

Radiative modes $K^+ \rightarrow \pi^+ \gamma^* \gamma^{(*)}$ and $K^+ \rightarrow \pi^+ \ell^+ \ell^- (\gamma)$ decays

Tomáš Husek

Charles University, Prague and University of Birmingham

Cairns, Australia

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FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University



UNIVERSITY OF
BIRMINGHAM



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Radiative decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

Motivation

Flavor-changing (strangeness-changing) neutral-current weak transitions

↔ absent at **tree** level in Standard Model

↔ manifest in radiative non-leptonic kaon decays like $K^+ \rightarrow \pi^+ \ell^+ \ell^- (\gamma)$, $\ell = e, \mu$

↔ interesting probe of SM quantum corrections and beyond

Underlying long-distance-dominated radiative modes (transitions) $K^+ \rightarrow \pi^+ \gamma^* (\gamma)$ studied before

↔ calculated in Chiral Perturbation Theory (ChPT) enriched with electroweak perturbations

↔ *Ecker, Pich, de Rafael, NPB 291 (1987), 303 (1988)*, at leading order (LO) (at one-loop level)

beyond LO: including the dominant unitarity corrections from $K \rightarrow 3\pi$

↔ *D'Ambrosio, Ecker, Isidori, Portolés, JHEP 08 (1998)*

↔ *Gabbiani, PRD 59 (1999)*

$K^+ \rightarrow \pi^+ \gamma^*$ transition

$$\begin{aligned} \mathcal{M}_\rho(K^+(P) \rightarrow \pi^+(r)\gamma_\rho^*(k)) &\equiv i \int d^4x e^{ikx} \langle \pi(r) | T [J_\rho^{\text{EM}}(x) \mathcal{L}^{\Delta S=1}(0)] | K(P) \rangle \\ &= \frac{e}{2} F(k^2) [(P-r)^2 (P+r)_\rho - (P^2 - r^2)(P-r)_\rho] \end{aligned}$$

Radiative decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

Form-factor parametrization

LO appears at $\mathcal{O}(p^4)$ + unitarity loop correction from $\pi\pi$ rescattering

↪ universal parametrization: $F(s) \propto W_+(s/M_K^2)$, $W_+(z) = G_F M_K^2 (a_+ + b_+ z) + W_+^{\pi\pi}(z)$

↪ *Ecker et al., NPB 291 (1987)*, *D'Ambrosio et al., JHEP 08 (1998)*

$$\frac{d\Gamma_+}{dz} = \frac{G_F^2 \alpha^2 M_K^5}{3(4\pi)^5} \lambda^{3/2}(z) \sqrt{1 - \frac{4r_\ell^2}{z} \left(1 + \frac{2r_\ell^2}{z}\right)} |W_+(z)|^2$$

LFU

↪ a_+ and b_+ should be the same for both (e and μ) channels

↪ discrepancy due to new physics via short-distance effects

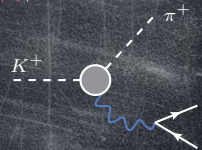
Moreover, the ratio deviates significantly from the VMD ansatz

$$\text{VMD: } \frac{b_+}{a_+} = \frac{M_K^2}{M_\rho^2} \approx 0.4, \quad \text{exp.: } \frac{b_+}{a_+} \approx 1.25$$

Measurement of quadratic term $c_+ z^2$ may further test the VMD hypothesis

ℓ	a_+	b_+	exp.
e	-0.587(10)	-0.655(44)	E865
e	-0.578(16)	-0.779(66)	NA48/2
μ	-0.575(39)	-0.813(145)	NA48/2
μ	-0.575(13)	-0.722(43)	NA62(2022) ← JHEP 11 (2022) 011

improve precision → radiative corrections, studied earlier: *Kubis and Schmidt, EPJC 70 (2010)*



Radiative decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

Bremsstrahlung

↪ LO (scalar) QED contributions where the real photon is radiated from lepton (meson) legs



↪ radiation from effective $K^+ \rightarrow \pi^+ \gamma^*$ vertex \rightarrow gauge invariant

↪ represented in terms of $F(s)$?

