In-medium heavy-quark interactions (from lattice QCD)

Johannes H. Weber (they/them)

[Humboldt-Universität zu Berlin & RTG2575]

XVIth Quark Confinement and the Hadron Spectrum,
Cairns, Australia, 08/23/2024

Heavy quarks in ultrarelativistic heavy-ion collisions. . .

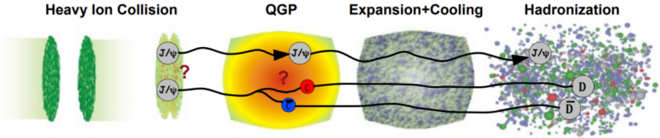

Figure: Steffen Bass, mod. by O. Kaczmarek

- HIC produce **mini big bangs**: expanding, dynamical, different stages.
- **Hard processes** that produce **HQ** occur only in pre-equilibrium stage.

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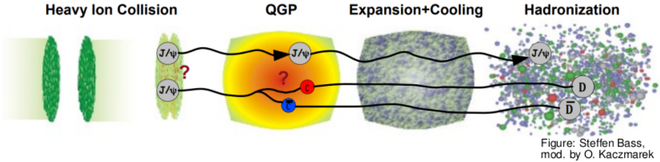


- HIC produce **mini big bangs**: expanding, dynamical, different stages.
- **Hard processes** that produce **HQ** occur only in pre-equilibrium stage.
- **HQ** pair in color-octet state: fly apart before medium forms $\Rightarrow$ **transport**.
- **HQ** pair in color-singlet state: bind before medium forms $\Rightarrow$ **quarkonia**.

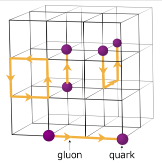
Lattice gauge theory in a nutshell

- **Non-perturbative lattice regularization** of QCD path integral: IR or UV via finite volume or lattice spacing ⇒ infinite volume and **continuum limit**
- Gauge invariance w/o Leibniz rule: change of variables $A_\mu(x) \Rightarrow U_\mu(x)$

$$U_\mu(x) = \mathrm{e}^{\mathrm{i}aA_\mu(x)} \qquad \text{gauge link}$$

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x) \qquad \text{plaquette}$$

$$S_{\mathrm{QCD}}[U,\bar{\psi},\psi] = a^4 \sum_{x,y\in V_4} \sum_f \bar{\psi}^f(x) M_f[U](x,y)\psi^f(y) - \sum_{x\in V_4}\sum_{\mu<\nu} \frac{1}{g_0^2} \mathrm{Re}\ \mathrm{Tr}\{1 - U_{\mu\nu}(x)\}$$



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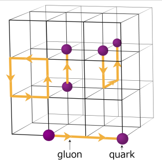
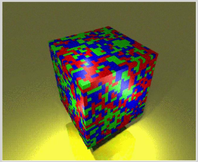
- **MCMC evaluation** of path integral $\Rightarrow$ quantum nature as **fluctuations**



HPC $\Longrightarrow$



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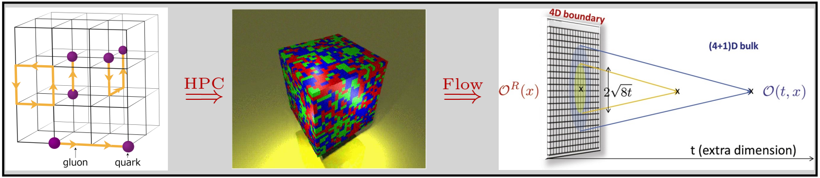
- **Gradient flow** at flow depth t tames **UV fluctuations** within radius $\sqrt{8t}$:

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t)\;, \qquad B_\mu(x,t=0) = A_\mu(x)$$

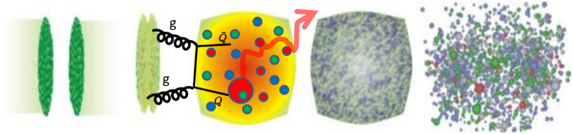
$$D_\nu X(x,t) = \partial_\nu + [B_\nu(x,t), X(x,t)]\;, \qquad G_{\nu\mu}(x,t) = \Big[\partial_\mu B_\nu - \partial_\nu B_\mu + [B_{|\mu}, B_\nu\Big](x,t)$$



Overview

Heavy-quark transport

- **Open heavy flavors** in QGP may arise either from the dissociation of quarkonia, or by not binding into color-singlet states in the first place.
- Thermalized **HQ**: $E = \boldsymbol{p}^2/2m_\mathrm{h} \sim T \ll p \sim \sqrt{m_h T} \ll m_h$, undergo **Langevin-type** evolution w independent, small kicks $\Delta p \sim T$.



original figure by S. Bass

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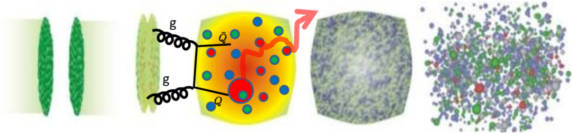

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- Three **HQ transport coefficients** are related by Einstein relations

$$D_\mathrm{s} = \frac{2T^2}{\kappa}\frac{\langle \boldsymbol{p}^2\rangle}{3m_\mathrm{h}T}, \quad \text{and} \quad \frac{\eta_\mathrm{D}}{\kappa} = \frac{1}{2m_\mathrm{h}T} = \frac{2}{3}\frac{1}{\langle \boldsymbol{p}^2\rangle} \quad \text{for} \quad m_\mathrm{h} \gg T \ .$$

- NLO hints at **poor convergence** of resummed perturbation theory, e.g.

$$\hat{\kappa} \equiv \frac{\kappa}{T^3} = \frac{16\pi}{3}\alpha_\mathrm{s}^2\left[\ln\frac{1}{g} + 0.07428 + 1.9026g\right] + \mathcal{O}(g^2, m_\mathrm{h}^{-1}) \quad \text{for} \quad m_\mathrm{h} \gg T \ .$$

[Caron-Huot et al., PRL 100, 2008]

Non-perturbative HQ correlators sensitive to transport?

