Flavor Structure of the proton gravitational form factors in a pion mean-field approach

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Gravitational form factors
of
the proton

Energy-Momentum Tensor operators

Hilbert-Einstein Action

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}L_M \qquad g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu}(\boldsymbol{r}) \qquad \lambda_{\mathrm{grav}} \gg M_N^{-1}$$

Changing the metric in the long-wave approximation,

we find the Energy-Momentum tensor that characterizes the response of the nucleon to the static variation of the space-time metric:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}} \qquad \partial_\mu T^{\mu\nu} = 0$$

Bellifante-Rosen type QCD EMT Current

$$T_q^{\mu\nu} = \frac{i}{4}\bar{\psi}_q\left(\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\right)\psi_q, \qquad T_g^{\mu\nu} = -F^{\mu\rho,a}F^{\nu,a}_{\ \ \rho} + \frac{1}{4}g^{\mu\nu}F^{\lambda\rho,a}F^a_{\lambda\rho}$$

Quark part

Gluon part

$$\overleftrightarrow{D}^\mu = \overleftrightarrow{\partial}^\mu - 2igA^\mu$$

$$\overleftrightarrow{\partial}^\mu = \overrightarrow{\partial}^\mu - \overleftarrow{\partial}^\mu$$

$$a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

Gravitational (EMT) form factors

- Gravitational form factors of the nucleon in QCD

Kobzarev et al. 1962; Pagels, 1966

D(Druck)-term Weiss & Polyakov, 1999

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu\sigma_\nu\}\rho}\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + \underline{M_N\bar{c}^a(t)g^{\mu\nu}}\right]u(p)$$

δg^{00} δg^{0i} δg^{ij}

Non-conservation of EMT pieces (cosmological constant)

O. V. Teryaev, Front. Phys. 11 (2016)
K.-F. Liu, PRD 104 (2021)

$\sum_a A^a(0) = 1$ Mass

Spin

$$\sum_a J^a(0) = \frac{1}{2}$$

Deformation of space
= mechanical properties of the nucleon

Pressure & Shear-force distributions (pressure anisotropy)

- Twist-4 operators MV Polyakov, HD Son, JHEP (2018)

$$\partial_\mu T^{\mu\nu} = 0 \rightarrow \sum_{q,g} \bar{c}^{q,g} = 0$$

$$\langle p's'|\frac{1}{2}ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p,s\rangle = M_N\bar{c}^Q\Delta^\beta\bar{u}(p',s')u(p,s)$$

$$\langle p's'|\frac{1}{2}\mathrm{tr}(G^{\beta\alpha}[iD^\sigma, F_{\sigma\alpha}])|p,s\rangle = M_N\bar{c}^g\Delta^\beta\bar{u}(p',s')u(p,s)$$

Twist-projected EMT currents

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

$$T_a^{\mu\nu} = \bar{T}_a^{\mu\nu} + \hat{T}_a^{\mu\nu}$$

Twist-4 EMT current: $\hat{T}_a^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^{\alpha}_{a,\alpha}$

Twist-2 EMT current: $\bar{T}_a^{\mu\nu} = T_a^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{a,\alpha}$

- Twist-2 baryon matrix elements

$$\langle B(p', J_3') | \bar{T}^a_{\mu\nu}(0) B(p, J_3) \rangle = \bar{u}(p', J_3') \bigg[A^a_B(t) \frac{P_\mu P_\nu}{M_B} + J^a_B(t) \frac{i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho}{2M_B} + D^a_B(t) \frac{\Delta_\mu \Delta_\nu - t g^{\mu\nu}}{4M_B}$$
$$- g_{\mu\nu} \left\{ \frac{t}{8M_B} J^a_B(t) - \frac{3t}{16M_B} D^a_B(t) + \frac{M_B}{4} \left(1 - \frac{t}{4M_B^2} \right) A^a_B(t) \right\} \bigg] u(p, J_3)$$

- Twist-4 baryon matrix elements

cbar form factor comes from the **twist-4** operator!

$$\langle B(p', J_3') | \hat{T}^a_{\mu\nu}(0) | B(p, J_3) \rangle = \bar{u}(p', J_3') \bigg[g_{\mu\nu} \bigg\{ M_B \bar{c}^a_B(t) + \frac{t}{8M_B} J^a_B(t)$$
$$- \frac{3t}{16M_B} D^a_B(t) + \frac{M_B}{4} \left(1 - \frac{t}{4M_B^2} \right) A^a_B(t) \bigg\} \bigg] u(p, J_3)$$

MV Polyakov, HD Son, JHEP (2018)
Y Hatta, A Rajan, K Tanaka, JHEP (2018)

Flavor decomposition of GFFs

- To decompose the GFFs, we need to compute the generalized EMT form factors for the flavor triplet & octet.

$$\begin{aligned} F_B^{\chi=0} &= F_B^u + F_B^d + F_B^s, \\ F_B^{\chi=3} &= F_B^u - F_B^d, \\ F_B^{\chi=8} &= \frac{1}{\sqrt{3}}\left(F_B^u + F_B^d - 2F_B^s\right) \end{aligned}$$

$$\sum_{a=q,g} F_B^a(t) = F_B(t), \quad \bar{c}_B(t) = 0$$

From the current conservation

- Naive effective EMT-like flavor nonsinglet currents

$$\hat{T}^{\rm eff}_{\mu\nu,\chi}(x) = \frac{i}{4}\bar{\psi}(x)\left(\gamma_\mu \overrightarrow{\partial}_\nu + \gamma_\nu \overrightarrow{\partial}_\mu - \gamma_\mu \overleftarrow{\partial}_\nu - \gamma_\nu \overleftarrow{\partial}_\mu\right)\lambda_\chi \psi(x)$$

Note that they are not conserved.

Extracting flavor-decomposed cbar form factors are the most challenging one!

The role of gluons

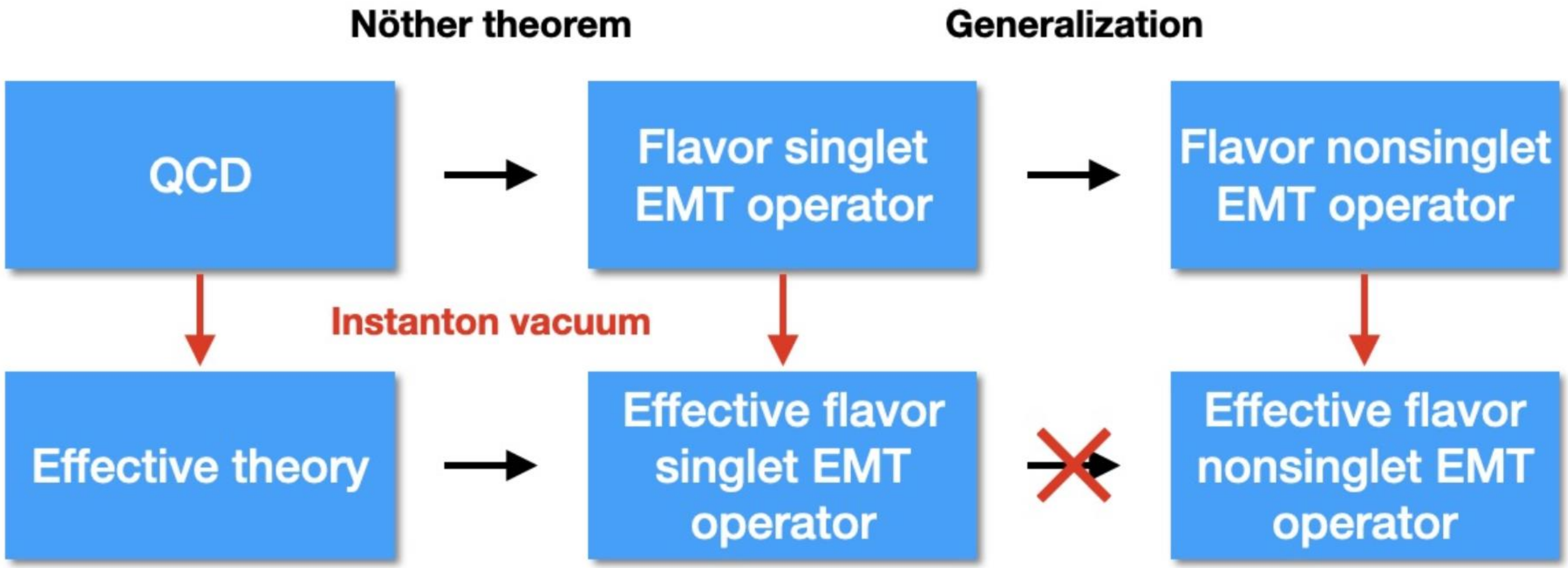
H. Y. Won, HChK, and J.-Y. Kim, PRD **108** (2023)

