Single Spin Asymmetry Among Hyperons Using the Scalar Diquark Model

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August 22, 2024

Outline

- *Internal Structure of the Hadrons*
- *Distribution functions*
  - Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)
  - Wigner Distributions (WDs)
  - Transverse Momentum-Dependent Parton Distributions (TMDs)
  - Generalized Parton Distributions (GPDs)
- *Transverse distortion*
- *Single-spin asymmetry*
- *Summary*

Internal Structure of the Hadrons

- Quantum Chromodynamics (QCD) provides a fundamental description of hadronic structure and dynamics in terms of their elementary quark and gluon degrees of freedom.
- Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.
- Many fundamental questions have not been resolved.

QCD: Theory of Strong Interactions

- At high energies, (α_s is small), QCD can be used perturbatively.
- At low energies, (α_s becomes large), one has to use other methods such as effective Lagrangian models to describe physics.
- New experimental tools are continually being developed to probe the nonperturbative structure of the theory: the hard diffractive reactions, semi-inclusive reactions, deeply virtual Compton scattering.
- **A lot of theoretical as well as experimental research work has been done to reveal the mysterious behavior of quarks inside the nucleons** - members of octet baryons with spin-parity quantum number as $J^P = (\frac{1}{2})^+$.
- Other octet baryon: **isospin partners Σ and Ξ have not been extensively studied**.

The chief reason of ignorance of other members of octet baryons

- Presence of **strange quark(s)**.
- Small life span of these strange baryons.
- Measurements also demand collision energy significantly above the hadronic scale to allow transfer of energy and momentum etc.

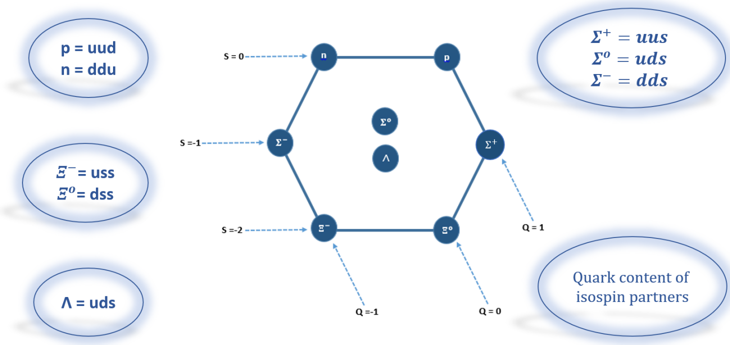
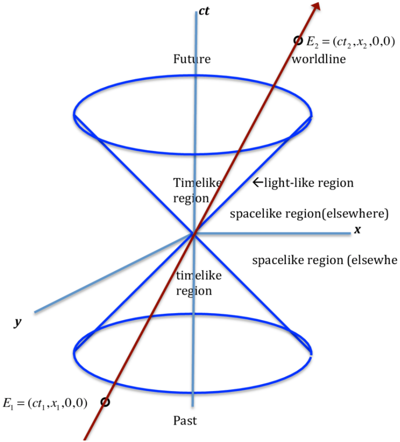


Figure 1: Octet Baryon

From Special Theory of Relativity:

- Space and time independently are not invariant quantities.
- Rather space-time is an invariant object.



Instant form v/s Front form

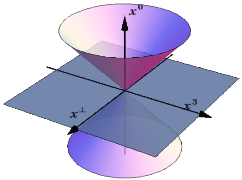


Figure 2: The instant form

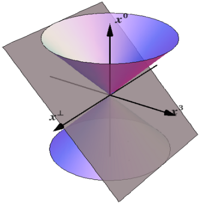


Figure 3: The front form

- All measurements are made at fixed t i.e. at $x^0 = 0$.
- All measurements are made at fixed light-cone time x^+ i.e. at $x^+ = x^0 + x^3 = 0$.

