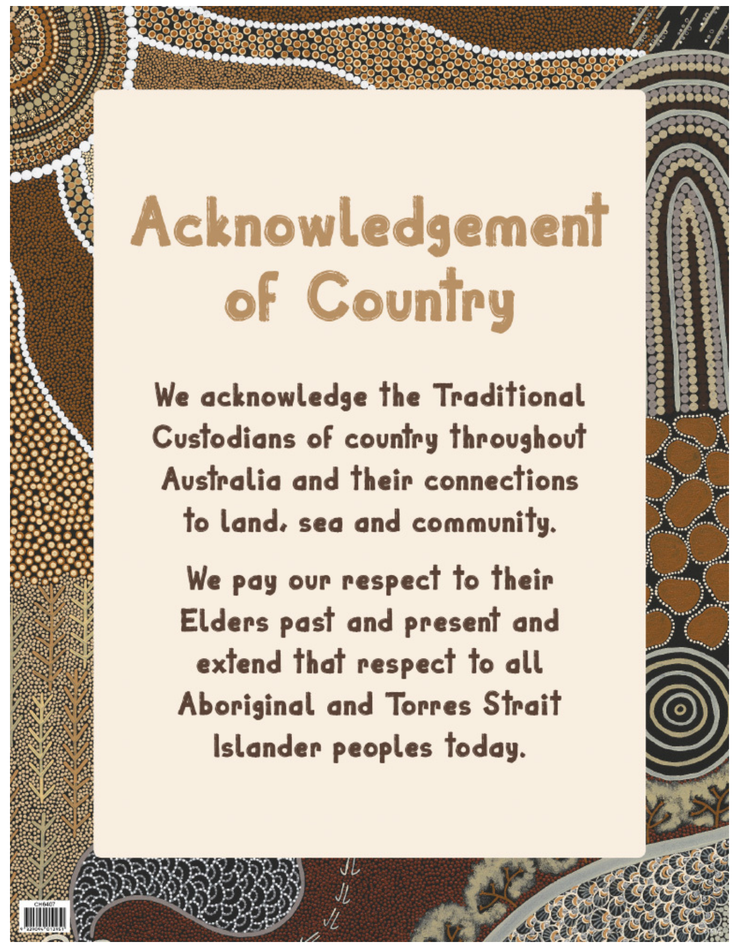
Scattering Amplitudes, double copies and equivalence theorems in Compactifed Warped Extra-Dimensions

Dipan Sengupta

With

R. Sekhar Chivukula (UCSD), Dennis Foren (UCSD-> Wolfram Computers), Kirtimaan A. Mohan (MSU) , Elizabeth H. Simmons (UCSD), X. Wang (UCSD -> INFN, Rome) and J. A. Gill (Adelaide -> UNSW), George Sanmayan (Adelaide) + Anthony Williams (Adelaide)



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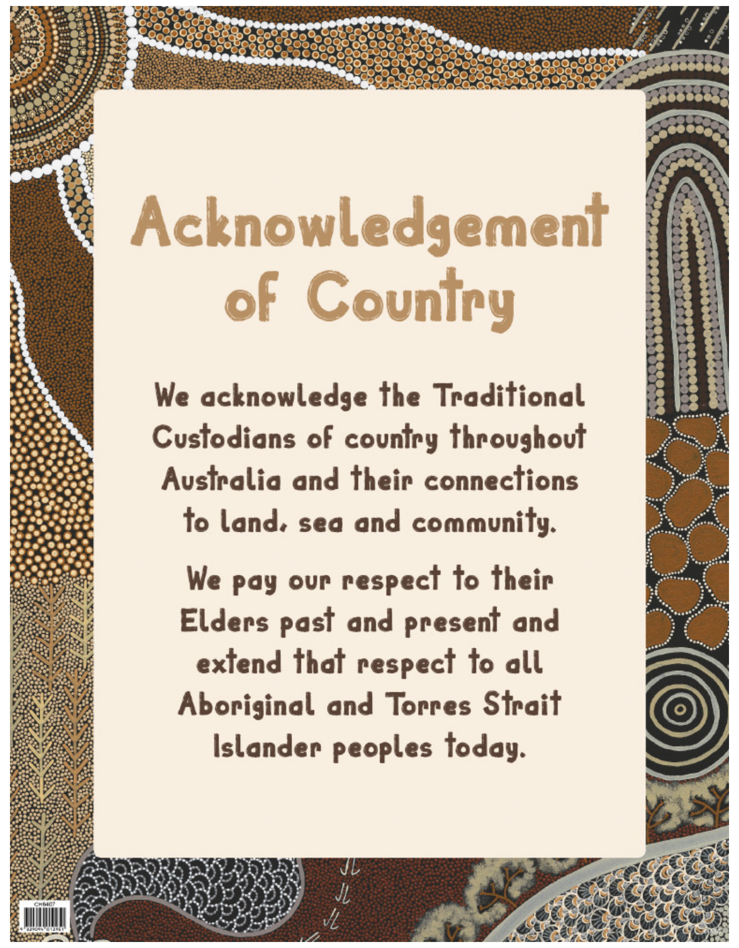