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Effective EMT operators

Effective EMT Operator

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)



Nöther theorem

Generalization

QCD → Flavor singlet EMT operator → Flavor nonsinglet EMT operator

Instanton vacuum

Effective theory → Effective flavor singlet EMT operator ✗ Effective flavor nonsinglet EMT operator

$$T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)\ [\mathrm{QCD}] \xrightarrow{\mathrm{eff}} T^{\mu\nu}_{\chi=0}(x)\ [\chi\mathrm{QSM}]$$

Effective operators: $T^{\mu\nu}_{\chi}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_{\chi}\psi(x)$?

Flavor nonsinglet operators require careful derivation!

$$T_3^{\mu\nu}(x)\quad T_8^{\mu\nu}(x)\quad T^{\mu\nu}_{1\pm 2i}(x)\quad T^{\mu\nu}_{4\pm 5i}(x)\quad T^{\mu\nu}_{6\pm 7i}(x)$$

Effective EMT Operator

Sum rules in QCD

$$\sum_{a=q,g} A_N^a(0) = A_N(0) = 1, \quad [\mathrm{QCD}]$$

$$\sum_{a=q,g} J_N^a(0) = J_N(0) = 1/2, \quad [\mathrm{QCD}]$$

$$\sum_{a=q,g} \frac{\langle N|T^{\mu}_{a,\mu}|N\rangle}{2M_N} = M_N A_N(0),$$

$$\text{with} \quad T^{\mu}_{q,\mu} = O(m_q), \quad [\mathrm{QCD}]$$

$$\sum_{a=q,g} \int d^3r\, p_N^a(r) = 0, \quad \sum_{a=q,g} \bar{c}_N^a(t) = 0, \quad [\mathrm{QCD}]$$

Sum rules in the effective theory

$$\sum_{q=u,d,\dots} A_N^q(0) = A_N(0) = 1 \quad [\mathrm{Eff.\ Theory}]$$

$$\sum_{q=u,d,\dots} J_N^q(0) = J_N(0) = 1/2, \quad [\mathrm{Eff.\ Theory}]$$

$$\frac{\langle N|T^{\mu}_{\chi=0,\mu}|N\rangle}{2M_N} = M_N \sum_{q=u,d,\dots} A_N^q(0),$$

$$\text{with} \quad T^{\mu}_{\chi=0,\mu} = T^{00}_{\chi=0} \quad [\mathrm{Eff.\ Theory}]$$

$$\sum_{a=q,g} \int d^3r\, p_N^a(r) = 0 \xrightarrow{\mathrm{eff}} \sum_{q} \int d^3r\, p_N^q(r) = 0$$

$$\sum_{a=q,g} \bar{c}_N^a(t) = 0 \xrightarrow{\mathrm{eff}} \sum_{q} \bar{c}_N^q(t) = 0$$

Gluons are integrated out.

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

EMT Operator from the instanton vacuum

A_μ → Effective quark operators → Pion mean fields

↑ Dilute instanton medium

↑ Bosonization

D. Diakonov et al. NPB 461 (1996)
J. Balla et al. NPB 510 (1998)

$$\pi^2 \left(\frac{\bar{\rho}}{\bar{R}}\right)^4 \approx 0.1$$

All low-energy theorems are satisfied (chiral anomaly, trace anomaly, etc).

- **The chiral-even twist-2 local operator** is generated by expanding the non-local vector current, which measures the vector GPDs, with respect to the space-time distance:

$$O_q^{\mu\nu_1\ldots\nu_n} := \bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{D}^{\nu_1}\overleftrightarrow{D}^{\nu_2}\ldots\overleftrightarrow{D}^{\nu_n\}}\lambda_\chi\psi(x) - \text{traces},$$
$$O_g^{\mu\nu_1\ldots\nu_n} := -F^{\{\mu\rho,a}\overleftrightarrow{D}^{\nu_1}\ldots\overleftrightarrow{D}^{\nu_{n-1}}F_\rho^{\nu_n\},a} - \text{traces}$$

Twist-2 gluon operators are suppressed with respect to the packing fraction.

$$\bar{T}_\chi^{\mu\nu}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_\chi\psi(x) - \text{traces}, \quad \bar{T}_g^{\mu\nu}(x) = 0$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

EMT Operator from the instanton vacuum

- Twist-3 gluon operators

The contributions from these operators have been found to be crucial for satisfying the QCD equation of motion.

→ Essential role in the decomposition of the nucleon spin

Spin-orbit correlations are also related to them.

- Twist-4 gluon operators should be also replaced by flavor-dependent quark operators.

→ Gluons should be considered when the flavor decomposition is considered, in particular, for the cbar form factors.

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

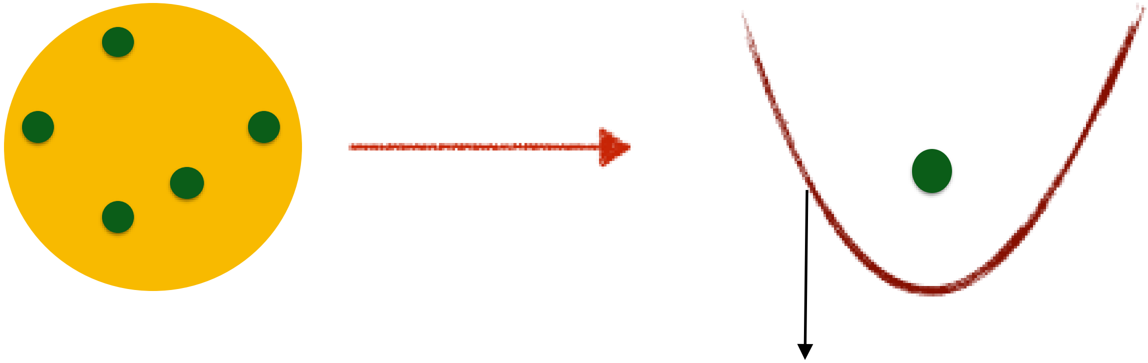
Pion Mean-field approach

Mean fields

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

This classical solution is regarded as a mean field.



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

Pion mean-field approach (Chiral Quark-Soliton model)

- Baryons as a state of Nc quarks bound by mesonic mean fields.