- Energy-momentum dispersion relation:

In the instant form,
$$p^0 = \sqrt{\vec{p}^2 + m^2}.$$

In the front form,
$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}.$$

No square-root for the Hamiltonian in light front form. Therefore, simplifes the dynamical structure.

Light-front vacuum is simple, as all the massive fluctuations in the ground state are absent. *Light-front provides the wavefunctions (LFWFs) required to describe the structure and dynamics of hadrons in terms of their constituents (quarks and gluons).*

- S. J. Brodsky, G. F. de Teramond, Phys. Rev. D 77, 056007 (2008).

Why Light Front?

- Ideal Framework to describe theoretically the hadronic structure. It can overcome many obstacles and has many advantages:
  - Simple vacuum structure ∼ vacuum expectation value is zero.
  - A dynamical system is characterized by ten fundamental quantities: energy, momentum, angular momentum and boost.
    ∼ seven out of which are kinematical. It allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents.

Light-Front Coordinates

- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

Light-Front in QCD

- Light Front QCD (LFQCD) is an *ab initio* approach to study the strongly interacting system. It is similar to perturbative and lattice QCD and is directly connected to the QCD Lagrangian.
- It is a Hamiltonian method, formulated in Minkowinski space rather than Euclidean space.
- The theory is quantized at fixed light-cone time $\tau = t + z/c$ rather than ordinary time t.

Outline

- Internal Structure of the Hadrons
- Distribution functions
  - Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)
  - Wigner Distributions (WDs)
  - Transverse Momentum-Dependent Parton Distributions (TMDs)
  - Generalized Parton Distributions (GPDs)
- Transverse distortion
- Single-spin asymmetry
- Summary

Distribution functions

(Mathematical tool to unfold the internal structure of hadrons)

Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)
Transverse Momentum-Dependent Parton Distributions (TMDs)
Generalized Parton Distributions (GPDs)
Parton Distribution Functions (PDFs)

$$W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, \Gamma\, \mathcal{W}(-\tfrac{1}{2}z, \tfrac{1}{2}z\,|\,n)\, \psi(\tfrac{1}{2}z)\, |p,\lambda\rangle\,.$$

Generalized Parton Correlation Function

- *S. Meissner, A. Metz and M. Schlegel, JHEP 08, 056 (2009).*

Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)

- Leading-twist GTMDs are the function of six variables $(x, \zeta, \mathbf{k}_\perp^2, \mathbf{k}_\perp.\mathbf{\Delta}_\perp, \mathbf{\Delta}_\perp^2)$.
- The GTMDs are accessible through double Drell-Yan processes.

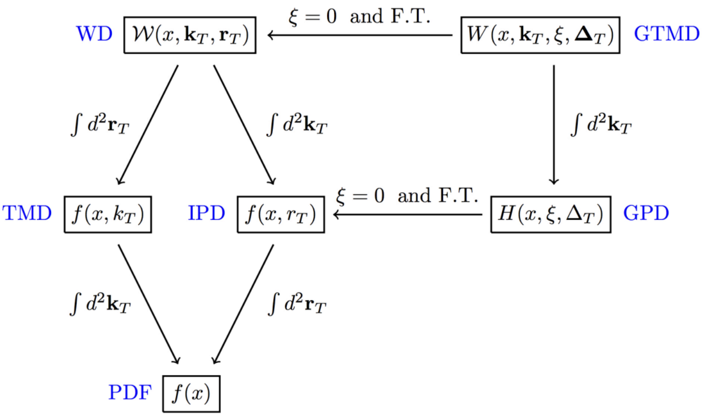
GTMDs

Quark polarization (columns); Nucleon polarization (rows)

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}(k_y H_{11} + \Delta_y H_{12})$	$-\frac{i}{M}(k_x H_{11} + \Delta_x H_{12})$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}(k_y F_{12} + \Delta_y \bar{F}_{13})$	$\cdots$	$\cdots$	$\frac{1}{M}(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13})$
T_y	$-\frac{i}{M}(k_x F_{12} + \Delta_x \bar{F}_{13})$	$\cdots$	$\cdots$	$\frac{1}{M}(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13})$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}(k_x H_{17} + \Delta_x H_{18})$	$\frac{1}{M}(k_y H_{17} + \Delta_y H_{18})$	G_{14}

-C. Lorce

Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)



- One can obtain GPDs and TMDs from GTMDs under certain kinematic limits.
- GTMDs are known as *mother distributions*.

