Mixing of heavy and light quarks in charmonium and light mesons

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FOR 5269

BERGISCHE UNIVERSITÄT WUPPERTAL

Outline

- Motivation
- A bit of introduction to lattice QCD
- Mixing of flavors, glueballs and 2-pion states in the scalar channel
- Conclusions

Part I

Motivation

Motivation

- Confinement predicts the existence of states made of gluons alone called glueballs which still await experimental confirmation; renewed interest e.g. M. Ablikim et al. (BESIII), Phys.Rev.Lett., 132, 2024
- . . . and more exotic states than “just” mesons and baryons called XYZ states; despite experimental discoveries since two decades still no understanding of their internal mechanics
- Glueballs are expected at energies close to charmonium; in QCD they are resonances with many decay channels e.g. into pions
- Several XYZ states have been discovered with charmonium content
- $\Rightarrow$ **study mixing of light and charm quarks**
- Lattice QCD provides ab-initio tool to study glueballs and exotics but it is challenging: signal-to-noise ratio, unstable states, . . .

Part II

A bit of introduction to lattice QCD

Spectroscopy on the Lattice

- Lattice QCD is very good at computing Euclidean correlation functions

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$$

where "operator" $\mathcal{O}$ is a (temporally) local combination of fields

- This path integral expression is related to an underlying QFT with Hamiltonian $\hat{H}$ and complete set of states $|n\rangle$, $\quad \hat{H}|n\rangle = E_n|n\rangle\,.$

$$\langle n|m\rangle = \delta_{n,m}, \sum_n |n\rangle\langle n| = 1$$

- A consequence of this relation is the "spectral decomposition"

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n |c_n|^2 e^{-E_n t}$$

with matrix elements called **overlaps**

$$c_n = \langle n|\mathcal{O}|\Omega\rangle$$

Excited States

GEVP method (Generalized eigenvalue problem) M. Lüscher & U. Wolff, Nuclear Physics B 339, 222–252, B. Blossier et al. Journal of High Energy Physics 2009, 094–094 :

- Consider several operators for the same symmetry channel $\mathcal{O}_1, \ldots, \mathcal{O}_{N_{\text{op}}}$
- Compute a correlation matrix with elements $C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$
- The generalized eigenvalues $\lambda_n(t, t_0)$ of

$$C(t)\vec{v}_n = \lambda_n C(t_0)\vec{v}_n$$

behave like

$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n} \times \left(1 \quad + \quad e^{-(t-t_0)\Delta_n}\right)$$

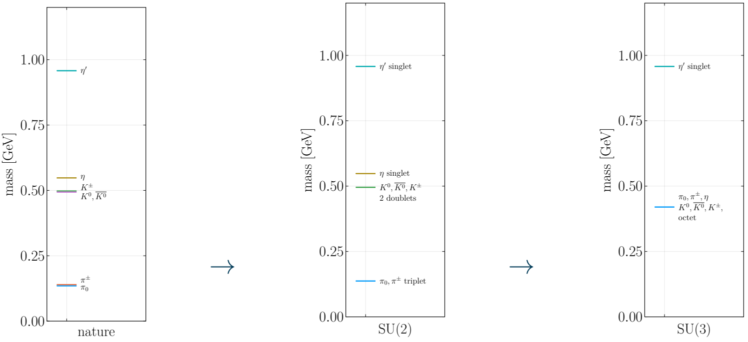
But we need a variational basis with operators that have good **overlaps** $\langle n| \mathcal{O}_i |\Omega\rangle \propto [C(t_0)v_n(t, t_0)]_i$ with all states.

Symmetries (Flavor)

Nature has no exact flavor symmetry, but an approximate $SU(2)$, or even $SU(3)$. In $N_\mathrm{f} = 3 + 1$ QCD

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to V \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad (\bar{u}, \bar{d}, \bar{s}) \to (\bar{u}, \bar{d}, \bar{s})V^\dagger, \qquad V \in SU(3)$$

is a symmetry $\to$ Energy eigenstates labeled by $|D, Y, I, I_3\rangle$



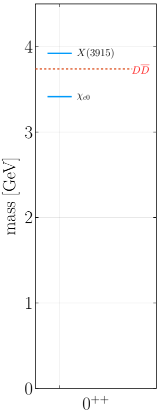
Lüscher's formalism

- A scattering process like: two particles $\longrightarrow$ two particles happens in real time
- In lattice QCD we work in Euclidean (imaginary) time
  - Perfect for spectroscopy
  - No direct access to scattering
- Lüscher's idea M. Lüscher, Nucl.Phys.B354, 531 (1991) : compute the two-particle spectrum in a finite volume and solve an equation to find the phase shifts and infer the resonance parameters, cf. talk by C. Thomas Tue 14:30 Session C

