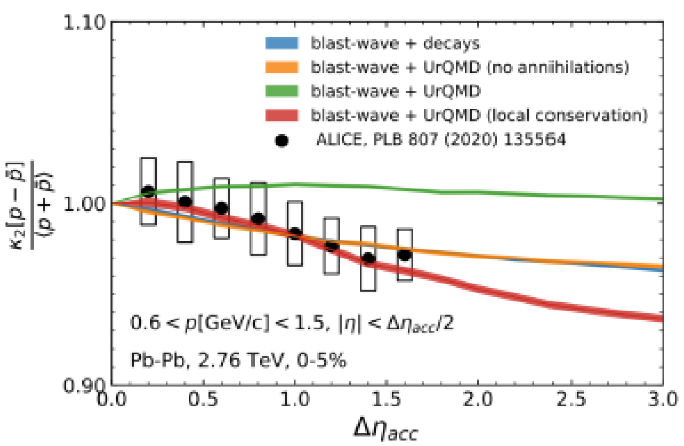


Bzdak et al review 1906.00936

Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

Topic 2: Chiral transition at $\mu_B = 0$

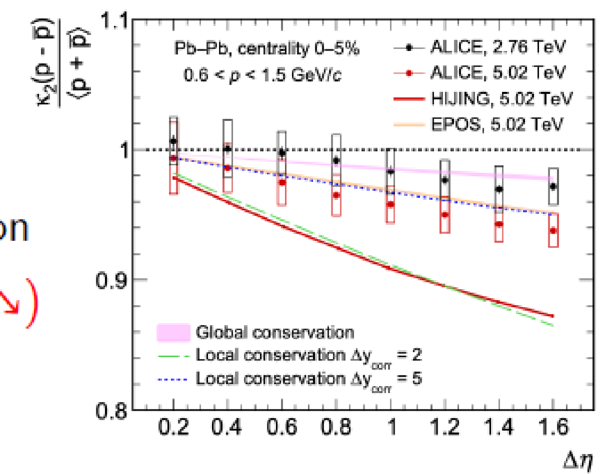
At LHC energies μ_B=0, the lower cumulants are well understood by baryon number conservation



$\kappa_2[p-\bar{p}]/\langle p+\bar{p}\rangle$:

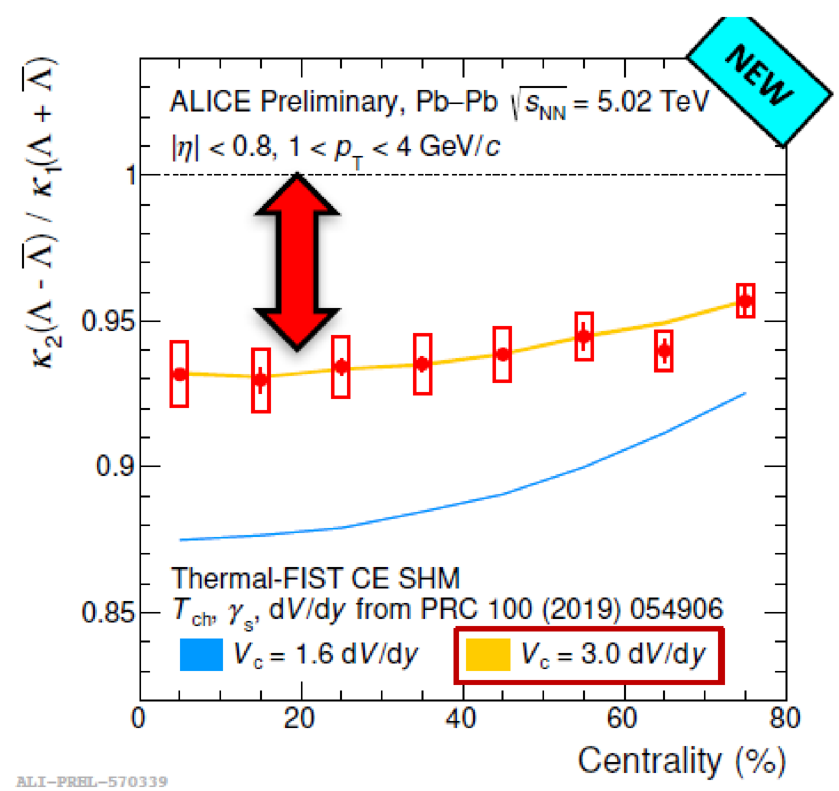
- Largely understood as (global) baryon conservation
  - Larger suppression at 5 TeV contrary to naïve expectation
- Interplay: baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of $\kappa_2[p+\bar{p}]$ can resolve it