- Transport coefficients are encoded in the **HQ vector current** $\mathcal{J}$.
- $\mathcal{J}\mathcal{J}$ correlators have narrow transport peak $\propto \frac{\eta_{\mathrm{D}}}{\omega^2+\eta_{\mathrm{D}}^2}$ suppressed by $\frac{T}{m_{\mathrm{h}}}$
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- Less difficult: **HQ momentum diffusion coefficient** κ via $\mathcal{F}\mathcal{F}$ correlators; $\mathcal{F}^i = m_K\, \partial_t \mathcal{J}^i$ **represents the forces** it experiences (m_K kinetic mass).

$$\kappa^{(m_K)} = \lim_{\omega\to 0} \kappa^{(m_K)}(\omega), \quad \kappa^{(m_K)}(\omega) = \frac{1}{3\chi}\int_{-\infty}^{+\infty} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \int d^3x \left\langle \frac{1}{2}\{\mathcal{F}^i(\boldsymbol{x},t), \mathcal{F}^i(0,0)\} \right\rangle ,$$

with QNS $\chi = 1/T \int d^3x \left\langle \mathcal{J}^0(\boldsymbol{x},t)\mathcal{J}^0(0,0)\right\rangle$. [Caron-Huot et al., JHEP 04, 2009]

- The leading term $\kappa_E \equiv \kappa^{(\infty)}$ is due to **chromoelectric** forces, while the HQ mass dependent correction $\langle \boldsymbol{v}^2\rangle\, \kappa_B$ is due to **chromomagnetic** forces

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New HQ diffusion results in this talk

- QCD with **two different including realistic sea quark masses**
- **Apples-to-apples comparison** of QCD or pure SU(3) theory

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- **Apples-to-apples comparison** of QCD or pure SU(3) theory
- Still **insufficient statistics in the crossover region**

Extended lattice data set

- Tree-level improved gauge action (Lüscher-Weisz) w (2+1) flavors of HISQ
- LCPs: $m_\pi = 161$ MeV $(m_l/m_s = 1/20)$ or $m_\pi = 322$ MeV $(m_l/m_s = 1/5)$
- Two spatial volumes (for some temperatures): 64^3 or 96^3

T [MeV]	$10/g_0^2$	am_s	m_l/m_s	N_σ	N_τ
153					32
163					30
174	7.8250	0.01640	1/20	64	28
188					26
204					24
133					30
143					28
154	7.5960	0.02020	1/20	64	26
167					24
182					22
200					20
137					24
149					22
164	7.3730	0.02500	1/20	64	20
182					18
205					16

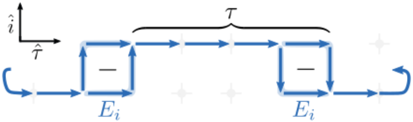
T [MeV]	$10/g_0^2$	am_s	m_l/m_s	N_σ	N_τ
	8.2763	0.009861			18
400	8.400	0.00887	1/5	64	20
	8.6165	0.007174			24
	8.2612	0.010004			16
444	8.400	0.00887	1/5	64	18
	8.6376	0.007036			22
	8.400	0.00887			16
500	8.5398	0.007703	1/5	64	18
	8.6647	0.006862			20
	8.8815	0.005626			24

T [MeV]	$10/g_0^2$	am_s	m_l/m_s	N_σ	N_τ
182	7.596	0.0202	1/20	64	22
200					20
188	7.825	0.0164	1/20	64	26
204					24
	7.570	0.01973		64	20
195	7.777	0.01601	1/5	64	24
	8.249	0.01011		96	36
222	7.825	0.0164	1/20	64	22
	7.704	0.01723		64	20
220	7.913	0.01400	1/5	64	24
	8.249	0.01011		96	32
250	7.596	0.0202	1/20	64	16
	7.857	0.01479		64	20
251	8.068	0.01204	1/5	64	24
	8.249	0.01011		96	28
271	7.825	0.0164	1/20	64	18
305					16
	8.036	0.01241		64	20
293	8.147	0.01115	1/5	64	22
	8.249	0.01011		96	24
286	8.400	0.00887	1/5	64	28
308					26
	8.126	0.01138	1/5	64	18
352	8.249	0.01011		96	20
	8.362	0.009095	1/5	64	22
333	8.400	0.00887	1/5	64	24
364					22

Heavy-quark momentum diffusion on the lattice

- Integrate out HQ: **Euclidean chromoelectric or -magnetic correlators**

$$G_{E,B}(\tau) = \frac{1}{3P}\int \mathrm{d}^3x \left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(1/T;\tau)\left\{\begin{array}{c} E_i(\tau) \\ B_i(\tau)\end{array}\right\} U(\tau;0)\left\{\begin{array}{c} E_i(0) \\ B_i(0)\end{array}\right\}\right]\right\rangle$$



