THE QCD PHASE STRUCTURE AND ITS SIGNATURES FROM FUNCTIONAL APPROACHES

Fabian Rennecke

CRC-TR 211
Strong-interaction matter under extreme conditions

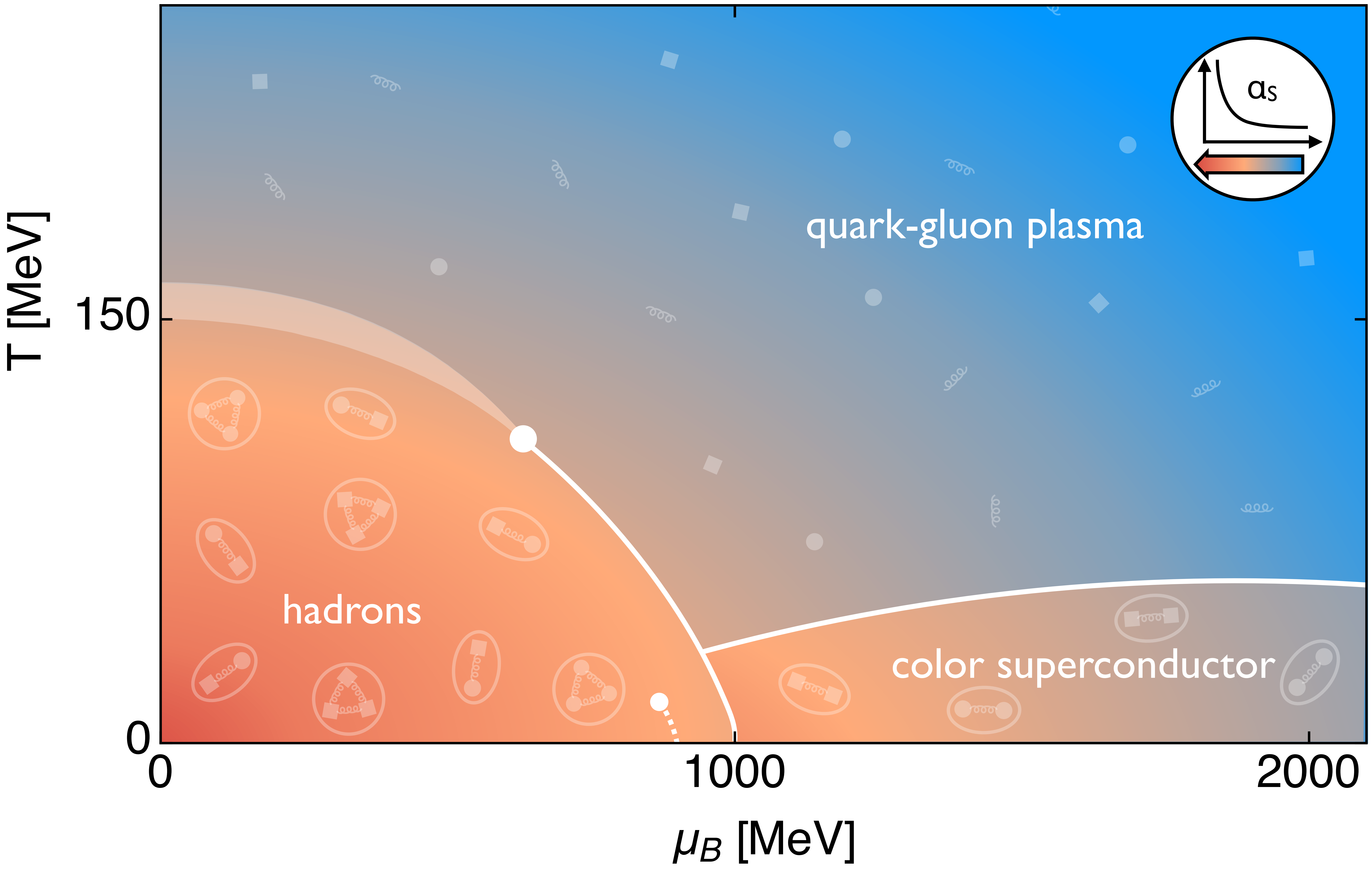
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF

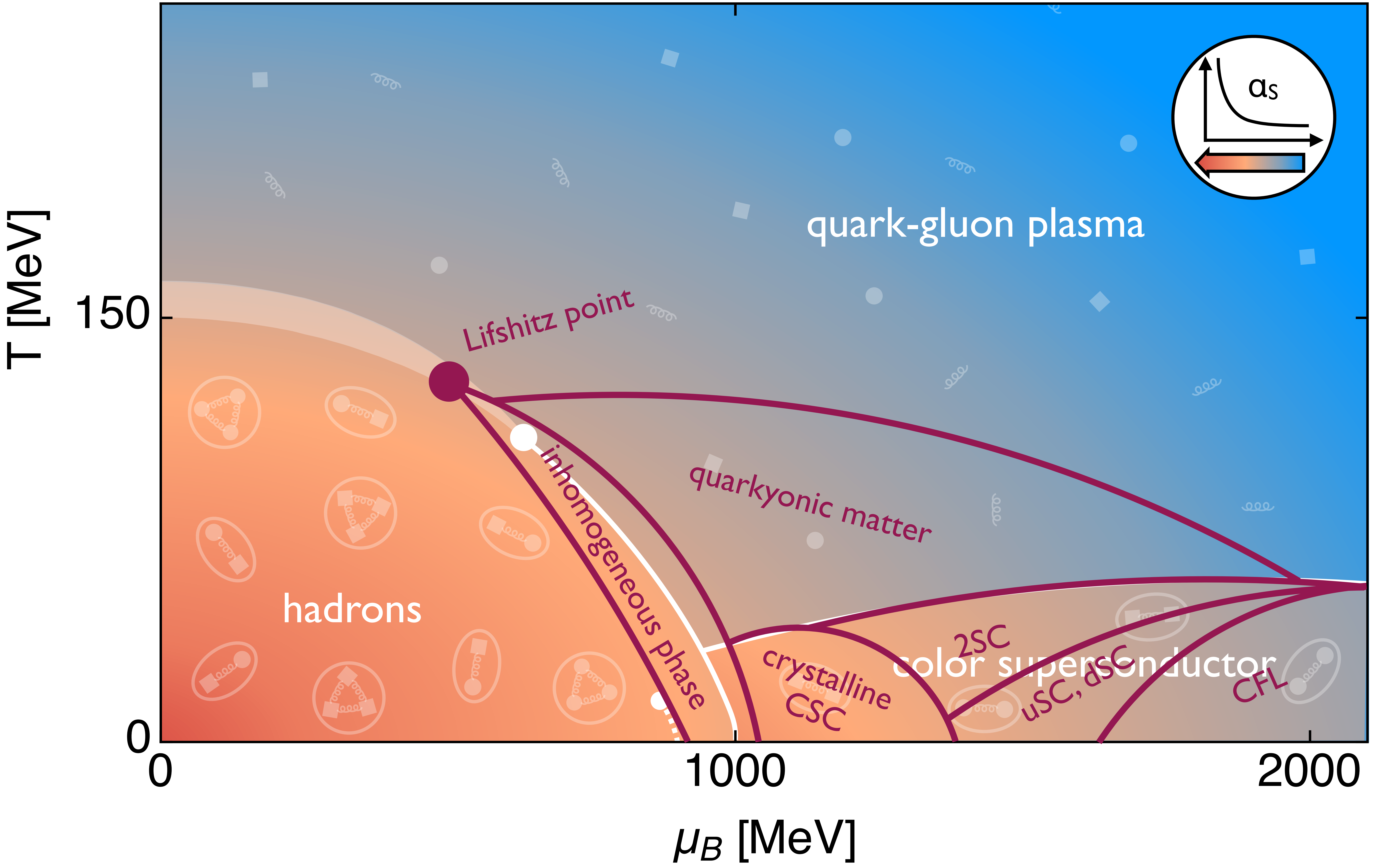
QUARK CONFINEMENT AND THE HADRON SPECTRUM

CAIRNS - 19/08/2024

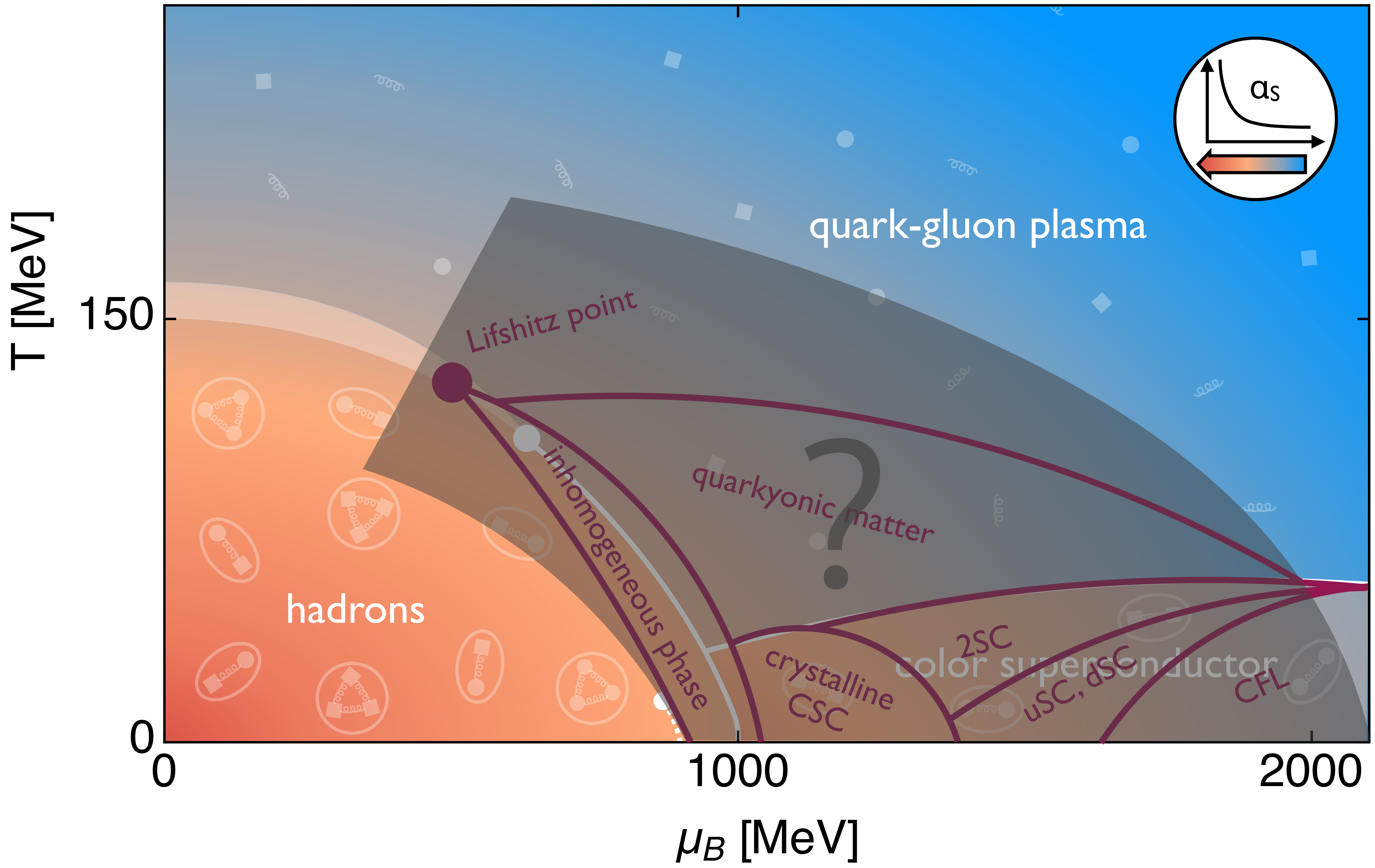
QCD PHASE DIAGRAM



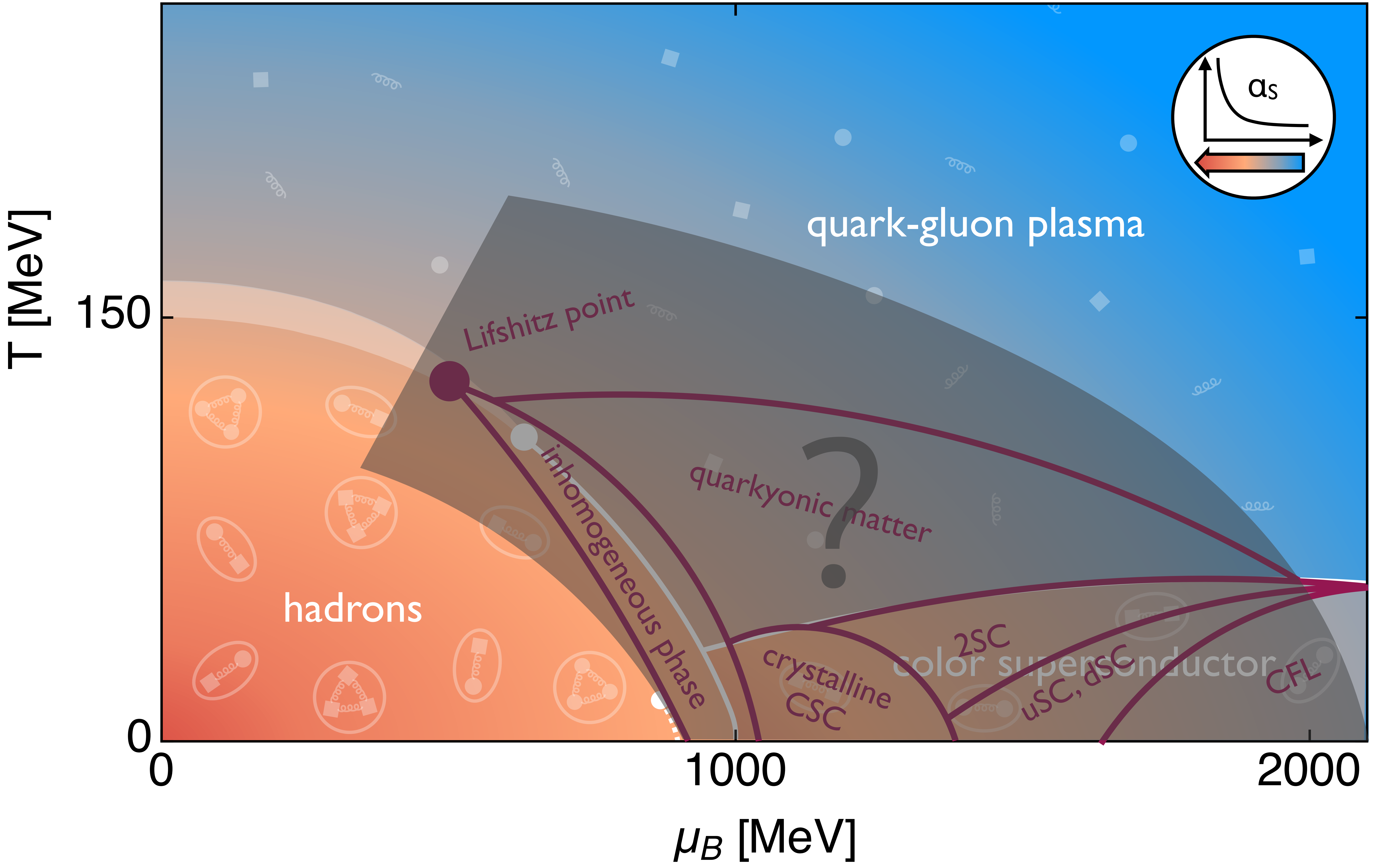
QCD PHASE DIAGRAM



QCD PHASE DIAGRAM



QCD PHASE DIAGRAM

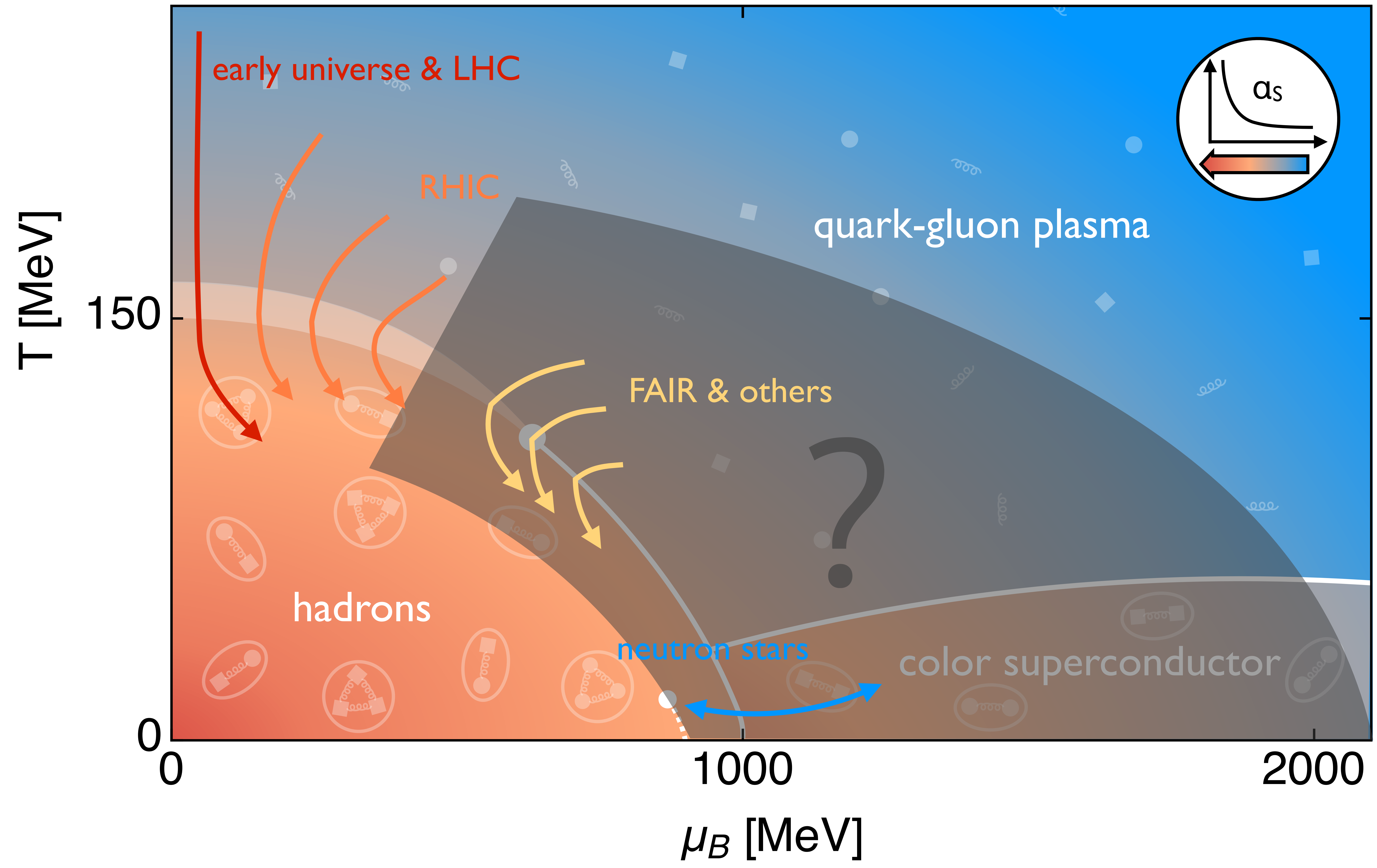


- nonperturbative physics
- lattice QCD cannot access finite μ directly due to sign problem
- quantum computing not feasible for QCD (yet)
- effective models only work in specific regimes; no "global" resolution