O. Savchuk et al., PLB 827, 136983 (2022)



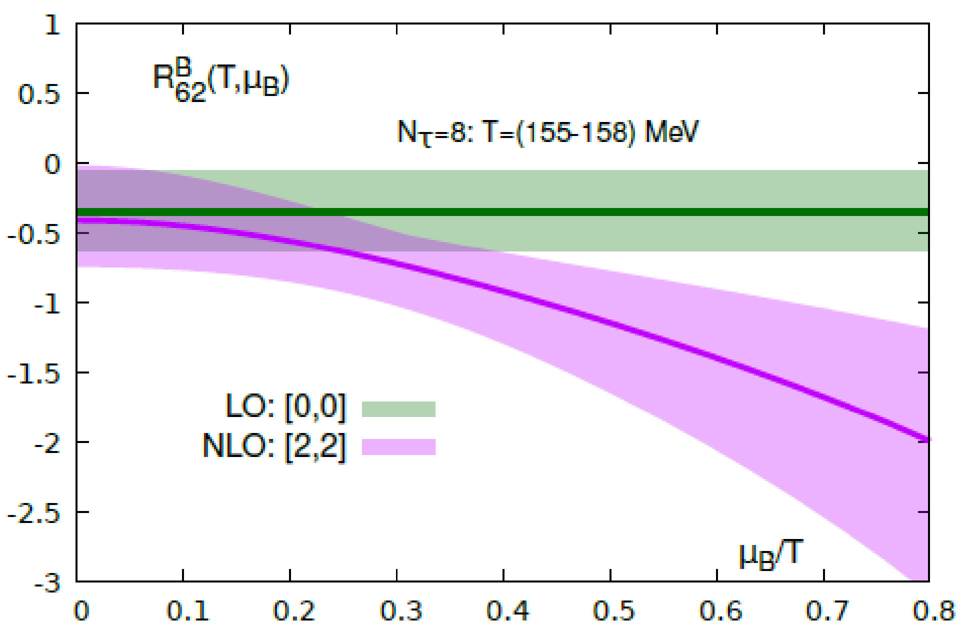
Deviation from Skellam allows you to calculate correlation volume. Large volume = early correlations

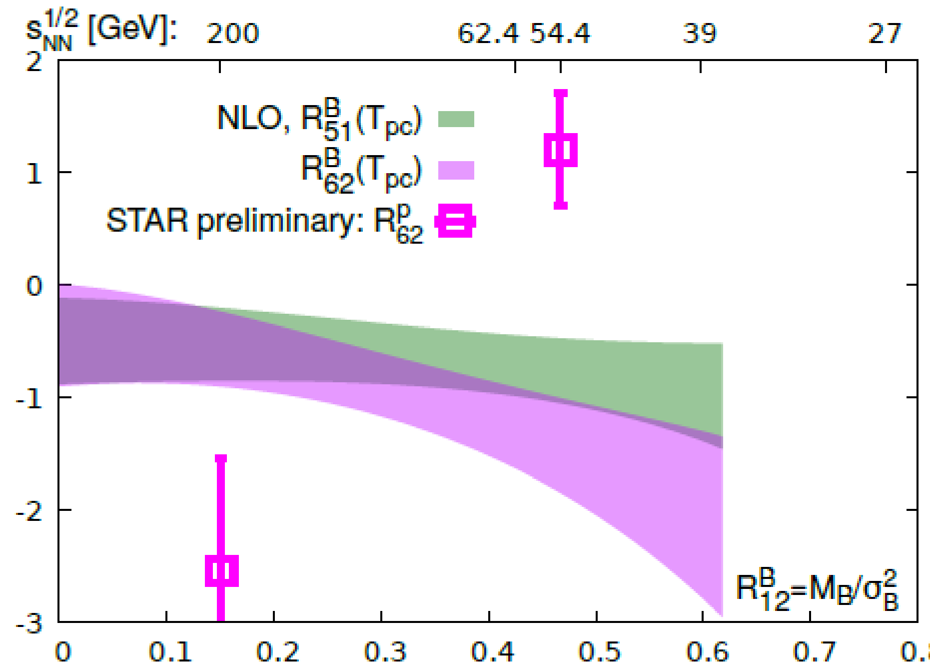
Strong early baryon correlations (Λ,p) (ALICE, SQM 2024)