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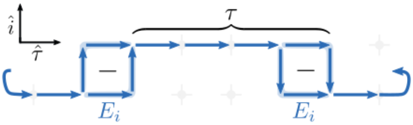

- $G_{E,B}(\tau)$ both require multiplicative renormalization; renormalized $G_{E,B}(\tau)$ are related via their spectral functions $\rho_{E,B}(\omega)$ to $\kappa_{E,B} = \lim_{\omega \to 0} \frac{2T}{\omega} \rho_{E,B}(\omega)$

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- $\rho_{E,B}(\omega)$ from $G_{E,B}(\tau)$: challenging **inverse problem** needs precise data!

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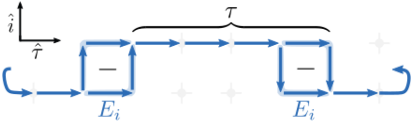

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- $\rho_{E,B}(\omega)$ from $G_{E,B}(\tau)$: challenging **inverse problem** needs precise data!
- **Very noisy** gluonic correlators $G_{E,B}(\tau)$: high statistics & noise reduction!
- **Multilevel algorithm** restricted lattice calculations to pure SU(3) theory.

  [Meyer, NJP 13, 2011]; [Banerjee et al., PRD 85, 2012]; [Francis et al., PRD 92, 2015];[Brambilla et al., PRD 102, 2020]
- **Gradient flow**: noise reduction & renormalization! [Altenkort et al., PRD 103, 2021]
- Gradient flow is **applicable in full QCD**, too! [HotQCD, PRL 130, 2023], [PRL 132, 2024]

Renormalization of the chromo-electric/-magnetic correlators

- $G_B(\tau)$ has **non-trivial anomalous dimension** $\gamma_0 = \frac{3}{8\pi^2}$**!** [Eichten, Hill, PLB 243, 1990]
- $\overline{\text{MS}}$ **renormalization factors** of $G_{E,B}(\tau)$ known at one-loop level in pQCD. [Christensen, Laine, PLB 755, 2016],[Laine, JHEP 06, 2021]
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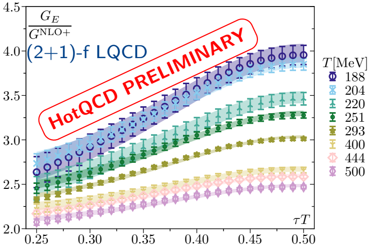
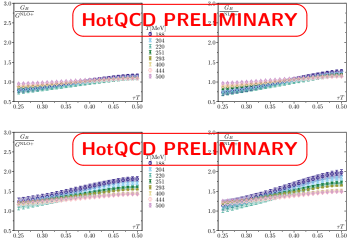
Chromo-electric



Chromo-magnetic
(different matching scales employed)



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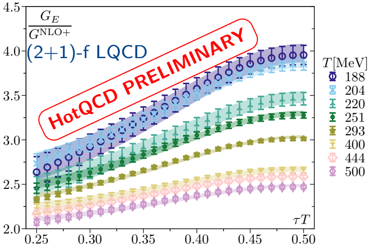

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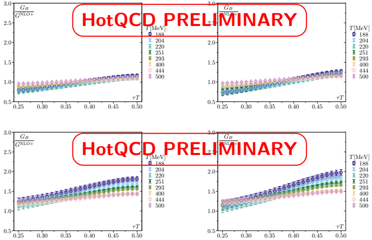

- Sea quark mass or volume dependence stat. irrelevant for $T \gtrsim 220$ MeV!
- Test better choices of matching and/or intermediate schemes for G_B

Reconstructing the spectral function

- Reconstruct **spectral functions** $\rho_{E,B}(\omega)$ and extract $\kappa_{E,B}$ via **models that incorporate known limiting behavior**. . . [Brambilla et al., PRD 102, 2020]

$$\rho^{\text{model}}_{E,B}(\omega) = \begin{cases} \rho^{\text{IR}}_{E,B}(\omega) \equiv \kappa_{E,B}\frac{\omega}{2T} & \omega \ll \omega_{\text{IR}} = T \\ \rho^{\text{match}}_{E,B}(\omega) = \textbf{?} & \text{for} \quad \omega_{\text{IR}} < \omega < \omega_{\text{UV}} \\ \rho^{\text{UV}}_{E,B}(\omega) \equiv K\rho^{\text{pert}}_{E,B}(\omega) & 2\pi T = \omega_{\text{UV}} \ll \omega \end{cases} .$$