Effective Lagrangian for $K^+ \rightarrow \pi^+ \gamma^*(\gamma)$ transition

$$\mathcal{L} = \underbrace{ieF(0)D_\mu K^+(w \partial_\nu F^{\mu\nu})\pi^-}_{\text{minimal term leading to gauge-invariant amplitude}} - \frac{e^2}{2} \tilde{\kappa} F(0) K^+ F_{\mu\nu} F^{\mu\nu} \pi^-$$

$$\mathcal{M}_{\rho\sigma}(K^+(P) \rightarrow \pi^+(r)\gamma_\rho^*(k_1)\gamma_\sigma(k_2)) = e^2 F(k_1^2) \left\{ k_1^2 \left(r_\rho \frac{P_\sigma}{P \cdot k_2} - P_\rho \frac{r_\sigma}{r \cdot k_2} + g_{\rho\sigma} \right) \right\} + e^2 \tilde{\kappa} F(k_1^2) [(k_1 \cdot k_2)g_{\rho\sigma} - k_{1\sigma}k_{2\rho}]$$

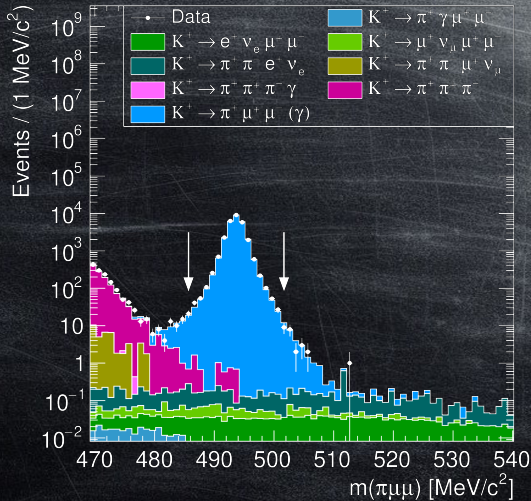
↪ term proportional to $\tilde{\kappa}$

↪ mimic the behavior of additional form factors adequately at given order

↪ ideally negligible effects in given set-up \rightarrow estimate of associated uncertainty

Radiative decays $K^+ \rightarrow \pi^+ l^+ l^-$

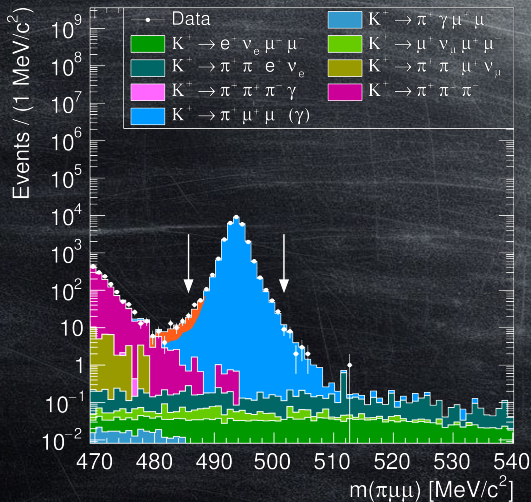
Spectra



(Thanks to *L. Bičian (NA62)*)

Radiative decays $K^+ \rightarrow \pi^+ l^+ l^-$

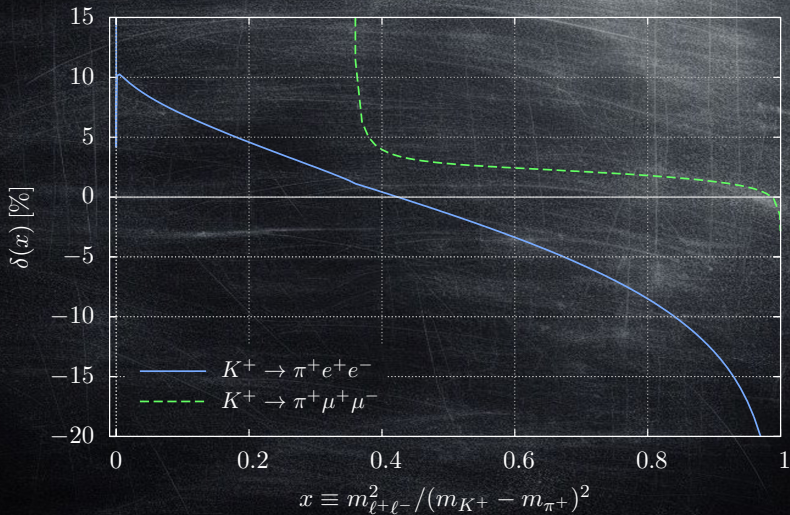
Spectra



(Thanks to *L. Bičian (NA62)*)

