Basis Light-Front Quantization: Advancing a First Principle Approach for Hadrons

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BLFQ Collaboration

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Overview

Introduction

Basis Light-Front Quantization (BLFQ) to

Proton : $(|qqq\rangle + |qqqg\rangle)$

Proton : $(|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle)$

Conclusions

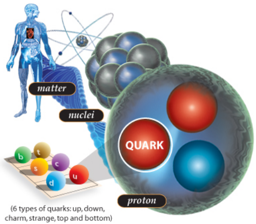
(PRD 108 094002 (2023), PLB 847 138305 (2023), PLB 855 138829 (2024), PLB 855 138831 (2024), work in progress)

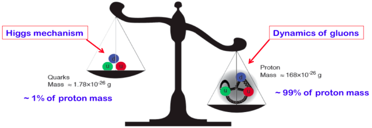
(Satvir Kaur, Thu 22/08, G, 11:00 : Properties of deuteron on the light front)

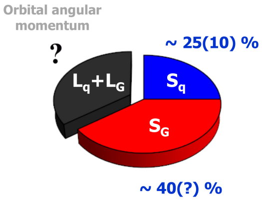
(Xingbo Zhao, Thu 22/08, B, 12:00 : Spatial imaging of the proton from a light-front Hamiltonian approach)

Fundamental Properties: Mass and Spin

- About 99% of the visible mass is contained within nuclei
- Nucleon: composite particles, built from nearly massless quarks ($\sim$ 1% of the nucleon mass) and gluons
- *How does 99% of the nucleon mass emerge?*
- Quantitative decomposition of *nucleon spin* in terms of quark and gluon degrees of freedom is not yet fully understood.
- *To address these fundamental issues → nature of the subatomic force between quarks and gluons, and the internal landscape of nucleons.*

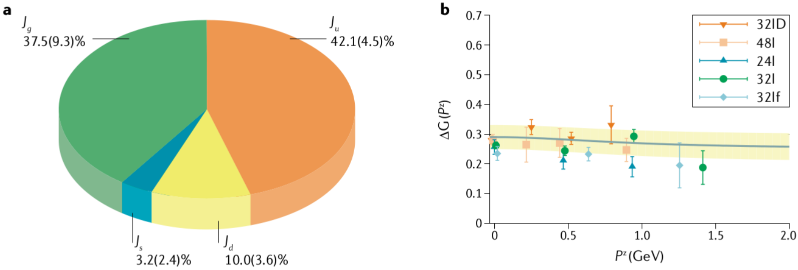






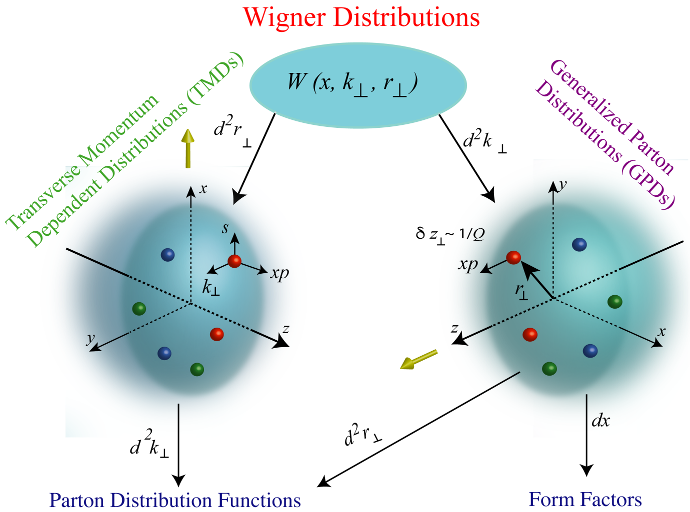
[1] Pictures (top to bottom) adopted from A. Signori, J. Qiu, C. Lorce

Spin sum rule	Formula	Terms	Characteristics
Frame independent (Ji)[30]	$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$	$\Delta\Sigma/2$ is the quark helicity L_q^z is the quark OAM J_g is the gluon contribution	The quark and gluon contributions, J_q and J_g, can be obtained from the GPD moments. A similar sum rule also works for the transverse angular momentum and has a simple parton interpretation
Infinite-momentum frame (Jaffe–Manohar)[31]	$\frac{1}{2}\Delta\Sigma + \Delta G + \ell_q + \ell_g = \frac{\hbar}{2}$	ΔG is the gluon helicity ℓ_q and ℓ_g are the quark and gluon canonical OAM, respectively	All terms have partonic interpretations; ℓ_q and ℓ_g are twist-three quantities. ΔG is measurable in experiments, including the RHIC spin and the EIC; ℓ_q and ℓ_g can be extracted from twist-three GPDs



[1] X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)
[2] Y.-B. Yang, R.S. Sufian, A. Alexandru et al., Phys. Rev. Lett. 118, 102001 (2017)

Hadron Tomography



- $x \to$ longitudinal momentum fraction; $k_\perp \to$ parton transverse momentum; $r_\perp \to$ transverse distance from the center.

Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories

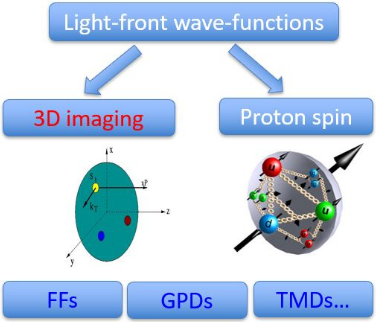
$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^- \equiv P^0 - P^3$: light-front Hamiltonian
- $P^+ \equiv P^0 + P^3$: longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principle / effective Hamiltonian as input
- Evaluate observables

$$O \sim \langle \Psi | \hat{O} | \Psi \rangle$$

- direct access to light-front wavefunction of bound states

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

- Fock expansion of baryonic bound states:

$$|\text{Proton}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots,$$

Solution proposed by BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^\perp)$ in the transverse plane	$\sum_i (2n_i + \lvert m_i\rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

Large $N_{\max}$ and K → High UV cutoff & low IR cutoff

- Exact factorization between center-of-mass motion and intrinsic motion

[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Nucleon within BLFQ

- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b} \kappa^4 \left[x_a x_b (\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a + m_b)^2} \right]$$
$$+ \frac{1}{2}\sum_{a\neq b} \frac{C_F 4\pi\alpha_s}{Q_{ab}^2} \bar{u}_{s_a'}(k_a')\gamma^\mu u_{s_a}(k_a)\bar{u}_{s_b'}(k_b')\gamma^\nu u_{s_b}(k_b) g_{\mu\nu}$$

Publications:

- Mondal et al., Phys. Rev. D 102, 016008 (2020) : Form Factors, PDFs,...
- Xu et al., Phys. Rev. D 104, 094036 (2021) : Nucleon structure, ...
- Liu et al., Phys. Rev. D 105, 094018 (2022) : Angular Momentum, ...
- Hu et al., Phys. Lett. B 833, 137360 (2022) : TMDs, ...
- Kaur et al., Phys. Rev. D 109, 014015 (2024) : Chiral-odd GPDs, ...
- Zhang et al., Phys. Rev. D 109, 034031 (2024) : Twist-3 GPDs, ...
- Liu et al., Phys. Lett. B 855, 138809 (2024) : Skewed GPDs, ...
- Nair et al., arXiv: 2403.11702 [hep-ph] : GFFs, ...
- Peng et al., coming soon : Double parton correlations, ...