E. Witten (1979)

Effective chiral action:

$$S_{\rm eff}[\pi^a] = -N_c \mathrm{Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

D. Diakonov & V. Petrov (1986)

D. Diakonov, V. Petrov, P. Pobylitsa (1988)

- Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\, n^a = x^a/r, & a = 1,\,2,\,3 \\ 0, & a = 4,\,5,\,6,\,7,\,8. \end{cases}$$

It breaks spontaneously $\mathrm{SU(3)_{flavor}} \otimes \mathrm{O(3)_{space}} \to \mathrm{SU(2)_{isospin+space}}$

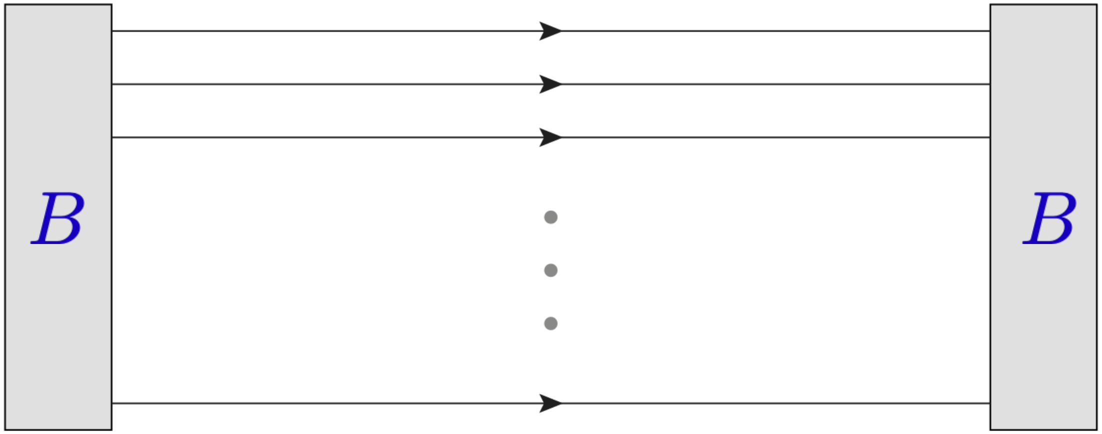
Witten's trivial embedding

$$U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)
D. Diakonov hep-ph/9802298

Baryon correlation function

Baryon as Nc valence quarks bound by pion mean fields



$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

$$\Pi_N(\vec{x},t) \;=\; \Gamma_N^{\{f\}}\Gamma_N^{\{g\}*}\frac{1}{Z}\int dU \prod_{i=1}^{N_c}\left\langle 0,T/2\left|\frac{1}{D(U)}\right|0,-T/2\right\rangle_{f,g} e^{-S_{\rm eff}}$$

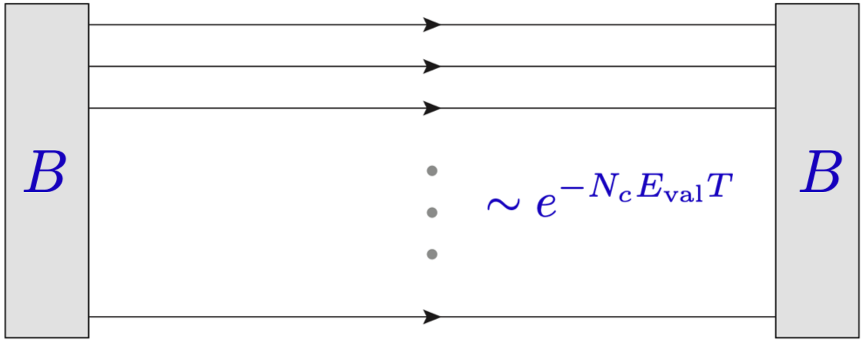
Presence of Nc quarks will polarize the vacuum or create mean fields.

Nc valence quarks → Vacuum polarization or meson mean fields

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)
D. Diakonov hep-ph/9802298

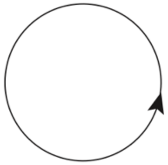
Baryon correlation function

Baryon as Nc valence quarks bound by pion mean fields



$\sim e^{-N_c E_{\text{val}} T}$

$$E_{\text{cl}} = N_c E_{\text{val}} + E_{\text{sea}}$$



$\sim e^{-E_{\text{sea}} T}$

Classical Nucleon mass is described by the Nc valence-quark energy and sea-quark energy.

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

$$\begin{tikzcd} \frac{\delta E_{cl}}{\delta U} = 0 \arrow[r] & M_{\text{cl}} \arrow[r] & P(r) \arrow[l, bend right] \end{tikzcd}$$

P(r): Soliton profile function
or Soliton field

Zero-mode(collective) quantization

- Rotational & Translational zero modes

$$\int \mathcal{D}U\mathcal{F}[U(\boldsymbol{x})] \to \int d^3\boldsymbol{X} \int \mathcal{D}A\,\mathcal{F}\left[TAU_{\mathrm{cl}}(R\boldsymbol{x})A^\dagger T^\dagger\right]$$

- Collective Hamiltonian & Wavefunctions in flavor SU(3) symmetry

$$H_{\mathrm{coll}} = M_{\mathrm{sol}} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2$$

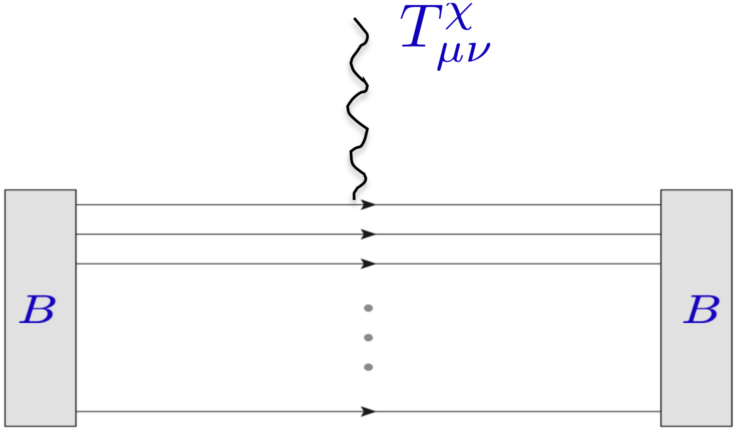
$$\Psi^{(\mu)}_{(YTT_3)(Y_RJJ_3)}(A) = \sqrt{\mathrm{dim}(\mu)}(-1)^{J_3-Y_R/2}D^{(\mu)*}_{(YTT_3)(Y_RJ-J_3)}(A)$$

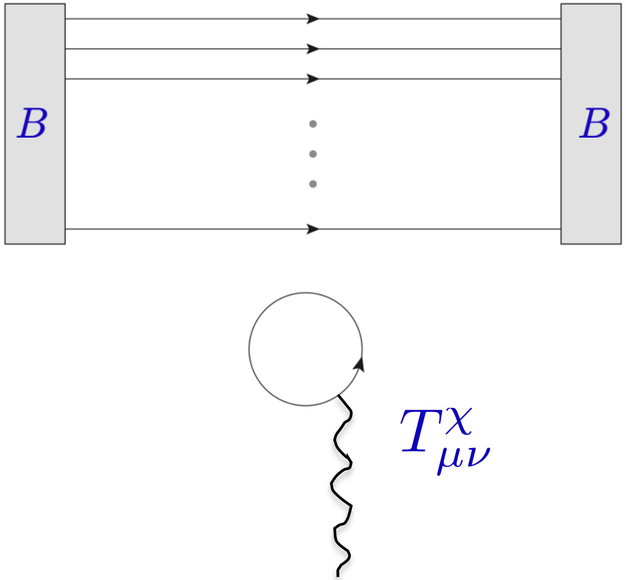
Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