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Why should I care about the scale of effective field theories

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Effective Field Theories

A systematic expansion in powers of momentum to extract the validity of Quantum field theories.

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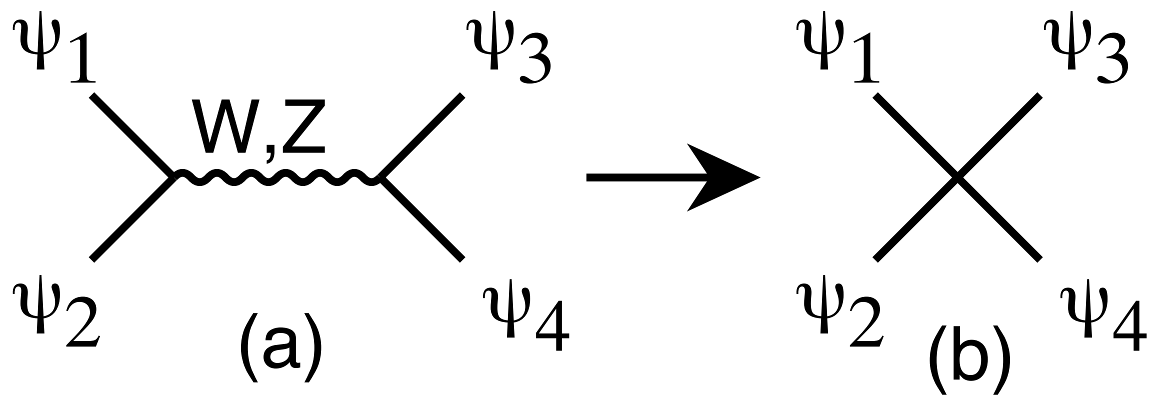
Four Fermi Theory



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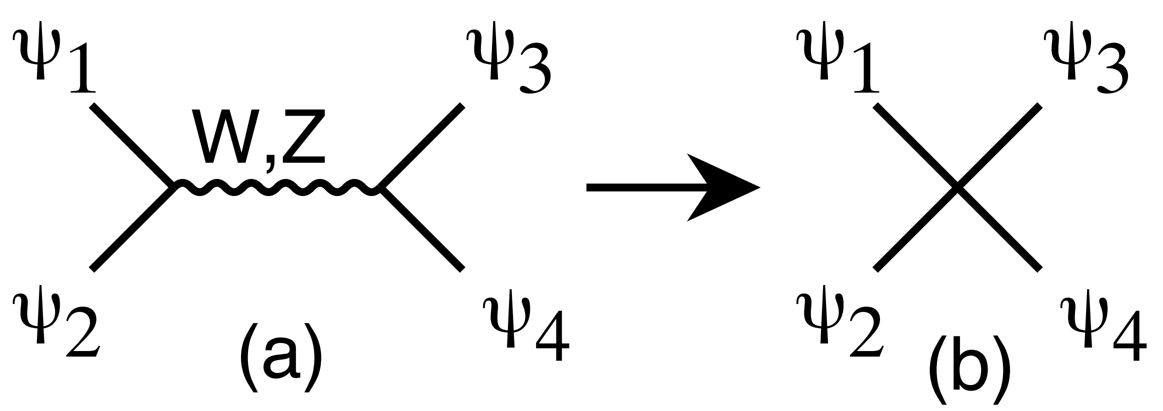

$$i\mathcal{A} = \left(-i\frac{e}{\sin\theta_w}\right)^2 J_-^\mu J_+^\nu \frac{-ig_{\mu\nu}}{q^2 - M_W^2} = -i\frac{e^2}{\sin^2\theta_w M_W^2} J_-^\mu J_{\mu+} + O\left(\frac{q^2}{M_W^2}\right)$$