Lattice QCD predictions on chiral criticality in higher order cumulants

A. Bazavov et al. (HotQCD), Phys.Rev.D 101 (2020) 7, 074502: [c6/c2 goes negative]

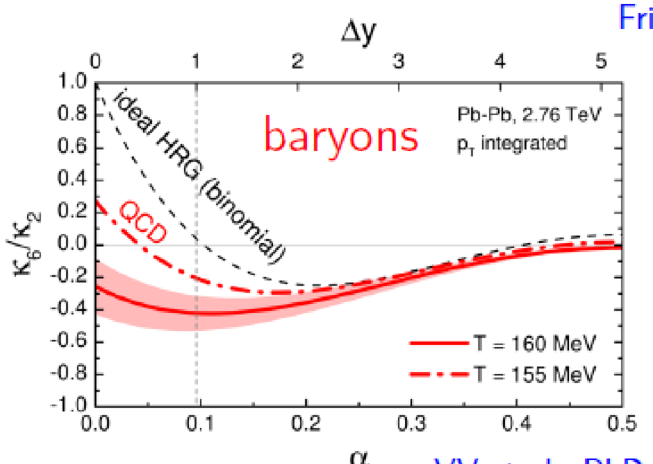




ALICE Collaboration, PLB 844, 137545 (2023)

High-order cumulants: probe remnants of chiral criticality

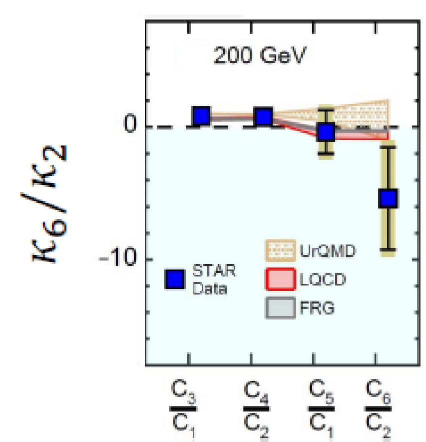
Friman et al., EPJC 71, 1694 (2011)



VV et al., PLB 811, 135868 (2020)

- negative κ_6 of baryons

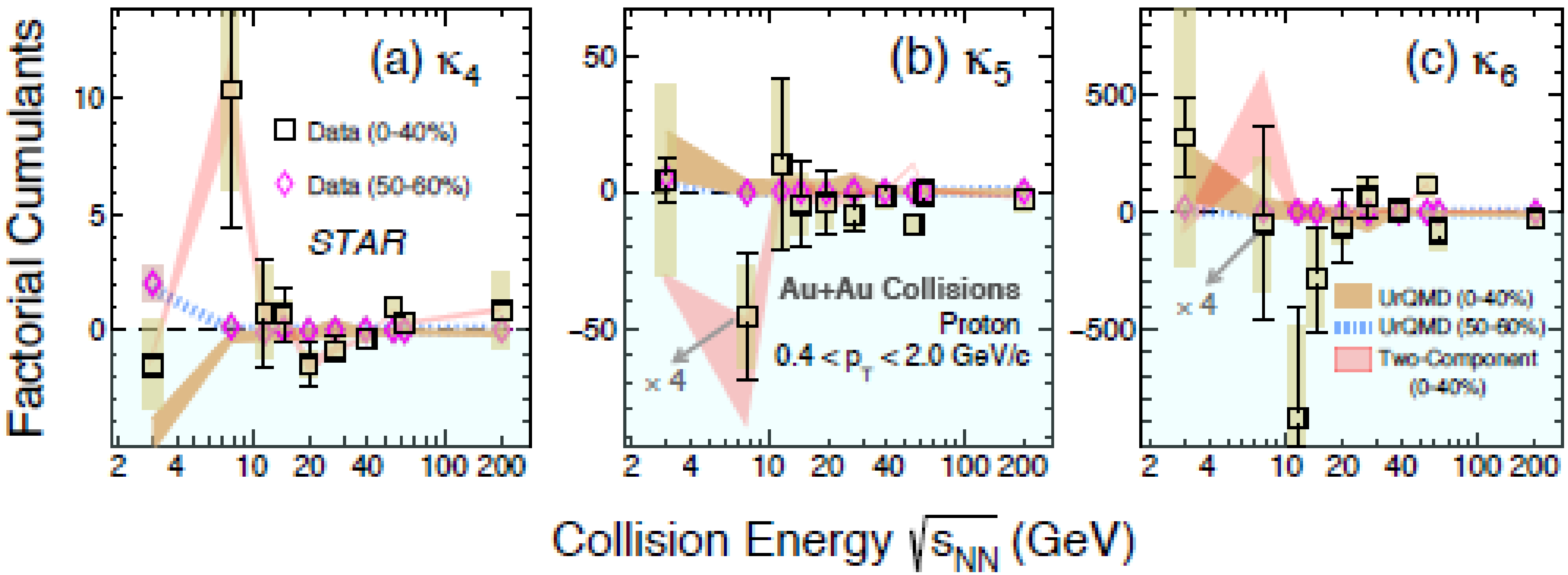
RHIC 200 GeV: hints of negative $\kappa_6 < 0$ (protons)