Proton with One Dynamical Gluon

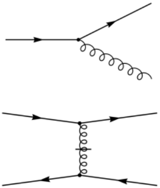
$$P^+P^-|\Psi\rangle = M^2|\Psi\rangle$$

$$|\text{proton}\rangle = \psi_{uud}|uud\rangle + \psi_{uudg}|uudg\rangle$$

$$P^- = P^-_{\text{QCD}} + P^-_C$$

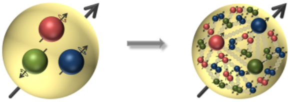
QCD Interaction:

$$P^-_{\text{QCD}} = \int \mathrm{d}x^- \mathrm{d}^2 x^\perp \Big\{ \frac{1}{2}\bar{\psi}\gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+}\psi - \frac{1}{2}A^i_a \left[m_g^2 + (i\partial^\perp)^2\right] A^i_a + g_s \bar{\psi}\gamma_\mu T^a A^\mu_a \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi \Big\},$$



Confinement only in leading Fock:

$$P^-_C P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \Big\{ \{\vec{r}^{\,2}_{ij\perp} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \Big\}$$



Parameters:

Truncation: Nmax=9, K=16.5
HO parameters: b=0.7GeV, b_{inst}=3GeV

m_u	m_d	m_g	κ	m_f	g
0.31GeV	0.25GeV	0.50GeV	0.54GeV	1.80GeV	2.40

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016); M. Burkardt, Phys. Rev. D 58, 096015 (1998).

Proton with One Dynamical Gluon

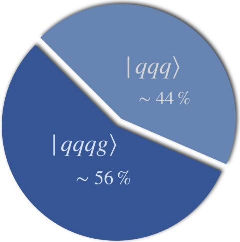
Fock expansion:

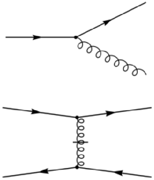
$$|\text{Proton}\rangle = a\,|uud\rangle + b\,|uudg\rangle + \ldots$$

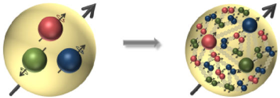
Light-front effective Hamiltonian :

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + H_{\text{confinement}} + H_{\text{vertex}} + H_{\text{inst}}$$

Fock Sector Decomposition



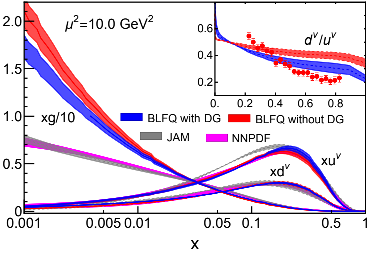


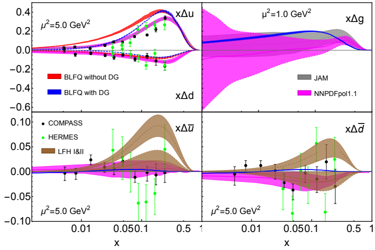


[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108 (2023) 094002.

Unpolarized and Helicity PDFs

BLFQ: PRD 108 (2023) 094002





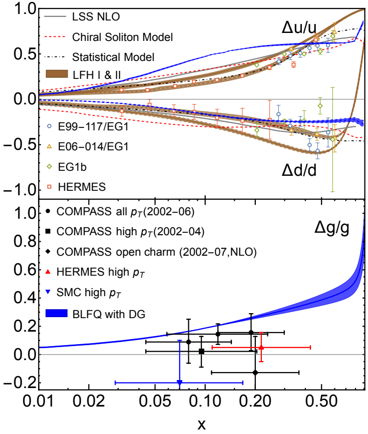
Diagonalizing H_{eff} ⇒ LF wavefunction ⇒ Initial PDFs ⇒ Scale evolution

- Model scale $\mu_0^2 = 0.24 \pm 0.01$ GeV^2.
- Quark momentum : $\langle x \rangle_u = 0.261 \pm 0.005, \quad \langle x \rangle_d = 0.109 \pm 0.005$ at 10 GeV^2.
- Quark spin: $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004, \quad \frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.
- Gluon spin: $\Delta G = 0.131 \pm 0.003$, PHENIX: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$.
- Sea quarks: solely generated from the QCD evolution.

[1] LFH: PRL 124 (2020), 082003; PHENIX: PRL 103 (2009) 012003.

Helicity Asymmetries

BLFQ: PRD 108 (2023) 094002



- Experimentally, the expected increase of $\Delta u/u$ is observed.
- For d quark: remains negative in the experimentally covered region.
- Global analyses favor negative values of $\Delta d/d$ at large-x.

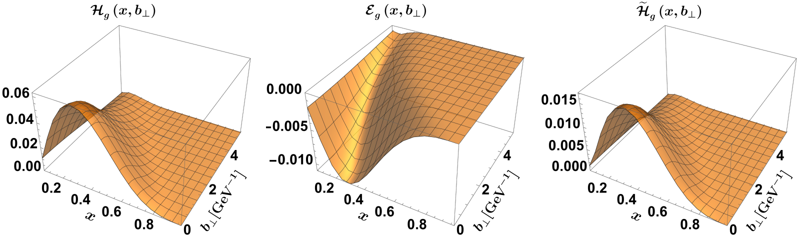
Gluon GPDs

BLFQ : PLB 847 (2023) 138305

$$F^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+ H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^g(x,\xi,t)\right)u(p,\lambda)\,,$$

$$\tilde{F}^g(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\,\bar{u}(p',\lambda')\left(\gamma^+\gamma_5 \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M} \tilde{E}^g(x,\xi,t)\right)u(p,\lambda)\,.$$

Non-skewed GPDs $(x,0,t)$ → FT → GPDs $(x,b_\perp)$



- Model scale $\mu_0^2 = 0.24 \pm 0.01$ GeV^2.
- Total Angular Momentum: $J = \frac{1}{2}\int \mathrm{d}x\, x[H(x,0) + E(x,0)]$;
  $J_g = 0.066$, 13.2% of the proton TAM.