EWSB and Unitarity

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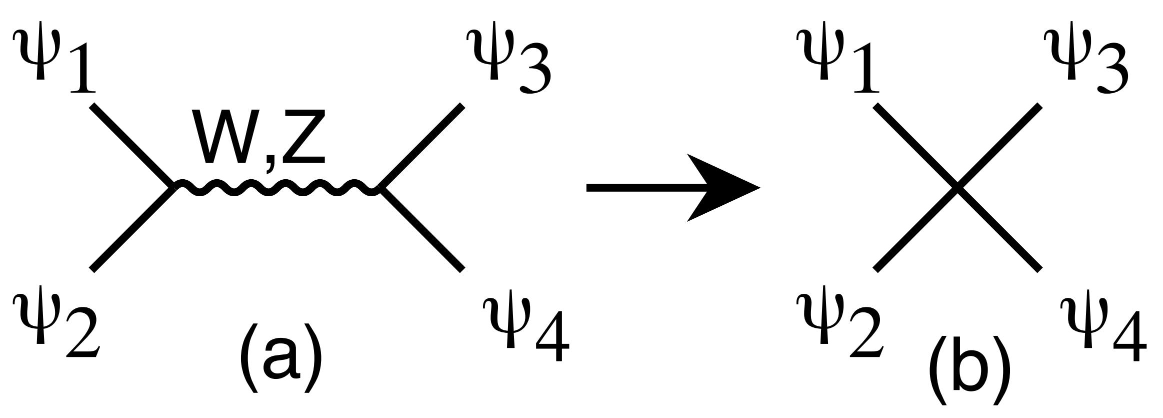

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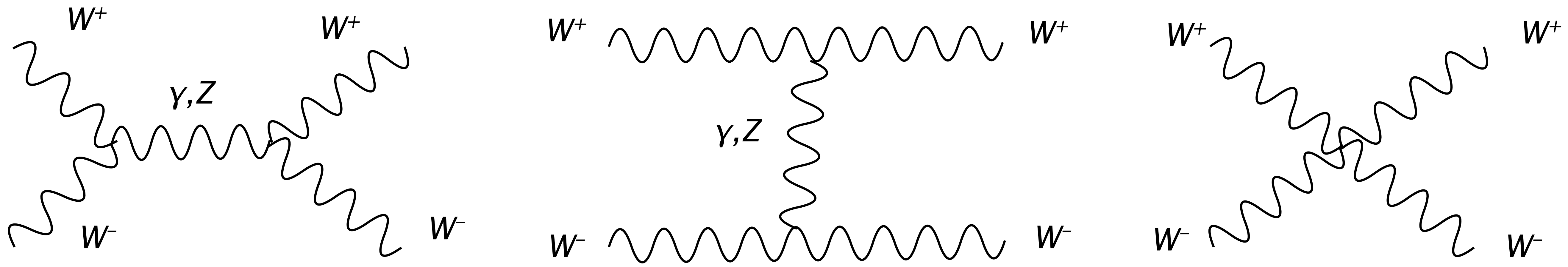

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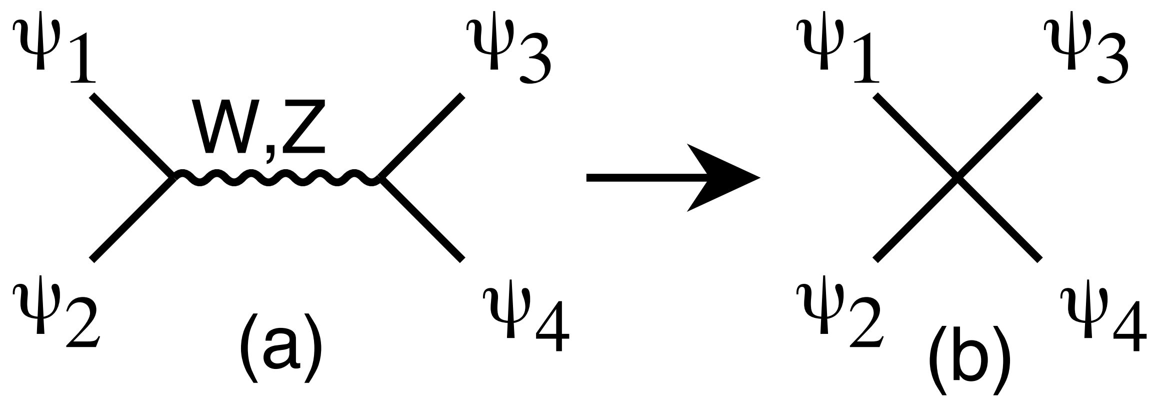