Wigner Distributions (WDs) I

- To understand the hadron structure, *the joint position and momentum distributions* (quantum analog to the classical phase-space distributions) Wigner distributions were introduced.
- Wigner distributions were first introduced by E. Wigner in 1932.

  -E. Wigner Phys. Rev. 70, 749 (1932)
- These distributions are the *quasi-probabilistic distributions*.
- No experiments yet.

- In QCD, Wigner distributions were first introduced by Xiangdong Ji

-X. -d. Ji, Phys. Rev. Lett. 91, 062001 (2003).

$$\rho^{[\Gamma]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} W^{[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{k}_\perp, x),$$

where $W^{[\Gamma]}(\mathbf{\Delta}_\perp, \mathbf{k}_\perp, x)$ in the proton state at fixed light-cone time $z^+ = 0$ is defined as

$$W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{p}_\perp, x) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ip\cdot z} \langle P_f | \bar{\psi}(-z/2) \Gamma \mathcal{W}_{[-\frac{z}{2},\frac{z}{2}]} \psi(z/2) | P_i \rangle.$$

Here, Γ indicates the Dirac γ-matrix, specifically for twist-2

- γ^+ : *corresponding to unpolarized quark,*
- $\gamma^+\gamma_5$: *corresponding to longitudinally-polarized quark,*
- $i\sigma^{j+}\gamma_5$: *corresponding to transversely-polarized parton,* where $j = 1$ or 2, depending upon the polarization direction of quark.

Transverse Momentum-Dependent Parton Distributions (TMDs) I

To get the information of hadron structure in momentum space, transverse momentum-dependent parton distributions (TMDs) were introduced.

- TMDs describe the probability to find a parton with longitudinal momentum fraction x and transverse momentum with respect to the direction of the parent hadron momentum in a hadron.
- TMDs $f(x, \vec{k}_\perp)$, are function of longitudinal momentum fraction carried by the active quark $x = \frac{k^+}{P^+}$ and the quark transverse momentum $\vec{k}_\perp$.

Transverse Momentum-Dependent Parton Distributions (TMDs) II

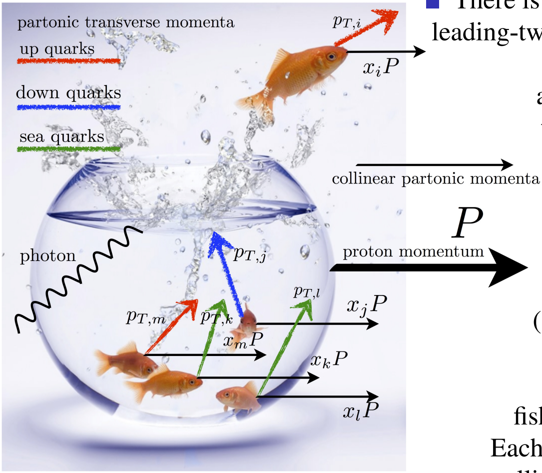
- TMDs are also of particular importance because they give rise to single spin asymmetries (SSAs).
- TMDs represent three-dimensional hadron picture in *momentum space*.
- They can be measured in a variety of reactions in lepton-proton and proton-proton collisions as semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan production where a final-state particle is observed with a transverse momentum.