D. Diakonov hep-ph/9802298

GFFs from the XQSM

- Rotational & Translational zero modes



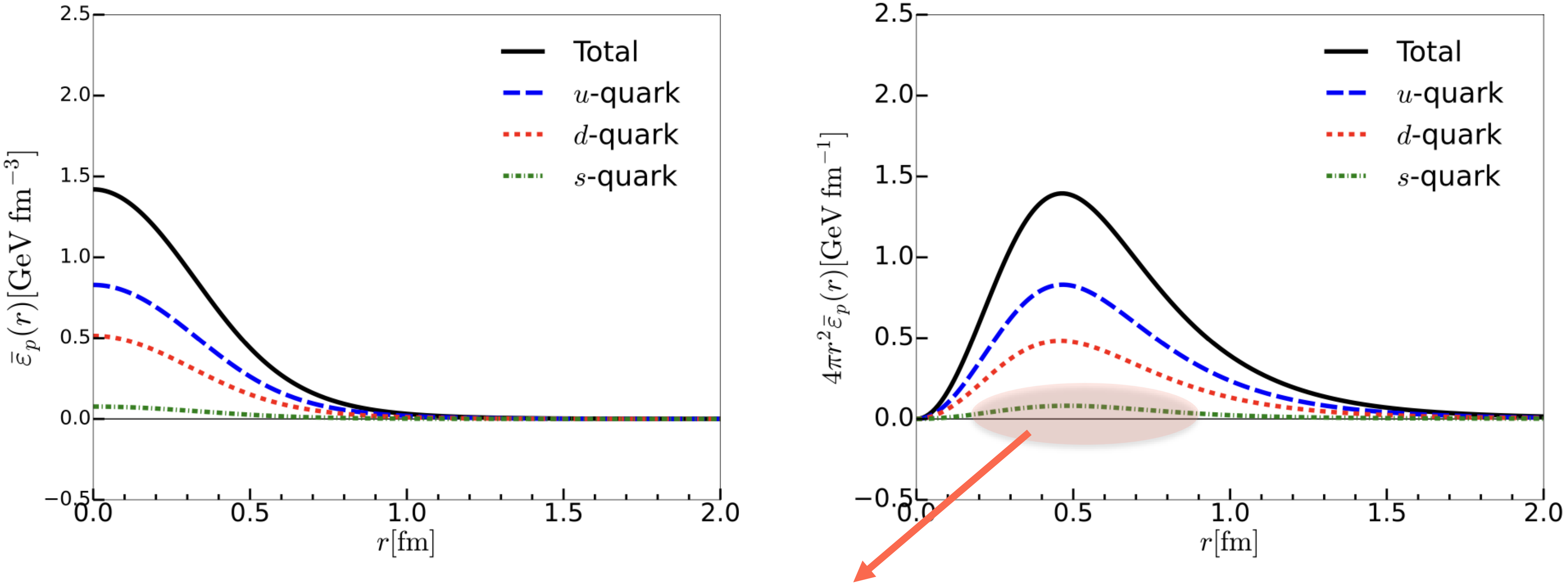


$$\langle B(p',J_3')|\hat{T}^{\rm eff}_{\mu\nu,\chi}(0)|B(p,J_3)\rangle = \lim_{T\to\infty}\frac{1}{Z_{\rm eff}}\mathcal{N}^*(p')\mathcal{N}(p)e^{ip_4\frac{T}{2}-ip_4'\frac{T}{2}}\int d^3\boldsymbol{x}\,d^3\boldsymbol{y}e^{(-i\boldsymbol{p}'\cdot\boldsymbol{y}+i\boldsymbol{p}\cdot\boldsymbol{x})}$$
$$\times\int\mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U\,J_B(\boldsymbol{y},T/2)\hat{T}^{\rm eff}_{\mu\nu,\chi}(0)J_B^\dagger(\boldsymbol{x},-T/2)\exp\left[-S_{\rm eff}\right]$$

For detailed results, see the refs. H. Y. Won, HChK, J.-Y. Kim JHEP (2024) & PRD 108 (2023)

Results & Discussion

Mass distributions



Contribution from the s quark is negligible.

$$\frac{3}{4}A_p^{\chi}(0) = \frac{1}{M_{\mathrm{sol}}}\int d^3r\,\bar{\varepsilon}_p^{\chi}(r)$$

$$A_p^0(0) = 1, \quad A_p^3(0) = 0.25, \quad A_p^8(0) = 0.47, \quad [\,\mathrm{SU}(3)\,]$$

$$A_p^0(0) = 1, \quad A_p^3(0) = 0.24 \qquad\qquad [\,\mathrm{SU}(2)\,]$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Mass distributions

- The gluon contributions to the leading-twist operators are parametrically suppressed with respect to the instanton packing fraction.

$$A_B^g = 0, \quad J_B^g = 0$$

J. Balla et al. NPB 510 (1998)
M. Polyakov &. H. Son, JHEP 09 (2018)

$$\bar{\varepsilon}_p^{u,d,s}(r) > 0$$

$$\bar{\varepsilon}_p^u(0) = 0.83 \text{ GeV/fm}^3, \quad \bar{\varepsilon}_p^d(0) = 0.51 \text{ GeV/fm}^3,$$
$$\bar{\varepsilon}_p^s(0) = 0.08 \text{ GeV/fm}^3$$

In the neutron,
u for d and d for u.

$$\bar{\varepsilon}_p^u(r) = \bar{\varepsilon}_n^d(r)$$
$$\bar{\varepsilon}_p^s(r) = \bar{\varepsilon}_n^s(r)$$

$$\langle r^2 \rangle_{\text{mass}}^p = 0.54\,\text{fm}^2 \quad [\text{SU(3)}]$$

$$\langle r^2 \rangle_{\text{mass}}^p < \langle r^2 \rangle_{\text{charge}}^p \qquad \langle r^2 \rangle_{\text{charge}}^p \approx 0.75\,\text{fm}^2$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Mass distributions

$$A_p^u(0) = 0.59, \quad A_p^d(0) = 0.35, \quad A_p^s(0) = 0.06, \quad [\,\mathrm{SU}(3)\,]$$

$$A_p^u(0) = 0.62, \quad A_p^d(0) = 0.38, \quad [\,\mathrm{SU}(2)\,]$$

- These numbers can be understood as the second Mellin moments of the PDFs. We list the predictions of the proton momentum fraction carried by the u-, d-, and s-quarks:

$$[\langle x \rangle_u : \langle x \rangle_d : \langle x \rangle_s] = [59\% : 35\% : 6\%]$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Angular momentum distribution

$$J_p^0(0) = \int d^3r\, \rho^0_{J,p}(r) = \frac{1}{2}$$

$$J_p^0 = 0.50, \quad J_p^3 = 0.58, \quad J_p^8 = 0.22, \quad [\,\mathrm{SU}(3)\,].$$
$$J_p^0 = 0.50, \quad J_p^3 = 0.55, \quad [\,\mathrm{SU}(2)\,]$$

Strange quark contribution is negligible.

$$J_p^u = 0.52, \quad J_p^d = -0.06, \quad \boxed{J_p^s = 0.04,} \quad [\,\mathrm{SU}(3)\,].$$
$$J_p^u = 0.53, \quad J_p^d = -0.03, \quad [\,\mathrm{SU}(2)\,]$$

$J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q + J_g$: Ji's relation X. Ji, PRL 78 (1997)

$J_g \approx 0$ Suppressed by the instanton packing fraction.

$$J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q \quad \Rightarrow \quad \frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L^q = 0.23 + 0.27$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Problem of the naïve decomposition