Zhao's talk: Thu 22/08, B, 12:00

BLFQ Predictions for Spin Decomposition

Fock expansion:

$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + ...$$

Quark and gluon helicities :

$$\Delta\Sigma_q = \int \mathrm{d}x\,\Delta q(x)$$

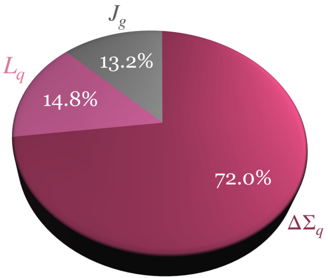
$$\Delta\Sigma_g = \int \mathrm{d}x\,\Delta G(x)$$

Total AM[2] :

$$J_i = \frac{1}{2}\int \mathrm{d}x\,x\,[H_i(x,0,0) + E_i(x,0,0)]$$

Kinetic OAM[2] :

$$L_q = \frac{1}{2}\int \mathrm{d}x\,[x\,\{H_q(x,0,0)+E_q(x,0,0)\}-\widetilde{H}_q(x,0,0)]$$



Kinetic[1]

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, Phys.Rev.D 108, 094002 (2023).
[2] X. Ji, Phys.Rev.Lett. 78, 610 (1997).

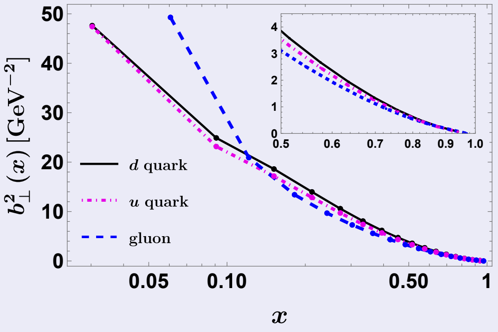
x-Dependent Squared Radius

$$\langle b_\perp^2 \rangle^i(x) = \frac{\int d^2\vec{b}_\perp b_\perp^2 \mathcal{H}^i(x, b_\perp)}{\int d^2\vec{b}_\perp \mathcal{H}^i(x, b_\perp)},$$

- Transverse squared radius:

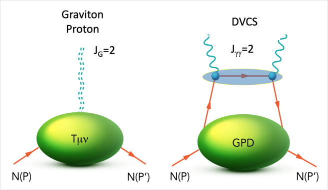
$$\langle b_\perp^2 \rangle = \sum_i e_q \int_0^1 \mathrm{d}x f^i(x) \langle b_\perp^2 \rangle^i(x)$$

- BLFQ[1]: $\langle b_\perp^2 \rangle = 0.47 \pm 0.04\ \mathrm{fm}^2$
- Experimental data [2]: $\langle b_\perp^2 \rangle_{\exp} = 0.43 \pm 0.01\ \mathrm{fm}^2$



[1] B. Lin, S. Nair, S.Xu, CM, X. Zhao, J. P. Vary, PLB 847, 138305 (2023).
[2] R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

Proton Gravitational Form Factors



- Parametrization of matrix element in terms of GFFs

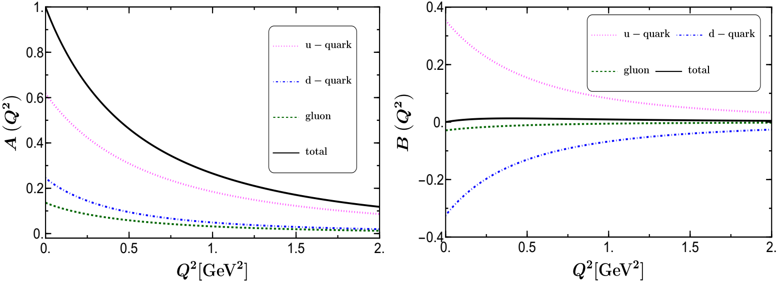
$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big]U$$

- Momentum sum rule : $\sum_i A^i(0) = 1$
- Gravitomagnetic moment sum rule : $\sum_i B^i(0) = 0$
- Spin sum rule: $J^i = \frac{1}{2}\left[A^i(0) + B^i(0)\right]$
- $4C(q^2) = D(q^2)$ provides shear forces and the pressure distributions

[1] Burkert *et. al.*: Rev. Mod. Phys. 95, 041002 (2023)
[2] Ji, Phys. Rev. Lett. 78, 610 (1997)

$A(Q^2)$ and $B(Q^2)$

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$



- $A(Q^2)$ and $B(Q^2)$: T^{++} component
- Spin sum rule: $J^i = \frac{1}{2}\left(A^i(0) + B^i(0)\right)$

$$\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0$$

- $D(Q^2) = 4C(Q^2)$: T^{ij} components

[1] S. Nair, CM, *et. al.* coming soon...

Overview of TMDs for Spin-1/2 Target

Quark correlator

$$\Phi_q^{[\Gamma]}\left(P,S;x=\frac{k^+}{P^+},\vec{k}_\perp\right)=\frac{1}{2}\int\frac{\mathrm{d}z^-\mathrm{d}z^\perp}{2(2\pi)^3}e^{ik\cdot z}\langle P,S|\,\bar{\Psi}_q(0)\Gamma\mathcal{W}(0^\perp,z^\perp)\Psi_q(z)\,|P,S\rangle\Big|_{z^+=0},$$

Parameterization:

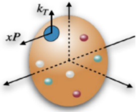
8 twist-2 TMDs:

6 T-even terms
2 T-odd terms

$$\Phi^{[\gamma^+]}=f_1-\frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M}f_{1T}^\perp,$$

$$\Phi^{[\gamma^+\gamma^5]}=\Lambda g_{1L}+\frac{k_\perp\cdot \boldsymbol{S}_\perp}{M}g_{1T},$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}=S_\perp^j h_1+\Lambda\frac{k_\perp^j}{M}h_{1L}^\perp+S_\perp^i\frac{2k_\perp^i k_\perp^j-(k_\perp)^2\delta^{ij}}{2M^2}h_{1T}^\perp+\frac{\epsilon_\perp^{ji}k_\perp^i}{M}h_1^\perp,$$



16 twist-3 TMDs:

8 T-even terms
8 T-odd terms

$$\Phi^{[1]}=\frac{M}{P^+}\left[e-\frac{\epsilon_T^{\rho\sigma}k_{\perp\rho}S_{T\sigma}}{M}e_T^\perp\right],$$

$$\Phi^{[i\gamma_5]}=\frac{M}{P^+}\left[S_L e_L-\frac{k_\perp\cdot S_T}{M}e_T\right],$$

$$\Phi^{[\gamma^\alpha]}=\frac{M}{P^+}\left[-\epsilon_T^{\alpha\rho}S_{T\rho}f_T-S_L\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}f_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}\epsilon_{T\rho\sigma}S_T^\sigma f_T^\perp+\frac{k_\perp^\alpha}{M}f^\perp\right],$$

$$\Phi^{[\gamma^\alpha\gamma_5]}=\frac{M}{P^+}\left[S_T^\alpha g_T+S_L\frac{k_\perp^\alpha}{M}g_L^\perp-\frac{k_\perp^\alpha k_\perp^\rho-\frac{1}{2}k_\perp^2 g_T^{\alpha\rho}}{M^2}S_{T\rho}g_T^\perp-\frac{\epsilon_T^{\alpha\rho}k_{\perp\rho}}{M}g^\perp\right],$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]}=\frac{M}{P^+}\left[\frac{S_T^\alpha k_\perp^\beta-k_\perp^\alpha S_T^\beta}{M}h_T^\perp-\epsilon_T^{\alpha\beta}h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}=\frac{M}{P^+}\left[S_L h_L-\frac{k_\perp\cdot S_T}{M}h_T\right].$$

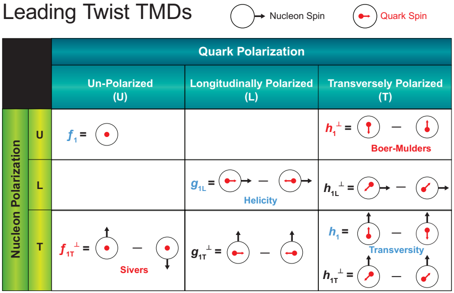
Jaffe-Ji notation:

f, e → unpolarized quarks
g → longitudinally polarized quarks
h → transversely polarized quarks

1 → the leading twist
L → longitudinally polarized hadron
T → transversely polarized hadron
⊥→ existing $k_\perp$ with a non-contracted index

[1] Meißner, et. al. JHEP08 (2009) 056.

Quark TMDs

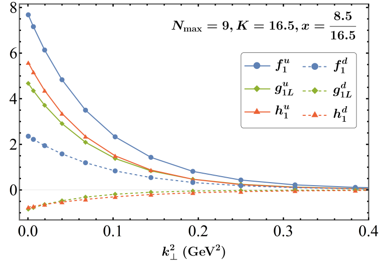


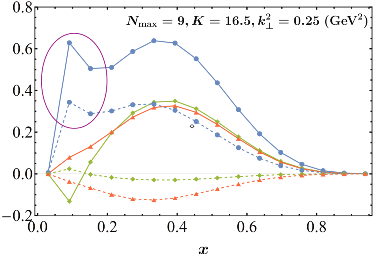
- Positivity bounds

$$f_1^g(x,\boldsymbol{k}_\perp^2) > 0, \quad f_1^g(x,\boldsymbol{k}_\perp^2) \geq |g_{1L}^g(x,\boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x,\boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M}|g_{1T}^g(x,\boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x,\boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2}|h_1^{\perp g}(x,\boldsymbol{k}_\perp^2)|$$





[1] Hongyao Yu, *et. al.* coming soon...

[2] A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.

Gluon TMDs

- Small-x limit

$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 |\boldsymbol{k}_\perp^2| h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)}{2M^2 \int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 1$$

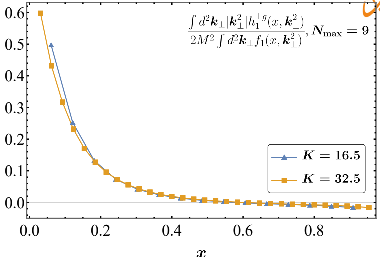
- Helicity asymmetry:

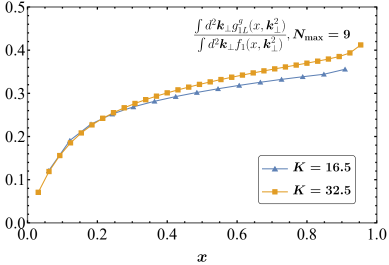
$$\lim_{x\to 0} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x, \boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 0,$$

$$\lim_{x\to 1} \frac{\int d\boldsymbol{k}_\perp^2 g_{1L}^g(x, \boldsymbol{k}_\perp^2)}{\int d\boldsymbol{k}_\perp^2 f_1^g(x, \boldsymbol{k}_\perp^2)} = 1$$

.

- With larger truncation K, satisfies the limiting cases.





[1] Hongyao Yu, *et. al.* Phys.Lett.B 855, 138831 (2024)

Gaussian Ansatz Compatibility?

- To check compatibility of BLFQ results with the Gaussian ansatz :

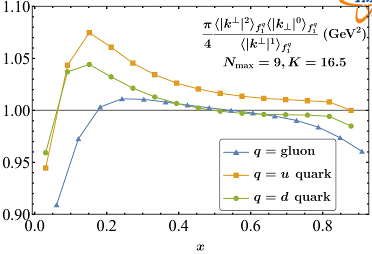
$$f_1^i(x, k_\perp^2) \approx a \frac{\exp\left(-\frac{|k_\perp|^2}{r}\right)}{\pi r}$$

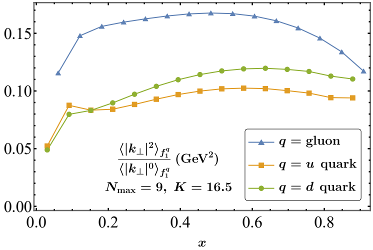
where $a = \langle |k_\perp|^0 \rangle_{f_1^i}$ and $r = \langle |k_\perp|^2 \rangle_{f_1^i}$

- If the Gaussian ansatz holds :

$$\frac{\langle |k_\perp|^2 \rangle_{f_1^i} \times \langle |k_\perp|^0 \rangle_{f_1^i}}{(\langle |k_\perp|^1 \rangle_{f_1^i})^2} \times \frac{\pi}{4} = 1$$

BLFQ results do not support Gaussian ansatz





[1] Hongyao Yu, *et. al.* in preparation

[2] Hongyao Yu, *et. al.* Phys.Lett.B 855, 138831 (2024)

Twist-3 Quark TMDs

Equation of motion relation: $$e = \tilde{e} + \frac{m}{M}\frac{f_1}{x}$$

twist-3 TMD = genuine twist-3 TMD + twist-2 TMD

$$f_1(x, k_\perp) \sim \langle P|\bar{\psi}\gamma^+\psi|P\rangle \sim \int [\mathrm{D}]\psi^*_{uud}\psi_{uud} + \int [\mathrm{D}]\psi^*_{uudg}\psi_{uudg}$$

$$\tilde{e}(x, k_\perp) \sim \langle P|\bar{\psi}\sigma^{j+}A_j\psi|P\rangle \sim \int [\mathrm{D}]\left[\psi^*_{uud}\psi_{uudg} + \mathrm{h.c.}\right]$$

Higher-twist distributions:

- ❑ Reflecting the physics of multi-parton correlations.
- ❑ Requiring the LFWFs of different Fock sectors.