Scalar Channel(s)

Goals of FOR5269:

- Decay of scalar glueball into pions
- Mixing of Charmonia with light hadrons
- E.g. the 0^{++}, flavor singlet channel
  - Charmonia



courtesy T. Korzec

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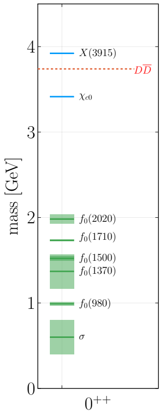
    Particle Data Group, Prog. Theor. Exp. Phys. 2022



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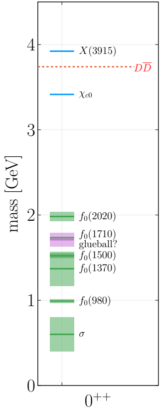

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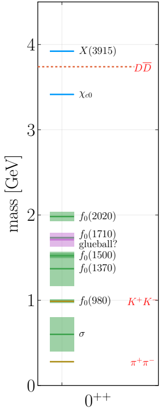
  - 2-meson states $\pi\pi$, KK, $\ldots$



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  - 2-mesons with momentum $\pi(\vec{p})\pi(-\vec{p})$
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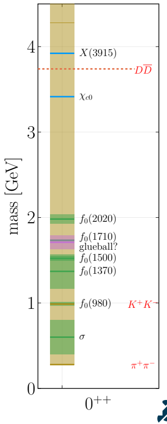

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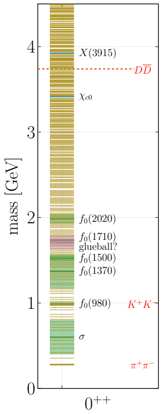
- Finite volume helps!
  Momenta are quantized $\vec{p} = \vec{n}\; 2\pi/L$ (free case)



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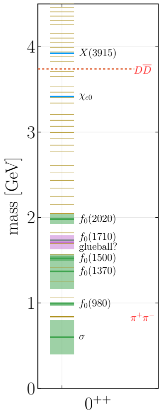
- Heavier pions help, e.g. SU(3) flavor symmetric point $m_\pi = 420$ MeV



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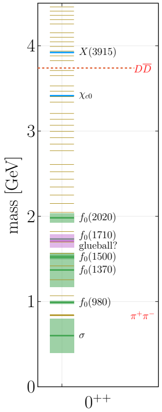


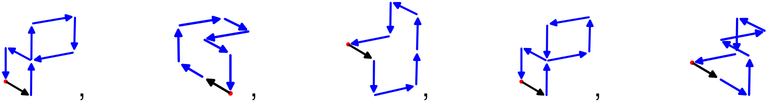
courtesy T. Korzec

Strategy: Start with $N_\mathsf{f} = 3+1, m_\pi \approx 1$ GeV, then $\searrow 420$ MeV

Operators

- scalar glueball 

  $\mathcal{O}_g$ is a linear combination of Wilson loops of different shapes and lengths, e.g.

- charmonium 

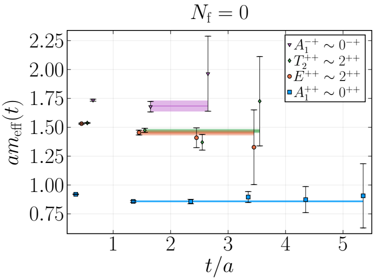
  $\mathcal{O}_{\chi_{c0}} = \bar{c}c$

- light scalar 
  - singlet $\mathcal{O}_{f_0} = \frac{1}{\sqrt{3}}\left(\bar{u}u + \bar{d}d + \bar{s}s\right)$
  - octet $\mathcal{O}_{a_0} = \frac{1}{\sqrt{2}}\left(\bar{u}u - \bar{d}d\right)$

    and 7 more operators

Signal-to-noise problem

- Example: gluonic operators, pure gauge, $\beta = 5.85$, 9000 gauge configurations
- effective masses $am_{\rm eff} = \ln(C_{ij}(t)/C_{ij}(t+a))$ approach ground state energy E_0 at large t
- Exponential increase of statistical errors



→ Need operators with large overlaps onto eigenstates to get early plateaus already at small t (large variational basis, smearing)

→ Similar issue affects quark-line disconnected diagrams: we use quark smearing called **distillation** M.Peardon et al. Phys.Rev.D 80 (2009) with profiles F.Knechtli, T.Korzec, M.Peardon, J.A.Urrea-Niño, Phys.Rev.D 106 (2022)

Part III

Mixing of flavors, glueballs and 2-pion states in the scalar channel

Mixing

Calculate the low-lying meson spectrum in $N_f = 3 + 1$ QCD.
This includes different states related to $SU(3)_F$:

- Flavor-singlet: f_0, η_c, glueballs, 2-pion states, etc...
- Flavor-octet: π, a_0, octet 2-pion states, etc...