- Decomposition of the isotriplet J

$$J_p^{u-d} = L_p^{u-d} + S_p^{u-d} + \boxed{\delta J_p^{u-d}}$$

M. Wakamatsu & H. Nakakoji, PRD 71 (2005)

Violation of Ji's sum rule (X.D. Ji, PRL 78 (1997))

- Origin of δJ_p^{u-d} : role of gluons

- The second moment of the chiral-odd **twist-3** quark distribution

$$\int_{-1}^{1} dx\,x\,e^{u+d}(x) = \frac{m}{M_N} N_c + \boxed{\frac{M}{M_N}\beta}$$

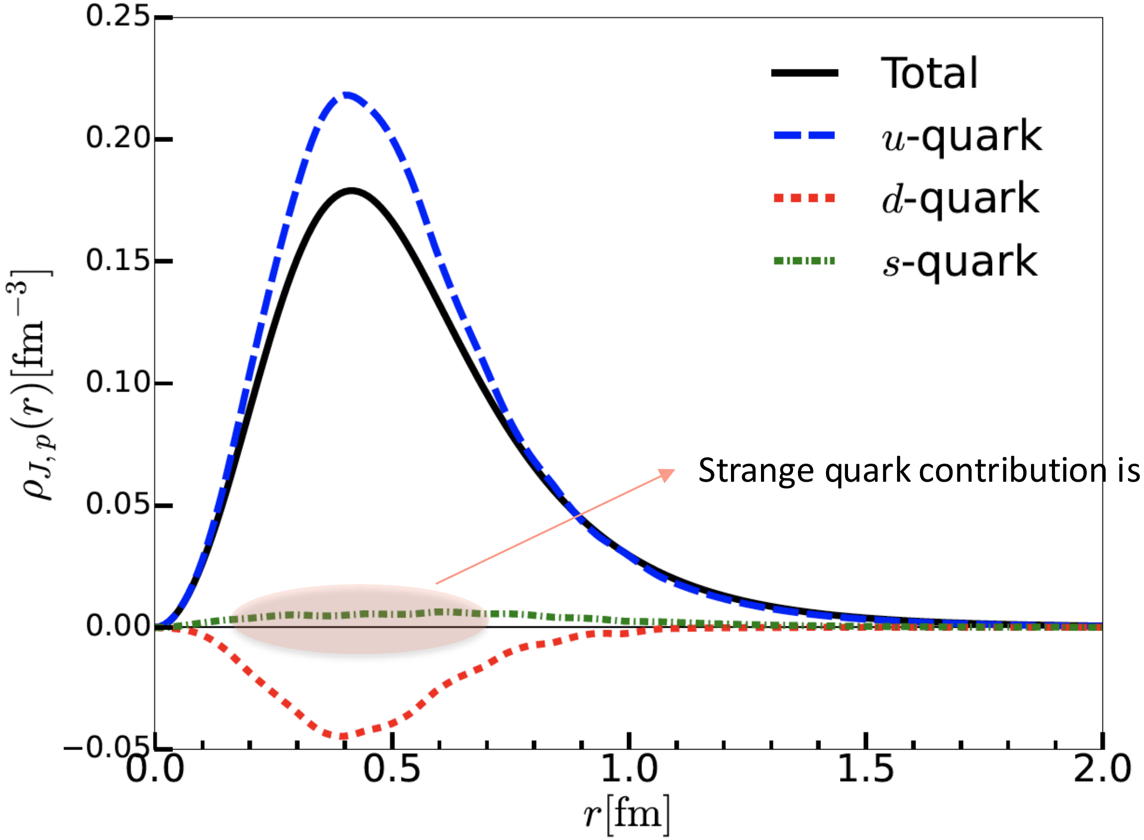
P. Schweitzer, PRD 67 (2005)
Ohnishi & M. Wakamatsu, PRD 69 (2004)

This makes the second moment deviate from QCD.

- ○ Indication: If the covariant derivatives had been used, these discrepancies would have been resolved.
- ○ Spin-orbit correlations are also very important to consider.

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

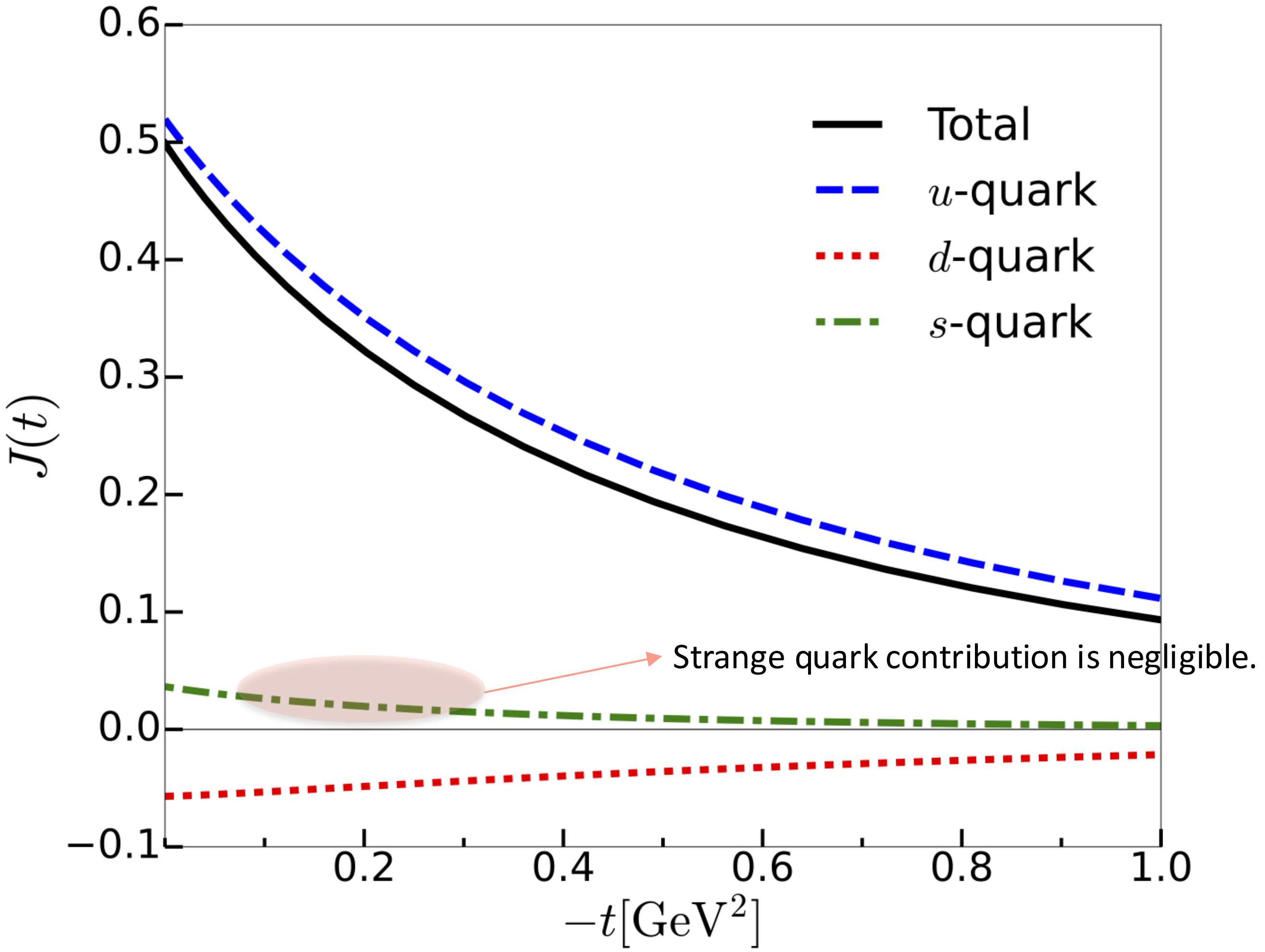
Angular momentum distribution



Strange quark contribution is negligible.