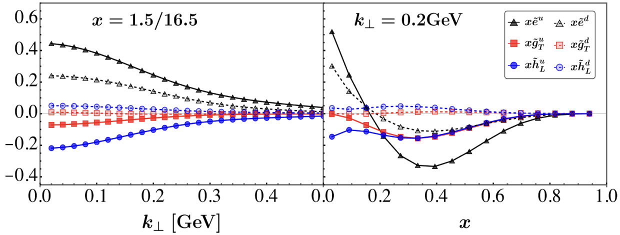
❖ **Multi-parton distributions** correspond to the interference of hadron wave functions between **different** Fock sectors.

❖ The physics of **quark-gluon correlations:**

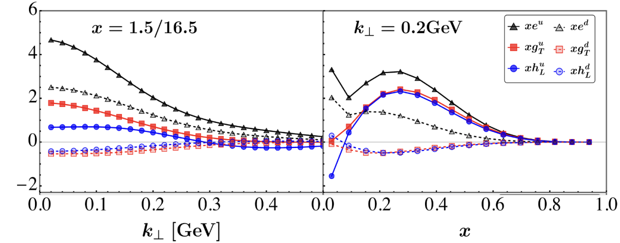
- Color Lorentz force
- Average transverse force
- Partonic transverse AM [X.D. Ji, et al. NPB 969 (2021) 115440]
- ……

[1] Zhimin Zhu, *et. al.* in preparation

Genuine twist-3 TMD:



twist-3 TMD:



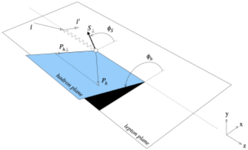
twist-3 TMD = genuine twist-3 TMD + twist-2 TMD

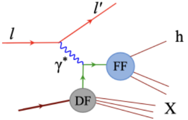
[1] Zhimin Zhu, *et. al.* Phys.Lett.B 855 (2024) 138829

Semi-inclusive DIS

$$\frac{d\sigma}{dxdydzdP_{hT}^2 d\varphi_h d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\times$$

$$\left\{\begin{array}{l}
1+\cos\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UU}^{\cos\phi_h}\right)+\cos 2\phi_h\left(\varepsilon A_{UU}^{\cos 2\phi_h}\right) \\
+\ \lambda\sin\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LU}^{\sin\phi_h}\right) \\
+\ S_L\left[\sin\phi_h\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UL}^{\sin\phi_h}\right)+\sin 2\phi_h\left(\varepsilon A_{UL}^{\sin 2\phi_h}\right)\right] \\
+\ S_L\lambda\left[\sqrt{1-\varepsilon^2}A_{LL}+\cos\phi_h\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LL}^{\cos\phi_h}\right)\right] \\
+\ S_T\left[\begin{array}{l}
\sin(\phi_h-\phi_S)\left(A_{UT}^{\sin(\phi_h-\phi_S)}\right) \\
+\ \sin(\phi_h+\phi_S)\left(\varepsilon A_{UT}^{\sin(\phi_h+\phi_S)}\right) \\
+\ \sin(3\phi_h-\phi_S)\left(\varepsilon A_{UT}^{\sin(3\phi_h-\phi_S)}\right) \\
+\ \sin\phi_S\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin\phi_S}\right) \\
+\ \sin(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1+\varepsilon)}A_{UT}^{\sin(2\phi_h-\phi_S)}\right)
\end{array}\right] \\
+\ S_T\lambda\left[\begin{array}{l}
\cos(\phi_h-\phi_S)\left(\sqrt{(1-\varepsilon^2)}A_{LT}^{\cos(\phi_h-\phi_S)}\right) \\
+\ \cos\phi_S\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos\phi_S}\right) \\
+\ \cos(2\phi_h-\phi_S)\left(\sqrt{2\varepsilon(1-\varepsilon)}A_{LT}^{\cos(2\phi_h-\phi_S)}\right)
\end{array}\right]
\end{array}\right\}$$





Factorization Theorem:

Twist-2

Twist-3

$$A_{UT}^{\sin(\phi_h-\phi_S)} \propto f_{1T}^{\perp}\otimes D_1$$

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_1\otimes H_1^{\perp}$$

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T}^{\perp}\otimes H_1^{\perp}$$

$$A_{UT}^{\sin(\phi_S)} \propto \frac{M}{Q}\left(f_T\otimes D_1 + h_1\otimes H_1^{\perp}+\cdots\right)$$

$$A_{UT}^{\sin(2\phi_h-\phi_S)} \propto \frac{M}{Q}\left(h_T\otimes H_1^{\perp} + h_T^{\perp}H_1^{\perp}+\cdots\right)$$

$$A_{LT}^{\cos(\phi_h-\phi_S)} \propto g_{1T}\otimes D_1$$

$$A_{LT}^{\cos(\phi_S)} \propto \frac{M}{Q}\left(g_T\otimes D_1 + e_T\otimes H_1^{\perp}+\cdots\right)$$

$$A_{LT}^{\cos(2\phi_h-\phi_S)} \propto \frac{M}{Q}\left(e_T\otimes H_1^{\perp} + e_T^{\perp}\otimes H_1^{\perp}+\cdots\right)$$

......

[1] Bacchetta, et al, JHEP 02 (2007) 093

[1] Zhimin Zhu, *et. al.* Phys.Lett.B 855 (2024) 138829

$A_{LT}^{\cos\phi_S}$ in π^+-produced SIDIS process in EicC

Structure functions after integrating over $P_{h\perp}$,

$$F_{UU} = x\sum_q e_q^2 f_1^q(x) D_1^q(z)$$

$$F_{LL} = x\sum_q e_a^2 g_1^a(x,Q^2) D_1^a(z,Q^2),$$

$$F_{LT}^{\cos\phi_S} = -x\sum_q e_q^2 \frac{2M_h}{Q}\left[x g_T^q(x) D_1^q(z) + \frac{M_h}{M} h_1^q(x)\frac{\tilde{E}^q(z)}{z}\right].$$

Input:

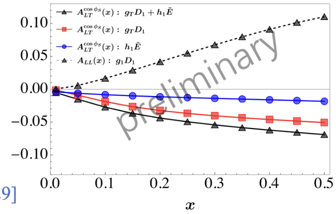
- PDFs: $f_1(x)$, $h_1(x)$ and $g_T(x)$ from [PLB 855 (2024) 138829]
- FF: $D_1(z)$ from [PRD, 75, 114010, 2007]
- Model quark-gluon-quark FF, $\tilde{E}^a(z) \approx \frac{m_a}{M_h}\frac{z}{1-z}D_1^a(z)$

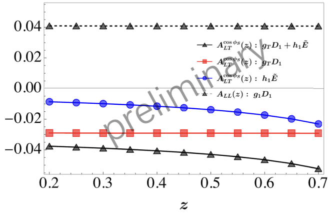
Kinematical region of EicC:

$$0.005 < x < 0.5, \quad 0.07 < y < 0.9,$$

$$0.2 < z < 0.7, \quad \text{fixed} \quad Q^2 = 5\text{GeV}^2.$$

- ✓ Twist-3 DSA $A_{LT}^{\cos\phi_s}$ is sizable.
- ✓ EicC has potential advantages in studying high-twist spin asymmetries.