↓

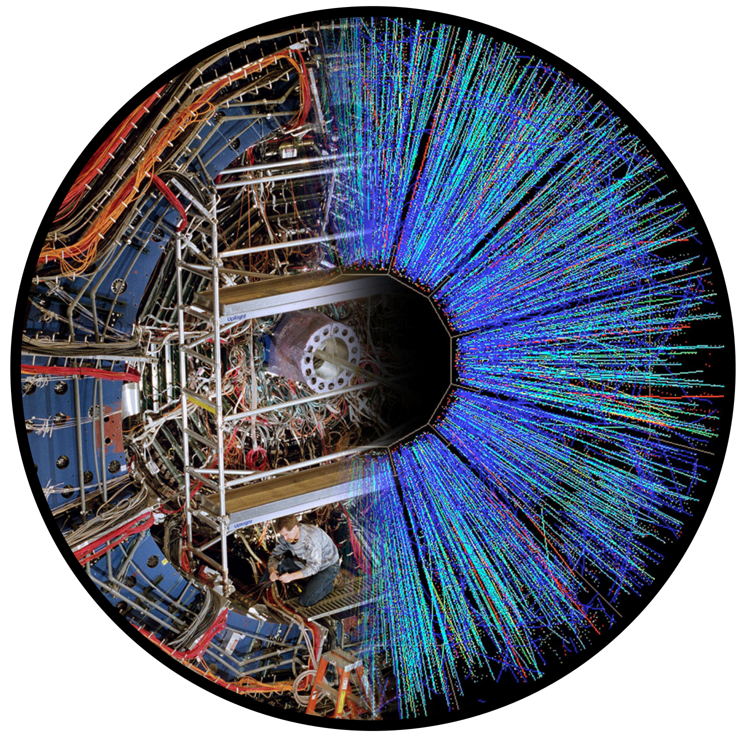
use functional methods

QCD PHASE DIAGRAM

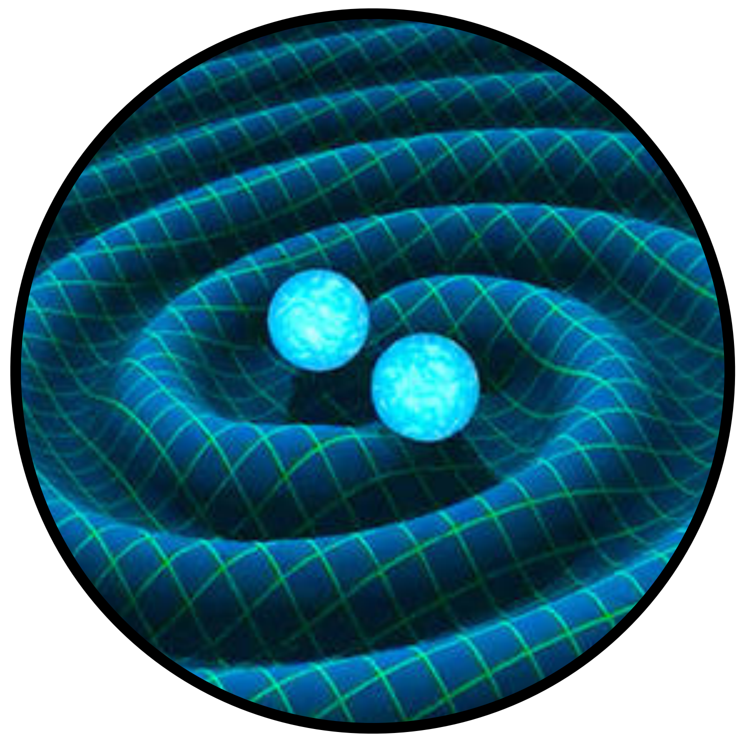


Experiments:

heavy-ion collisions



e.g. gravitational waves



FUNCTIONAL METHODS

The path integral encodes all possible correlation functions of a QFT

$$Z[J] = \int \mathcal{D}\varphi \, e^{iS[\varphi] + i\int_x J(x)\varphi(x)} \qquad \langle \varphi \cdots \varphi \rangle \sim \frac{1}{Z[0]} \frac{\delta}{\delta J} \cdots \frac{\delta}{\delta J} Z[J] \bigg|_{J=0}$$

Solving a QFT ⇔ knowing all correlation functions. There are two popular (= practical) strategies to do this

Dyson-Schwinger equations (DSE)

"quantum equations of motion"

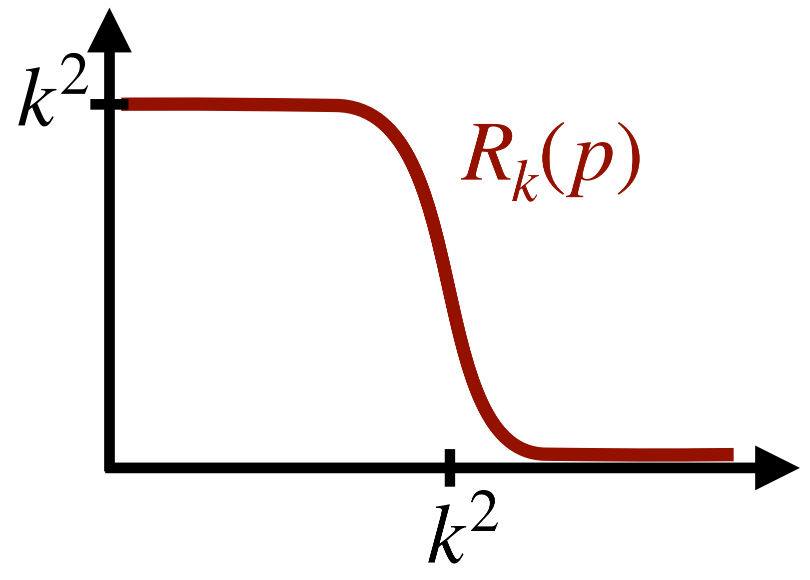
$$\int \mathcal{D}\varphi \left(\frac{\delta S[\varphi]}{\delta \varphi(x)} + J(x) \right) e^{iS[\varphi] + i\int_x J(x)\varphi(x)} = 0$$

functional renormalization group (FRG)

successively integrate out quantum fluctuations

$$Z_k[J] = \int \mathcal{D}\varphi \, e^{i\left(S[\varphi] + \Delta S_k[\phi]\right) + i\int_x J(x)\varphi(x)}$$

$$\Delta S_k[\varphi] = \int_p \frac{1}{2} \varphi(p) \, R_k(p) \, \varphi(-p)$$



- J-derivatives: tower of **exact** relations between correlation function (requires truncations)
- FRG: convert to differential equation through k-derivative: RG flow from UV to IR $(k = 0)$
- no sign problems: finite density and real time directly accessible

QCD-RELATED REVIEWS

FRG

[Pawlowski, arXiv:0512261]
[Gies, arXiv:0611146]
[Rosten, arXiv:1003.1366]
[Braun, arXiv:1108.4449]
[Dupuis at al., arXiv:2006.04853]
[Fu, arXiv:2205.00468]

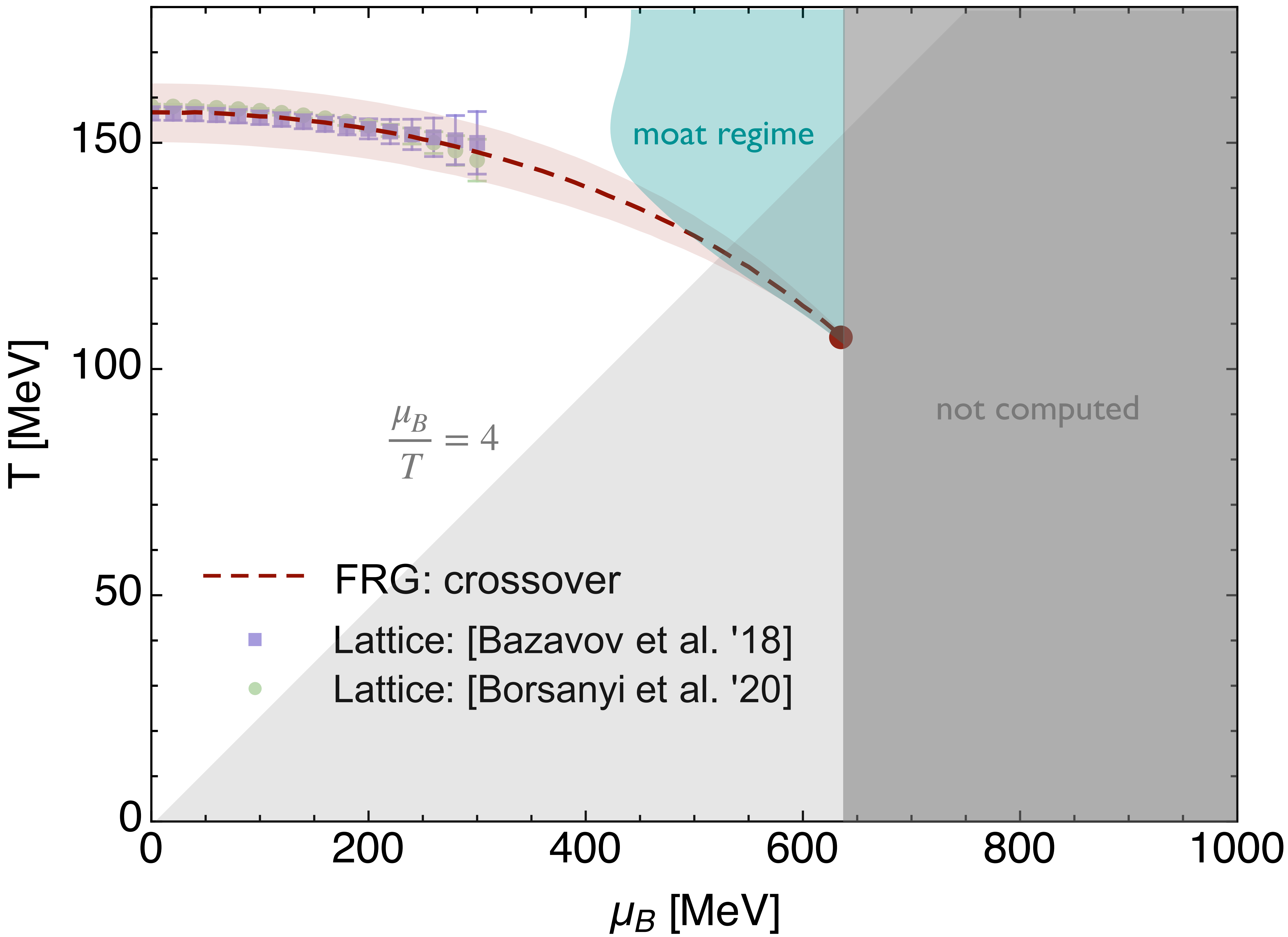
DSE

[Alkofer, von Smekal, arXiv:0007355]
[Fischer, arXiv:0605173]
[Roberts, Schmidt, arXiv:0005064]
[Eichmann at al, arXiv:1606.09602]
[Fischer, arXiv:1810.12938]
[Huber, arXiv:1808.05227]

QCD PHASE DIAGRAM

[Fu, Pawlowski, FR, PRD 101 (2019)]

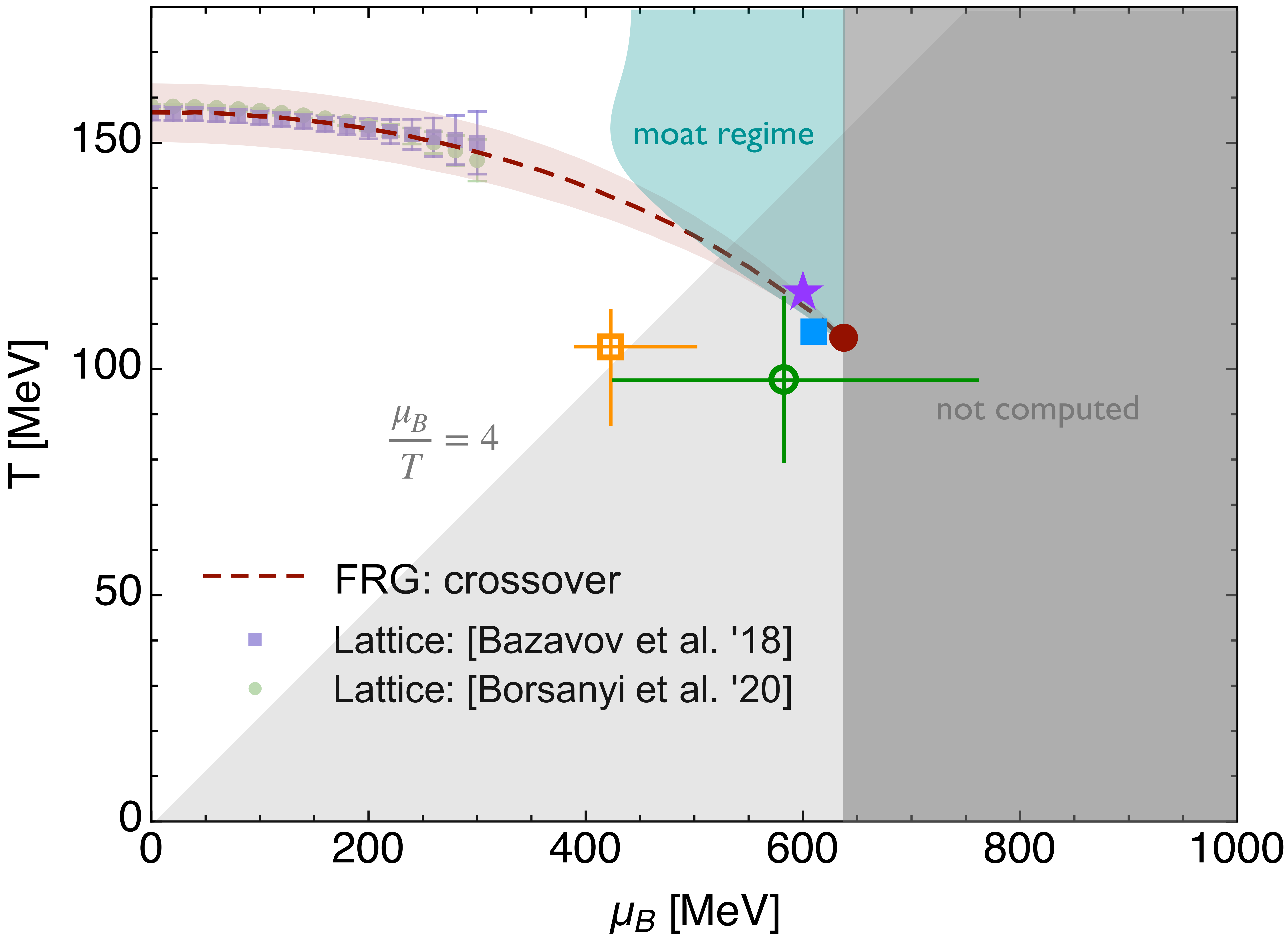
First result for the chiral transition with $N_f = 2 + 1$ flavors at finite T and μ_B



- need to improve systematics at large μ (WIP)
- **CEP at $(T, \mu_B) = (107, 635)$ MeV**
- indications for a moat regime

CRITICAL ENDPOINT

FRG result corroborated by subsequent direct computations & extrapolations in QCD



direct computations in QCD

(that agree with available lattice data)

- ● FRG [Fu, Pawlowski, FR, PRD 110 (2019)]
- ■ DSE [Gao, Pawlowski, PLB 820 (2020)]
- ★ DSE [Gunkel, Fischer, PRD 104 (2021)]

Lattice extrapolations

(using reconstructions of YLEs)

- ○ conformal Padé [Basar, PRC 110 (2024)]
- ☐ multi-point Padé [Clarke et al., 2405.10196]