Radiative decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

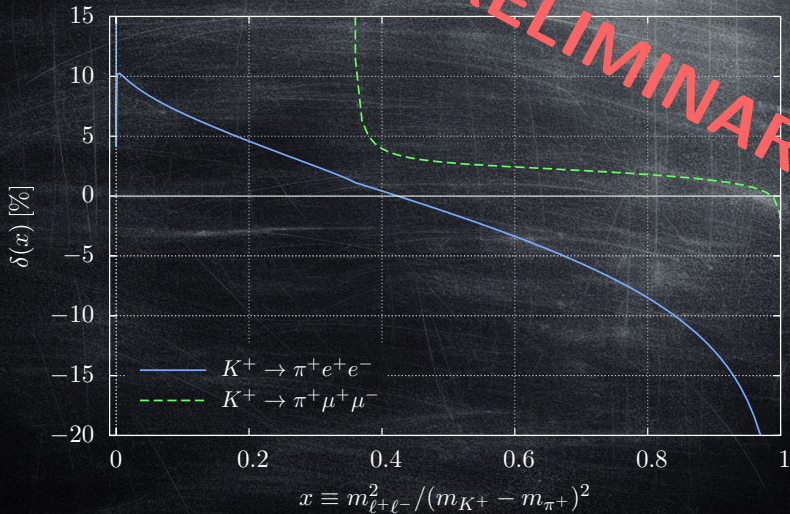
Radiative corrections



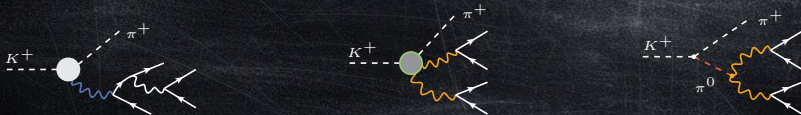
Radiative decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$

Radiative corrections

PRELIMINARY



$K^+ \rightarrow \pi^+ 4e$ decay



$K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$

Introduction

The long-distance-dominated $K^+ \rightarrow \pi^+ \gamma^*$ transition essential also for $K^+ \rightarrow \pi^+ 4e$
 \hookrightarrow one also needs to consider $K^+ \rightarrow \pi^+ \gamma^* \gamma^*$ transition

Challenging to observe $K^+ \rightarrow \pi^+ 4e$ away from $m_{4e} \simeq M_{\pi^0}$

\hookrightarrow suppressed decay rate \longrightarrow attractive to study possible effects of BSM physics

\hookrightarrow to identify new-physics-scenario contribution \longrightarrow need for (rough) estimate of SM rate

\hookrightarrow new-physics effects spotted as deviations from such SM predictions

$$B(K^+ \rightarrow \pi^+ 4e, \text{ non-res.}) = 7.2(7) \times 10^{-11}$$

\longrightarrow possible BSM scenarios are being explored

Hostert, Pospelov, PRD 105 (2022)

\hookrightarrow $K \rightarrow \pi 4e$ decays proceed via $K \rightarrow \pi (X' \rightarrow XX)$ intermediate states

\hookrightarrow cascade of dark-sector particles $X^{(\prime)}$

\hookrightarrow underlying dynamics potentially significantly enhanced compared to the SM case

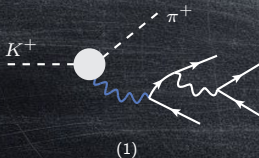
\longrightarrow searches in suitable experiments

\hookrightarrow more precise knowledge of SM background essential

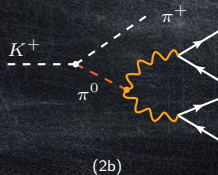
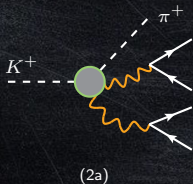
\hookrightarrow ideally at level suited for Monte Carlo (MC) implementation

$K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$
Standard Model prediction: Topologies