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- The **NLO vacuum spectral function** scales w $\rho^{\text{LO}}(\omega) = \frac{g^2(\bar{\mu}_\omega)C_F\omega^3}{6\pi}$:

$$\frac{\rho^{\text{NLO}}_{E;0}(\omega,\bar{\mu}_\omega)}{\rho^{\text{LO}}(\omega)} = \left\{1 + \frac{g^2(\bar{\mu}_\omega)}{(4\pi)^2}\left(N_c\left[\frac{11}{3}\ln\frac{\bar{\mu}^2_\omega}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3}\right] - N_f\left[\frac{2}{3}\ln\frac{\bar{\mu}^2_\omega}{4\omega^2} + \frac{20}{9}\right]\right)\right\},$$

$$\frac{\rho^{\text{NLO}}_{B;0}(\omega,\bar{\mu}_\omega)}{\rho^{\text{LO}}(\omega)} = c^2_B(\bar{\mu}_\omega,\bar{\mu}_T)\left\{1 + \frac{g^2(\bar{\mu}_\omega)}{(4\pi)^2}\left(N_c\left[\frac{5}{3}\ln\frac{\bar{\mu}^2_\omega}{4\omega^2} + \frac{134}{9} - \frac{2\pi^2}{3}\right] - N_f\left[\frac{2}{3}\ln\frac{\bar{\mu}^2_\omega}{4\omega^2} + \frac{26}{9}\right]\right)\right\}$$

[Burnier et al., JHEP 08, 2010]; [Banerjee et al., JHEP 08, 2022]

- **Anomalous scaling:** $\ln c^2_B(\bar{\mu}_\omega,\bar{\mu}_T) = \gamma_0\int_{\bar{\mu}^2_T}^{\bar{\mu}^2_\omega} g^2(\mu)\frac{d\mu^2}{\mu^2}$. [HotQCD, PRL 132, 2024]
- N.B.: **Thermal contributions** $\propto C_F g^4\omega^3(T/\omega)^{2n}$ omitted in $\rho^{\text{pert}}_{E,B}(\omega)$.

Reconstructing the spectral function

- Reconstruct **spectral functions** $\rho_{E,B}(\omega)$ and extract $\kappa_{E,B}$ via **models that incorporate known limiting behavior**... [Brambilla et al., PRD 102, 2020]

$$\rho^{\text{model}}_{E,B}(\omega) = \begin{cases} \rho^{\text{IR}}_{E,B}(\omega) \equiv \kappa_{E,B}\frac{\omega}{2T} & \omega \ll \omega_{\text{IR}} = T \\ \rho^{\text{match}}_{E,B}(\omega) = \textbf{?} & \text{for} \quad \omega_{\text{IR}} < \omega < \omega_{\text{UV}} \\ \rho^{\text{UV}}_{E,B}(\omega) \equiv K\rho^{\text{pert}}_{E,B}(\omega) & 2\pi T = \omega_{\text{UV}} \ll \omega \end{cases} .$$

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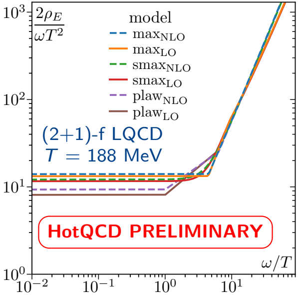
- N.B.: **Thermal contributions** $\propto C_{\text{F}}g^4\omega^3(T/\omega)^{2n}$ omitted in $\rho^{\text{pert}}_{E,B}(\omega)$.
- No scale $(\bar{\mu}_T,\bar{\mu}_\omega)$ dependence in all-order result; minimize it at NLO.
- **"Optimal" scales**: w $\bar{\mu}_{\text{DR}} \simeq 9.08222T$ in EQCD @ $N_{\text{f}} = 3$. [Kajantie et al., NPB 503, 1997]
  - $\mu^{\text{opt}}_E \equiv 14.7427\omega$: eliminates $\rho^{\text{NLO}}_{E;0}(\omega,\mu^{\text{opt}}_E) = 0$. [Burnier et al., JHEP 08, 2010]
  - $(\mu^{\text{opt}}_B)^2 \equiv (c_{\text{opt}}\omega)^2 + \bar{\mu}^2_{\text{DR}}$: $\frac{\rho^{\text{NLO}}_{B;0}(\omega,\mu^{\text{opt}}_B)}{\rho^{\text{LO}}(\omega)}$ small at $\omega \to \infty$. [Altenkort et al., PRD 109, 2024]

Spectral function modeling uncertainty due to matching region

- Three different models $\rho^{\text{match}}_{E,B}(\omega)$ match IR and UV [Altenkort et al., PRD 103, 2021]

$$\rho^{\text{max}}(\omega) = \max\left(\rho^{\text{IR}}, \rho^{\text{UV}}\right)(\omega)\ ,$$
$$\rho^{\text{smax}}(\omega) = \sqrt{\rho^{\text{IR}\ 2} + \rho^{\text{UV}\ 2}}(\omega)\ ,$$
$$\rho^{\text{plaw}}(\omega) = a\omega^{b}, \quad a = \frac{\rho^{\text{IR}}(\omega_{\text{IR}})}{(\omega_{\text{IR}})^{b}}, \quad b = \frac{\ln \rho^{\text{IR}}(\omega_{\text{IR}}) - \ln \rho^{\text{UV}}(\omega_{\text{UV}})}{\ln \omega_{\text{IR}} - \ln \omega_{\text{UV}}}.$$



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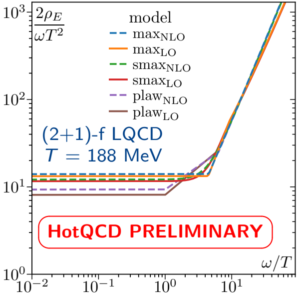
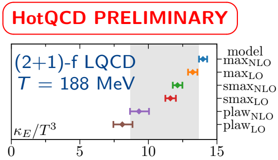


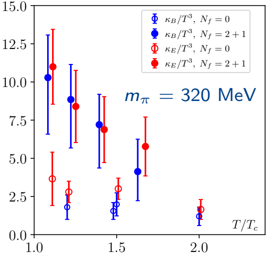


Central value/error of $\kappa_{E,B}$ via model averaging/spread

- **Model uncertainty of $\rho^{\text{match}}_{E,B}(\omega)$ dominates** the error budget for $\kappa_{E,B}$.
- **Uncertainty due to anomalous scaling** is clearly subleading for κ_B.