↑

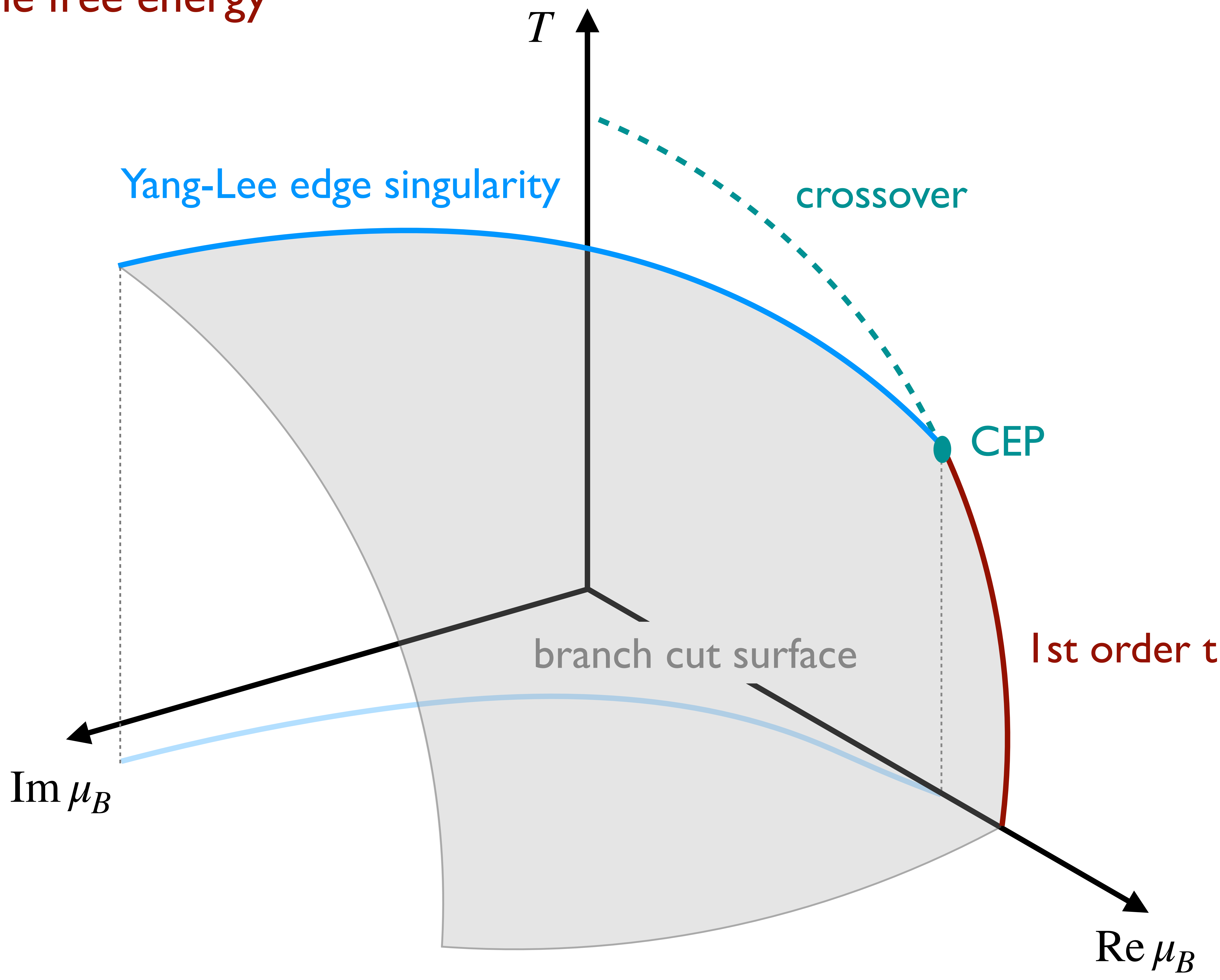
$N_\tau = 6$; expected to move to

$\mu_B \approx 650\,\mathrm{MeV}$ in the continuum limit

YANG-LEE EDGE SINGULARITIES

What happens to the CEP for $T > T_{\mathrm{CEP}}$? It moves into the complex μ plane and is called YLE!

YLE: branch point of the free energy



CEP RECONSTRUCTION USING YLES

How to find YLE on the lattice?

- no direct access due to sign problem
- reconstruct YLE locations from lattice data at $T > T_{\text{CEP}}$
- extrapolate to $\operatorname{Im}\mu_{\text{YLE}} = 0$

How to extrapolate?

Assuming that the data is in the scaling region of the CEP:

$$\operatorname{Im}\mu_{\text{YLE}} \sim \left(|z_c|/t\right)^{-\beta\delta}$$ [Stephanov, PRD 73 (2006)]

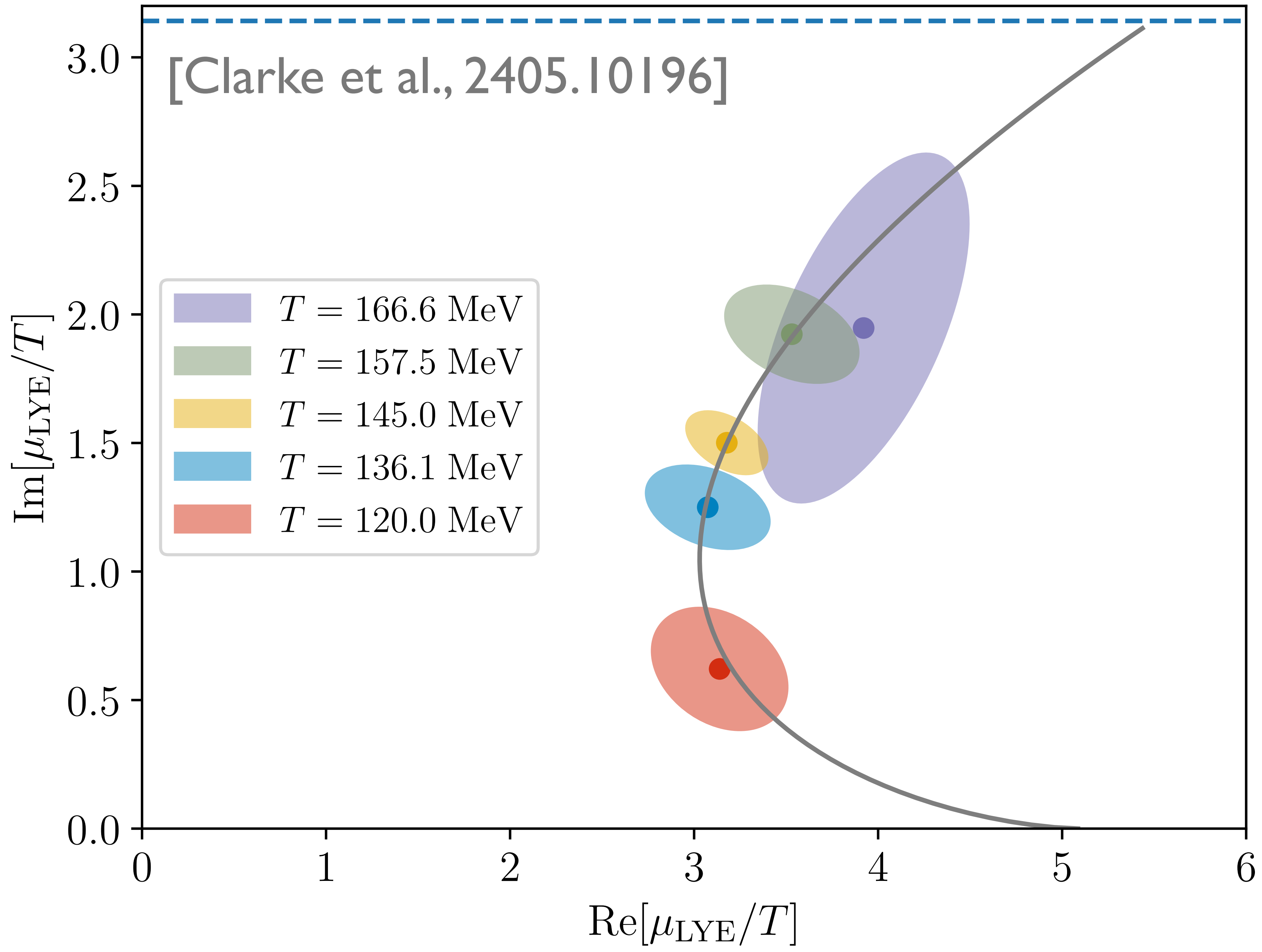
- in this case, YLE location z_c is universal as well
- directly available only from the FRG

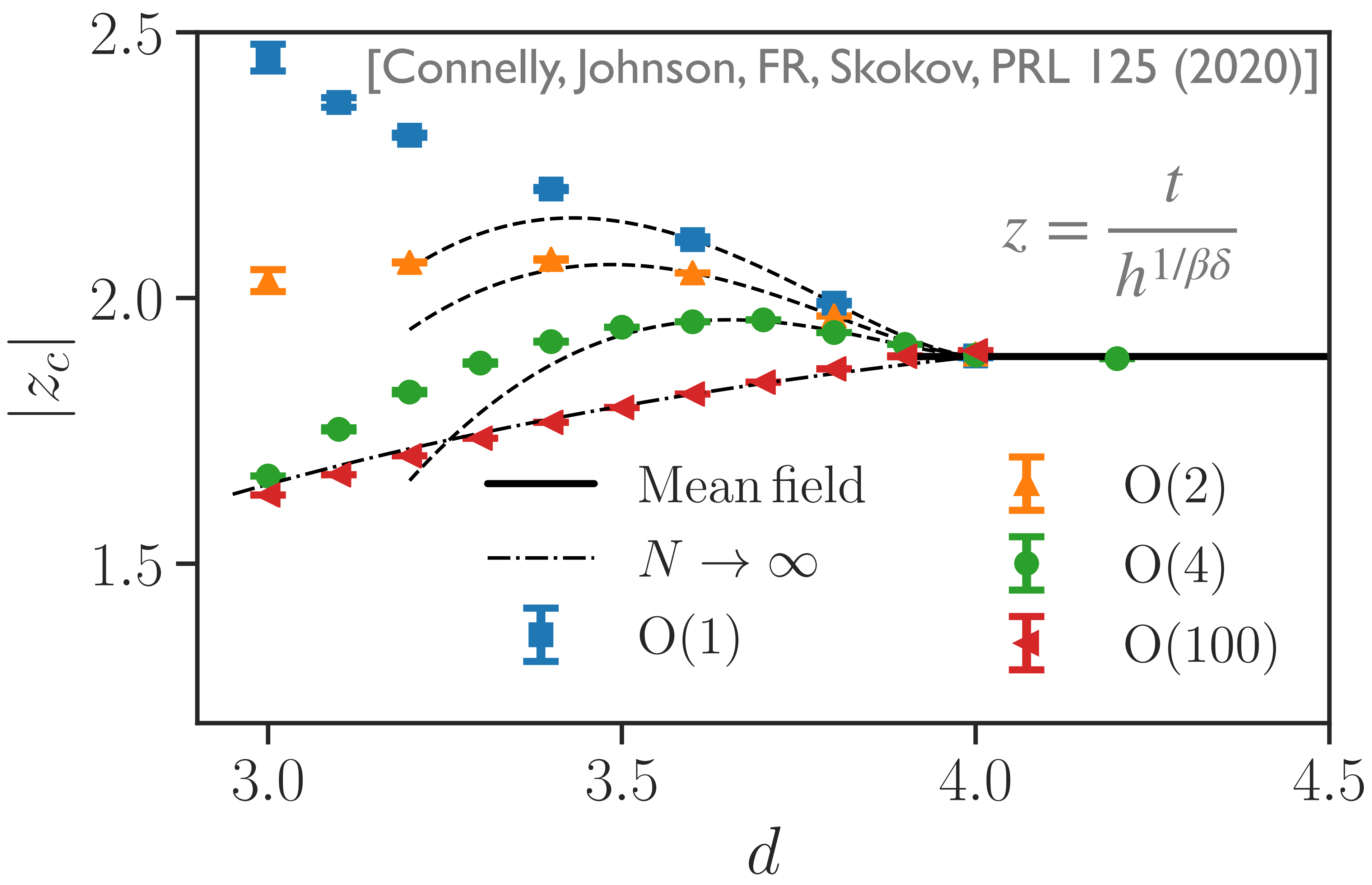
[Connelly, Johnson, FR, Skokov, PRL 125 (2020)]
[FR, Skokov, Annals Phys. 444 (2022)]
[Johnson, FR, Skokov, PRD 107 (2023)]

However, scaling regime most likely small, so accurate extrapolations require non-universal information

→ also use functional methods (WIP)

first exploratory DSE study: [Wan et al, 2401.04957]



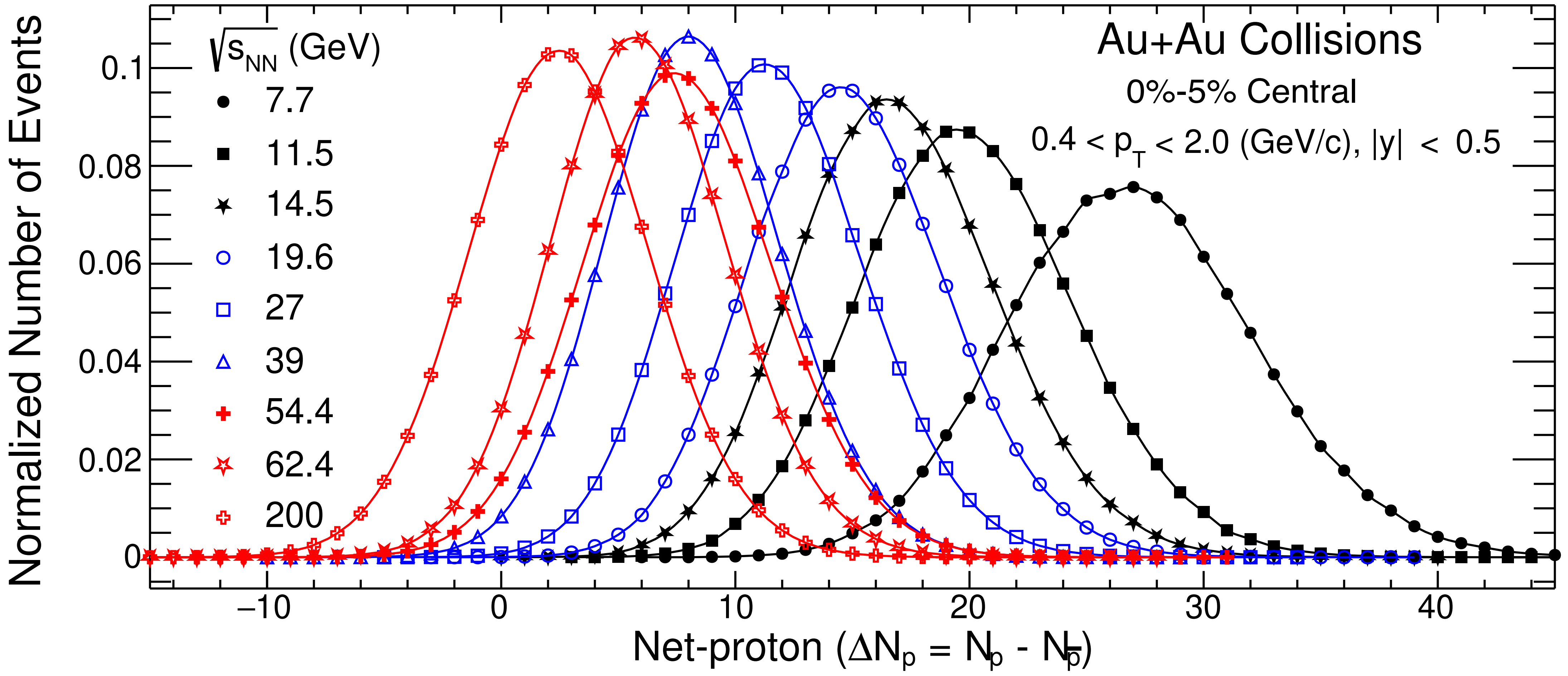


CAN WE MEASURE THE CEP?

experiment: heavy-ion collisions

- measure net-proton distributions $P(N_P)$
- smaller collision energy → larger μ_B

[STAR (2021)]



- susceptibilities from moments of the distribution

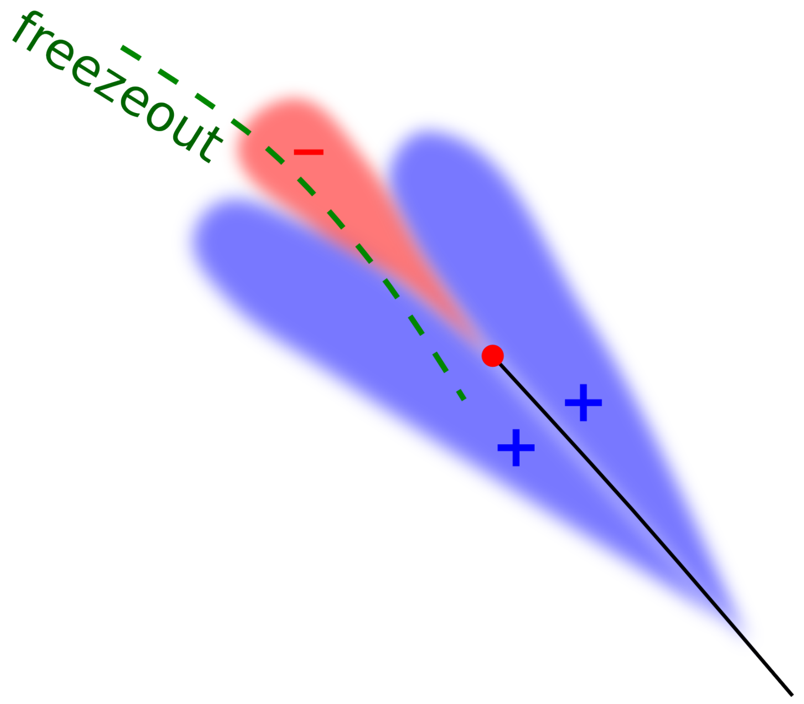
$$\left\langle \left(N_P - \langle N_P \rangle\right)^n \right\rangle = \sum_{N_P} \left(N_P - \langle N_P \rangle\right)^n P(N_P)$$