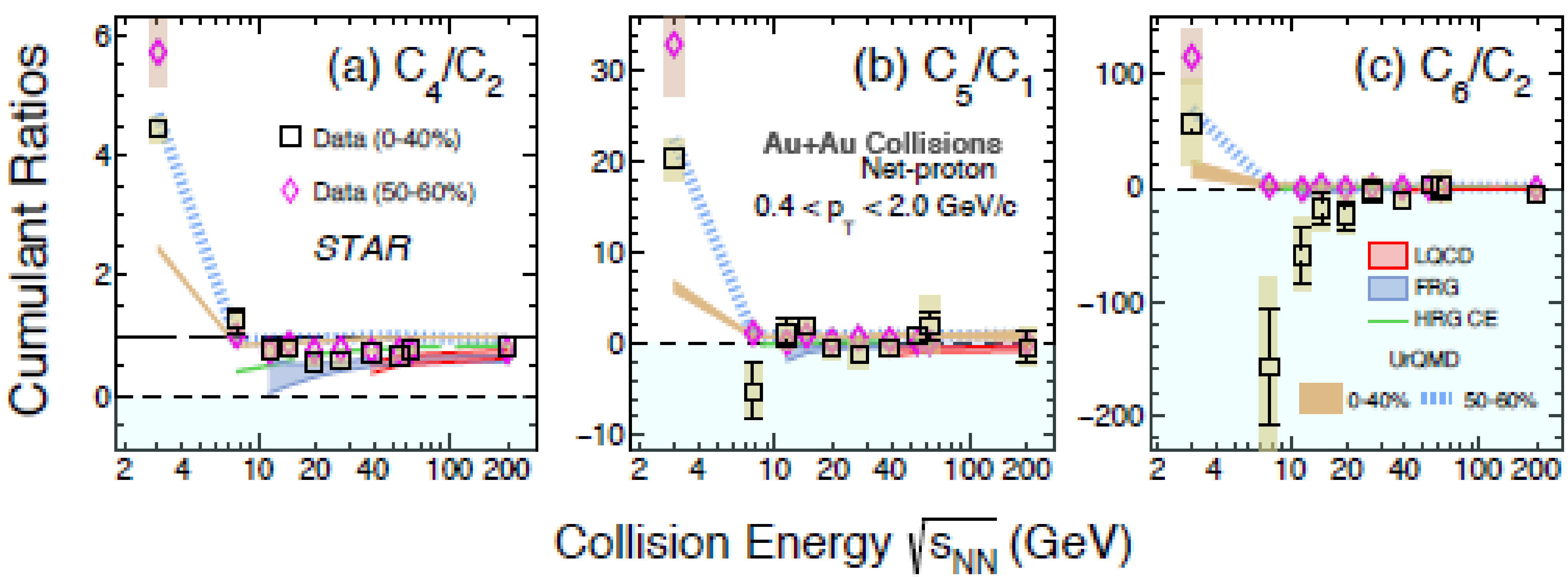


STAR Collaboration, PRL 130, 082301 (2023)

- are baryons even more negative?