Temperature and flavor dependence of heavy-quark diffusion

- $\hat{\kappa}_{E,B}$ is **larger in (2+1)-f QCD than in pure SU(3)** at the same T/T_c.



[HotQCD, PRL 132, 2024]

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- N_f-blind $\hat{\kappa}_{E,B} \simeq 0.40(3)\, g^4(2\pi T)$ by far exceeds LO of $0.0079\, g^4(2\pi T) + \ldots$

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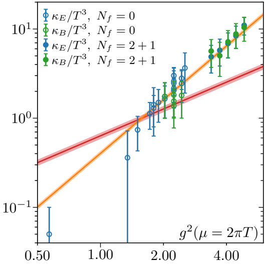

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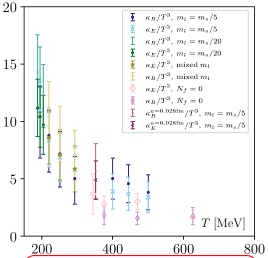

HotQCD PRELIMINARY

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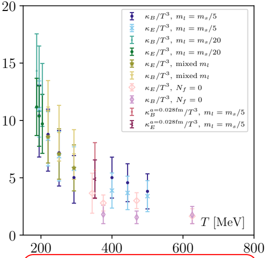
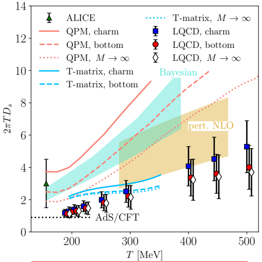
- Diffusion coefficient $D_s = \frac{2T^2}{\kappa}\frac{\langle \boldsymbol{p}^2\rangle}{3m_h T}$ is close to **AdS/CFT bound** $2/(\pi T\sqrt{\lambda})$ for certain λ [Casalderrey-Solana, Teany, PRD 74, 2006]
- Systematically below weak-coupling or T-matrix results [Tang et al., EPJ C 60, 2024]
- Phenomenological modeling disfavored [ALICE, JHEP 01, 174, 2022]; [Sambataro et al., PLB 849, 2024]
- $1/m_h$ **correction** via κ_B: rather modest effect for all T; cf. [HotQCD, PRL 132, 2024]



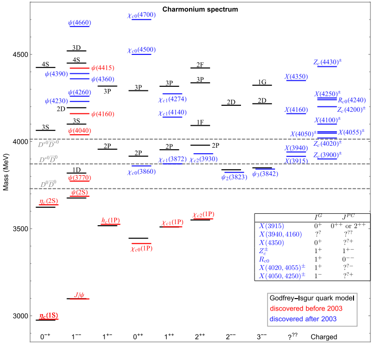
HotQCD PRELIMINARY



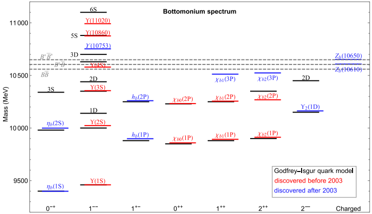
HotQCD PRELIMINARY

Quarkonia

- Plethora of non-relativistic QCD bound states, analogs of positronium.



Typical HQ velocities:
$v \sim \alpha_s(1/a_0) \approx 0.3 \ldots 0.1$



[Chen et al., FBS 61, 2020]

- Small v: integrate out the **hard mass scale** $m_h \gg p \sim m_h v \Rightarrow$ **NRQCD**.

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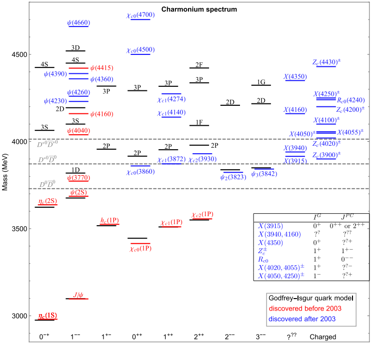

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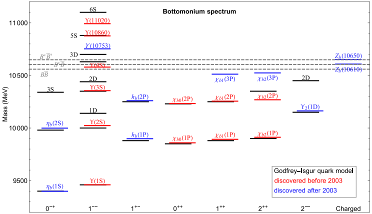

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- **Ultrasoft binding energy scale** $E_B \sim m_h v^2$ is even smaller: integrate out **soft momentum scale** $\frac{1}{r} \sim p \sim m_h v \gg E_B \sim m_h v^2 \Rightarrow$ **potential NRQCD**.
- Wilson coefficients of **pNRQCD** depend on distance $r \Rightarrow$ **potentials**.