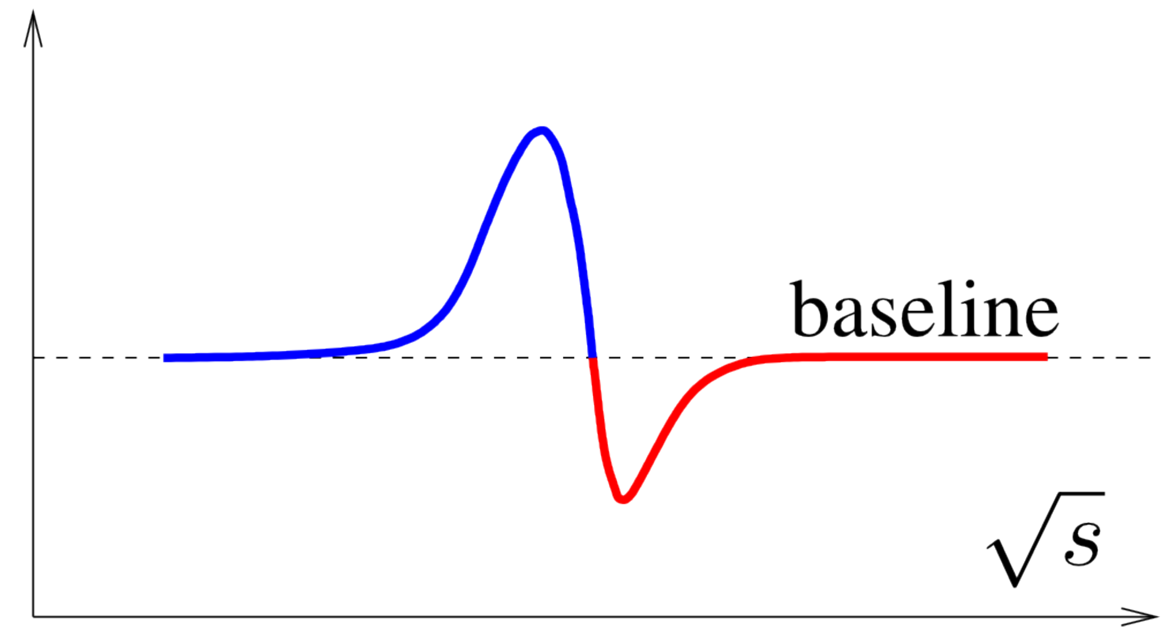
theory

- susceptibilities of the net-baryon distribution

$$\chi_n^B = T^{n-4} \frac{\partial^n p}{\partial \mu_B^n} \sim \left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^n \right\rangle$$

- χ_n scale near CEP
- scaling near the CEP: non-monotonic beam-energy dependence of kurtosis $\sim R_{42}^B = \chi_4/\chi_2$





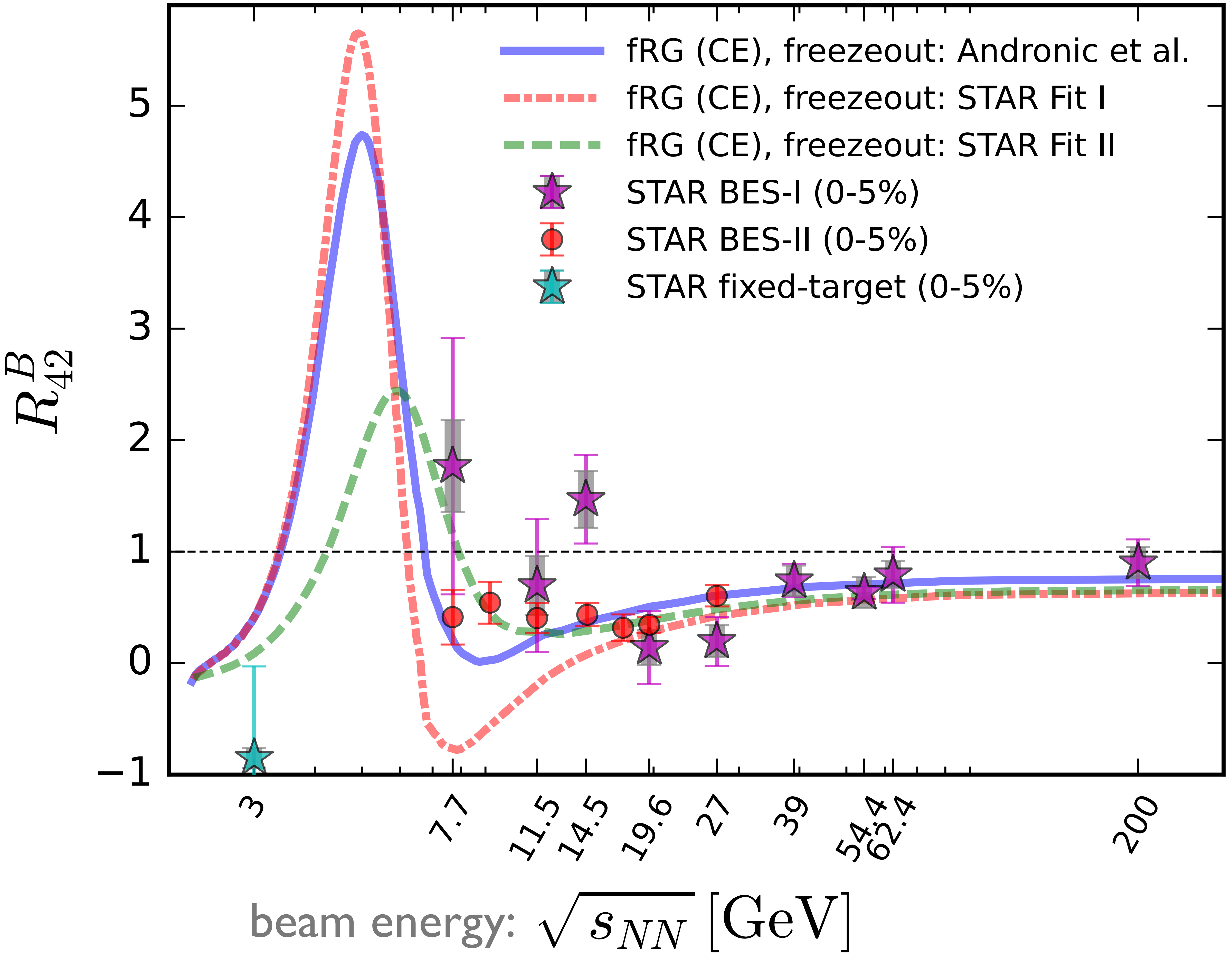
[Stephanov, PRL 102 (2009)]

→ measurements can be sensitive to critical fluctuations, but there are **many caveats and subtleties!**

RIPPLES OF THE CEP

low-energy model with QCD and HIC input:

[Fu, Luo, Pawlowski, FR, Yin, 2308.15508]



where the particles ~freely stream to the detector

- no criticality seen at (putative) freeze-out
- still, functional methods see pronounced non-monotonicity at low beam-energies

→ criticality not necessary for non-monotonic $\sqrt{s}$ dependence of R_{42}

- peak position only sensitive to freeze-out location, its height is sensitive to the distance from the CEP
- no sign of the CEP in experimental data

→ need data between $\sqrt{s} = 3 - 8\,\mathrm{GeV}$: **FAIR**

first (exploratory) QCD results using DSE:

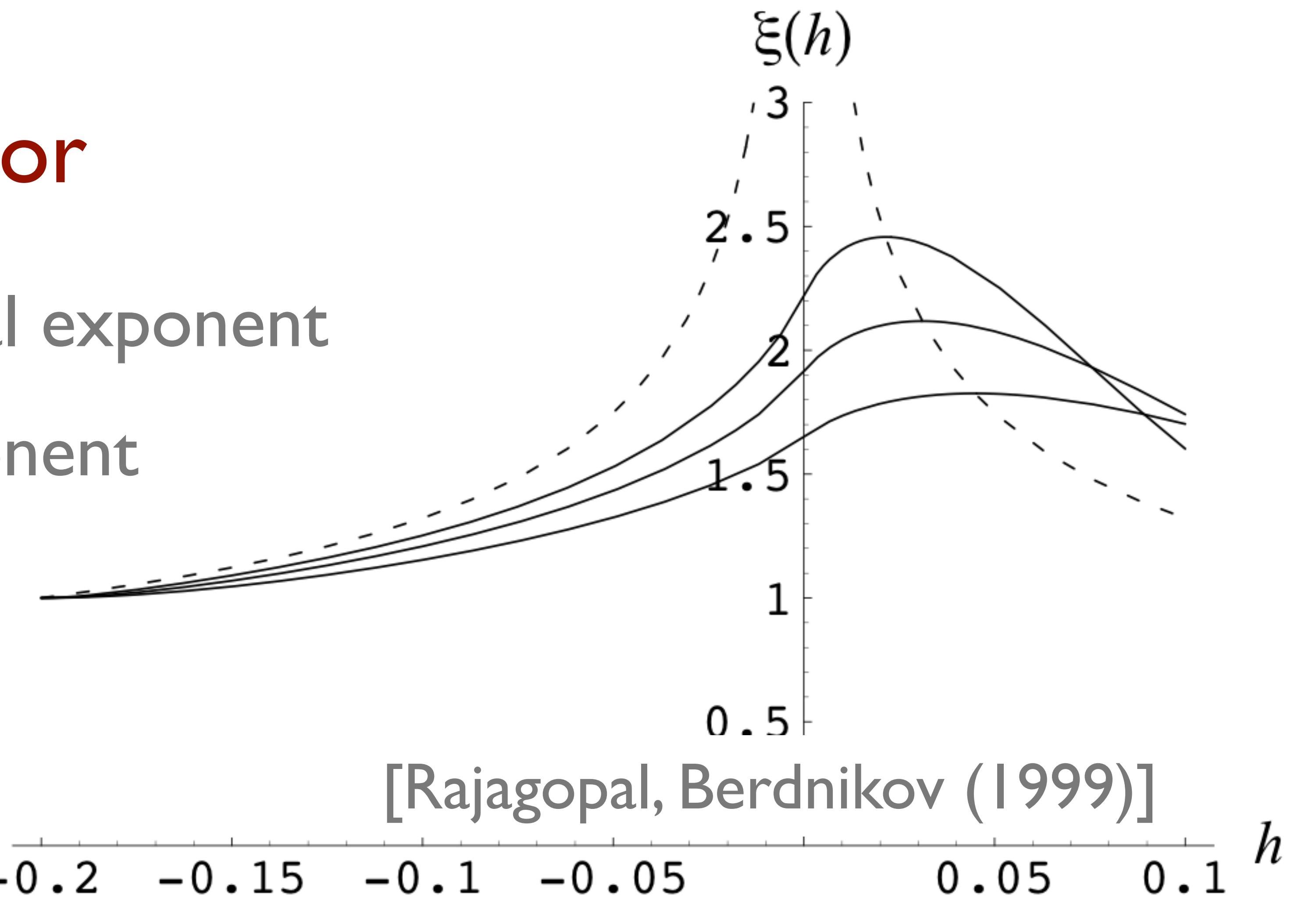
[Isserstedt et al., PRD 100 (2019)]

[Bernhardt, Fischer, Isserstedt, PLB 841 (2023)]

CRITICALITY

If the system "closely" passes the CEP, it is governed by universal critical behavior

- correlation length ξ diverges at CEP in equilibrium, $\xi \sim (T - T_c)^{-\nu}$ ⟵ static critical exponent
- critical slowing down: relaxation time also diverges, $\tau \sim \xi^z$ ⟵ dynamic critical exponent
- slow modes and conserved quantities determine dynamic universality



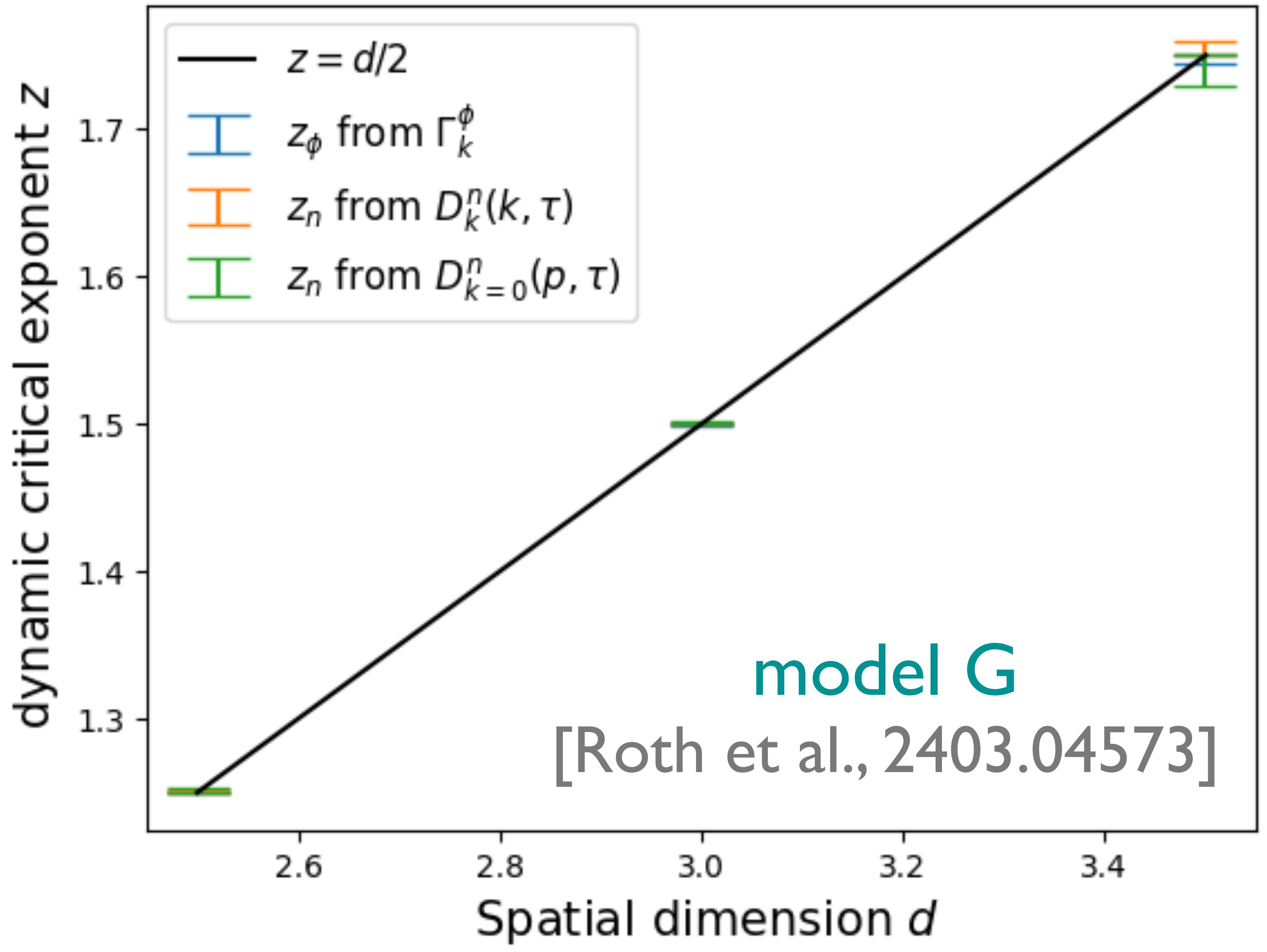
[Rajagopal, Berdnikov (1999)]

⟶
- ○ model G for the chiral transition
- ○ model H for the CEP

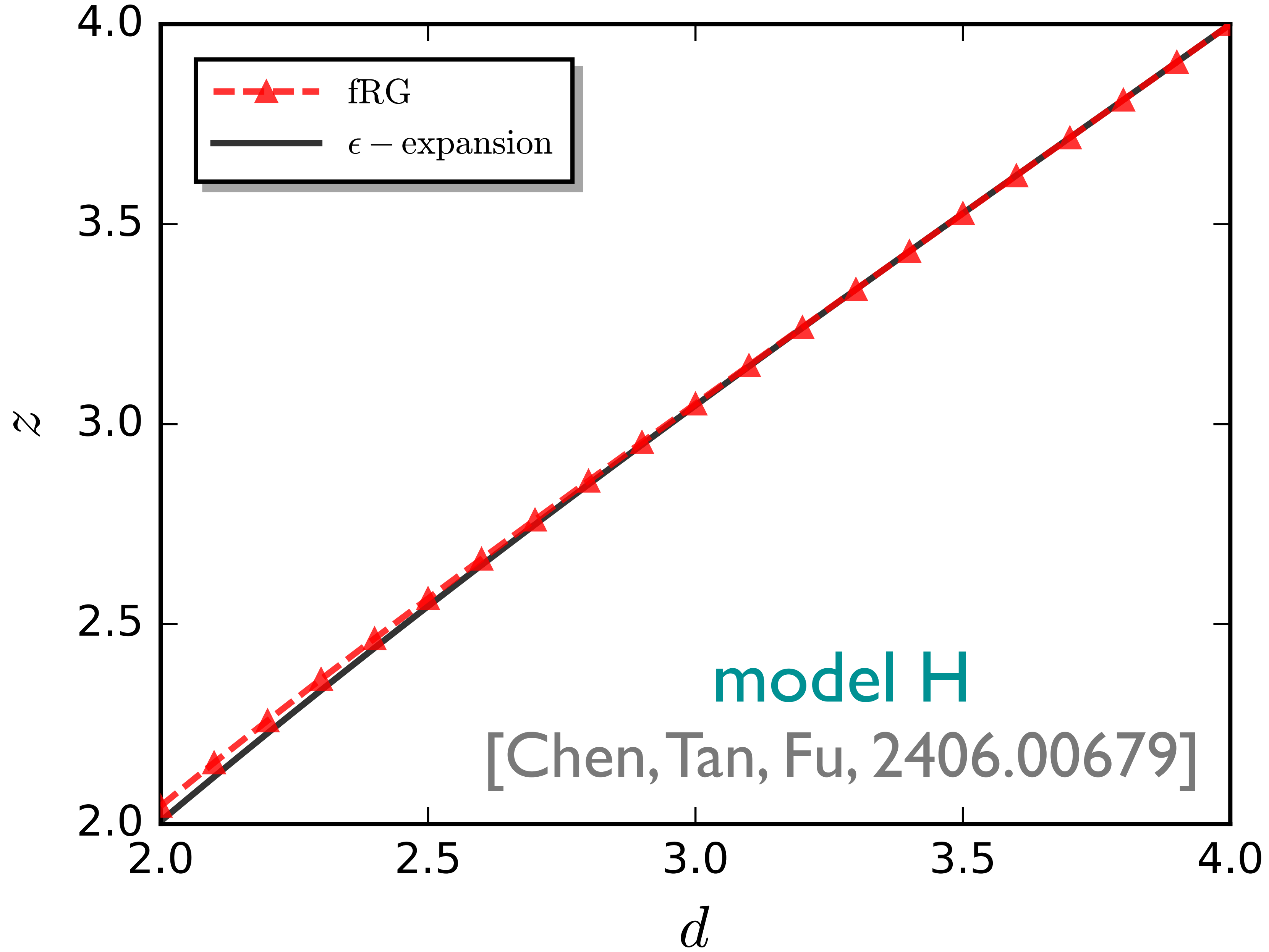
[Halperin, Hohenberg, RMP 49 (1977)]
[Son, Stephanov, PRD 70 (2004)]

Critical exponents are necessary to describe the system in the critical scaling regime. (F)RG is made for that.