One-photon-exchange topology



Two-photon-exchange topology



TH, PRD 106 (2022)

$$K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$$

Two-photon-exchange topology: Matrix element

The two-photon transition of topology (2a) can be written, approximately, as follows:

$$\begin{aligned} \mathcal{M}_{\rho\sigma}^{(a)}(K(P) \rightarrow \pi(r)\gamma_\rho^*(k_1)\gamma_\sigma^*(k_2)) \\ \simeq e^2 F(k_1^2) \left\{ (k_1^2 r_\rho - r \cdot k_1 k_{1\rho}) \frac{(2P - k_2)_\sigma}{2P \cdot k_2 - k_2^2} - (k_1^2 P_\rho - P \cdot k_1 k_{1\rho}) \frac{(2r + k_2)_\sigma}{2r \cdot k_2 + k_2^2} \right. \\ \left. + (k_1^2 g_{\rho\sigma} - k_{1\rho} k_{1\sigma}) \right. \\ \left. + \tilde{\kappa} [(k_1 \cdot k_2) g_{\rho\sigma} - k_{1\sigma} k_{2\rho}] \right\} \\ + \{k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma\} \end{aligned}$$

↪ in this model depends on a **single** form factor (the same $F(s)$)

↪ useful when measuring $F(s)$ → radiative corrections for the $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay
 ↪ one of the photons on-shell

Soft-photon regime → approximation **justified**

Hard photons → free parameter $|\tilde{\kappa}| \lesssim 1$ introduced to cover model uncertainty

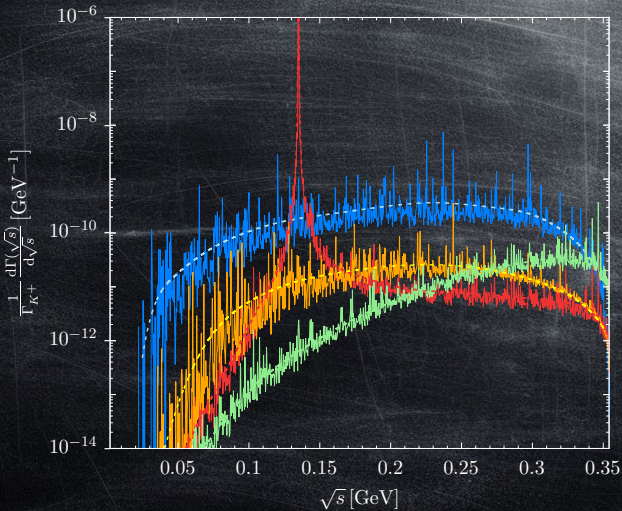
↪ physical results do not seem to be sensitive to this parameter

For $K^+ \rightarrow \pi^+ 4e$, we assume it is good enough (at least) as an order-of-magnitude guess

↪ numerically **negligible** (one order of magnitude) compared to the topology (1)

$K^+ \rightarrow \pi^+ e^+ e^- e^+ e^-$

Contributions to the branching ratio



[large MC samples generated by A. Shaikhiev, E. Goudzovski]

TH, PRD 106 (2022)

$K^+ \rightarrow \pi^+ \gamma^* \gamma^*$ transition

WORK IN PROGRESS

$K^+ \rightarrow \pi^+ \gamma^* \gamma^*$ transition

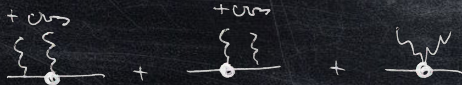
$K^+ \rightarrow \pi^+ \gamma^* \gamma^*$ transition at LO in ChPT

$K^+ \rightarrow \pi^+ \gamma^{(*)}$ transition



$$= ieF_{\text{ChPT}}^{(\text{LO})}(k^2) [k^2 r_\rho - (k \cdot r) k_\rho] + e\tilde{F}(k^2)(M_K^2 + M_\pi^2)(P + r)_\rho$$

$K^+ \rightarrow \pi^+ \gamma^{(*)} \gamma^{(*)}$ transition



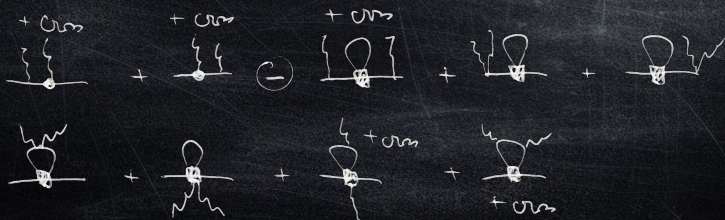
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$$= ieF_{\text{ChPT}}^{(\text{LO})}(k^2) [k^2 r_\rho - (k \cdot r) k_\rho] + e\tilde{F}(k^2)(M_K^2 + M_\pi^2)(P+r)_\rho$$