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Flavor-decomposed J form factors



Strange quark contribution is negligible.

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Mechanical properties: Twist-2 case

Stability condition in SU(3)

$$\int d^3r\, p_p^{u+d+s}(r) = 0$$

However, there is no proper way of constructing the effective flavor-triplet and -octet EMT currents by a global symmetry.

- Our strategy

$\xcancel{T_\chi^{\mu\nu}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_\chi\psi(x)}$: It contains both twist-2 & twist-4 operators.

We first consider the twist-2 EMT operator

$$\bar{T}_\chi^{\mu\nu}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_\chi\psi(x) - \text{traces}, \quad \bar{T}_g^{\mu\nu}(x) = 0$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Mechanical properties: Twist-2 case

Stability condition in SU(3)

$$\int d^3r\, p_p^{u+d+s}(r) = 0$$

Twist-2 case

$$\int d^3r\, \bar{p}_p^{u+d+s}(r) = \frac{1}{4}M_N \neq 0\,!$$

$$\bar{p}_B^{\chi}(r) = \frac{1}{3}\bar{\varepsilon}_B^{\chi}(r)$$

Twist-4 contribution

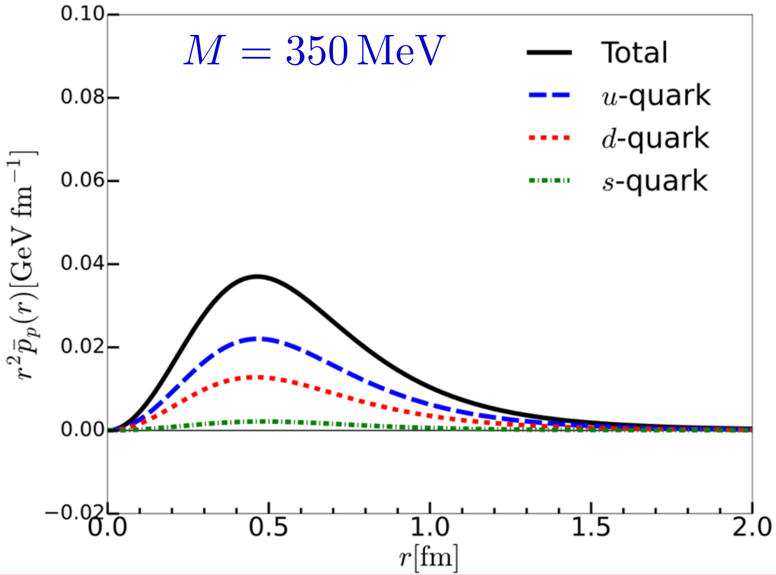
$$\int d^3r\, \hat{p}_p^{u+d+s}(r) = -\frac{1}{4}M_N$$

Both quark and gluon contribution should be considered.

- The derivation of the twist-4 EMT operators will later be mentioned.

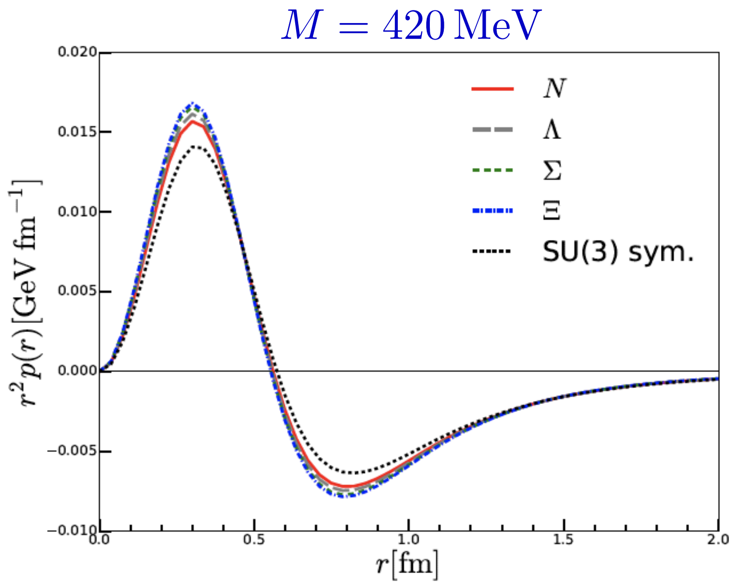
H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

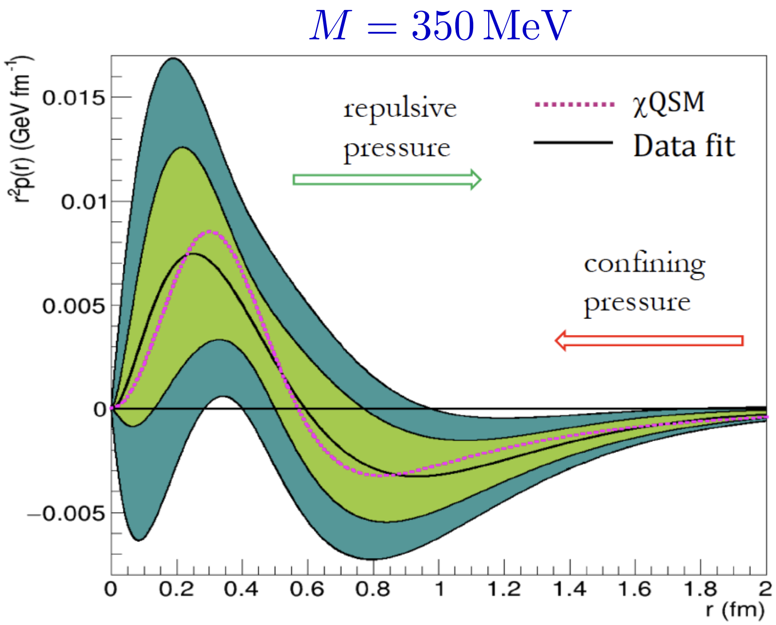
Mechanical properties: Twist-2 case



H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Twist-2 part of the pressure density.
No nodal point.

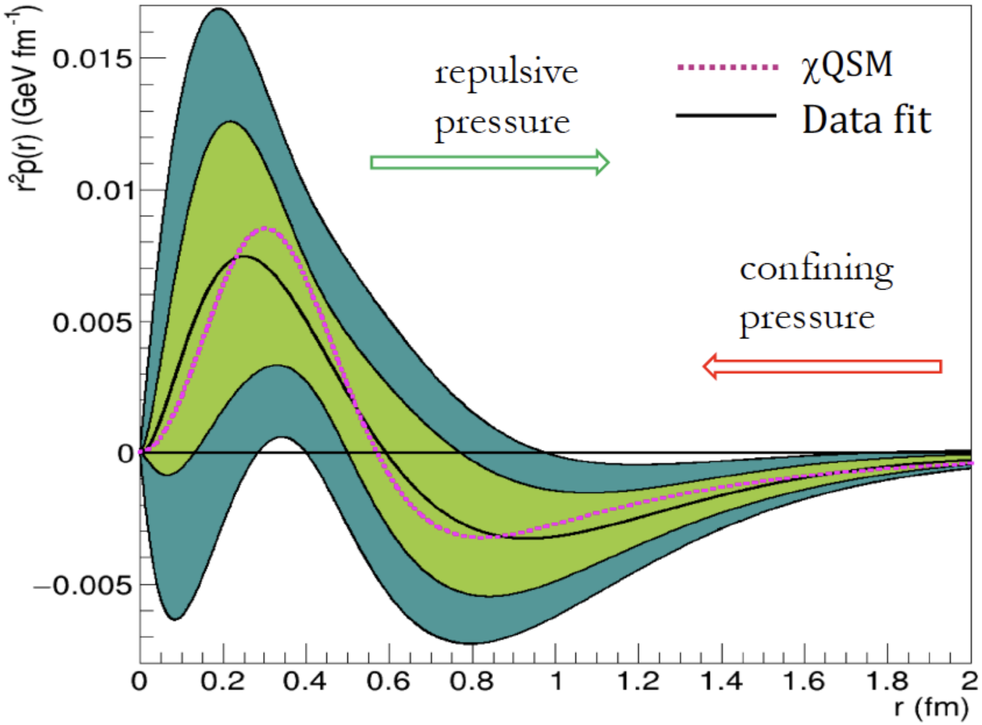




V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Color blindness in SU(3)