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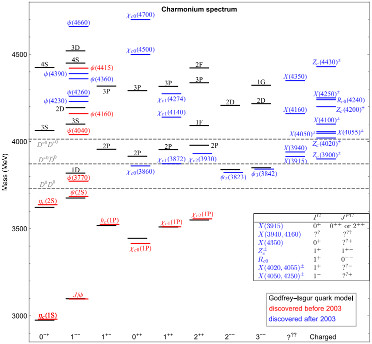

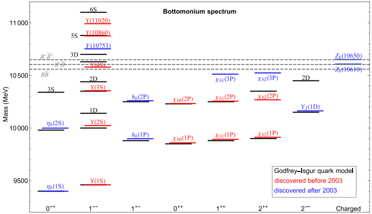

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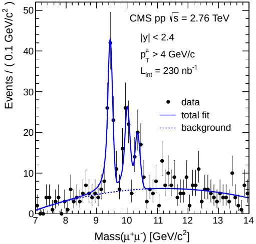
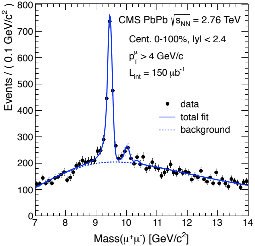
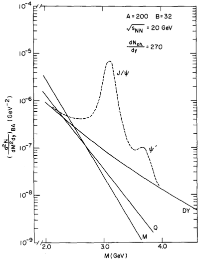
- Wilson coefficients of **pNRQCD** depend on distance r $\Rightarrow$ **potentials**.
- **Simultaneous expansion** in $1/m_h$ and $\alpha_s \sim v$. **pNRQCD** also in r.

Quarkonia suppression





[CMS, PRL 109, 2012]



[Matsui, Satz, PLB 178, 1986]

- The observed **rate of excited quarkonia** is smaller in **HIC** (than in *pp*).
- Quarkonia suppression as QGP signature in HIC suggested 37 years ago:

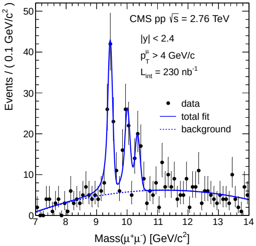
$$E(r) \sim -\frac{\alpha}{r} e^{-r m_D} + \text{const} \quad \text{at } T > T_c \; ,$$

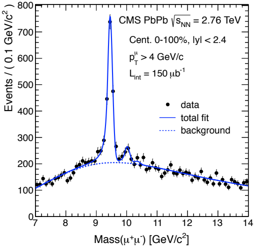
i.e., ad hoc **Debye screened Coulomb potential model** for charmonium.

- Thermal/electric/**Debye mass** is defined at leading order

$$m_D^2 = \frac{C_A + T_f N_f}{3} g^2 T^2 \; .$$

Quarkonia suppression





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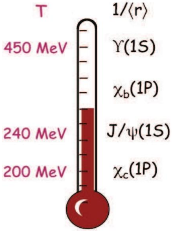

[Mocsy, EPJ C61, 2009]

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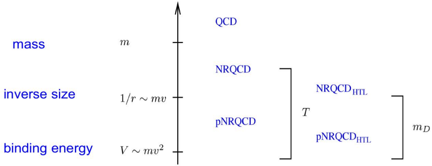
- **Sequential melting**: as quarkonia of various radii melt at various temperatures, their rates serve as a thermometer. [Karsch, et al., 1988]

In-medium static potential at weak coupling

NR hierarchy:
$v \sim \alpha_s(m_h)$



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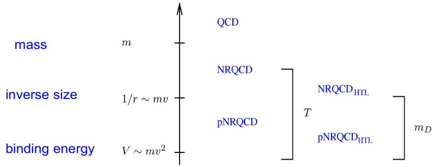

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  w gluon spectral fct $\sigma_r^{\mathrm{HTL}}(\omega,T) \Rightarrow \mathrm{Re}V_s^{\mathrm{HTL}} = F_S^{\mathrm{HTL}} + \mathcal{O}(g^4)$, $\mathrm{Im}V_s^{\mathrm{HTL}} \sim \mathcal{O}(g^2T)$.

  [Laine et al., JHEP 03, 2007]

- Assume $\Delta V \ll m_D \ll T \ll 1/r$: $\mathrm{Re}V_s = V_s + \mathcal{O}(g^4r^2T^3)$, $\mathrm{Im}V_s \sim \mathcal{O}(g^4r^2T^3, g^6T)$

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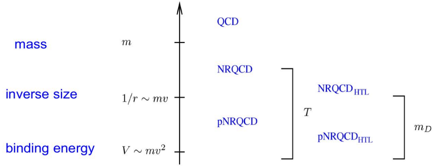

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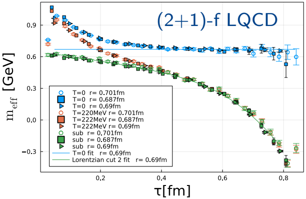
- Treatment as open quantum system $\Rightarrow$ [Ajaharul Islam, Th, 08/22, 12:00]

Spectral structure of the lattice correlator



[HotQCD, PRD 109, 2024]

- Use Wilson line correlator in Coulomb gauge to access **lowest, well-separated peak** in static $Q\bar{Q}$ spectrum: **static energy** $V(r)$.
- Real $V(r)$: large Euclidean time limit of
$$m_{\text{eff}}(\tau, r) = -\partial_\tau \ln W(\tau, r) \overset{\tau\to\infty}{\Longrightarrow} V(r) \ ,$$
- Complex $V(r, T)$: **no plateau** in $m_{\text{eff}}(\tau, r, T)$.