[1] Zhimin Zhu, *et. al.* in preparation

Effective Hamiltonian with Dynamical Gluon and Sea Quarks

Fock expansion:

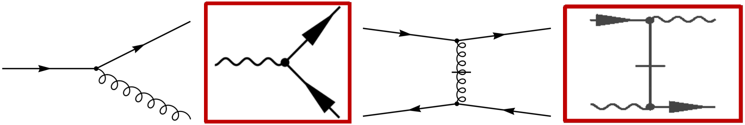
$$|\,\text{Proton}\rangle = a\,|\,uud\rangle + b\,|\,uudg\rangle + c_1\,|\,uudu\bar{u}\rangle + c_2\,|\,uudd\bar{d}\rangle + c_3\,|\,uuds\bar{s}\rangle + \ldots$$

Light-front QCD Hamiltonian :

$$H_{\text{LF}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + \sout{H_{\text{confinement}}} + H_{\text{vertex}} + H_{\text{inst}}$$

$$H_{\text{vertex}} + H_{\text{inst}} = g_s \bar{\psi}\gamma_\mu T^a A^\mu_a \psi + \frac{1}{2} g_s^2 \bar{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi}\gamma^+ T^a \psi$$

$$+ \frac{1}{2} g_s^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{(i\partial^+)} A_\nu \gamma^\nu \psi$$



[1] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

[2] Siqi Xu, Yiping Liu, CM, *et. al.*, coming soon ...

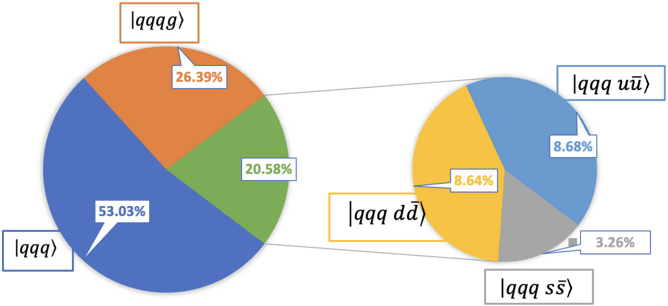
Fock Sector Decomposition

$$|P_{proton}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 16$

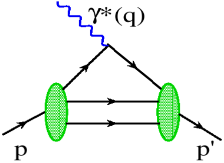
m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

In five quark Fock sector, we use current quark mass



[1] Siqi Xu, Yiping Liu, CM, *et. al.*, coming soon ...

Proton EM Form Factors



- EM current: $J^\mu = \bar{\psi}\gamma^\mu\psi$
- $\langle p';\uparrow|J^+(0)|p;\uparrow(\downarrow)\rangle \sim F_{1(2)}(q^2)$
- Two FFs: $F_{1(2)}(q^2 = -Q^2)$

Sach's form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2),$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2).$$

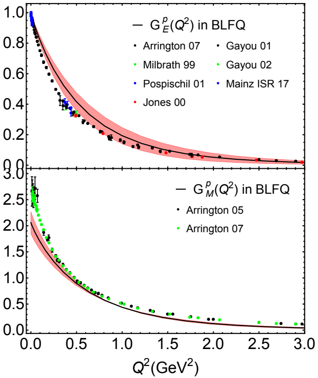
Proton Radii

$$\langle r_E^2\rangle = -6\frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0},$$

$$\langle r_M^2\rangle = -\frac{6}{G_M(0)}\frac{dG_M(Q^2)}{dQ^2}\bigg|_{Q^2=0}.$$

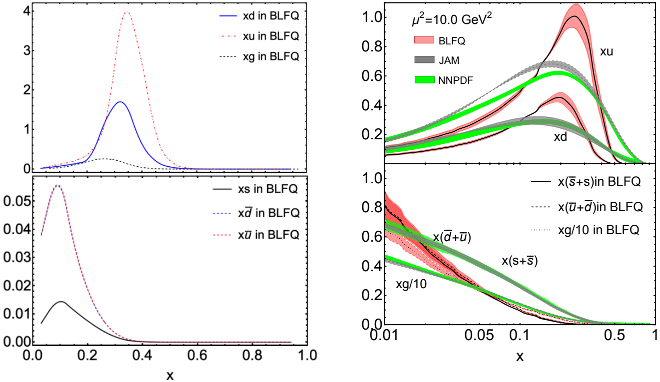
$$\sqrt{\langle r_E^2\rangle} = 0.72 \pm 0.05\,(0.840^{+0.003}_{-0.002})\text{ fm}$$

$$\sqrt{\langle r_M^2\rangle} = 0.73 \pm 0.02\,(0.849^{+0.003}_{-0.003})\text{ fm}$$



[1] Siqi Xu, Yiping Liu, CM, *et. al.*, coming soon ...

Unpolarized PDFs

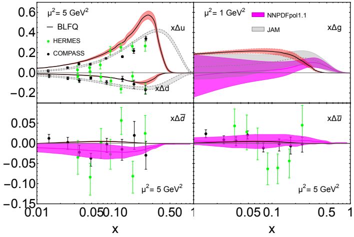


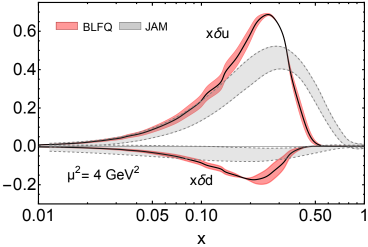
Diagonalizing H_{LFQCD} ⇒ LFWFs ⇒ Initial PDFs ⇒ Scale evolution

- Model scale $\mu_0^2 = 0.22 \pm 0.02$ GeV^2, $\langle x \rangle_{u+d} = 0.37 \pm 0.01$ 10 GeV^2.
- Longitudinal excitations challenging, in absence of confining potential.

[1] Siqi Xu, Yiping Liu, CM, *et. al.*, coming soon ...

Helicity and Transversity PDFs





- Gluon spin $\Delta G = 0.29 \pm 0.03$[1] for $x_g \in [0.05, 0.2]$ at 10 GeV^2
- NNPDF[2] analysis: $\Delta G = 0.23(6)$; lattice QCD[3]: $\Delta G = 0.251(47)(16)$
- Tensor Charges: $\delta u = 0.81 \pm 0.08,\ \delta d = -0.22 \pm 0.01$[1]
- JAM[4] analysis: $\delta u = 0.71(2)$, $\delta d = -0.200(6)$; lattice QCD[5]: $\delta u = 0.784(28)$, $\delta d = -0.204(11)$.