V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

Burkert et al. assumed the flavor blindness.

$$D^{u-d}(0) \approx 0$$

$D^{u-d}(0) = 0.29$ in SU(2)

$D^{u-d}(0) = 0.062$ in SU(3)

The flavor blindness is only valid in SU(3)!

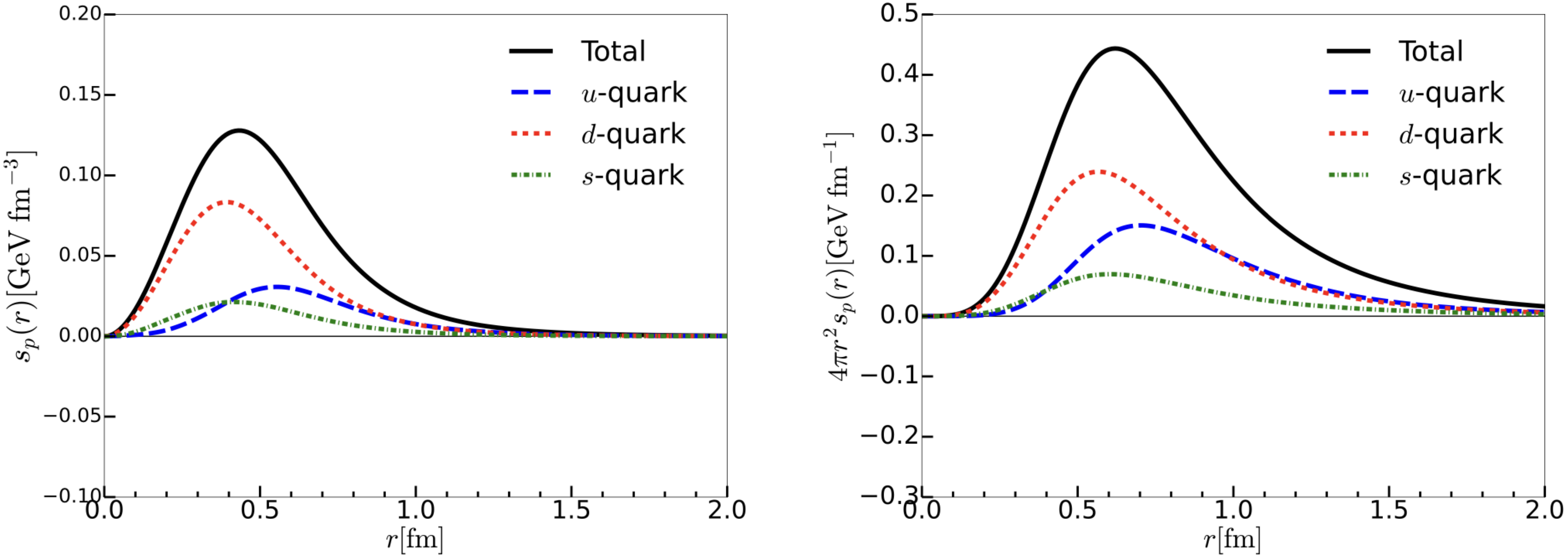
The strange quarks should essentially be considered in the proton!

Lattice QCD arrives at a similar conclusion.
(D. Hackett et al. 2310.08484)

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Shear force densities

ij off-diagonal component of the EMT: No twist-4 contribution



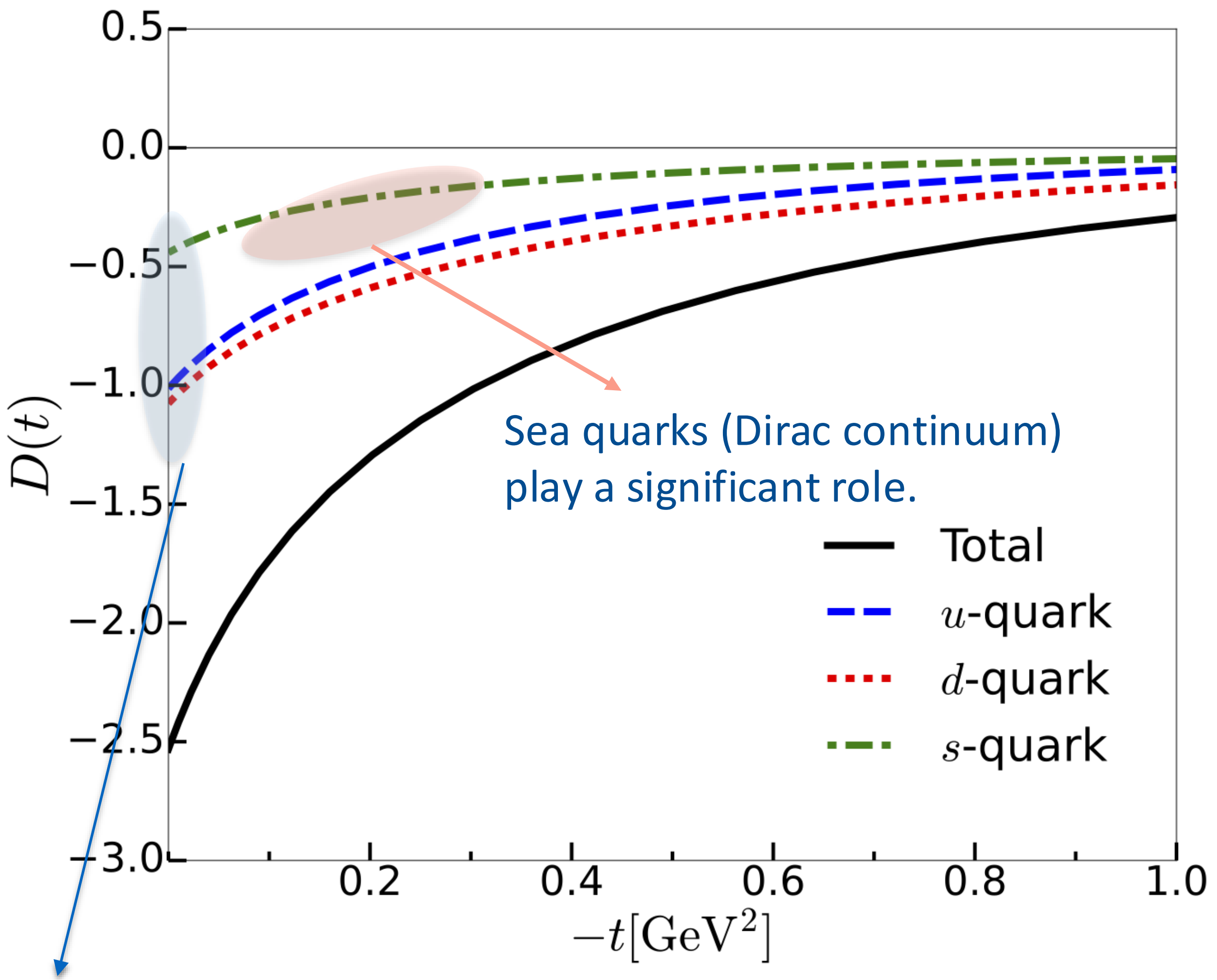
$\frac{2}{3}s_p(r) + p_p(r) > 0$ Local equilibrium condition

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Flavor-decomposed D-term form factors

$$D_B^\chi(t)\delta_{J_3'J_3} = 4M_{\rm sol}\int d^3r \frac{j_2(r\sqrt{-t})}{t} s_B^\chi(r)$$

D-term can be evaluated at the twist-2 level.