$K^+ \rightarrow \pi^+ \gamma^{(*)} \gamma^{(*)}$ transition



$K^+ \rightarrow \pi^+ \gamma^* \gamma^*$ transition at LO in ChPT

$$\begin{aligned}
 \mathcal{M}_{\rho\sigma}(K^+(P) \rightarrow \pi^+(r) \gamma_\rho^*(k_1) \gamma_\sigma^*(k_2)) & \\
 = e^2 F(k_1^2) T_{\rho\sigma}^{(0)} + (k_1 \leftrightarrow k_2, \rho \leftrightarrow \sigma) & \\
 + e^2 C_1 \epsilon_{\rho\sigma}(k_1)(k_2) & \\
 + e^2 \left[T_{\rho\sigma}^{(1)} A_{\gamma^* \gamma^*}^{(+,1)} - 2 T_{\rho\sigma}^{(2)} A_{\gamma^* \gamma^*}^{(+,2)} \right] + 4e^2 (k_1 \cdot k_2) T_{\rho\sigma}^{(1*)} \left[-\hat{c} + \kappa \frac{(k_1 + k_2)^2}{2k_1 \cdot k_2} \right] - 4e^2 T_{\rho\sigma}^{(2)} \kappa \frac{(k_1 + k_2)^2}{2k_1 \cdot k_2} &
 \end{aligned}$$

$$T_{\rho\sigma}^{(0)} = (k_1^2 r_\rho - r \cdot k_1 k_{1\rho}) \frac{(2P - k_2)_\sigma}{2P \cdot k_2 - k_2^2} + (P \leftrightarrow -r) + (k_1^2 g_{\rho\sigma} - k_{1\rho} k_{1\sigma})$$

$$T_{\rho\sigma}^{(1)} = g_{\rho\sigma} - \frac{(k_1 \cdot k_2)(k_{1\sigma} k_{2\rho} + k_{1\rho} k_{2\sigma}) - k_1^2 k_{2\rho} k_{2\sigma} - k_2^2 k_{1\rho} k_{1\sigma}}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2},$$

$$T_{\rho\sigma}^{(1*)} = g_{\rho\sigma} - \frac{k_{1\sigma} k_{2\rho}}{k_1 \cdot k_2}$$

$$T_{\rho\sigma}^{(2)} = \frac{[k_1^2 k_{2\rho} - (k_1 \cdot k_2) k_{1\rho}][k_2^2 k_{1\sigma} - (k_1 \cdot k_2) k_{2\sigma}]}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2},$$

$$T_{\rho\sigma}^{(2*)} = 0$$

$$A_{\gamma^* \gamma^*}^{(+,i)} = [M_K^2 - M_\pi^2 + s] \hat{A}_{\gamma^* \gamma^*}^{(i)}(M_\pi^2) - [M_K^2 - M_\pi^2 - s] \hat{A}_{\gamma^* \gamma^*}^{(i)}(M_K^2)$$

$K_L \rightarrow \pi^0 \gamma^* \gamma^*$ transition at LO in ChPT

$$\begin{aligned} \mathcal{M}_{\rho\sigma}(K_L(P) \rightarrow \pi^0(r)\gamma_\rho^*(k_1)\gamma_\sigma^*(k_2)) \\ = -2e^2 \left[T_{\rho\sigma}^{(1)} A_{\gamma^*\gamma^*}^{(0,1)} - 2T_{\rho\sigma}^{(2)} A_{\gamma^*\gamma^*}^{(0,2)} \right] - 2e^2 \kappa M_K^2 T_{\rho\sigma}^{(1)} \end{aligned}$$

$$T_{\rho\sigma}^{(0)} = (k_1^2 r_\rho - r \cdot k_1 k_{1\rho}) \frac{(2P - k_2)_\sigma}{2P \cdot k_2 - k_2^2} + (P \leftrightarrow -r) + (k_1^2 g_{\rho\sigma} - k_{1\rho} k_{1\sigma})$$

$$T_{\rho\sigma}^{(1)} = g_{\rho\sigma} - \frac{(k_1 \cdot k_2)(k_{1\sigma} k_{2\rho} + k_{1\rho} k_{2\sigma}) - k_1^2 k_{2\rho} k_{2\sigma} - k_2^2 k_{1\rho} k_{1\sigma}}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2},$$