Example: dynamic critical exponent from real time FRG (review: [Dupuis at al., Phys. Rep. 910 (2020)])



model G
[Roth et al., 2403.04573]



model H
[Chen, Tan, Fu, 2406.00679]

TO SCALE OR NOT TO SCALE

[FR ∈ fQCD Collaboration, 2310.19853]

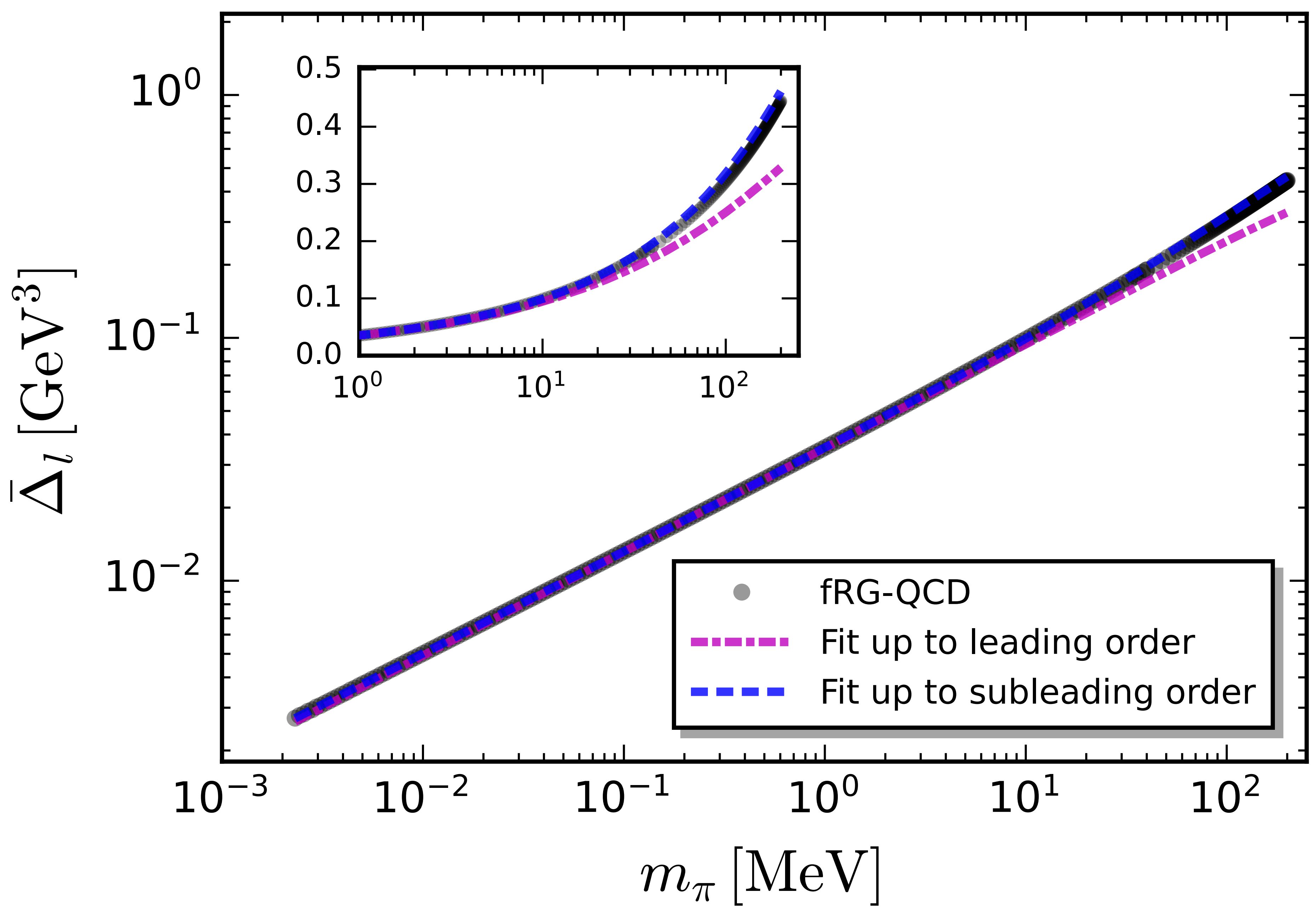
Universality is very powerful, but where does it apply?

- universality occurs if the system is scale invariant
- RG: scale invariance if system can be linearized around fixed point

→ critical region is uniquely defined

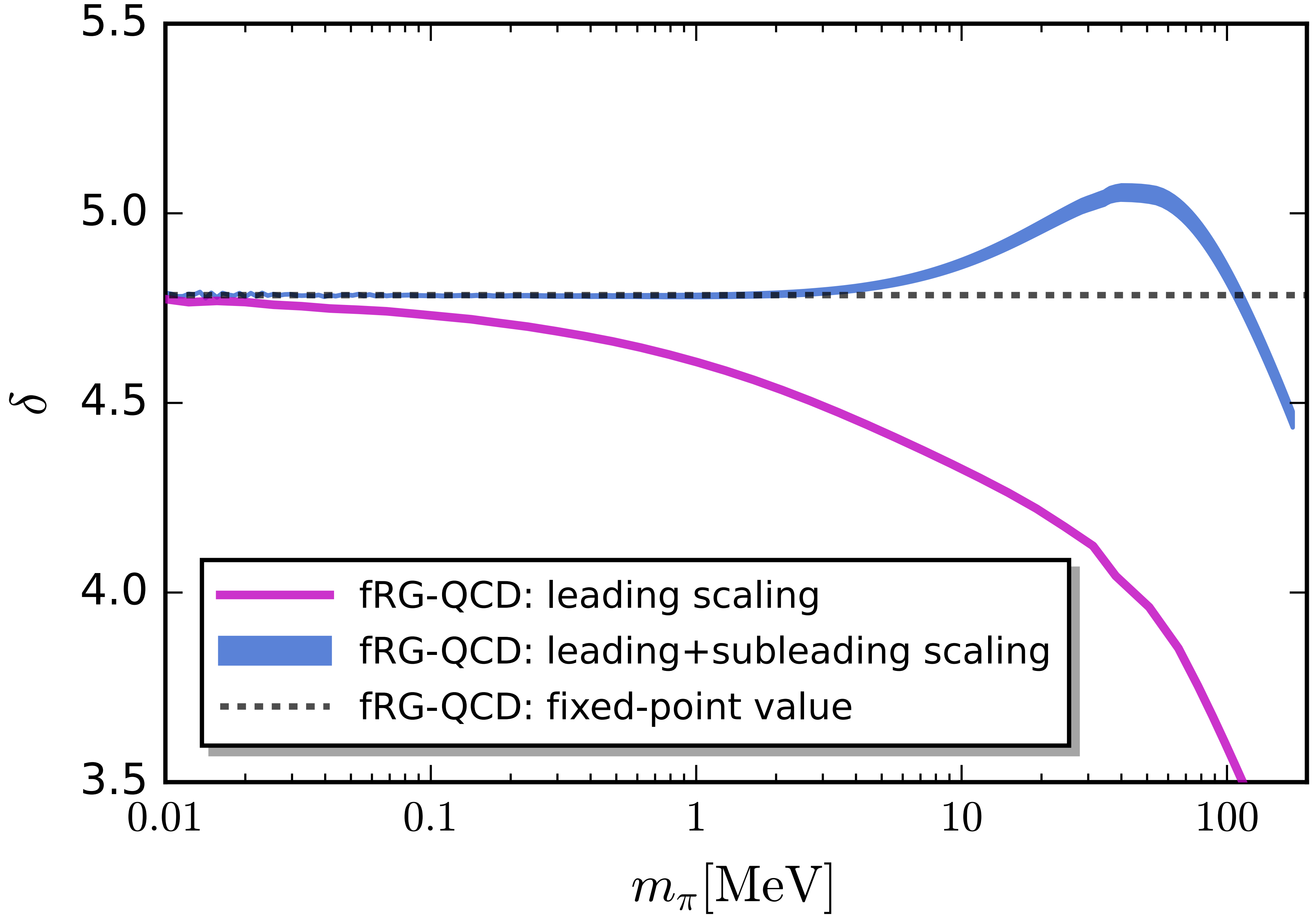
Example: size of the critical region of the chiral transition:

- chiral condensate for different quark masses



$$\bar{\Delta}_l(T, m_\pi) = m_\pi^{2/\delta} f_G(z) + f_{\mathrm{reg}}(T, m_\pi)$$

- determine breakdown of scaling from QCD data



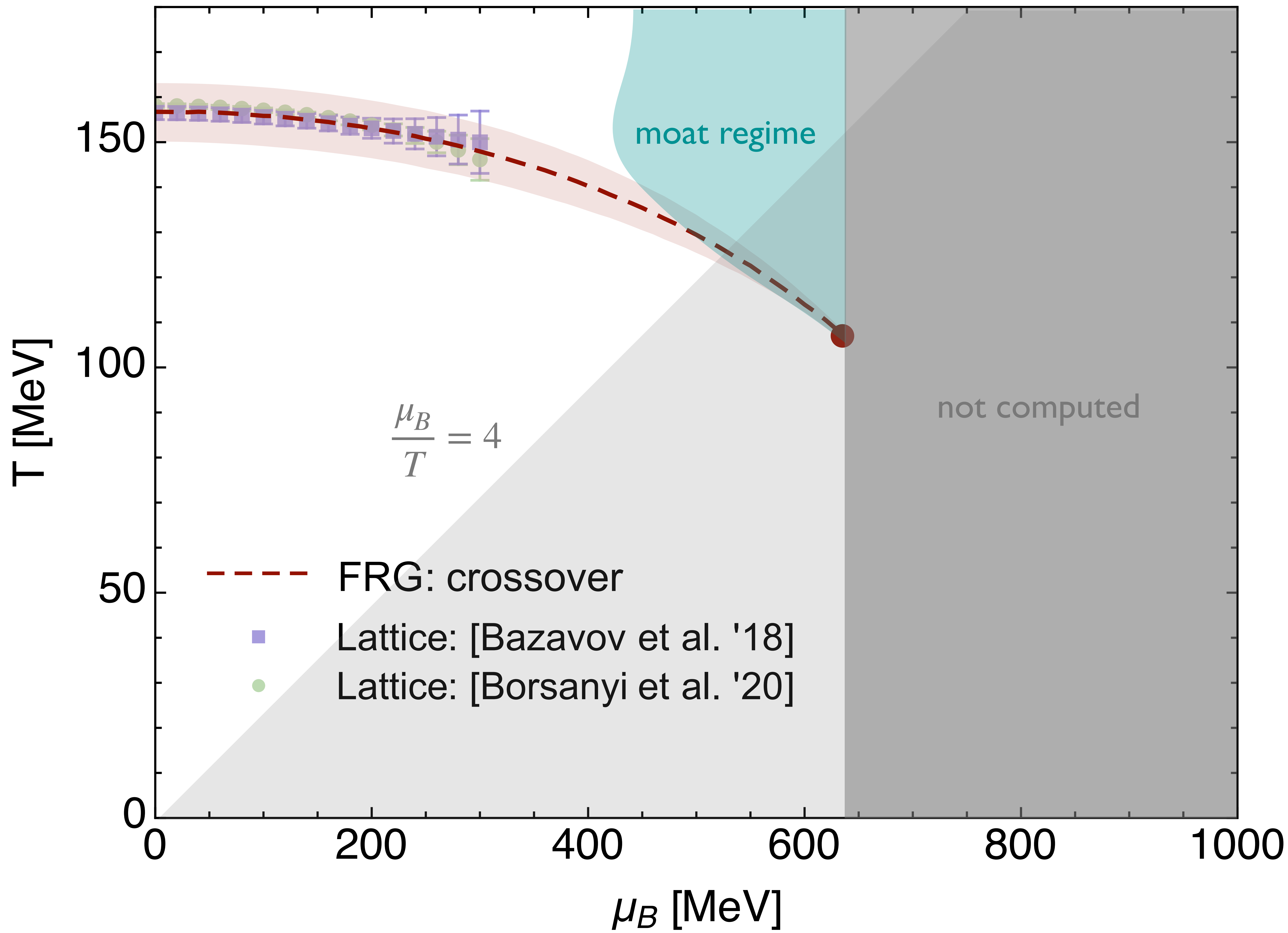
→ **tiny critical region:** $m_\pi \lesssim 5\,\mathrm{MeV}$

small critical regions typical for thermal phase transitions, most likely including the CEP (WIP)

QCD PHASE DIAGRAM

[Fu, Pawlowski, FR, PRD 101 (2019)]

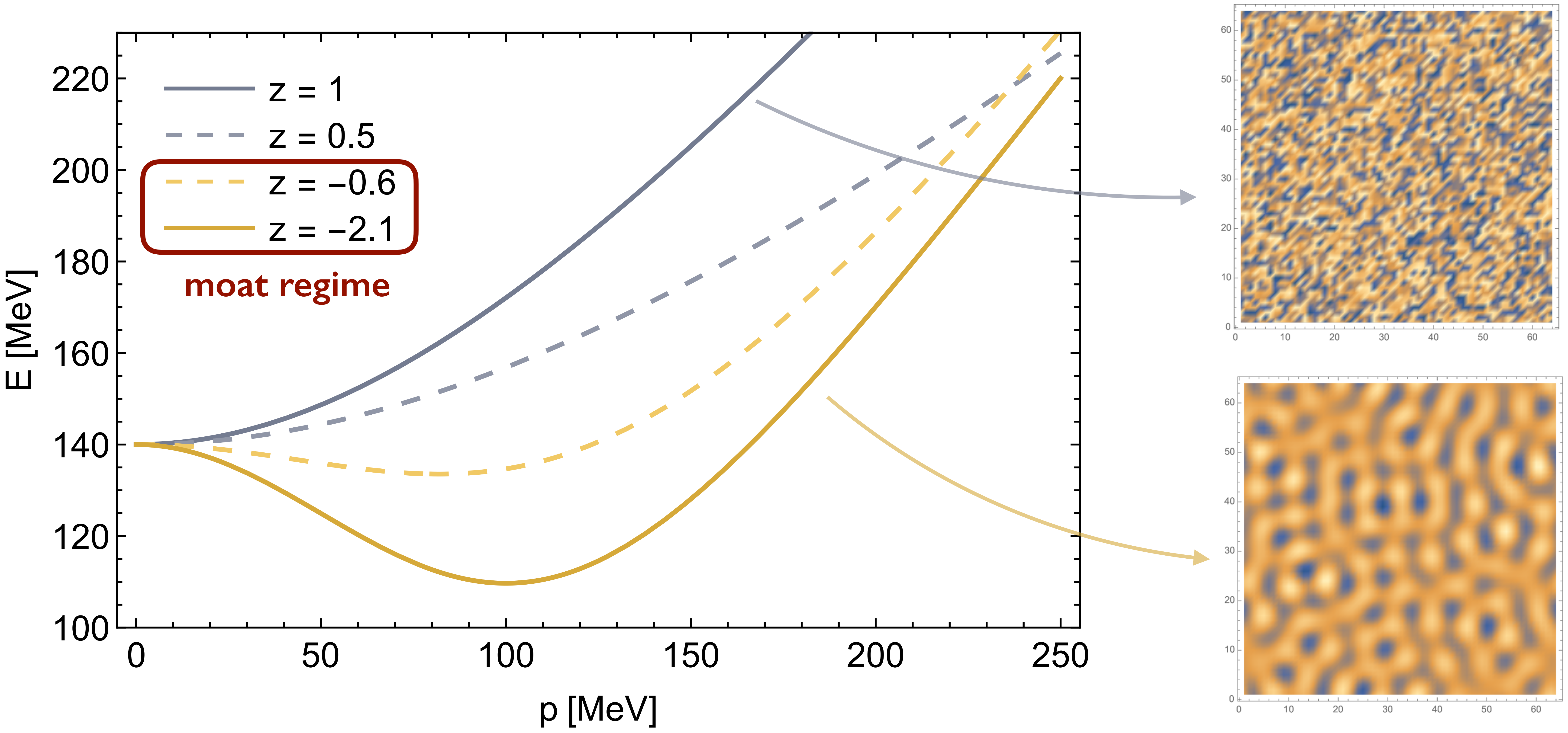
First result for the chiral transition with $N_f = 2 + 1$ flavors at finite T and μ_B



- need more 4-quark channels to improve systematics at large μ
- CEP at $(T, \mu_B) = (107, 635)$ MeV
- **indications for a moat regime**

THE MOAT REGIME

Modes with modified dispersion appear at large μ_B: $E^2(\mathbf{p}^2) = \sqrt{Z(\mathbf{p}^2)\,\mathbf{p}^2 + m^2} = \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p}^6) + m^2}$



position space:

homogeneous configuration

$\langle \phi(r)\phi(0) \rangle \sim e^{-mr}$

"homogeneous pattern"