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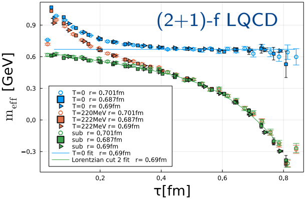
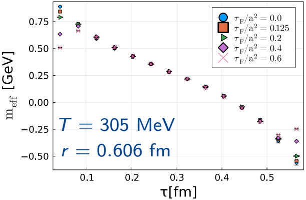


[HotQCD. PRD 109. 2024]



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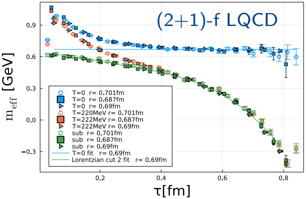
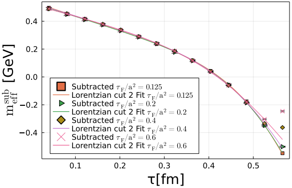
- In-medium static $Q\bar{Q}$ spectrum:
  - broad quasiparticle peak (operator indep.)
  - high-frequency part (operator dep.)
  - low-frequency part ($T > 0$, operator dep.)

$$\rho_r(\omega, T) = \rho_r^{\text{low}}(\omega, T) + \rho_r^{\text{peak}}(\omega, T) + \rho_r^{\text{high}}(\omega)$$

Spectral structure of the lattice correlator



[HotQCD. PRD 109. 2024]



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- In-medium static $Q\bar{Q}$ spectrum:
  - broad quasiparticle peak (operator indep.)
  - high-frequency part (operator dep.)
  - low-frequency part ($T > 0$, operator dep.)

$$\rho_r(\omega, T) = \rho_r^{\mathrm{low}}(\omega, T) + \rho_r^{\mathrm{peak}}(\omega, T) + \rho_r^{\mathrm{high}}(\omega)$$

- If HF part dominated by lattice cutoff: may subtract $T = 0$ HF part $\Rightarrow$ curve collapse!

In-medium static energy from the lattice

- **Quasiparticle peak** is locally a **Lorentzian**: potential parameters $\mathrm{Re}\,V(r,T), \mathrm{Im}\,V(r,T)$, and non-potential parameters $\sigma_\infty, t_{Q\bar{Q}}, c_i$

$$\rho_r^{\mathrm{peak}}(\omega) = \frac{1}{\pi} e^{\mathrm{Im}\sigma_\infty} \frac{\left(\omega - \mathrm{Re}V\right)\sin(\mathrm{Re}\sigma_\infty) + |\,\mathrm{Im}V|\cos(\mathrm{Re}\,\sigma_\infty)}{\left(\omega - \mathrm{Re}V\right)^2 + \left(\mathrm{Im}V\right)^2}$$

$$+ c_0 - c_1 t_{Q\bar{Q}}(\omega - \mathrm{Re}V) + c_2 t_{Q\bar{Q}}^2(\omega - \mathrm{Re}V)^2 + \cdots$$

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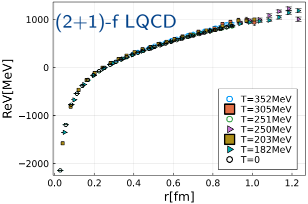

[HotQCD, PRD 109, 2024]

- Tails must be regularized, e.g. as **Gaussian** or as **cut Lorentzian** for similar outcomes!

$$\rho_r^{\text{cL}}(\omega) = \frac{1}{\pi} \frac{A_r \theta(|\omega - \mathrm{Re}\,V| - \mathrm{Cut})\Gamma_L}{(\omega - \mathrm{Re}\,V)^2 + \Gamma_L^2} .$$

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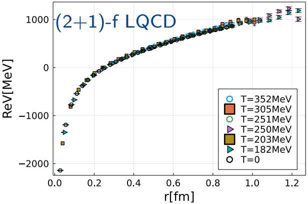
- **2nd cumulant** $c_2 = -\frac{dm_{\text{eff}}^{\text{sub}}}{d\tau}(\tau \to 0)$ is a quite model-independent proxy for **width** $|\mathrm{Im}\,V|$.
- At $rT \sim 1$ $|\mathrm{Im}\,V| \sim \sqrt{c_2} \sim 1.5T \gg V - \mathrm{Re}V$: **deconfinement by dissociation**.
- Consistent with [HotQCD, PRD 105, 2021] and supersedes [Burnier et al., PRL 114, 2014].



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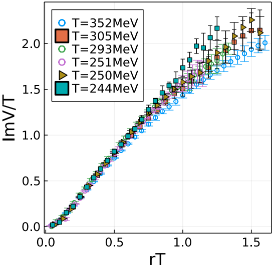
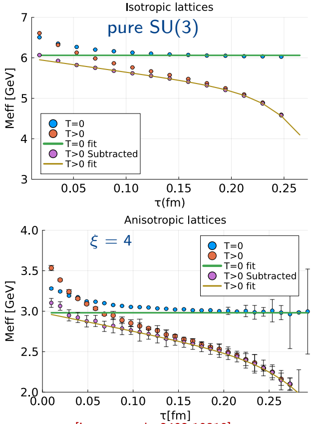


In-medium static energy in pure SU(3)

Previous Bayesian inference or HTL-inspired analyses: screening in pure SU(3).