Transverse Momentum-Dependent Parton Distributions (TMDs) III

- The quark-quark correlation to evaluate quark TMDs in proton is given by

$$\Phi^{[\Gamma]}(x, \mathbf{k}_\perp) = \frac{1}{2}\int \frac{dz^-}{2\pi}\frac{d^2\mathbf{z}_\perp}{(2\pi)^2} e^{ik.z/2}\langle P|\bar{\Psi}(0)\Gamma\Psi(z)|P\rangle\Big|_{z^+=0}.$$

Transverse Momentum-Dependent Parton Distributions (TMDs)



-Image courtesy: A. Signori

- There is one quark TMD at leading-twist in case of kaon, while 8 quark and gluon TMDs at the leading twist in case of nucleon (spin-1/2).
- In the figure, partons (quarks and gluons) are like fishes confined inside a fishbowl (the proton). Each parton has its own collinear and transverse velocity, indicated by black and colored arrow respectively.

Generalized Parton Distributions (GPDs) I

- GPDs provide a 3-D picture of the partonic nucleon structure. From 3-D we mean that GPDs encode information on the distribution of partons both in the transverse plane and longitudinal direction.

- Generalized Parton Distributions can be accessed through deep exclusive processes such as DVCS or DVMP. DVCS reaction $\gamma^* + p \rightarrow \gamma + p$ has extraordinary senstivity to fundamental features of the proton's structure.

- GPDs are much richer in content about the hadron structure than ordinary parton distributions.

Generalized Parton Distributions (GPDs) II

- GPDs allows us to access partonic configurations with a given longitudinal momentum fraction, but also at specific location (transverse) inside the hadron.

- GPDs depends on three variables x, ζ, t.
  - x is the fraction of momentum transfer.
  - ζ gives the longitudinal momentum transfer.
  - t is the square of the momentum transfer in the process.

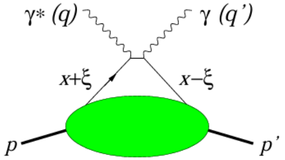
- Several experiments such as H1 collaboration, ZEUS collaboration and fixed target experiments at HERMES have finished taking data on DVCS. In the forward limit of zero momentum transfer, the GPDs reduce to ordinary parton distributions.

Generalized Parton Distributions (GPDs) III

- One can define the correlation to evaluate unpolarized GPD in proton $F^{[\Gamma]}(x, \zeta = 0, t)$ as

$$F^{[\Gamma]}(x, 0, t) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ixP^+z^-/2} \langle P_f | \bar{\Psi}(0) \Gamma \Psi(z) | P_i \rangle \Big|_{z^+ = \mathbf{z}_\perp = 0}.$$

- The GPDs explain through various exclusive processes such as Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP).



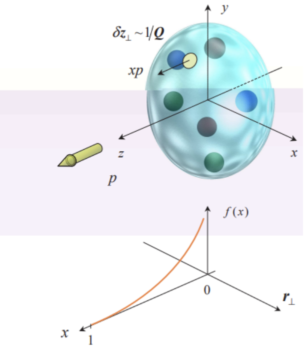
Generalized parton distributions (GPDs)
For zero skewness

- Reduced to PDFs,
  $H^{X_q}(x, 0, 0) = f^{X_q}(x) \Rightarrow$ PDFs
- Pathway to FFs
  $\int f^q(x, 0, t)\, dx \Rightarrow$ FFs.

$$f^q(x, 0, t) \xrightarrow{FT} f^q(x, 0, \mathbf{r}_\perp)$$

$$\int f^q(x, 0, \mathbf{r}_\perp)\, dx \Rightarrow \text{FT FFs.}$$

- 3D hybrid distribution
  - 1D momentum
  - 2D coordinate space



-*Phys. Rept.* 418, 1 – 387(2005).