Detailed experimental results (STAR, 2207.09837)





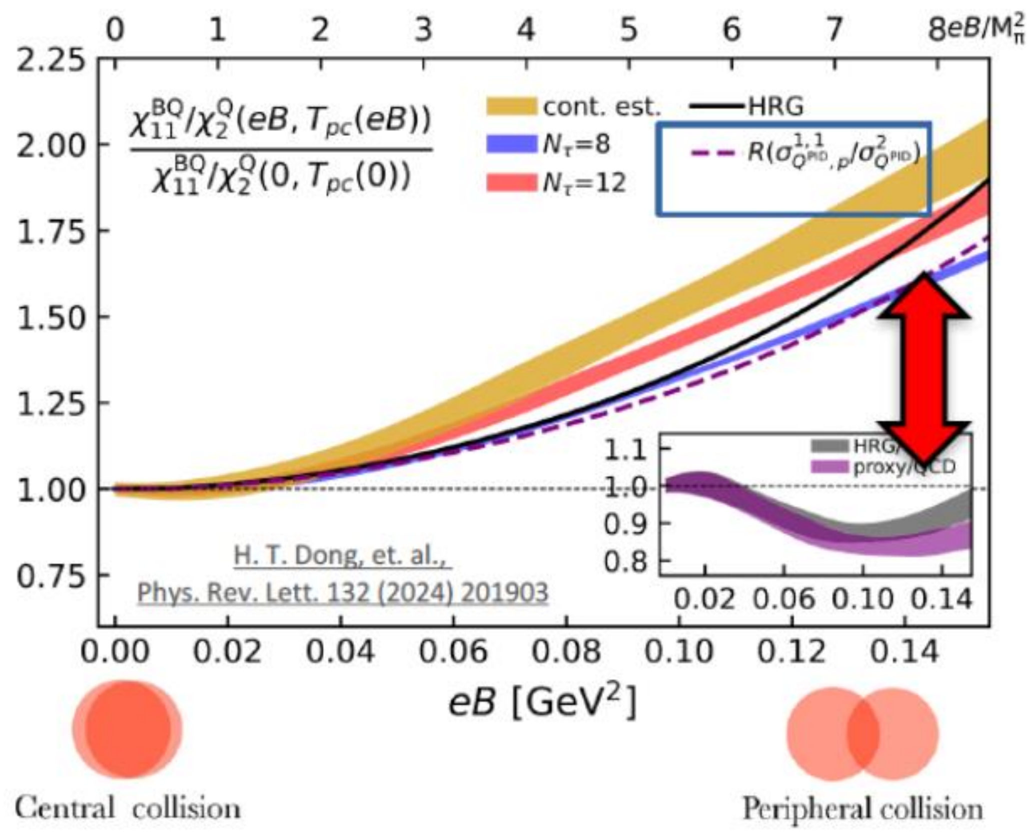
Topic 3: Conserved charge fluctuations to probe early magnetic field

H.-T. Ding et al., PRL 132, 201903 (2024)

Magnetic field + LQCD EOS → Any modification in chiral susceptibilities?