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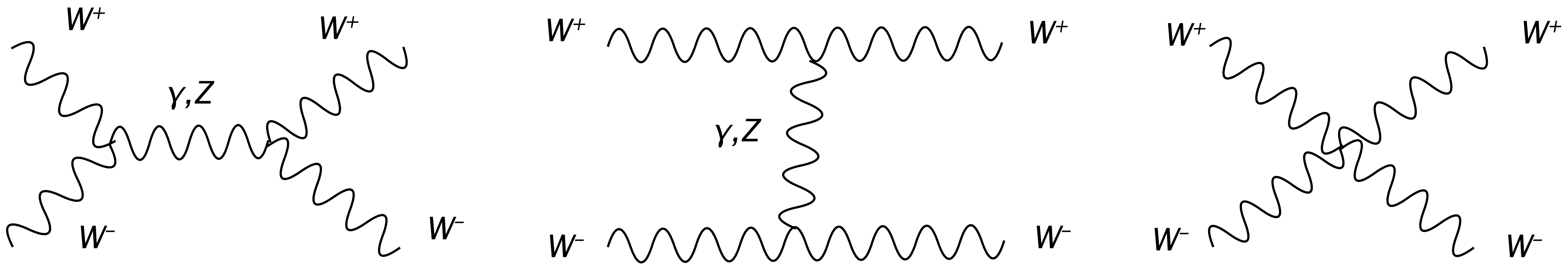
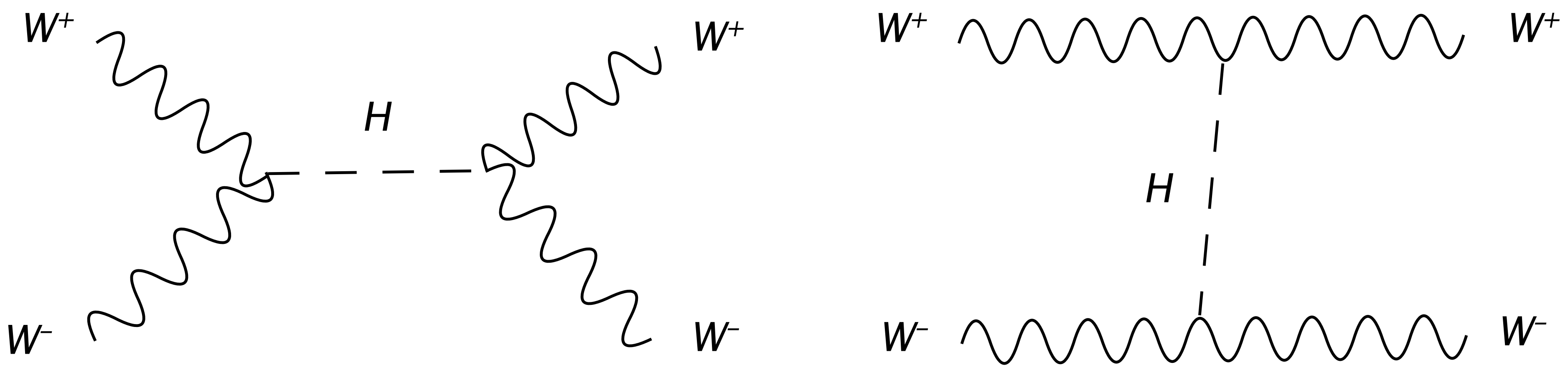


$$\epsilon_L^\mu = k^\mu / M_W + O(M_W^2/E_W^2)$$

$$\epsilon_L^{W^+}\cdot\epsilon_L^{W^-} \approx \frac{k_{W^+}\cdot k_{W^-}}{m_W^2} = \frac{s}{2m_W^2}$$