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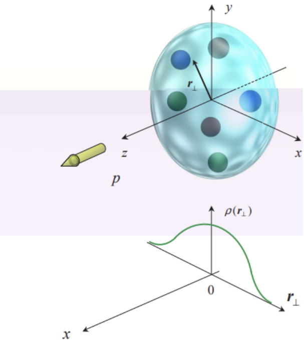

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  $\int f^q(x,0,\mathbf{r}_\perp)\,dx \Rightarrow$ FT FFs.
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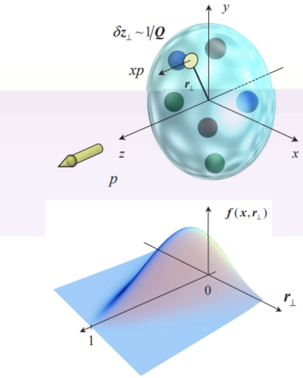

-*Phys. Rept.* 418, 1 – 387(2005).

Generalized Parton Distributions (GPDs)

- GPDs have **deep pockets** as it is a source to study the intrinsic properties of a baryon.
- Correlator corresponding to the unpolarized baryon with zero skewness is expressed as

$$\frac{1}{2}\int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \left\langle P' \left| \bar{\psi}\left(\frac{-y}{2}\right)\gamma^+ \psi\left(\frac{y}{2}\right)\right| P \right\rangle \Big|_{y^+=0,\mathbf{y}_\perp=\mathbf{0}}$$
$$= \frac{1}{2\bar{P}^+}\bar{u}(P')\left[H_X^q(x,0,\mathbf{\Delta}_\perp)\gamma^+ + E_X^q(x,0,\mathbf{\Delta}_\perp)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}\right]u(P) \quad (1)$$

- These $H_X^q(x,0,\mathbf{\Delta}_\perp)$ and $E_X^q(x,0,\mathbf{\Delta}_\perp)$ can be used to evaluate charge distribution and magnetization densities respectively.

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- *Single-spin asymmetry*
- *Summary*

Transverse Distortion

- Distortion in the parton distributions is always observed for a transversely polarized target.
- GPD $E_X^q(x, 0, \mathbf{\Delta}_\perp)$ involves baryon flip helicity.
- Unpolarized quark inside a transversely polarized baryon.

Transverse Distortion

- To investigate, consider a state that is polarized in the y-direction in the infinite momentum frame

$$|Y\rangle = \frac{1}{\sqrt{2}}\left[|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + i\,|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle\right].$$

- Unpolarized quark distribution for the above defined state can be written in terms of the impact parameter space coordinates as

$$q_{\hat{y}}^{X_q}(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp\cdot\mathbf{b}_\perp}\left[H_X^q(x,0,t) + i\frac{\Delta_\perp^x}{2M_X}E_X^q(x,0,t)\right]$$
$$= \mathcal{H}_X^q(x,\mathbf{b}_\perp) + \frac{1}{2M_X}\frac{\partial}{\partial b^x}\mathcal{E}_X^q(x,\mathbf{b}_\perp).$$

- Expression of GPD H_X^q in $\mathbf{b}_\perp$-space

$$\mathcal{H}_X^q(x, \mathbf{b}_\perp) = \frac{g^2}{16\pi^3(1-x)}\Big[(xM_X + m_q)^2[K_o(Z)]^2 + \mathcal{M}_X^{un}(x)[K_1(Z)]^2\Big].$$

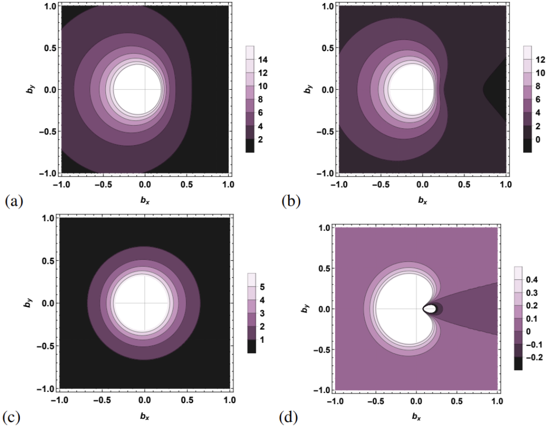
- Expression of GPD E_X^q in $\mathbf{b}_\perp$-space

$$\mathcal{E}_X^q(x, \mathbf{b}_\perp) = \frac{g^2}{16\pi^3} 2M_X(xM_X + m_q)^2[K_o(Z)]^2 .$$

-arXiv:2405.00445 [hep-ph].