Common approach:

- Study light meson states with $\bar{l}(t)\Gamma l(t)$ operators including disconnected correlations.
- Study charmonium states with $\bar{c}(t)\Gamma c(t)$ operators disregarding charm disconnected correlations.

!!! Mixing is artificially **eliminated**.

But $\mathcal{O}_c = \bar{c}\Gamma c$ and $\mathcal{O}_l = \bar{u}\Gamma u + \bar{d}\Gamma d + \bar{s}\Gamma s$ are in the same symmetry channel. E.g. flavor-singlet 0^{++} channel $(\Gamma = \mathbb{I})$

$$C_l(t) \overset{t\to\infty}{\approx} |\langle f_0| \mathcal{O}_l |\Omega\rangle|^2 e^{-E_{f_0} t}$$

$$C_c(t) \overset{t\to\infty}{\approx} |\langle f_0| \mathcal{O}_c |\Omega\rangle|^2 e^{-E_{f_0} t} \neq |\langle \chi_{c0}| \mathcal{O}_c |\Omega\rangle|^2 e^{-E_{\chi_{c0}} t}$$

If mixing is included do not need to rely on assumptions like
$|\langle \chi_{c0}| \mathcal{O}_c |\Omega\rangle| >> |\langle f_0| \mathcal{O}_c |\Omega\rangle| > 0$ to compute charmonium spectrum

Mixing contd

Mixing matrix $C(t) =$

$$\begin{pmatrix} \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_l(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_c(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_l(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_c(0)\rangle & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_c(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & * & \langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_{2\pi}(0)\rangle & \langle \mathcal{O}_{2\pi}(t)\bar{\mathcal{O}}_g(0)\rangle \\ * & * & * & \langle \mathcal{O}_g(t)\bar{\mathcal{O}}_g(0)\rangle \end{pmatrix}$$

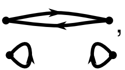
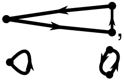
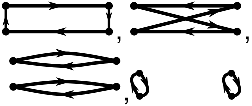
- $O_g(t)$: gluonic operators, e.g built from Wilson loops.
- $O_{2\pi}(t)$: flavor-singlet 2-pion operators at zero total momentum.

$\rightarrow \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0)\rangle \neq 0$ for $i \neq j$ means **no decoupling** between operators.
We solve a GEVP:

$$C(t)\vec{v}_n(t, t_0) = \lambda_n(t, t_0)C(t_0)\vec{v}_n(t, t_0)$$

and compute the overlaps $\langle n| \mathcal{O}_i |\Omega\rangle$
Noise in vectors and choice of t_0 play a large role here J.J. Dudek et al. Phys. Rev. D 77, 034501 (2008)

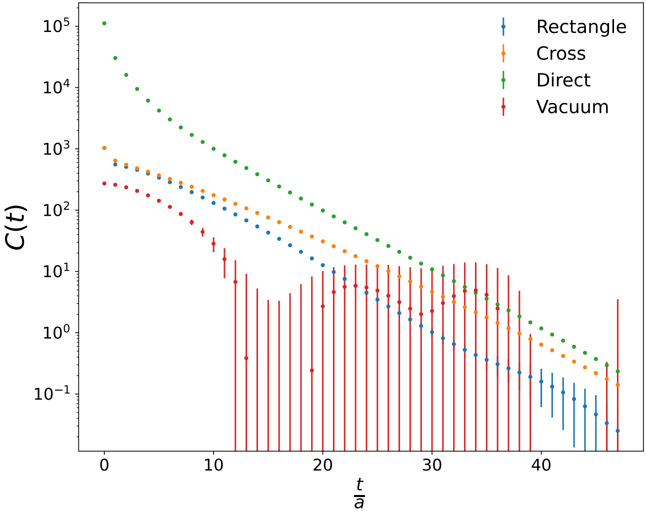
Diagrams after Wick contractions

	$\mathcal{O}_l$	$\mathcal{O}_c$	$\mathcal{O}_{2\pi}$	$\mathcal{O}_g$
$\mathcal{O}_l$				
$\mathcal{O}_c$	-			
$\mathcal{O}_{2\pi}$	-	-		
$\mathcal{O}_g$	-	-	-	