$$T_{\rho\sigma}^{(1*)} = g_{\rho\sigma} - \frac{k_{1\sigma} k_{2\rho}}{k_1 \cdot k_2}$$

$$T_{\rho\sigma}^{(2)} = \frac{[k_1^2 k_{2\rho} - (k_1 \cdot k_2) k_{1\rho}][k_2^2 k_{1\sigma} - (k_1 \cdot k_2) k_{2\sigma}]}{(k_1 \cdot k_2)^2 - k_1^2 k_2^2},$$

$$T_{\rho\sigma}^{(2*)} = 0$$

$$A_{\gamma^*\gamma^*}^{(0,i)} = [s - M_\pi^2] \hat{A}_{\gamma^*\gamma^*}^{(i)}(M_\pi^2) + [M_K^2 + M_\pi^2 - s] \hat{A}_{\gamma^*\gamma^*}^{(i)}(M_K^2)$$

$K \rightarrow \pi \gamma^{(*)} \gamma^{(*)}$ transition at LO in ChPT

Comparison with literature

The above results reduce to special cases present in literature:

$$K^+ \rightarrow \pi^+ \gamma \gamma$$

↪ *Ecker, Pich, de Rafael*, Nucl. Phys. B **291** (1987), B **303** (1988)

↪ *D'Ambrosio, Portolés*, Phys. Lett. B **386** (1996)

$$K^+ \rightarrow \pi^+ \gamma \gamma^*$$

↪ *Gabbiani*, Phys. Rev. D **59** (1999)

$$K_L \rightarrow \pi^0 \gamma \gamma$$

↪ *Ecker, Pich, de Rafael*, Nucl. Phys. B **291** (1987), B **303** (1988)

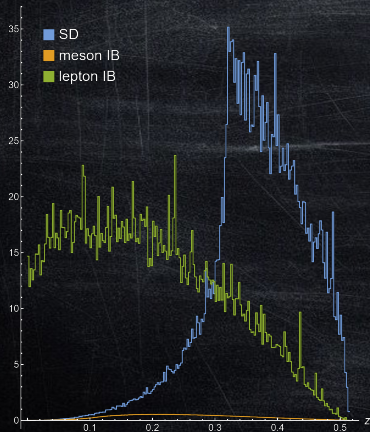
↪ *Gabbiani, Valencia*, Phys. Rev. D **64** (2001)

$$K_L \rightarrow \pi^0 \gamma \gamma^*$$

↪ *Donoghue, Gabbiani*, Phys. Rev. D **56** (1997), D **58** (1998)

$$K^+ \rightarrow \pi^+ e^+ e^- \gamma$$

$E_\gamma^* > 30 \text{ MeV}$

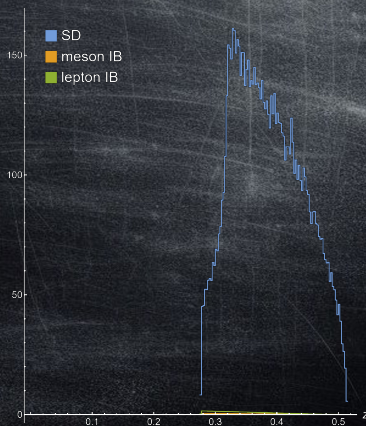
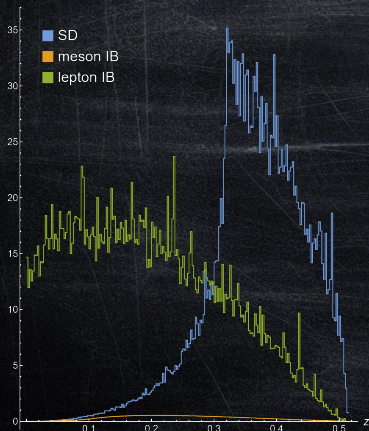


$$K^+ \rightarrow \pi^+ e^+ e^- \gamma$$

$E_\gamma^* > 30 \text{ MeV}$

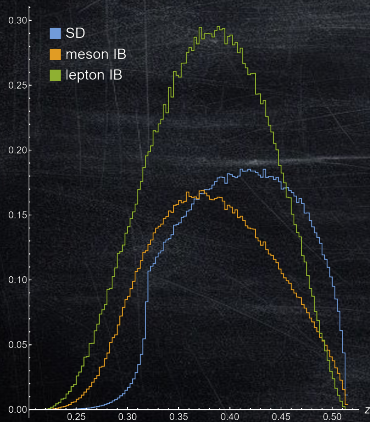
"NA48 cuts" in $e^+e^-\gamma$ rest frame [PLB 659 (2008) 493]

$$\Leftrightarrow z \equiv \frac{m_{ee\gamma}^2}{M_K^2} > 0.277, \quad \angle(e^+, e^-) < \min(\angle(e^\pm, \gamma))$$