Transverse distortion; $\Lambda\,(u - flavor)$

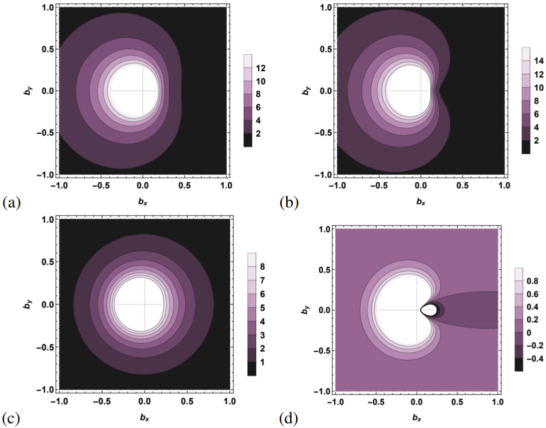
- Transversely distorted towards left.
- More spread at $x = 0.4$.
- Implies u quark flavor has the most capability to carry longitudinal momentum fraction ≈ 0.4.



For fixed longitudinal momentum fraction x
(a) 0.2, (b) 0.4, (c) 0.6 and (d) 0.8

Transverse distortion; $\Lambda\,(s-flavor)$

- Transversely distorted towards left.
- Spread at $x = 0.4$ decreases.
- Spread at $x = 0.6$ increases.
- Implies s quark flavor has the most capability to carry longitudinal momentum fraction ≈ 0.6.



For fixed longitudinal momentum fraction x
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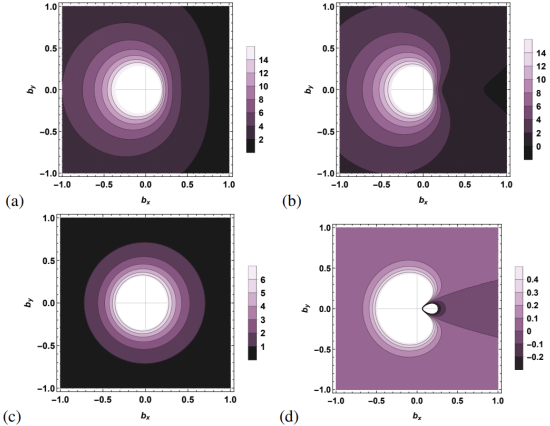
Transverse distortion; $\Sigma\,(u - flavor)$

- Transversely distorted towards left.
- Characteristics of u quark flavor are same as in Λ.
- Amplitude and spread is more.



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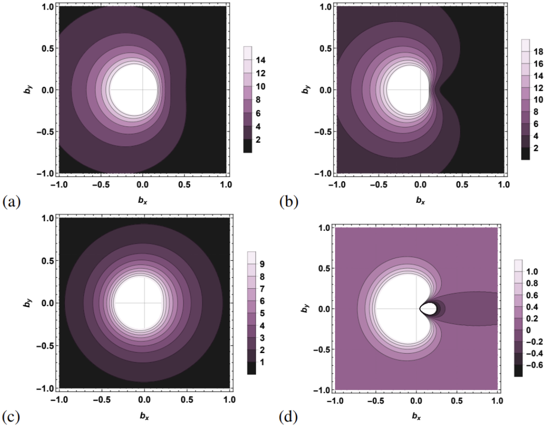
Transverse distortion; $\Sigma\,(s - flavor)$