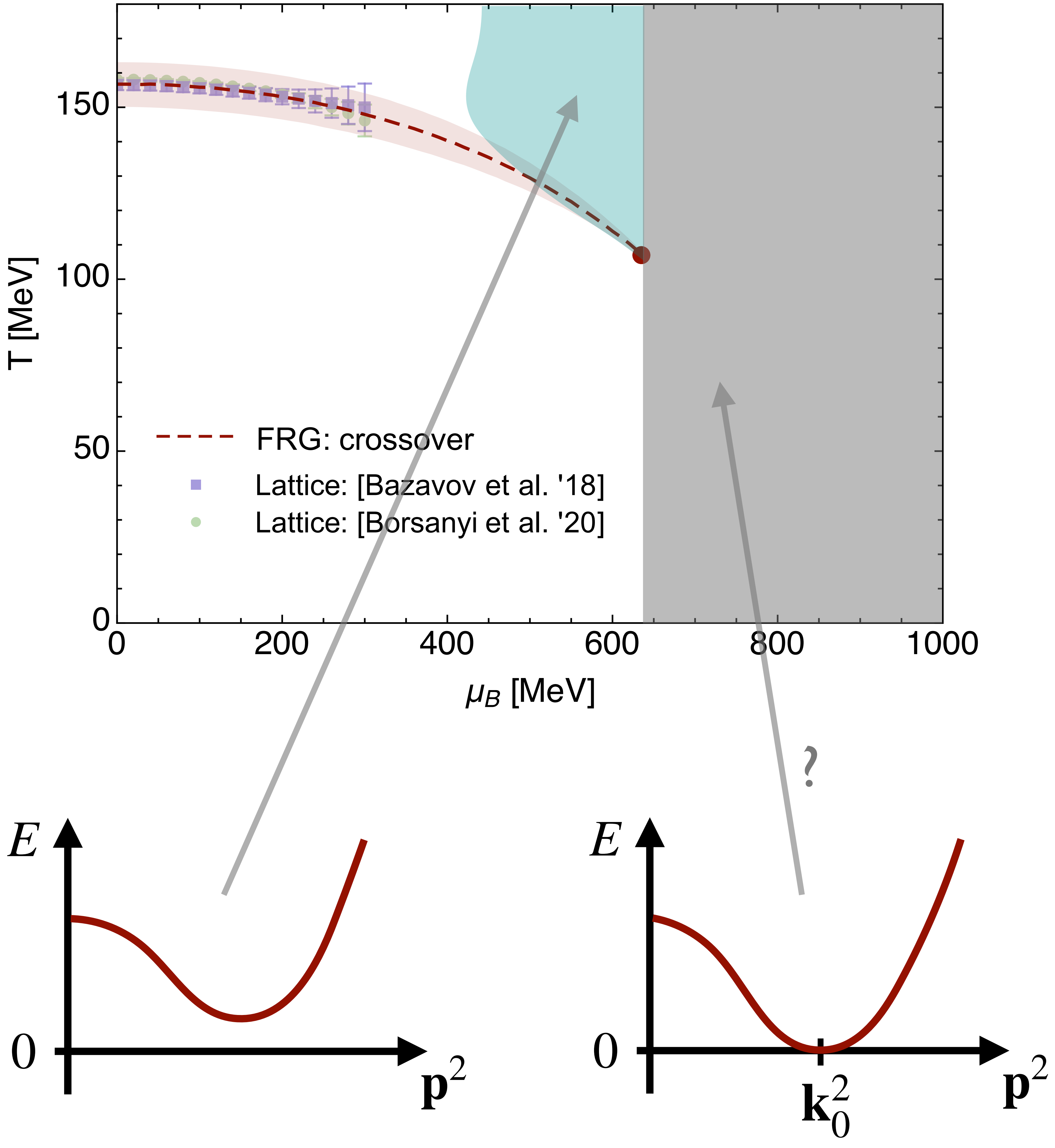
$\langle \phi(r)\phi(0) \rangle \sim \sin(k_0 r)\, e^{-mr}$

favored momentum (wavenumber) ↔ **spatial modulations (patterns)**

analogy in condensed matter:
Friedel oscillations
[FR, Yin (in preparation)]

PATTERN FORMATION

The energy gap might close:

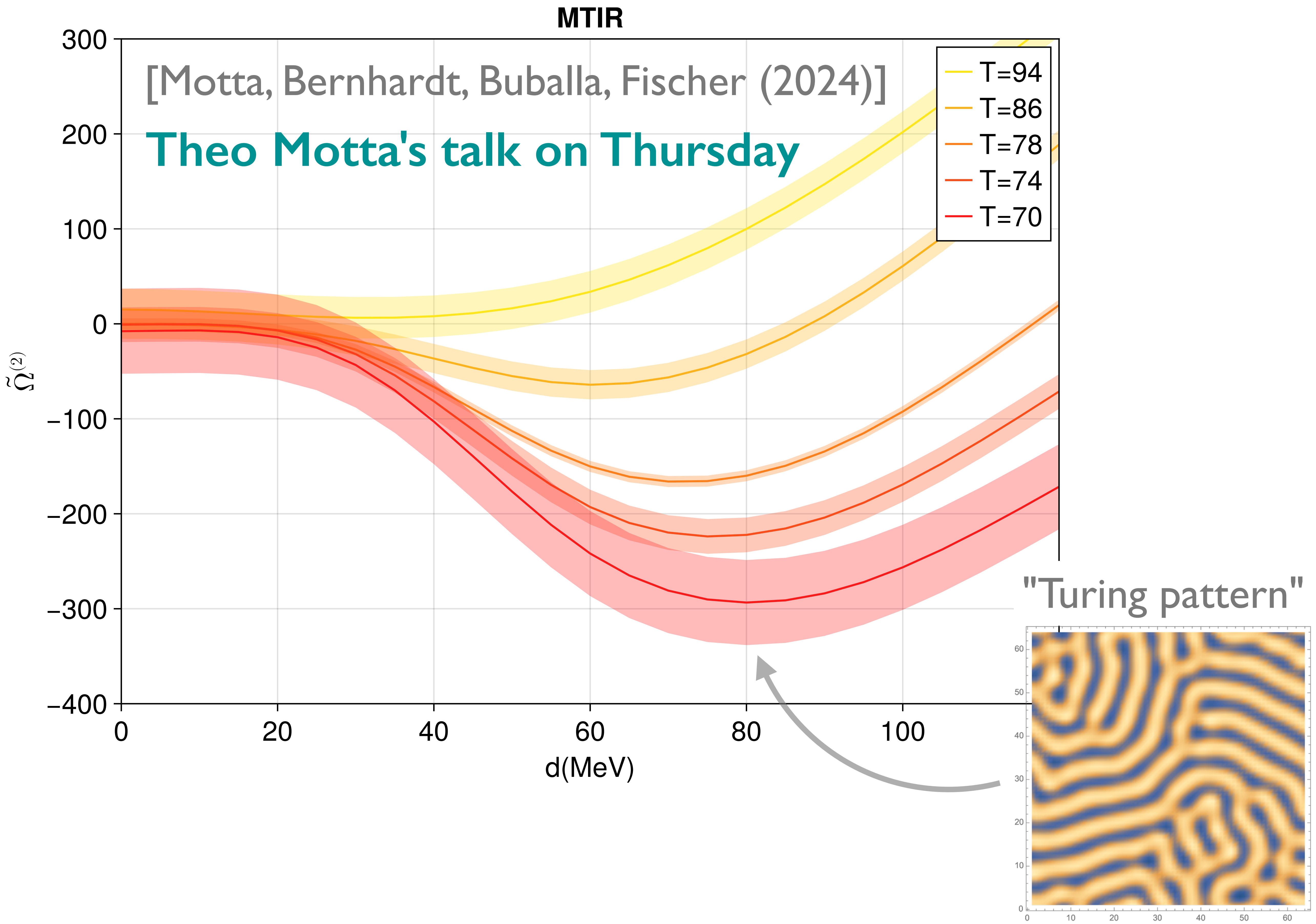


$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an **inhomogeneous condensate**

- common in low-energy models
- found in QCD model with DSE:


[Motta, Bernhardt, Buballa, Fischer (2024)]

Theo Motta's talk on Thursday

"Turing pattern"

TYPES OF PATTERNS

Is an inhomogeneous instability always possible/lead to Turing patterns?

No: formation of inhomogeneous phases depends on dynamics of soft (massless) modes (e.g., Goldsones from (spatial) symmetry breaking).

→ fluctuation-induced instabilities of inhomogeneous phases

→ other types of patterned phases possible

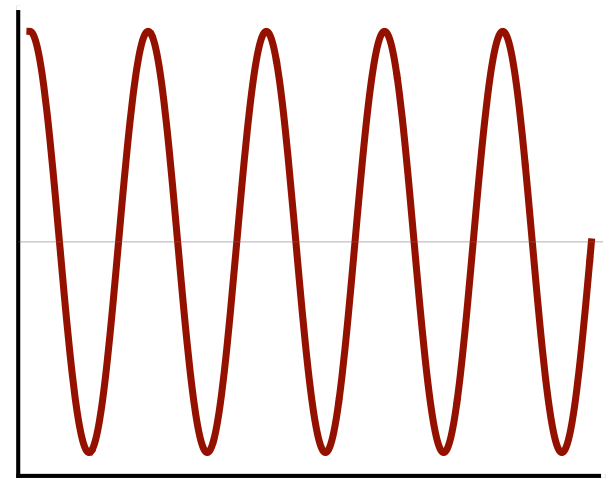
[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]
[Pisarski, Tsvelik, Valgushev, PRD 102 (2020)]

- inhomogeneous phases are a typical mean-field result (no Goldstone fluctuations; also true for the previous DSE result)
- FRG: inhomogeneous phase may only exist at intermediate scales

[Jeong, Murgana, Dash, Rischke, arXiv:2407.13589]

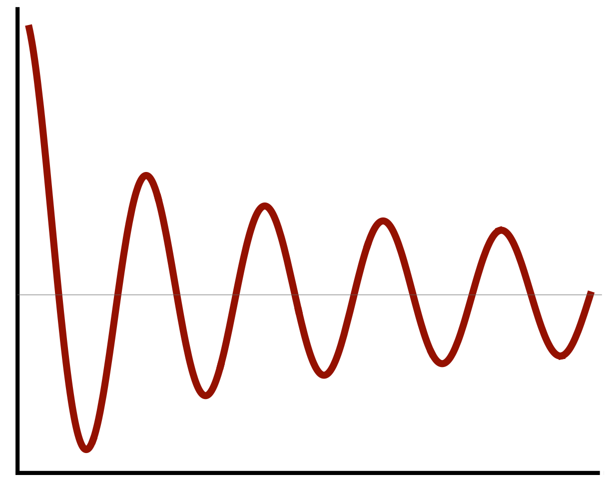
inhom. phase

$\langle\phi(r)\phi(0)\rangle \sim \sin(k_0\, r)$



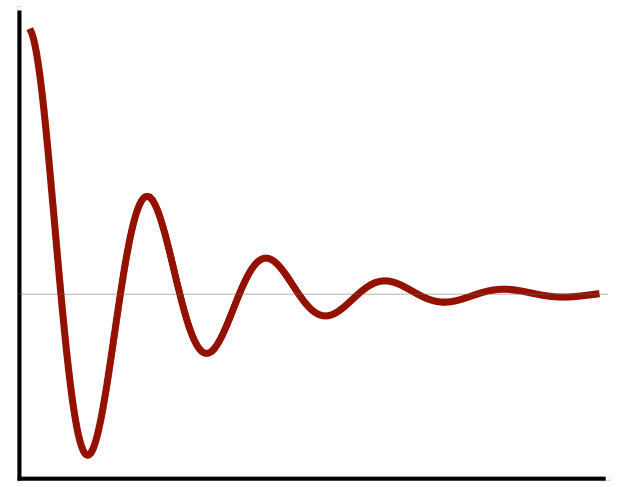
liquid crystal

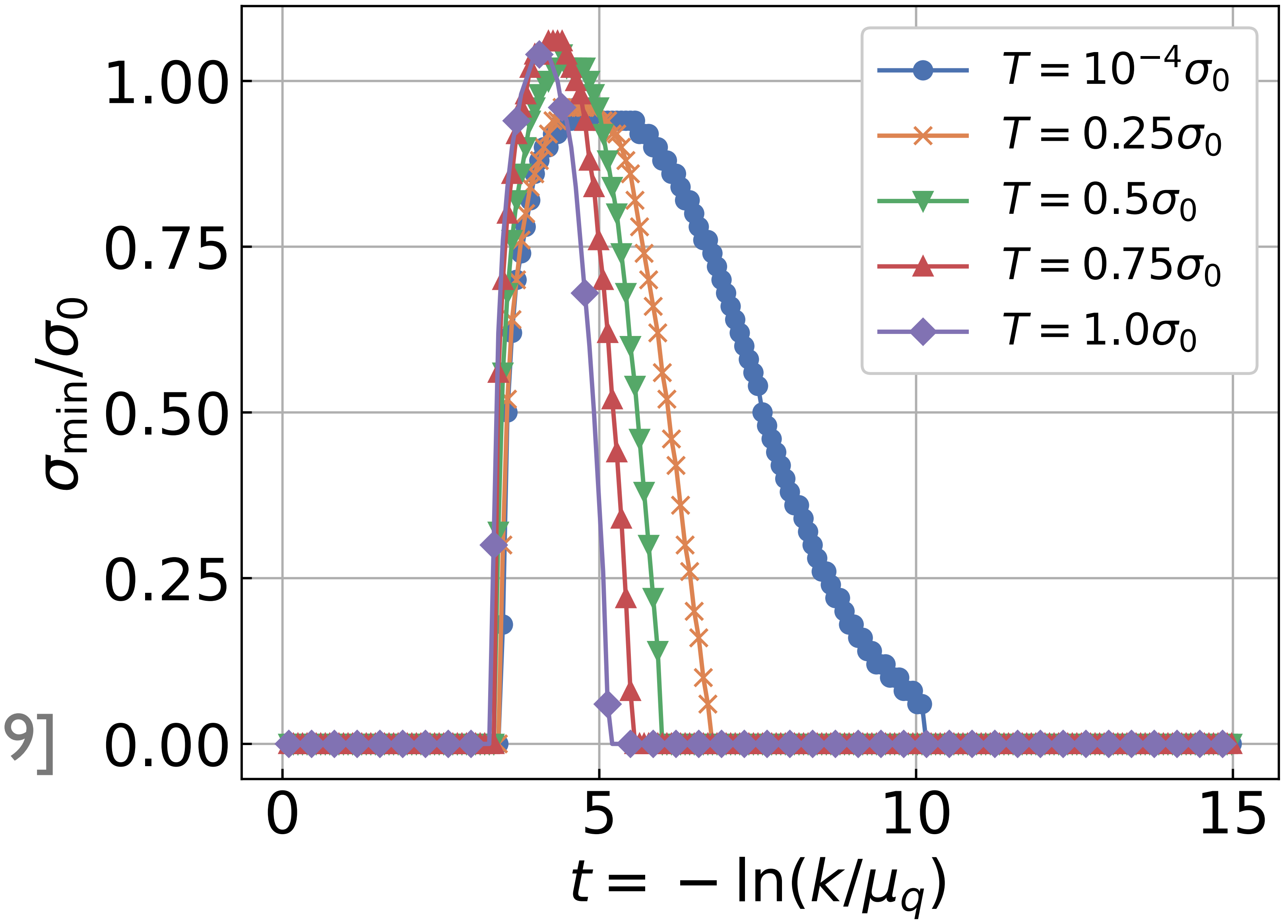
$\langle\phi(r)\phi(0)\rangle \sim \sin(k_0 r)\, r^{-\alpha}$



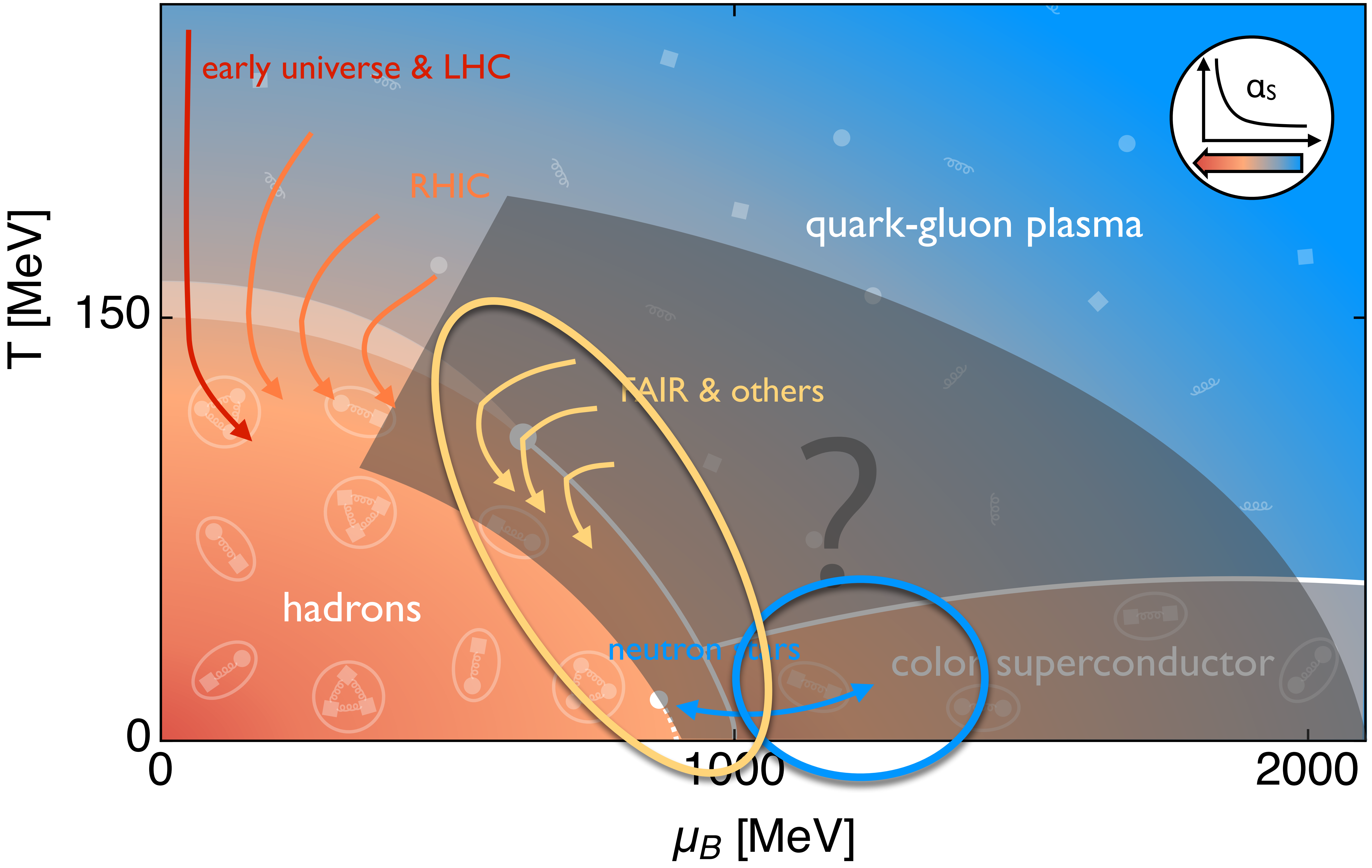
quantum pion liquid

$\langle\phi(r)\phi(0)\rangle \sim \sin(k_0 r)\, e^{-mr}$





WHERE DO WE EXPECT TO FIND PATTERNS?