---

[1] Siqi Xu, Yiping Liu, CM, *et. al.*, coming soon ...
[2] E. R. Nocera, *et. al.* (NNPDF), Nuclear Physics B 887, 276 (2014)
[3] Y.-B. Yang, *et. al.* (Lattice) Phys. Rev. Lett. 118, 102001 (2017)
[4] C. Cocuzza, *et. al.* (JAM), Phys. Rev. Lett. 132, 091901 (2024)
[5] R. Gupta, *et. al.* (Lattice), Phys. Rev. D 98, 091501 (2018)

Conclusions

- Basis Light-front Quantization : A non-perturbative approach based on light-front QCD Hamiltonian
- LF Hamiltonian ⇒ Wavefunctions ⇒ Observables.
- $|qqq\rangle + |qqqg\rangle$ $(P^- = P^-_{\rm QCD} + P^-_{\rm C})$ ⇒ Provides good description of data/global fits for various observables.
- $|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle$, $(P^- = P^-_{\rm QCD})$ ⇒ Provides qualitative description of data/global fits for mass, spin, EMFFs, PDFs . . . .
  Zhao's talk: Thu 22/08, B, 12:00

Outlook

- Include three-gluon and four-gluon interactions in the Hamiltonian.
- *This is not a complete picture . . . long way to go*.

Enormous amount of possibilities with future EICs . . . . . . Thank You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TMDs of Spin-1/2 Target

Gluon TMDs correlator :

$$\Phi^{g[ij]}(x,\vec{k}_\perp;S) = \frac{1}{xP^+}\int \frac{dz^-}{2\pi}\frac{d^2\vec{z}_\perp}{(2\pi)^2} e^{ikz}\langle P;S|F_a^{+j}(0)\mathcal{W}_{+\infty,ab}(0;z)F_b^{+i}(z)|P;S\rangle\,|_{z^+=0^+}$$

Parametrization

$$\Phi^g(x,\vec{k}_\perp;S) = \delta_\perp^{ij}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= f_1^g(x,\vec{k}_\perp^2) - \frac{\epsilon_\perp^{ij}k_\perp^i S_\perp^j}{M} f_{1T}^{\perp g}(x,\vec{k}_\perp^2)$$

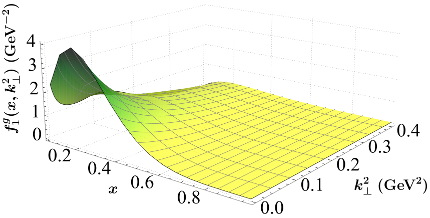
$$\tilde{\Phi}^g(x,\vec{k}_\perp;S) = i\epsilon_\perp^{ij}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= S^3 g_{1L}^g(x,\vec{k}_\perp^2) + \frac{\vec{k}_\perp\cdot\vec{S}_\perp}{M} g_{1T}^g(x,\vec{k}_\perp^2)$$

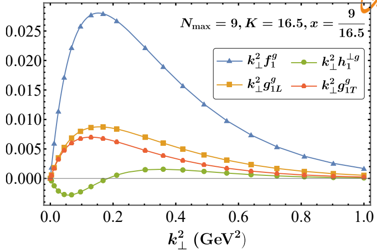
$$\Phi_T^{g,ij}(x,\vec{k}_\perp;S) = -\hat{\mathbf{S}}\Phi^{g[ij]}(x,\vec{k}_\perp;S)$$
$$= -\frac{\hat{\mathbf{S}}k_\perp^i k_\perp^j}{2M^2} h_1^{\perp g}(x,\vec{k}_\perp^2) + \frac{S^3\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}k_\perp^k}{2M^2} h_{1L}^{\perp g}(x,\vec{k}_\perp^2)$$
$$+ \frac{\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}S_\perp^k}{2M}\left(h_{1T}^g(x,\vec{k}_\perp^2) + \frac{\vec{k}_\perp^2}{2M^2}h_{1T}^{\perp g}(x,\vec{k}_\perp^2)\right)$$
$$+ \frac{\hat{\mathbf{S}}k_\perp^i\epsilon_\perp^{jk}(2k_\perp^k\vec{k}_\perp\cdot\vec{S}_\perp - S_\perp^k\vec{k}_\perp^2)}{4M^3} h_{1T}^{\perp g}(x,\vec{k}_\perp^2),$$

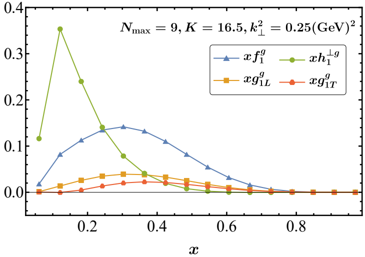
	PARTON SPIN		
GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta},\ldots$
TARGET SPIN U	f_1^g		$h_1^{\perp g}$
TARGET SPIN L		g_1^g	$h_{1L}^{\perp g}$
TARGET SPIN T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$

[1] A. Accardi *et al.*, Eur.Phys.J.A 52 (2016) 9, 268.
[2] Meißner, et. al. PRD D 76 (2007), 034002.
[3] Pisano's, Khatiza's...talks

Gluon TMDs







- Positivity bounds

$$f_1^g(x, \boldsymbol{k}_\perp^2) > 0, \quad f_1^g(x, \boldsymbol{k}_\perp^2) \geq |g_{1L}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|}{M} |g_{1T}^g(x, \boldsymbol{k}_\perp^2)|,$$

$$f_1^g(x, \boldsymbol{k}_\perp^2) \geq \frac{|\boldsymbol{k}_\perp|^2}{2M^2} |h_1^{\perp g}(x, \boldsymbol{k}_\perp^2)|$$

- Satisfies Mulders-Rodrigues relations

[1] Hongyao Yu, *et. al.* coming very soon...

GPDs and GFFs

- The second Mellin's moment of GPDs:

$$\int \mathrm{d}x\, x\, H(x,\xi,t) = A(t) + \xi^2 D(t)$$

$$\int \mathrm{d}x\, x\, E(x,\xi,t) = B(t) - \xi^2 D(t)$$

- GPDs in terms of the Compton Form Factors :

$$\mathrm{Re}\mathcal{H}(\xi,t) + i\,\mathrm{Im}\mathcal{H}(\xi,t) = \int_0^1 \mathrm{d}x \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x + i\epsilon}\right] H(x,\xi,t)$$

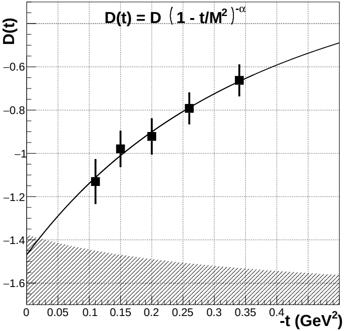
- Compton Form Factors are directly related to the observables we can experimentally determine in DVCS measurements.