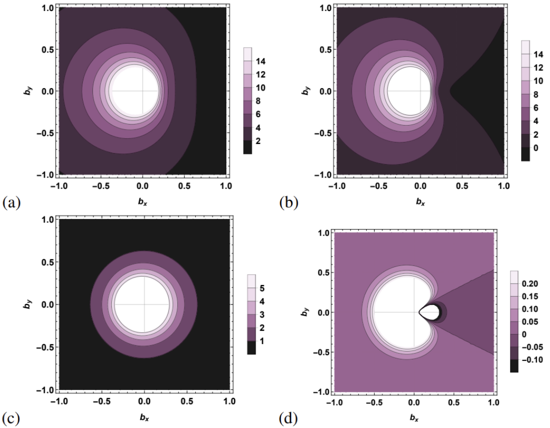
- Transversely distorted towards left.
- Characteristics of s quark flavor are same as in Λ.
- Amplitude and spread is more.
- Reason of carrying more longitudinal momentum fraction lies in the respective masses of constituent an active quark and spectator.



For fixed longitudinal momentum fraction x
(a) 0.2, (b) 0.4, (c) 0.6 and (d) 0.8

Transverse distortion; $\Xi\,(u - flavor)$

- Transversely distorted towards left.
- Characteristics of u quark flavor are same as in Λ and Σ.
- Amplitude is same as in Σ.
- Comparatively less spread is observed than Σ.
- Being a partner of a heavy di-quark ss, less spread and small x.



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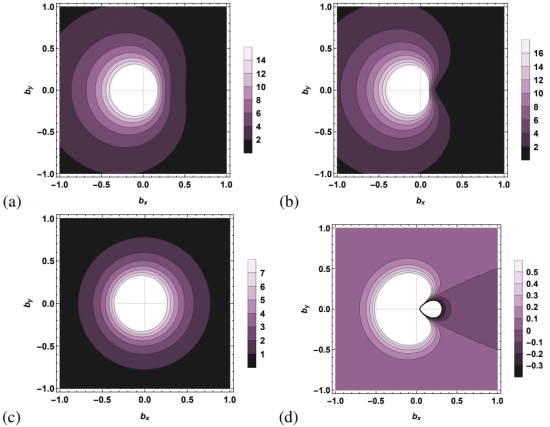
Transverse distortion; $\Xi\,(s - flavor)$

- Transversely distorted towards left.
- Characteristics of s quark flavor are same as in Λ and Σ.
- Amplitude is smaller than Σ.
- But comparatively less spread is observed than Σ.
- Being a partner of a heavy di-quark us, got less spread.



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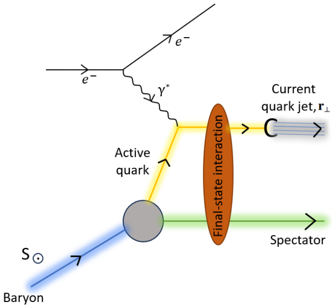
Single-spin asymmetry

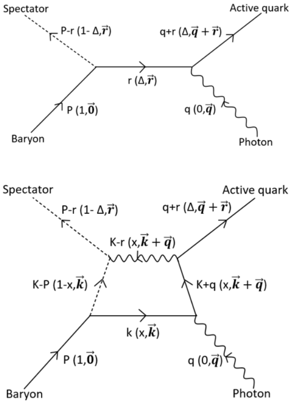
- Presence of transverse distortion implies the presence of single-spin asymmetry (SSA) as per "$SSA = GPD * FSI$" relation.
- Accessible in SIDIS.



Final-state interaction in the semi-inclusive deep inelastic scattering.

Single-spin asymmetry

- Presence of transverse distortion implies the presence of single-spin asymmetry (SSA) as per "$SSA = GPD * FSI$" relation.
- Amplitudes of an interaction can be written by Feynman diagrams.



Tree and one-loop Feynman diagrams for $X + \gamma^* \rightarrow q + \mu_n^0$.