patterns are expected in the "unknown" region of the phase diagram

this is/will be covered by FAIR/CBM and other fixed target experiments

→ **search for patterns in heavy-ion collisions!**

SEARCH FOR PATTERNS IN HICS

intuitive idea

[Pisarski, FR, PRL 127 (2021)]

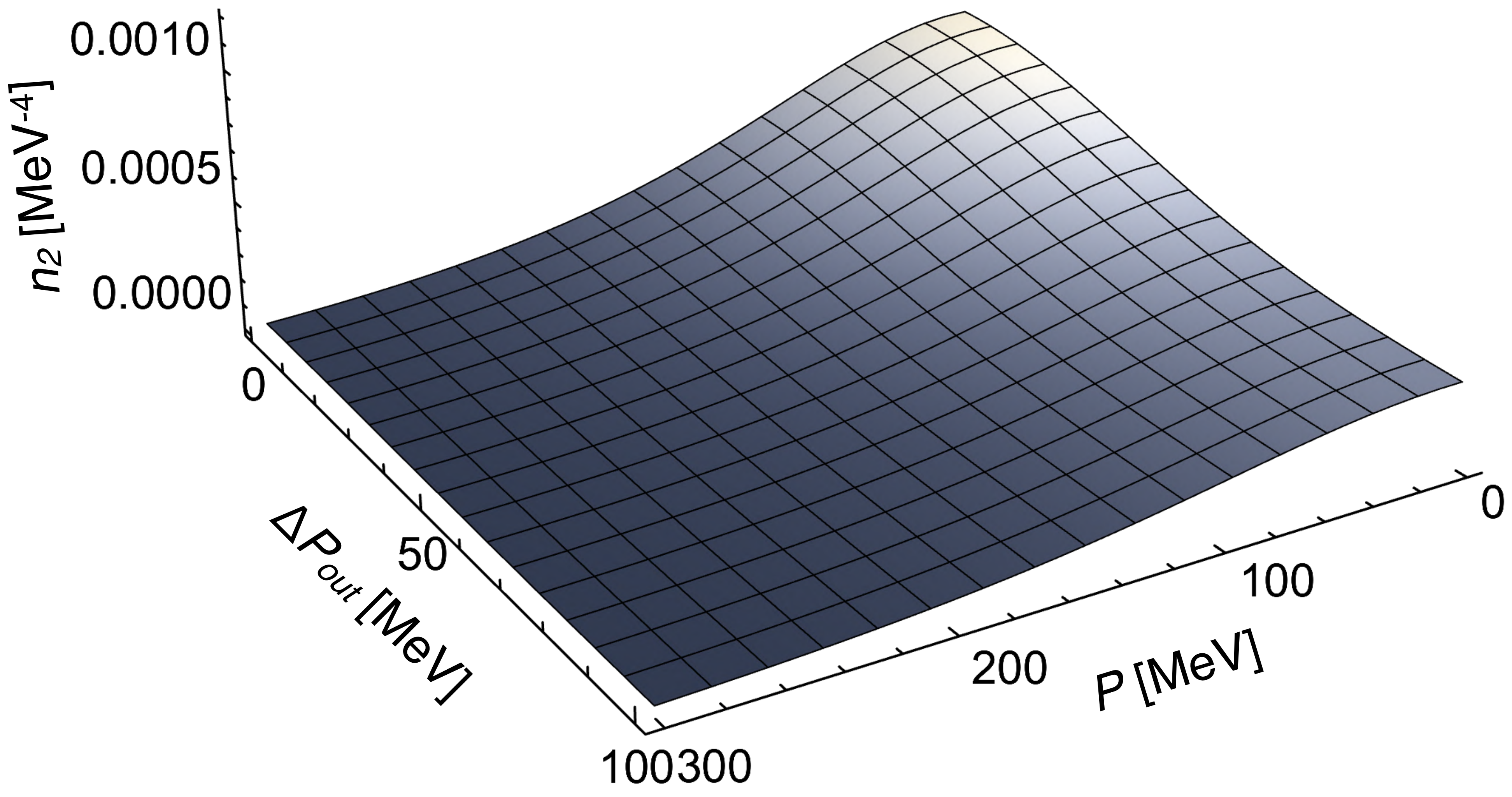
Characteristic feature of patterned regimes: modes with minimal energy at nonzero momentum

⇒ **enhanced particle production at nonzero momentum**

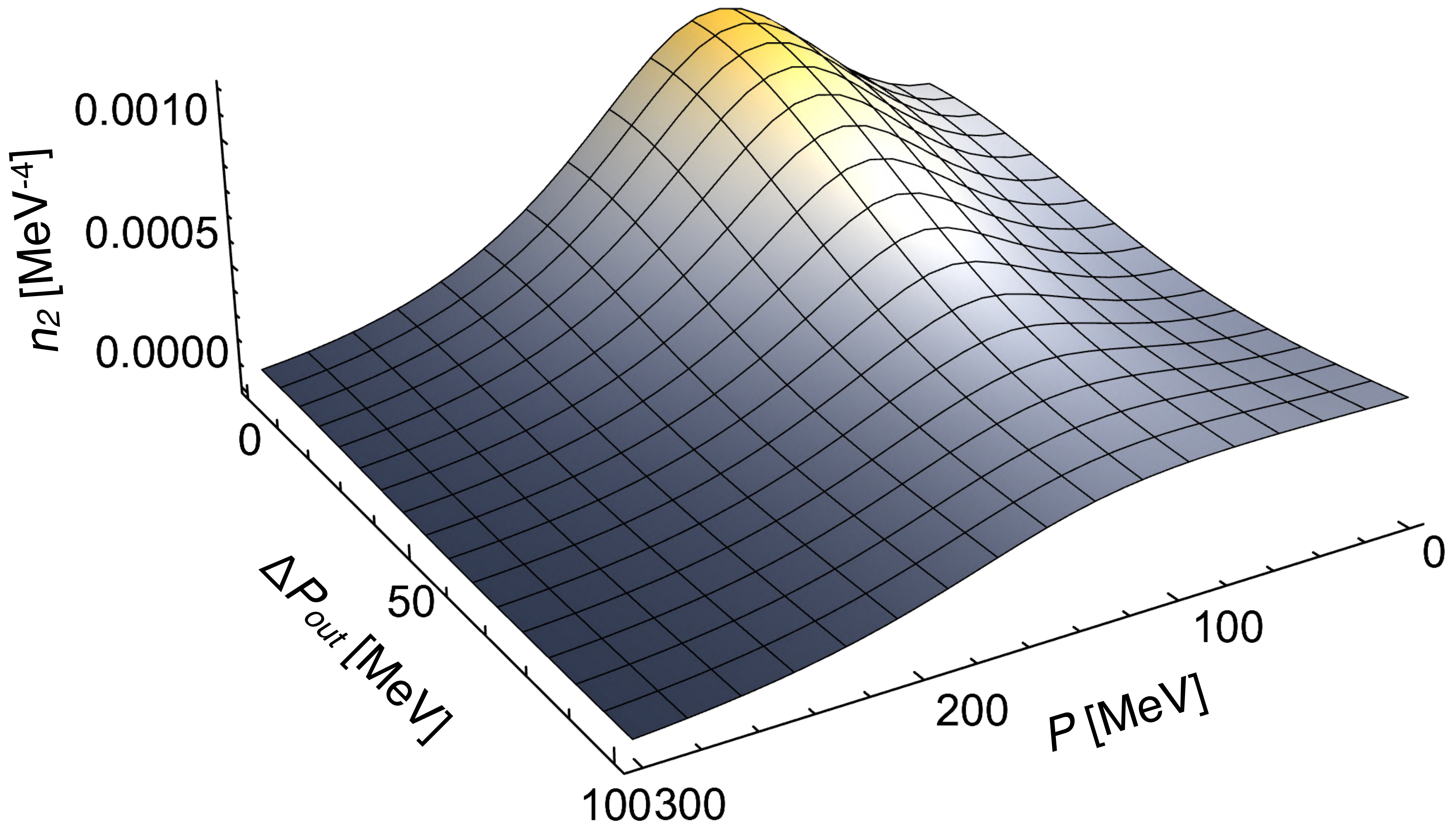
⟶ look for signatures in the **momentum dependence** of particle correlations

Example: pion interference (HBT correlations) [FR, Pisarski, Rischke, PRD 107 (2023)]

no moat



moat



PARTICLE CORRELATIONS AND SPECTRAL FUNCTIONS

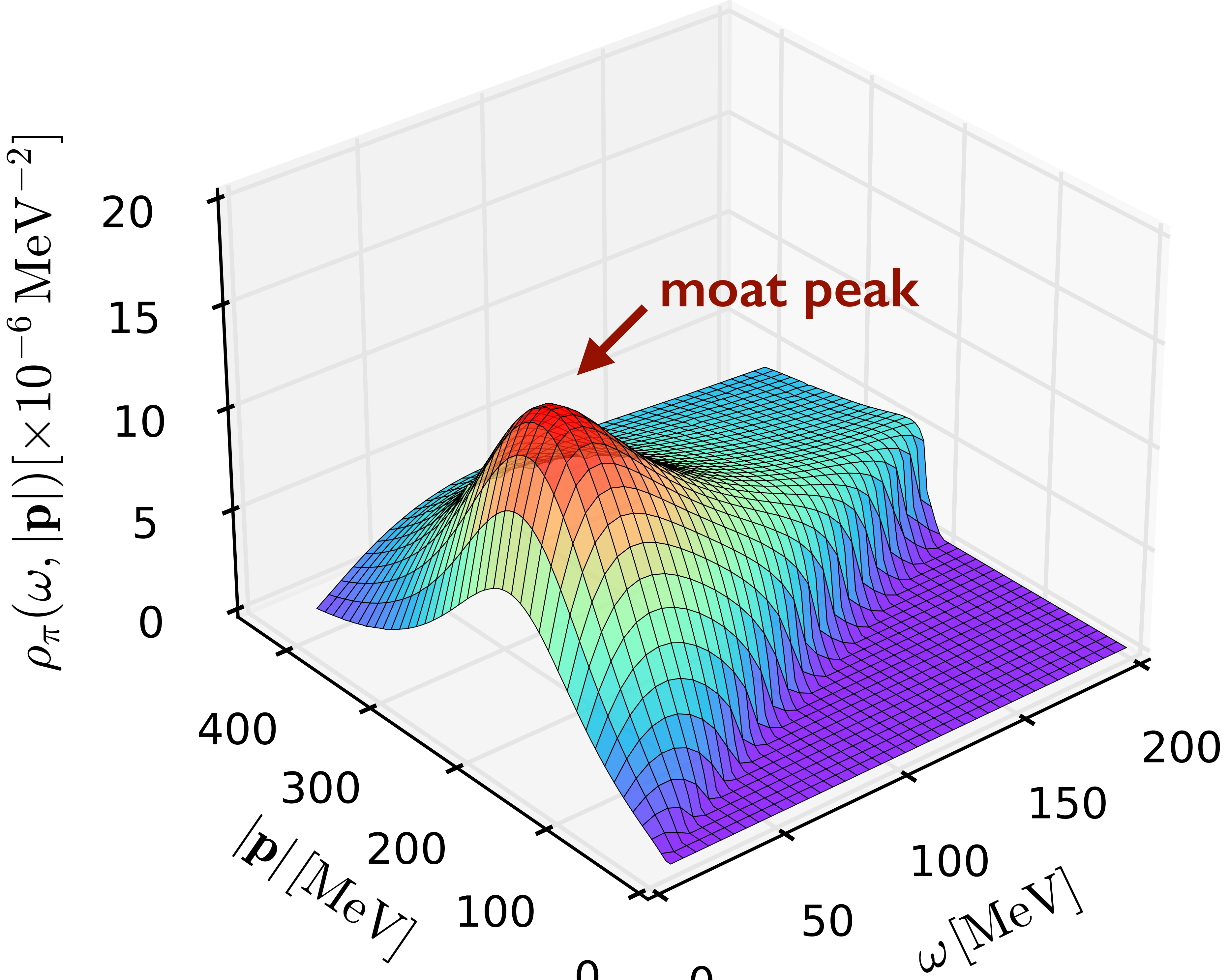
Two-particle correlations measured in experiment determined by in-medium spectral functions

$$C(\mathbf{P}, \mathbf{\Delta P}) = \int_X e^{-i\mathbf{\Delta}P\cdot X} f(X, \mathbf{P})\, \rho(X, \mathbf{P})$$

[Pisarski, FR, PRL 127 (2021)]
[FR, Pisarski, Rischke, PRD 107 (2023)]

- spectral functions can be computed directly with the FRG & DSE
- example: pion channel spectral function in QCD in the moat regime

[Floerchinger, JHEP 1025 (2012)]
[Kamikado et al., EPJ C74 (2014)]
⋮
[Horak, Pawlowski, Wink, PRD 102 (2020)]

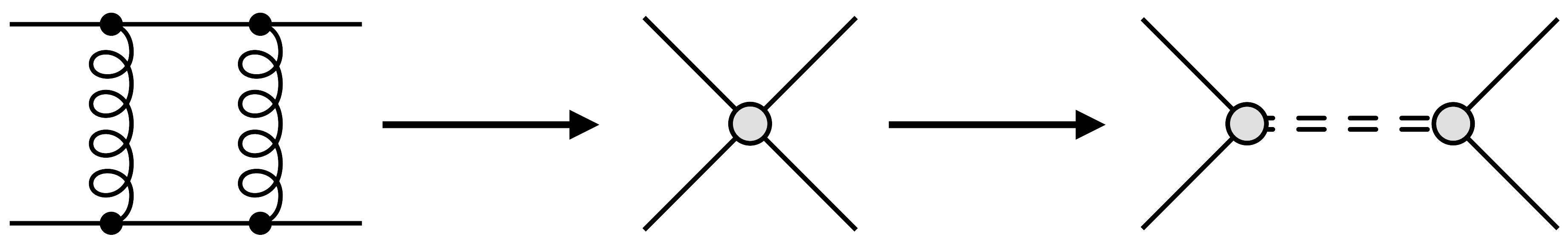


→ experimental signatures of patterns from functional QCD input (WIP)

[Fu, Pawlowski, Pisarski, FR, Wen, Yin (in preparation)]

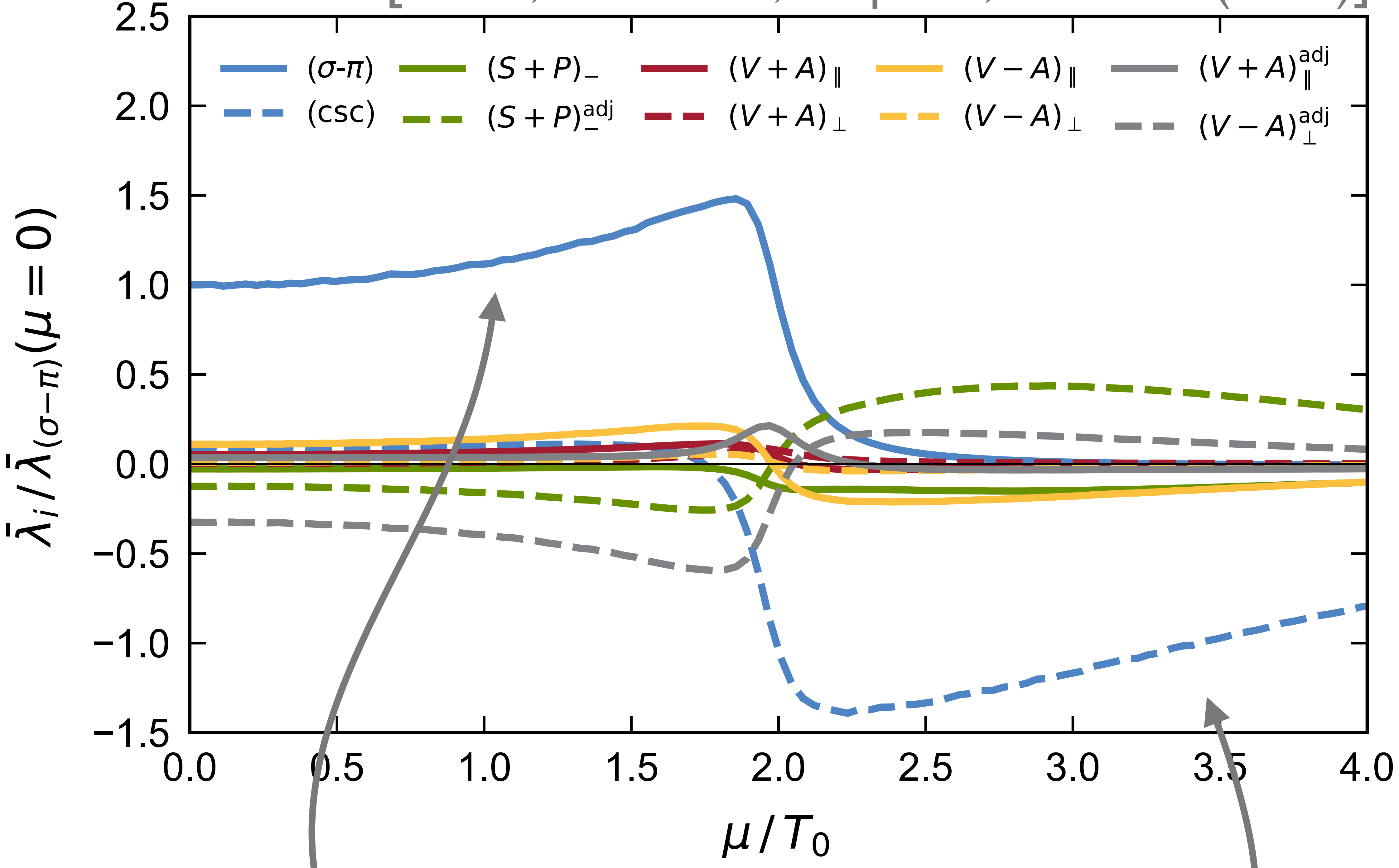
GOING TO LARGE DENSITY. SYSTEMATICALLY

- formation of new phases signaled by resonances in 4-quark scattering



- complete sets of 4-quark scattering channels can be constructed
  e.g., [Braun, J. Phys. G 39 (2012)]
- large-μ EoS and color-superconducting gap directly accessible
  [Müller, Buballa, Wambach, EPJ A 49 (2013)]
  [Leonhardt et al. PRL 125 (2019)]
  [Geissel, Gorda, Braun, PRD 110 (2024)]
  [Lu et al., arXiv:2310.16345]
  ⋮
- momentum dependence can be used to study emergent bound states in detail (directly related to Bethe-Salpeter equations)
  [Eichmann et al., PPNP 91 (2016)]
  [Fu et al., 2401.07638]
  [Fukushima et al., arXiv:2308.16594]