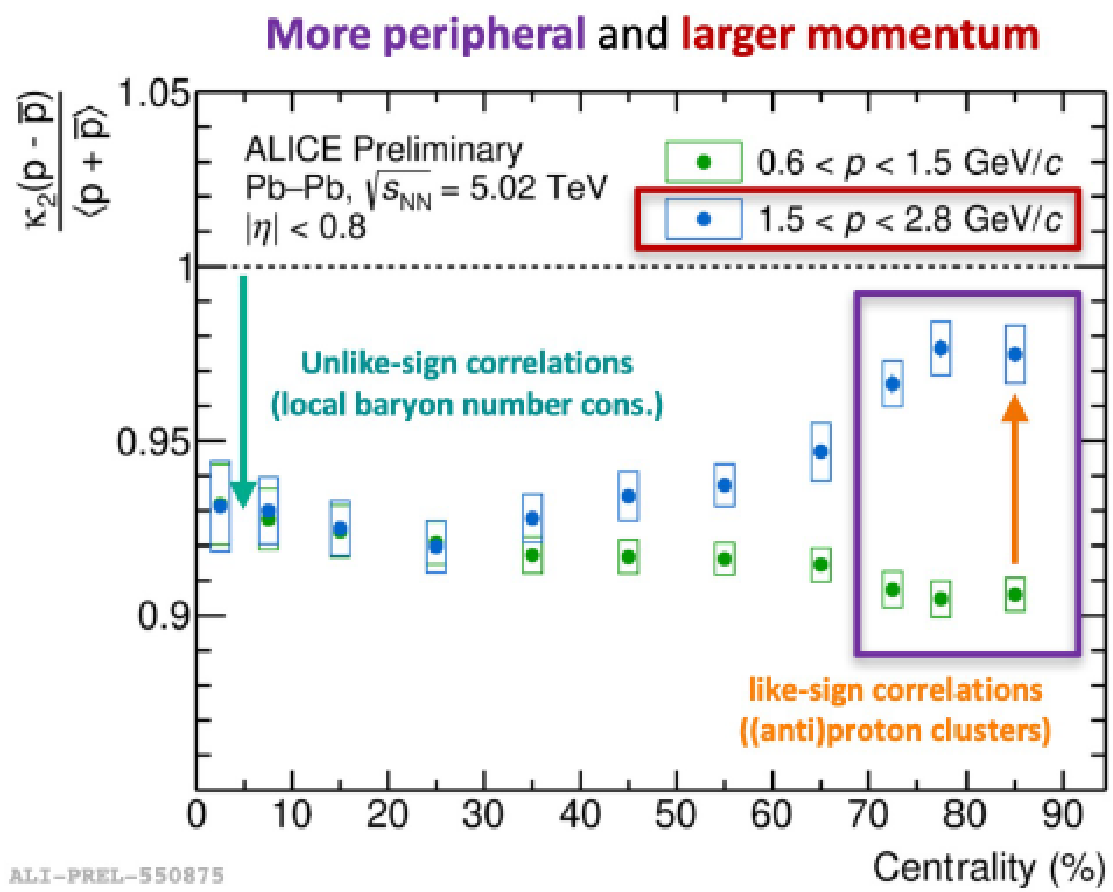
$$\chi^{lmn}_{\mathrm{B,S,Q}} = \frac{1}{VT^3}\sigma^{lmn}_{\mathrm{B,S,Q}}$$

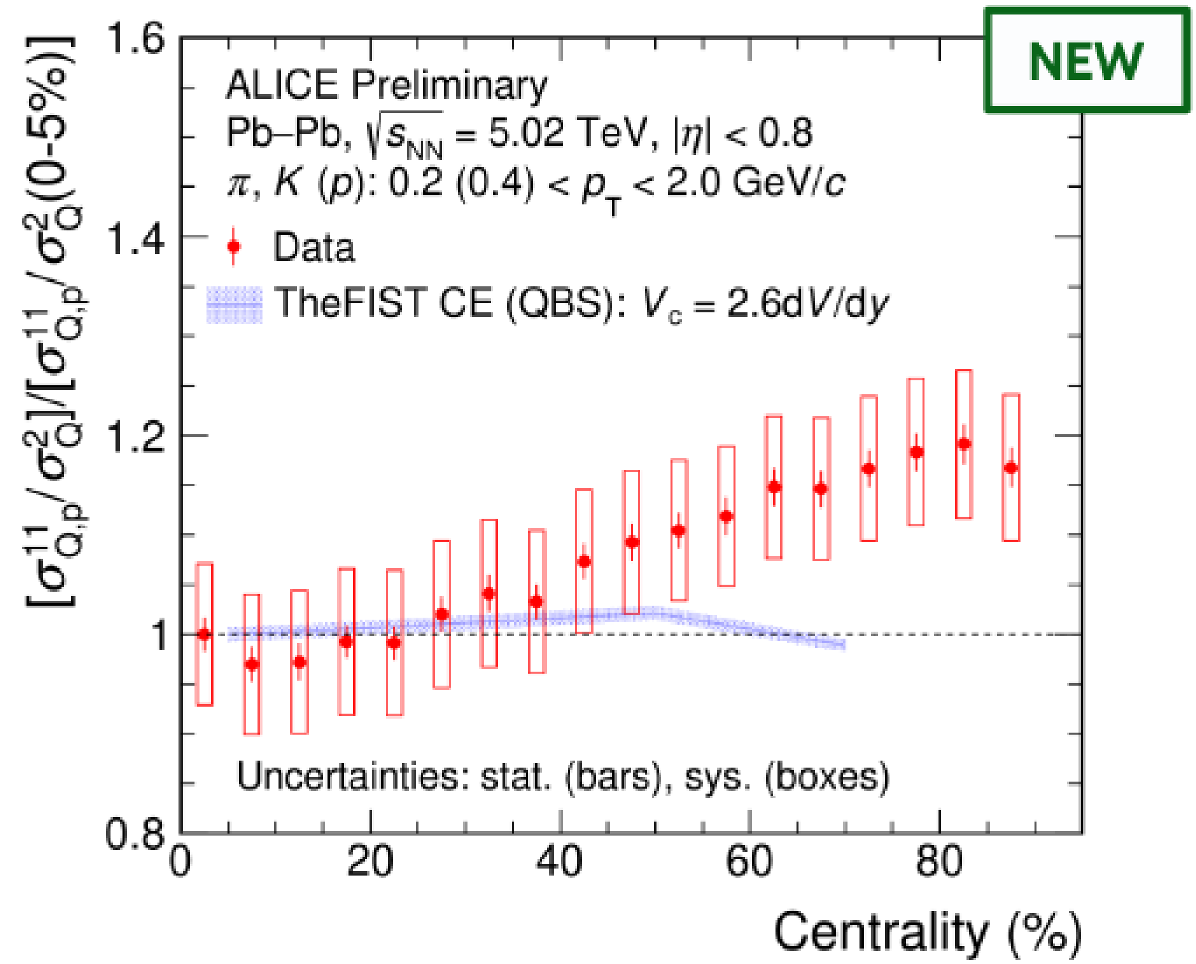
Proxies:
Charge: K, π, p
Baryon: p
Strangeness: K