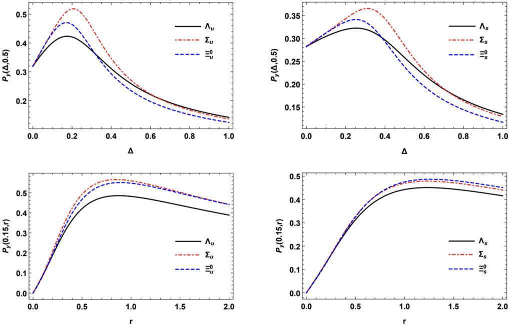
Single-spin asymmetry

- Presence of transverse distortion implies the presence of single-spin asymmetry (SSA) as per "$SSA = GPD * FSI$" relation.
- Amplitudes of an interaction can be written by Feynman diagrams.
- SSA in terms of amplitudes can be written as

$$\begin{aligned}\mathcal{P}_y^{X(q)}(\Delta, r) = [&i\,(\mathcal{A}_X^q(\Uparrow\rightarrow\uparrow)^*\,\mathcal{A}_X^q(\Downarrow\rightarrow\uparrow) \\ &-\mathcal{A}_X^q(\Uparrow\rightarrow\uparrow)\,\mathcal{A}_X^q(\Downarrow\rightarrow\uparrow)^*) \\ &+i\,(\mathcal{A}_X^q(\Uparrow\rightarrow\downarrow)^*\,\mathcal{A}_X^q(\Downarrow\rightarrow\downarrow) \\ &-\mathcal{A}_X^q(\Uparrow\rightarrow\downarrow)\,\mathcal{A}_X^q(\Downarrow\rightarrow\downarrow)^*)]1/C\end{aligned}$$

Single-spin asymmetry

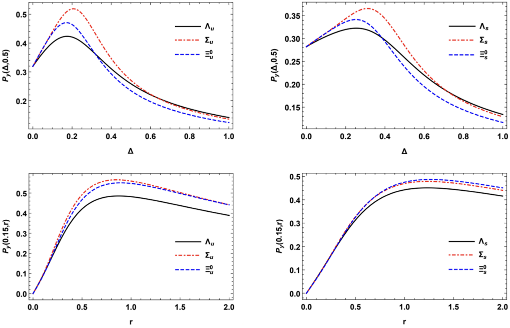
- SSA v/s Δ
  As amplitude of transverse distortion was more for the case of Σ, the amplitude of SSA is also found to be more for Σ.
  Amplitudes fall off fast for heavy baryons.
  Peak value for u quark distributions ia at small Δ than s quark.



Δ = quark light-front momentum fraction
r = magnitude of an active quark momentum jet relative to the virtual photon direction.

Single-spin asymmetry

- SSA v/s r
  More heavier the baryon, more amplitude it will have.
  No significant difference in the amplitudes has been found for heavy baryons.
  Peak value for u quark distributions is at small Δ than s quark.



Δ = quark light-front momentum fraction
r = magnitude of an active quark momentum jet relative to the virtual photon direction.

Outline

- *Internal Structure of the Hadrons*
- *Distribution functions*
  - Generalized Transverse Momentum-Dependent Parton Distributions (GTMDs)
  - Wigner Distributions (WDs)
  - Transverse Momentum-Dependent Parton Distributions (TMDs)
  - Generalized Parton Distributions (GPDs)
- *Transverse distortion*
- *Single-spin asymmetry*
- *Summary*

Summary

- Presence of transverse distortion in transversely polarized baryon give a hint of single-spin asymmetry (SSA) existence.
- Among Λ and Σ with same quark content (2 lighter quarks and 1 strange quark), more the mass of baryon, more is the SSA.
- Among Σ and Ξ, diquark is heavier for the case of Ξ and tends to have more amplitude than an active quark.
- More the mass of a constituent quark, less is the amplitude of SSA.
- Heavier the quark flavor, more share of longitudinal momentum it carries.

Thank you!