- In DVCS experiments, GPDs are not directly accessible in the full x-space, but only at $x = \pm\xi$

D-term

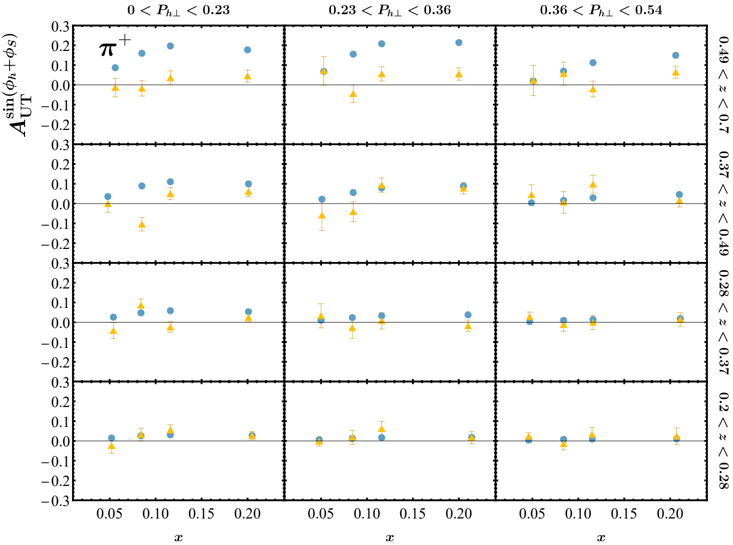
- Only $D(t) = 4C(t)$ GFF can be extracted via DVCS
- $D(t)$ can be determined from the dispersion relation :

$$D(t) = \mathrm{Re}\mathcal{H}(\xi, t) - \frac{1}{\pi}\mathcal{P}\int_0^1 \mathrm{d}x \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] \mathrm{Im}\mathcal{H}(\xi, t)$$



[Fig: Burkert *et. al.*: 2310.11568]

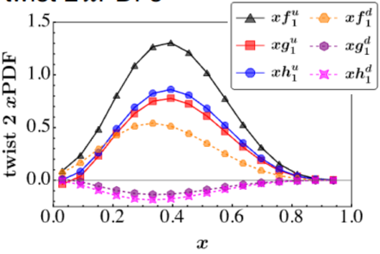
Spin Asymmetry in SIDIS



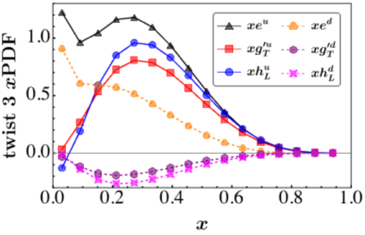
[1] Honhyao, *et. al.* in preparation

xPDFs: Twist-2 vs Twist-3

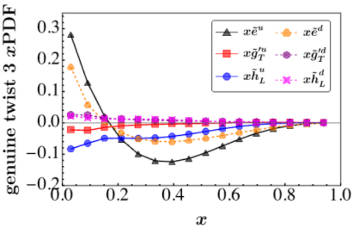
twist-2 xPDFs



twist-3 xPDFs



genuine twist-3 xPDFs



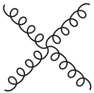
$$\int \frac{d^2k_\perp}{(2\pi)^2} f(x, k_\perp) = f(x)$$

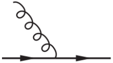
Twist-3 PDFs:

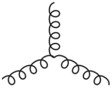
- more concentrating in small x
- similar magnitude to twist-2 PDFs

[1] Zhimin Zhu, *et. al.* in preparation

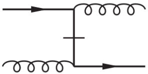
Light-Front QCD with Light-Cone Gauge ($A^+ = 0$)

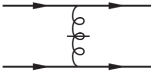


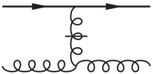


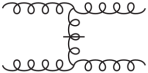


$$
\begin{aligned}
\hat{P}^-_{\rm LFQCD} =& \frac{1}{2}\int \mathrm{d}x^- \mathrm{d}^2x^\perp \;\; \overline{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\psi + A^{ia}(i\partial^\perp)^2 A^{ia} \\
&+ g_s \int \mathrm{d}x^- \mathrm{d}^2x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a} T^a \psi \\
&+ \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2x^\perp \; \overline{\psi}\gamma_\mu A^{\mu a} T^a \frac{\gamma^+}{i\partial^+}\Big(\gamma_\nu A^{\nu b} T^b \psi\Big) \\
&+ \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2x^\perp \; \overline{\psi}\gamma^+ T^a \psi \frac{1}{(i\partial^+)^2}\Big(\overline{\psi}\gamma^+ T^a \psi\Big) \\
&- g_s^2 \int \mathrm{d}x^- \mathrm{d}^2x^\perp \; if^{abc}\, \overline{\psi}\gamma^+ T^c \psi \frac{1}{(i\partial^+)^2}\Big(i\partial^+ A^{\mu a} A^b_\mu\Big) \\
&+ g_s \int \mathrm{d}x^- \mathrm{d}^2x^\perp \; if^{abc}\, i\partial^\mu A^{\nu a} A^b_\mu A^c_\nu \\
&+ \frac{g_s^2}{2}\int \mathrm{d}x^- \mathrm{d}^2x^\perp \; if^{abc}\, if^{ade}\, i\partial^+ A^{\mu b} A^c_\mu \frac{1}{(i\partial^+)^2}\Big(i\partial^+ A^{\nu d} A^e_\nu\Big) \\
&- \frac{g_s^2}{4}\int \mathrm{d}x^- \mathrm{d}^2x^\perp \; if^{abc}\, if^{ade} A^{\mu b} A^{\nu c} A^d_\mu A^e_\nu.
\end{aligned}
$$









[1] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, Phys. Rep. 301, 299-486 (1998).

Parameters

$$|P,\Lambda\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle$$

- **We use following observables to fix the parameters in the first two Fock sectors**
  - **Nucleon mass**
  - **Nucleon electromagnetic form factors**

m_u	m_d	m_f	g	b	b_{inst}
0.99 GeV	0.94 GeV	5.9 GeV	3.0	0.6 GeV	2.7 GeV

- **The parameters effectively parameterize certain non-perturbative dynamics**

- **In five-quark Fock component, the quark masses are equal to current quark masses**

m_u	m_d	m_s
0.00216 GeV	0.00467 GeV	0.0934 GeV

Truncation parameters: $N_{\max} = 7$ **and** $K_{\max} = 16$