Sea quarks (Dirac continuum) play a significant role.

Similar situation in the Electric quadrupole (E2) transitions of the delta isobar

Y Hatta, M Strikman, PLB (2021)

$$-1.3 < D_s < 0.4$$

The strange-quark contributions are essential for flavor blindness!

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Estimation of mechanical Radius

$$\langle r^2\rangle_{\mathrm{mech}} = \frac{\int d^3r\; r^2 \;\left[\frac{2}{3}s(r)+p(r)\right]}{\int d^3r\;\left[\frac{2}{3}s(r)+p(r)\right]} = \boxed{\frac{6D}{\int_{-\infty}^{0} dt\; D(t)}}$$

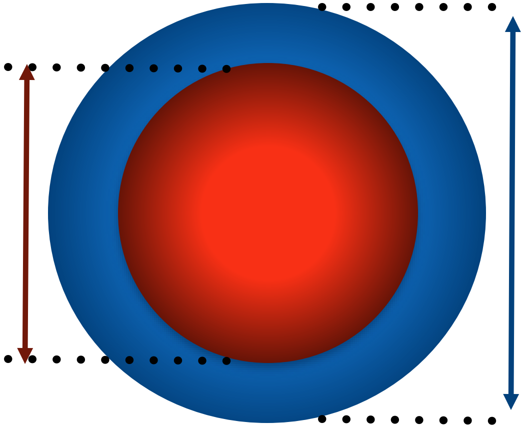
$$\sqrt{\langle r^2\rangle_{\mathrm{mech}}} = 0.69\,\mathrm{fm}$$

$$\sqrt{\langle r^2\rangle_{\mathrm{mech}}} = (0.63 \pm 0.06 \pm 0.13)\,\mathrm{fm}$$

V. Burkert et al. (2022)

$\sqrt{\langle r^2\rangle_{\mathrm{mech}}} = 0.73\,\mathrm{fm}$ in SU(3)

$$\sqrt{\langle r^2\rangle_{\mathrm{mech}}} < \sqrt{\langle r^2\rangle_{\mathrm{ch}}}$$



Mechanical radius

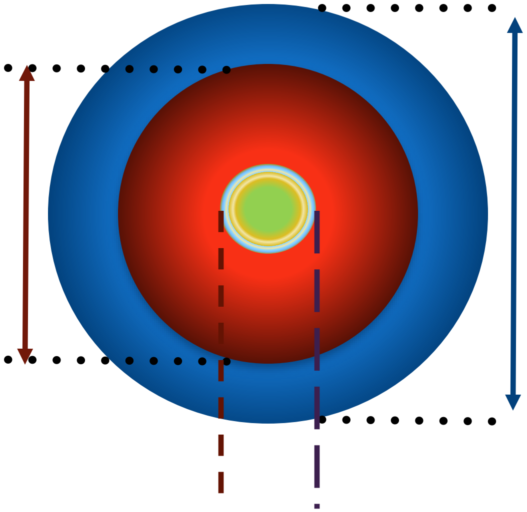
$\sim 0.6\,\mathrm{fm}$

Charge radius

$\sim 0.8\,\mathrm{fm}$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Estimation of mechanical radius



Mechanical radius $\sim 0.6\,\mathrm{fm}$

Charge radius $\sim 0.8\,\mathrm{fm}$

Mass radius $\sim 0.5\,\mathrm{fm}$

$$\sqrt{\langle r^2\rangle_{\mathrm{mass}}} < \sqrt{\langle r^2\rangle_{\mathrm{mech}}} < \sqrt{\langle r^2\rangle_{\mathrm{ch}}}$$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Outlook: Twist-4 operator

Twist-4 effective operator

Decomposition of the quark and gluon contributions in cbar ff.

- Regularization and renormalization scheme dependence
- Discrepancy between Hatta et al. and Polyakov & Son

Hatta et al. JHEP 12, 008 (2018)

$$\bar{c}_q(0,\mu) = \frac{1}{4}\Big[- A_q(0,\mu) + \frac{\alpha_s}{4\pi}\Big\{ \frac{\langle F^2\rangle_R}{3M_N} \Big\} \Big],$$

$$\bar{c}_g(0,\mu) = \frac{1}{4}\Big[- A_g(0,\mu) + \frac{\alpha_s}{4\pi}\Big\{ -\frac{11N_c}{6}\frac{\langle F^2\rangle_R}{M_N} \Big\} \Big]$$

Ratio of the quark and gluon contribution

$$[\text{quark} : \text{gluon}] = [1 : -\frac{11N_c}{2}]$$

$$\bar{c}_Q \simeq -0.124 \; [\mu = 2\,\text{GeV}]$$

$$\bar{c}_Q \simeq -0.146 \; [\mu = \infty] \; \text{pQCD}$$

Dimensional regularization

Polyakov & Son JHEP 09, 156 (2018)

$$\langle p's'|\frac{1}{2}ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p,s\rangle = M_N\bar{c}^Q\Delta^\beta\bar{u}(p',s')u(p,s)$$

$$\langle p's'|\frac{1}{2}\text{tr}(G^{\beta\alpha}[iD^\sigma,F_{\sigma\alpha}])|p,s\rangle = M_N\bar{c}^g\Delta^\beta\bar{u}(p',s')u(p,s)$$

$$\bar{c}_{\text{quark}} \sim \frac{1}{6}(M\bar{\rho})^2\log\frac{1}{M\bar{\rho}}$$

$$\bar{c}_Q \simeq 1.4\times 10^{-2} \; [\mu = .6\,\text{GeV}]$$

$$\Rightarrow \quad \bar{c}_Q \simeq 0, \quad \bar{c}_g \simeq 0$$

Twist-4 effective operator

Quark part of the twist-4 operators

Isovector part: $T^{(4),\chi=3}_{\mu\nu,Q} \sim (M\bar{\rho})^2 \sim 0$

Derivation of the isoscalar part (EMT) is under way

Gluon part of the twist-4 operators

$T^{(4)}_{\mu\nu,g} \sim (M\bar{\rho})^2 \sim 0$ Polyakov & Son JHEP 09, 156 (2018)

JY Kim, Ch. Weiss, in progress

HY Won, JY Kim, HChK, in progress

Summary

- Flavor decomposition of GFFs requires the flavor nonsinglet EMT operators that cannot be constructed without any ambiguity so far.

- Both the twist-2 and twist-4 EMT operators should be considered.

- Twist-4 operator also contribute to the GFFs, in particular, to the cbar FF.

- Gluons come into essential play in describing the GFFs
  even with the effective theory.

- The flavor decomposition of the nucleon mass and pressure requires information on cbar form factors.

- Cbar form factors may interplay between the quarks and gluons.

- As far as the total gravitational form factors are concerned,
  the results from the effective theory are still OK.

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Many thanks to J-Y. Kim and H-Y. Won for wonderful collaborations!

Thank you very much for the attention!