[Braun, Leonhardt, Pospiech, PRD 101 (2020)]



(closely above the phase transition temp.)

chiral condensate $\sim \langle\bar{q}q\rangle$:
hadronic phase

quark Cooper-pair $\sim \langle qq\rangle$:
color-superconductor

→ "dynamical decision" about favored ground state

SUMMARY

functional methods can be used to study the QCD from first principles

a lot of progress at finite density and real time in recent years

a lot more needs to be done

a lot I couldn't cover here

and I apologize for that

fQCD Collaboration:

Braun, Chen, Fu, Gao, Ihssen, Geissel, Huang, Lu, Pawlowski, FR, Sattler, Schallmo, Stoll, Tan, Toepfel, Turnwald, Wen, Wessely, Yin, Zorbach

BACKUP

FUNCTIONAL METHODS FOR QCD

The starting point is always the same: the classical QCD action. (Usually) with gauge fixing

$$S_{\mathrm{QCD}}[\varphi] = \int_x \left\{ \bar{q}\left(\gamma_\mu D_\mu - m_q\right) q - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 \right\}$$

- covariant derivative and field strength:

$$D_\mu = \partial_\mu - i g A^a_\mu T^a$$

$$F^a_{\mu\nu} T^a = [D_\mu, D_\nu]$$

The end point can be expressed in terms of the effective action Γ

$$\Gamma[\phi] = \sup_J \left\{ \int_x J(x)\,\phi(x) - \ln Z[J] \right\} \qquad \phi = \frac{\delta \ln Z}{\delta J} = \langle \varphi \rangle$$

→ contains all fully dressed correlation functions; quantum analogue of action S

Gauge-invariance is encoded in non-Abelian Ward-identities (Slavnov-Taylor identities)

$$\int_x \frac{\delta \Gamma[\Phi, Q]}{\delta Q(x)} \cdot \frac{\delta \Gamma[\Phi, Q]}{\delta \Phi(x)} = 0$$

+ (R_k-dependent terms for the FRG)

[Ellwanger, PLB 335 (1994)]

→ symmetry relations between correlation functions

↗ BRST charges

↖ $\Phi = q, \bar{q}, A, c, \bar{c}$

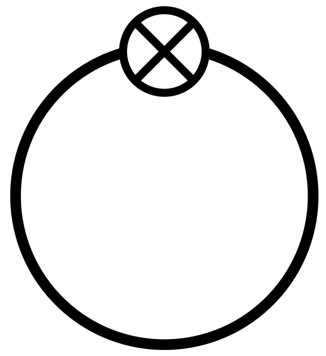
FRG application: [Pawlowski, Schneider, Wink, 2202.11123]

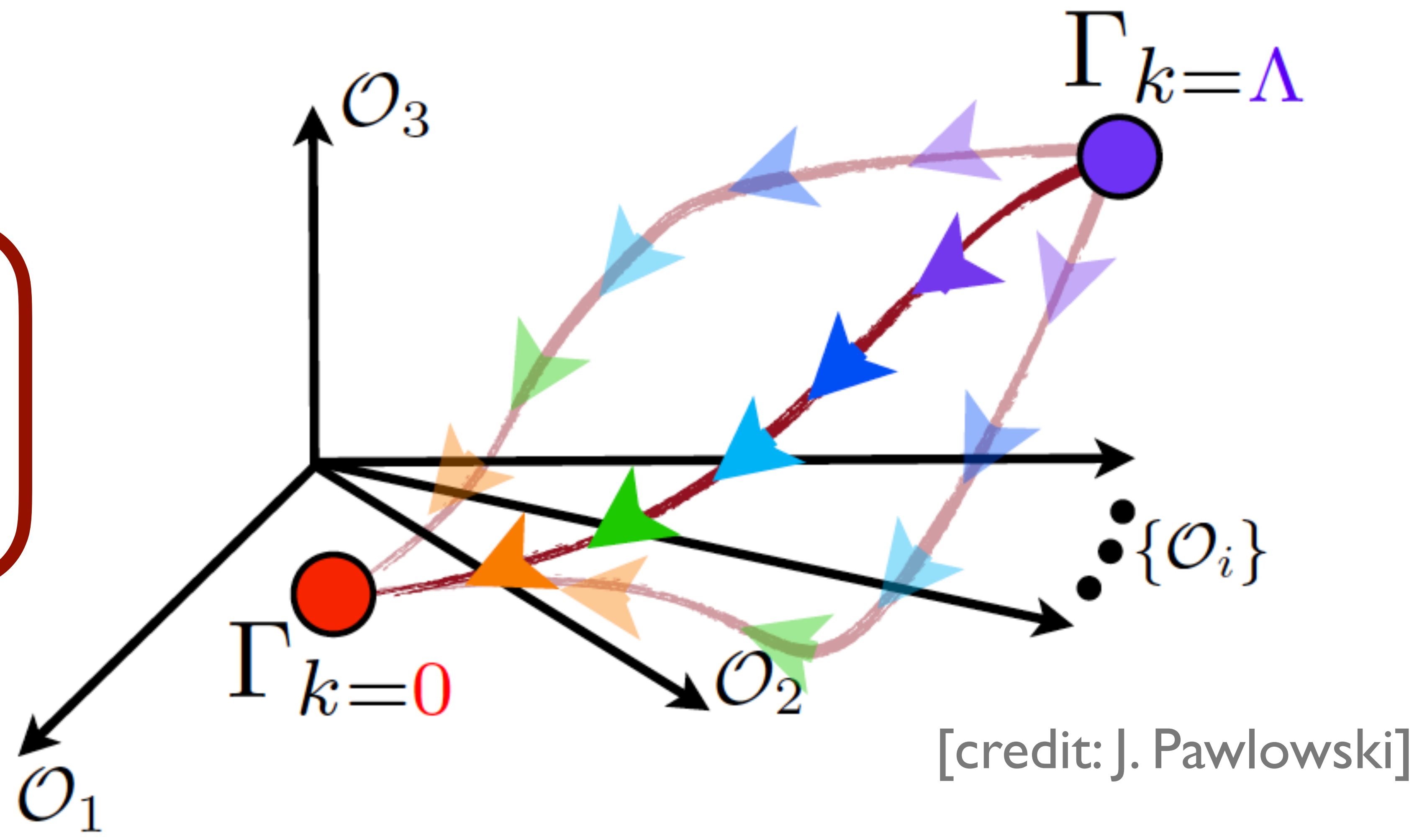
FUNCTIONAL RENORMALIZATION GROUP

Turn the path integral into a differential equation through the regulator-induced RG-scale k-dependence

→ **flow equation**
[Wetterich (1993)]

$$\boxed{\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\Big[\big(\Gamma_k^{(2)} + R_k\big)^{-1} \cdot \partial_k R_k\Big] = \frac{1}{2}}$$

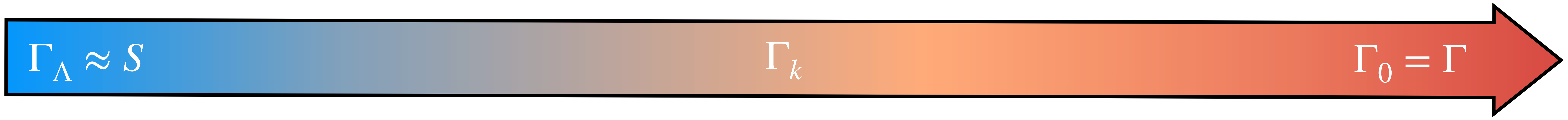




[credit: J. Pawlowski]

- start from small length scale/large energy scale $\Lambda \gg \Lambda_{\mathrm{QCD}}$: $\Gamma_\Lambda \approx S_{\mathrm{QCD}}$
- successively incorporate quantum corrections by lowering k

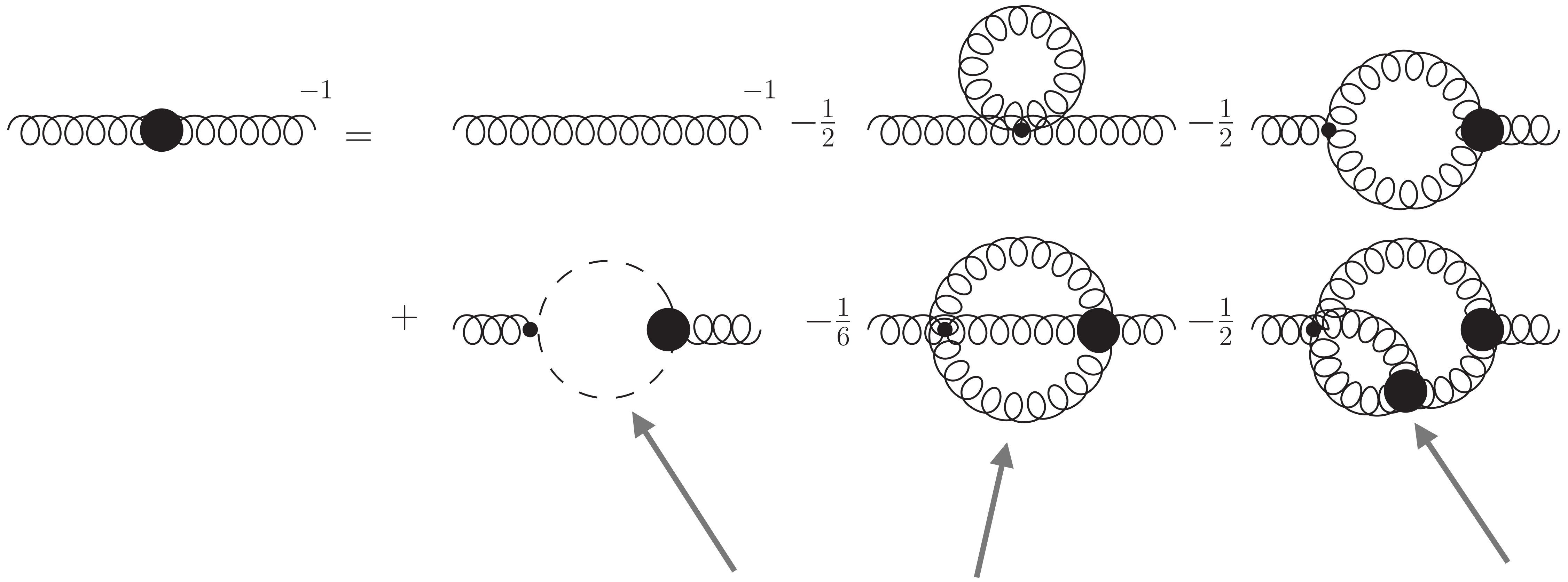
→ lowering k = zooming out/coarse-graining



CORRELATION FUNCTIONS

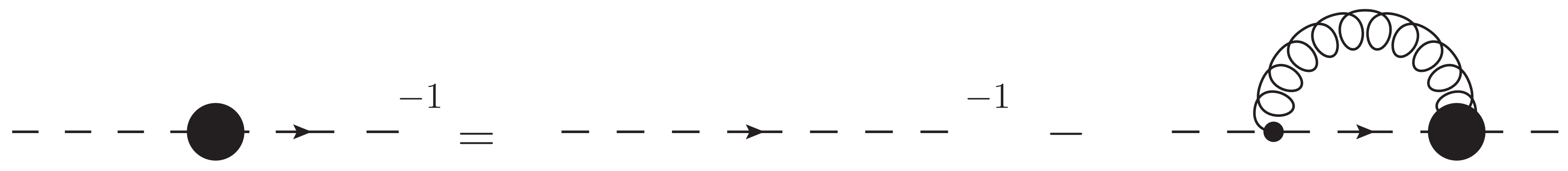
Example: DSE for the gauge sector

[Huber, Maas, von Smekal, JHEP 11 (2012)]



fully dressed propagators

fully dressed vertices



CORRELATION FUNCTIONS

Example: DSE for the gauge sector

[Huber, Maas, von Smekal, JHEP 11 (2012)]

+ ∞ more equations ... ⟶ truncation necessary

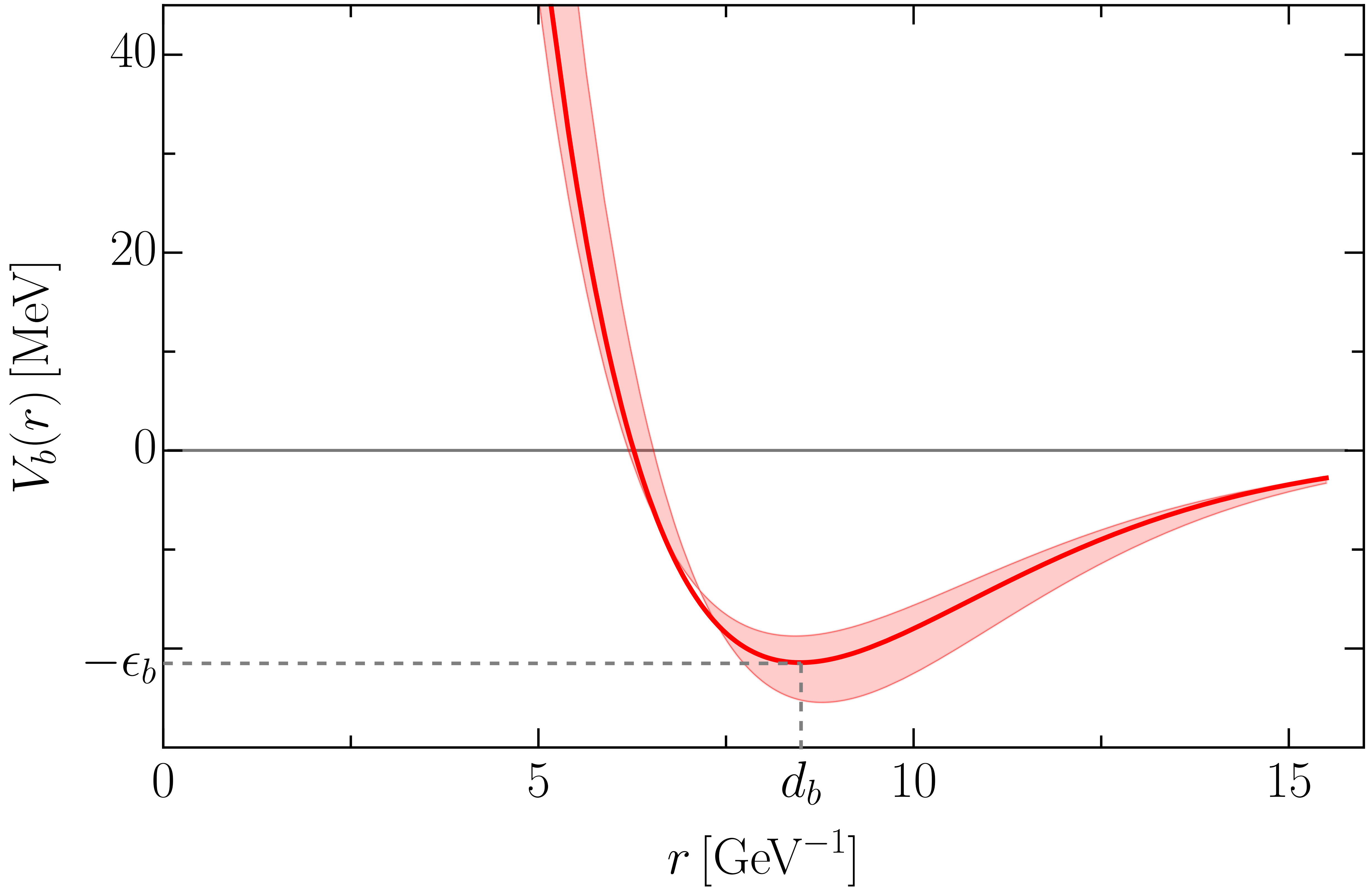
No small parameter to control the truncations at strong coupling (exception: FRG near 2nd-order transitions).

Still, apparent (and perhaps natural) hierarchy of relevance from low- to high-order correlations.

⟶ aim for "apparent convergence"

NUCLEAR MATTER FROM THE FRG

Nuclear matter properties from density-channel interactions $\sim \lambda_V (\bar{q}\gamma^0 q)^2$ [Fukushima et al., arXiv:2308.16594]



- short-distance nuclear repulsion from first principles
- first estimates for nuclear matter properties very promising
    - saturation density: $n_0 \approx 0.21(16)\,\mathrm{fm}^{-3}$ $(0.16\,\mathrm{fm}^{-3})$
    - binding energy: $\epsilon_b \lesssim 21(5)\,\mathrm{MeV}$ $(\epsilon_b = 16\,\mathrm{MeV})$

HIGH-DENSITY EOS

Many properties of neutron stars and their mergers are sensitive to the QCD EoS at high density.

- EoS and diquark gap in the non-perturbative high-density regime from first principles

[Müller, Buballa, Wambach, EPJ A 49 (2013)]
[Müller, Buballa, Wambach, arXiv:1603.02865]
[Leonhardt et al. PRL 125 (2019)]
[Braun, Schallmo, PRD 105 (2021)]
[Braun, Schallmo, PRD 106 (2022)]
[Geissel, Gorda, Braun, PRD 110 (2024)]
[Lu et al., arXiv:2310.16345]