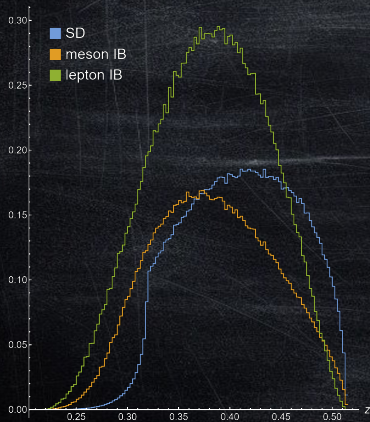
$$K^+ \rightarrow \pi^+ \mu^+ \mu^- \gamma$$

$E_\gamma^* > 30 \text{ MeV}$

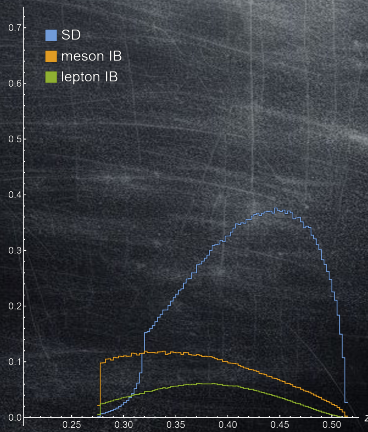


$$K^+ \rightarrow \pi^+ \mu^+ \mu^- \gamma$$

$E_\gamma^* > 30 \text{ MeV}$



"NA48 cuts"



Rare electromagnetic decay $\pi^0 \rightarrow e^+e^-$



$\pi^0 \rightarrow e^+e^-$ update

Short review paper on radiative corrections

Neutral-pion decay into an electron-positron pair: A review and update

↪ Phys. Rev. D 110 (2024) 033004

↪ accompanies the latest branching-ratio measurement at NA62 (preliminary results)

↪ revises, discusses, and summarizes radiative correction for $\pi^0 \rightarrow e^+e^-$ in one spot

↪ describes relation between the theoretical and experimental observables

$$\hookrightarrow B(\pi^0 \rightarrow e^+e^-(\gamma), x > x_{\text{cut}}) = [1 + \delta(x_{\text{cut}})] B(\pi^0 \rightarrow e^+e^-)$$

↪ updates the critical NLO QED correction $\delta_+(x_{\text{cut}})$ that relates these

$$\hookrightarrow \delta_+(0.95) = [-6.06(7)_\xi - 0.08\tilde{\chi}] \% = -6.1(2)\%$$

↪ calculates the overall correction $\delta = [10.67(7)_\xi - 0.25\tilde{\chi}] \% = 10.7(1)_\xi(2)_\chi \%$

↪ derives other related useful experimental quantities / ratios

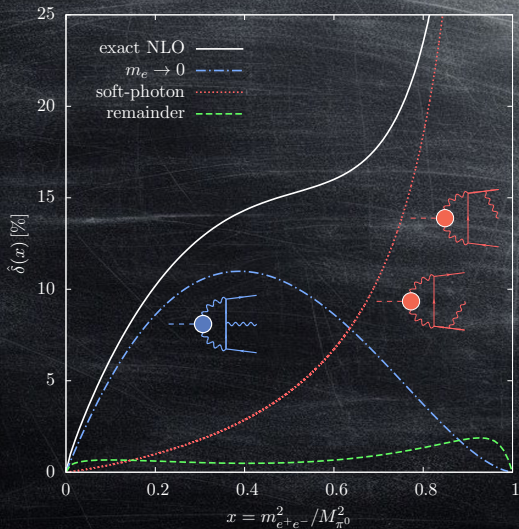
↪ discusses the latest measurements (KTeV, NA62)

↪ properties of the $\pi^0 \rightarrow e^+e^-(\gamma)$ amplitude in various limits

↪ discusses its relation with the Dalitz-decay radiative corrections



Properties of $\pi^0 \rightarrow e^+e^-(\gamma)$ amplitude in various limits



Thank you for listening!