Disconnected diagrams arise for flavor-singlets; they have large statistical errors

- Basis of different distillation profiles for 1-particle states $(\Gamma = \mathbb{I})$
- 2-pion operator with standard distillation *for now*

$\pi\pi$ Diagrams and their error in A1



Singlet $\langle \mathcal{O}_{|1,0,0,0\rangle} \mathcal{O}^\dagger_{|1,0,0,0\rangle} \rangle = -\frac{32}{3}\mathcal{R} + \frac{2}{3}\mathcal{C} + 2\mathcal{D} + 8\mathcal{V}$

$N_f = 3 + 1$ ensembles

Wilson fermion action with non-perturbatively determined clover improvement + Lüscher-Weisz gauge action P. Fritzsch et al. J. High Energ. Phys. 2018, 25 (2018); R. Höllwieser et al. Eur. Phys. J. C 80, 349.

A1	A1h
96×32^3	96×32^3
$a \approx 0.054$ fm	$a \approx 0.069$ fm
$m_\pi \approx 420$ MeV	$m_\pi \approx 800$ MeV
$N_v^{light} = 100$	$N_v^{light} = 200$
$N_v^{charm} = 200$	$N_v^{charm} = 200$

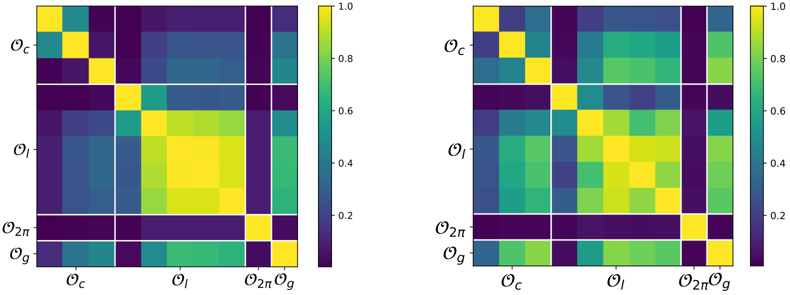
Varying m_π we can control the decay thresholds:

- Quenched 0^{++} glueball $\approx$ 1800 MeV
- ensemble A1: 0^{++} Glueball $\rightarrow \pi\pi$, $\pi\pi\pi\pi$
- ensemble A1h: 0^{++} Glueball $\rightarrow \pi\pi$

Following results were presented by J.A. Urrea-Niño at Lattice 2024

0^{++} flavor-singlet correlation matrix at $t = a$

$C_{ij}(t) \to C_{ij}(t)/\sqrt{C_{ii}(a)C_{jj}(a)}$.



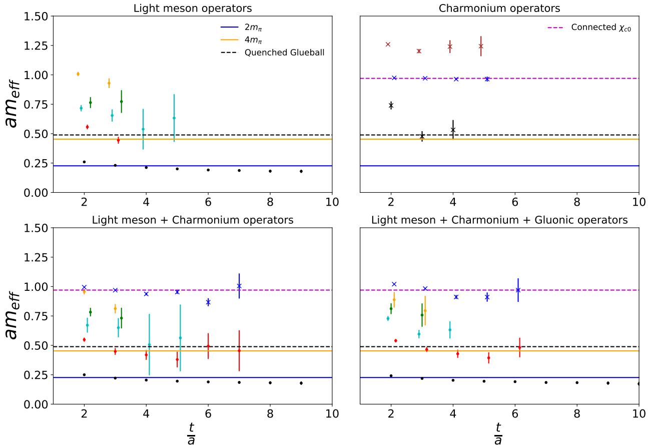
A1 ensemble

A1h ensemble

All operators "talk" to each other to some degree.