1) Increase towards peripheral collisions

New preliminary ALICE data (SQM 2024)





1. A rise of χ_2^B, χ_{11}^{BQ} in peripheral collisions due to magnetic field suggested by LQCD
2. **Hint of magnetic field in peripheral collisions in data?**

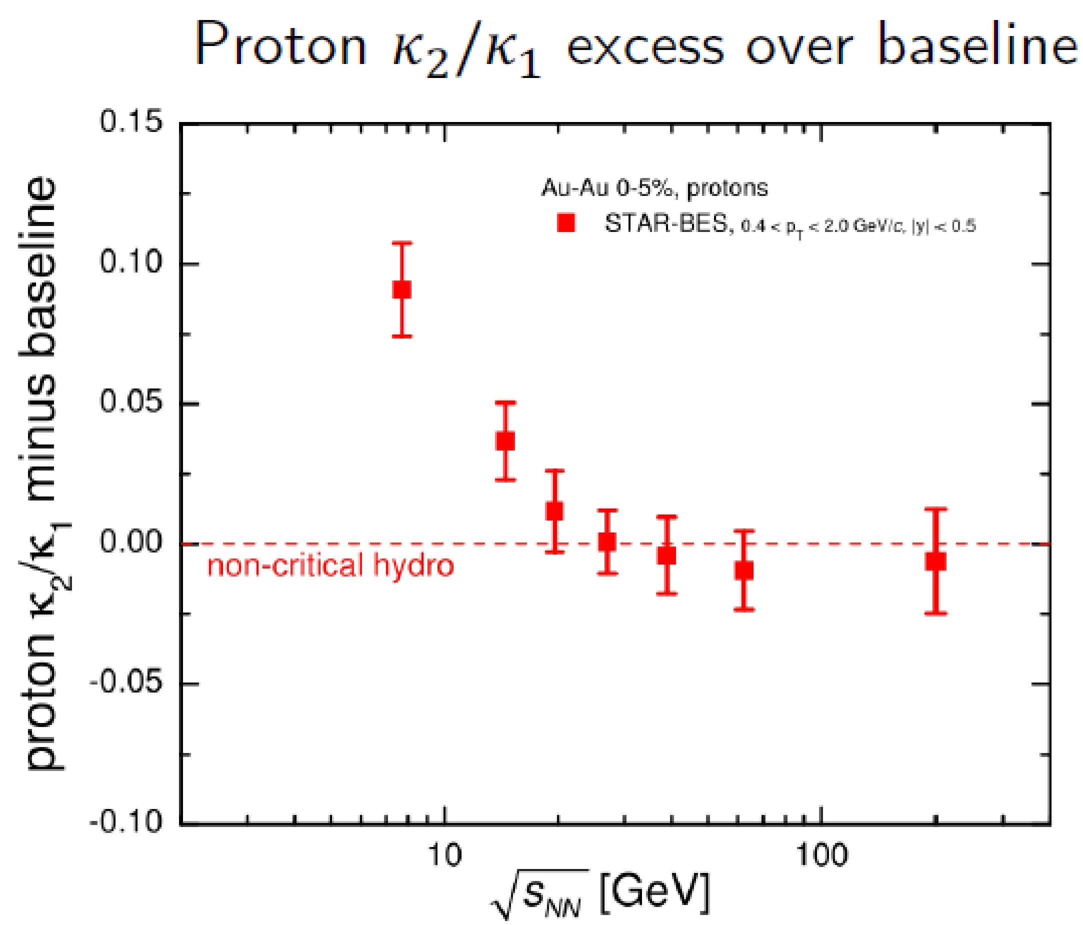
1) Increase towards peripheral collisions
2) **Similar behavior** is also observed in **net-p fluctuations** for the larger momentum range