[Rothkopf et al., PRL 108, 2012]; [Burnier, Rothkopf, PRD 95, 2017]; [Bala, Datta, PRD 101, 2020]



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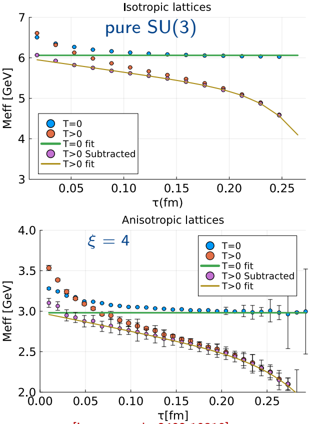

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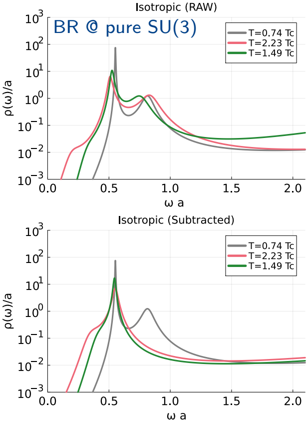

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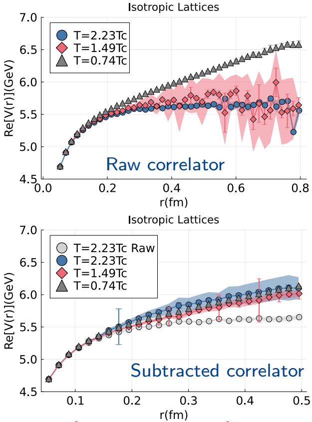

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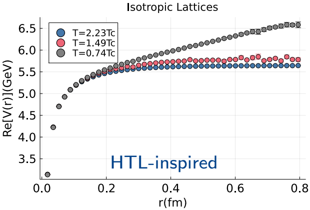

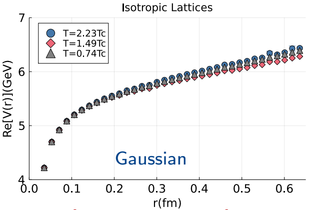

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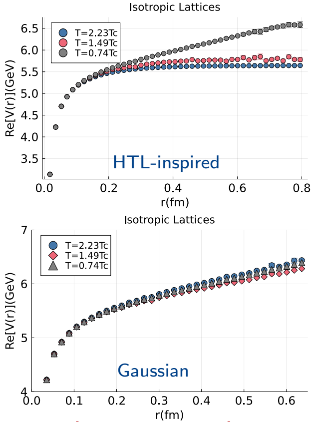

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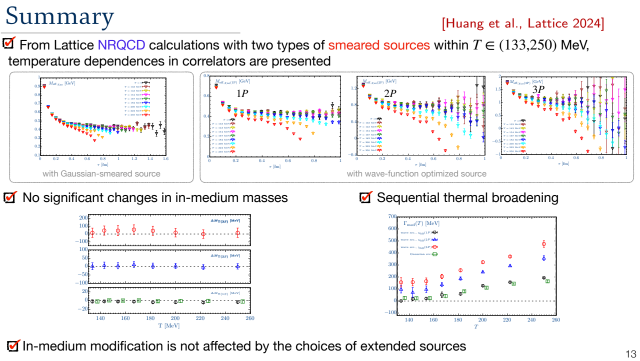
(Non-)subtraction of the HF part is the key model choice here.

Recent news on in-medium bottomonia with NRQCD (HotQCD)



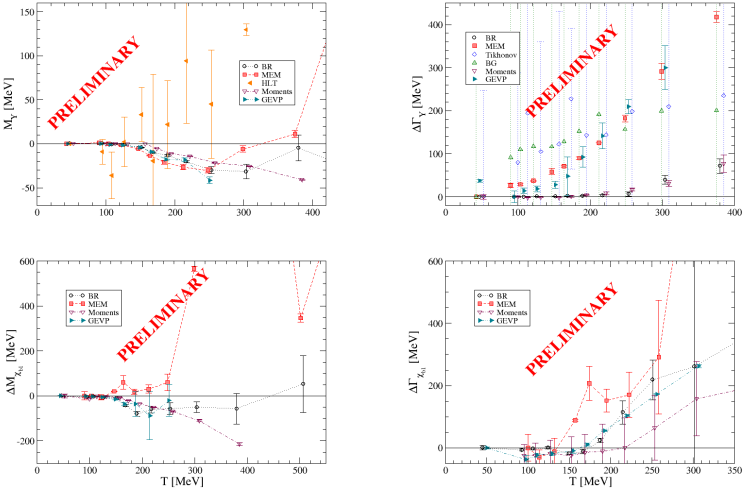
- Extended sources suppress/select excited states, but distort the HF part.
- Similar Ansatz as for the potential: **subtract ρ^{high} (from $T = 0$)**,

$$\rho(\omega, T) = \rho^{\text{low}}(\omega, T) + \rho^{\text{peak}}(\omega, T) + \rho^{\text{high}}(\omega) \; ;$$

  peak modeled as Gaussian; **low-energy tail** with a well-separated delta.
- Mass shift compatible w zero and large width ⇒ consistent w potential.

Recent news on in-medium bottomonia with NRQCD (FASTSUM)

[Jon-Ivar Skullerud, Wed 15:30]



- Analysis with many different and complementary approaches.
- Mild thermal mass shift w/o clear T dependence for most approaches.

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Thank you for your attention!