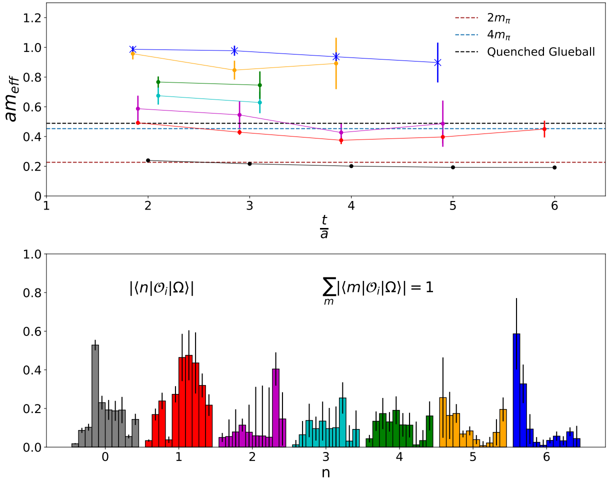
- $\mathcal{O}_g$: Sum of Laplacian eigenvalues. C. Morningstar et al. Phys. Rev. D 88, 014511
- $\mathcal{O}_{l/c}$: From SVD pruning of sub-blocks. J. Balog et al., Phys. Rev. D 60, 094508, F. Niedermayer et al., Nuclear Physics B 597, 413–450

0^{++} single-particle operator mixing in A1



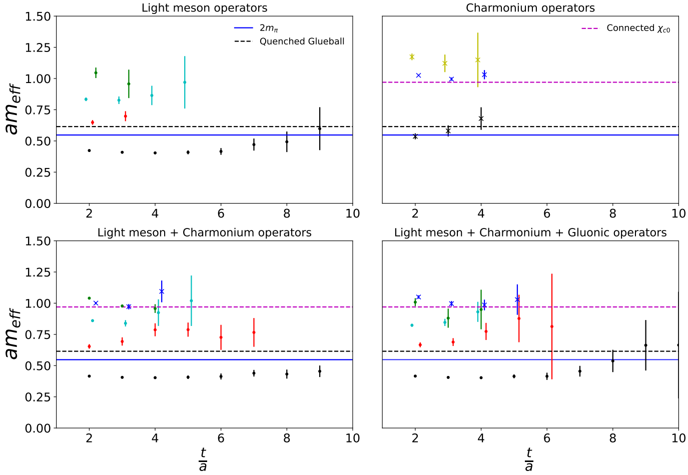
- Charmonium operators alone see a light state
- Including $\mathcal{O}_g$ does not change the low-lying spectrum; similar findings to R. Brett et al. 1909.07306

0^{++} full operator mixing in A1



- 10 operators: $3 \times \mathcal{O}_c$, $5 \times \mathcal{O}_l$, $\mathcal{O}_{2\pi}$ and $\mathcal{O}_g$
- 2-pion operator introduces an additional state $(n = 2)$

0^{++} single-particle operator mixing in A1h



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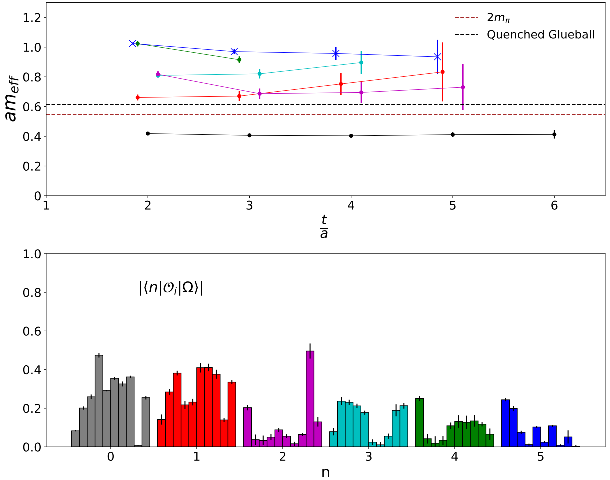

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Part IV

Conclusions

Conclusions and Outlook

- Observation of mixing of light and heavy quarks, glueball and 2-pion states in flavor singlets through lattice simulations of $N_\mathrm{f} = 3 + 1$ QCD at two values of the pion mass
- 2-pion operator has the smallest mixing with one-particle states; it yields an additional state very close to quenched 0^{++} glueball
- Inclusion of disconnected diagrams in charmonium correlators leads to a state in the region of light mesons
- Inclusion of glueball operators in the GEVP basis does not lead to a new state
- Statistical noise from glueball and disconnected correlations is a major problem

Outlook:

- More 1-particle operators to better sample radial excitations
- More 2-pion operators: bact-to-back momentum + profiles
- Better sampling methods for disconnected diagrams, e.g. multi-level sampling with fermions Cè, Giusti and Schaefer, Phys.Rev.D 95 (2017) ; cf. quenched glueballs L. Barca et al, 2406.12656
- Apply Lüscher's method at different volumes to study scattering