$\Longrightarrow$ Magnetic field or ?

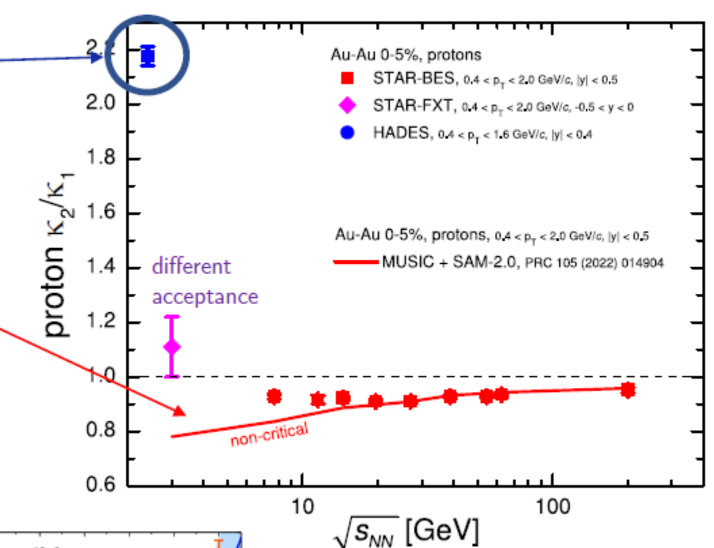
Conclusions and Summary

- New expansion schemes for lattice QCD extend the region of validity to values of $\mu_B/T = 3.5$
- Indication of critical point in Lee-Yang singularities.
- Location of point in agreement with DSE, FRG, and holography approaches that use lattice QCD as anchor.
- Location in the range of collision energies around 3-5 A GeV (potentially measurable at RHIC, FAIR, NICA, HPARC). Data still inconclusive, but more to come.
- Indication of chiral criticality at μ_B=0 and strong early magnetic fields from higher order fluctuations

Backup: Factorial results



- Intriguing hints from HADES@2.4 GeV and STAR-FXT@3GeV: huge excess of two-proton correlations!
  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]
  [STAR Collaboration, Phys. Rev. Lett. 128, 202303 (2022)]
- No change of trend in the non-critical reference
- Additional mechanisms:
  - Nuclear liquid-gas transition (the other QCD critical point)
  - Light nuclei formation/fragmentation
  - Stronger initial state, volume, and baryon stopping fluctuations
  Talk by A. Bzdak, Wed 14:20; Poster by A. Rustamov
- Difference in acceptance (-0.5<y<0 vs |y|<0.5)
- Improved modeling of lower energies required



VV, Phys. Rev. C 106, 064906 (2022)

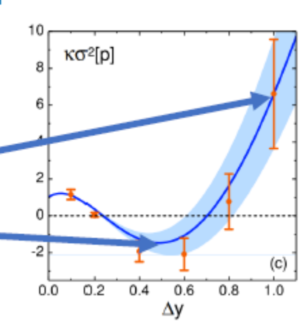


Figure from O. Savchuk et al., PLB 835, 137540 (2022)