EFT breaks down at W mass
Goldstone Equivalence theorem



The Higgs kicks in and cancels the divergence to a constant

Why should I care about the scale of effective field theories

Power Counting in Momenta

$$S = \int \mathrm{d}^{\mathsf{d}}x \, \mathscr{L}(x) \qquad [\mathscr{L}(x)] = \mathsf{d} \qquad \mathscr{L}(x) = \sum_i c_i \, O_i(x)$$

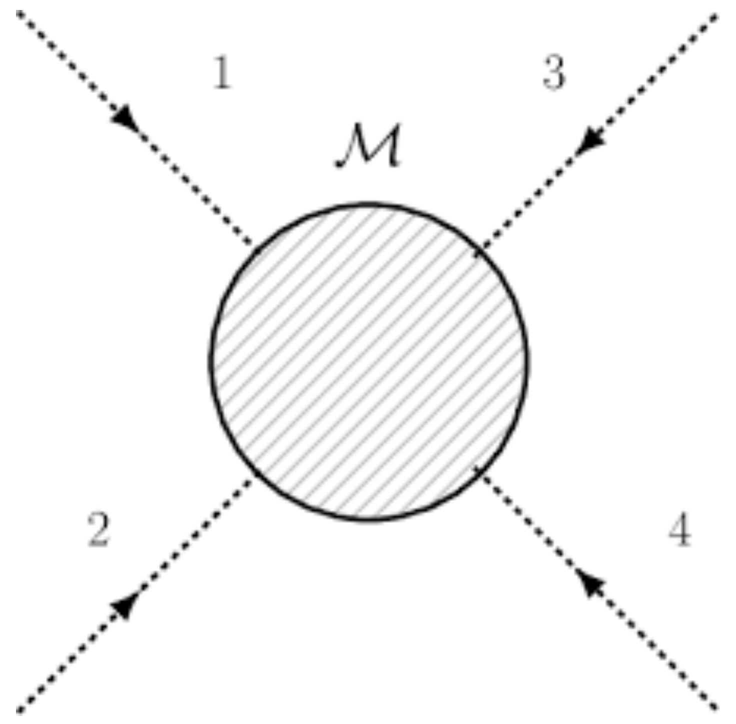
$$S = \int \mathrm{d}^{\mathsf{d}}x \, \bar{\psi} \, i\partial\!\!\!/ \, \psi \qquad S = \int \mathrm{d}^{\mathsf{d}}x \, \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi$$

$$[\psi] = \frac{1}{2}(\mathsf{d}-1) \qquad [\phi] = \frac{1}{2}(\mathsf{d}-2)$$

A generic EFT in D dimensions

$$\mathscr{L}_{\mathrm{EFT}} = \sum_{\mathscr{D}\geq 0, i} \frac{c_i^{(\mathscr{D})} O_i^{(\mathscr{D})}}{\Lambda^{\mathscr{D}-d}} = \sum_{\mathscr{D}\geq 0} \frac{\mathscr{L}_{\mathscr{D}}}{\Lambda^{\mathscr{D}-d}}$$

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots$$



A scattering amplitude with some Momentum scale

$$\mathscr{A} \sim \left(\frac{p}{\Lambda}\right)^{\mathscr{D}-\mathsf{d}}$$

See for example , Manohar, Tasi lectures 2022

Gravity as an Effective Field Theory

The Classical Action for Gravity

$$S_g = \int d^4x \sqrt{g} \frac{2}{\kappa^2} R$$

Einstein's Equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

Conserved Source

$$\sqrt{g} T^{\mu\nu} \equiv -2 \frac{\partial}{\partial g_{\mu\nu}} (\sqrt{g} \mathcal{L}_m)$$

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The Ricci is a two derivative object

Einstein Hilbert Action is a dimension 6 operator with a cut-off M_{Pl}

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1} \epsilon^{\nu}_{\pm 1}$$

$$\epsilon^{\mu}_{\pm 1} = \pm \frac{e^{\pm i\phi}}{\sqrt{2}} (0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp i c_\phi, s_\theta)$$

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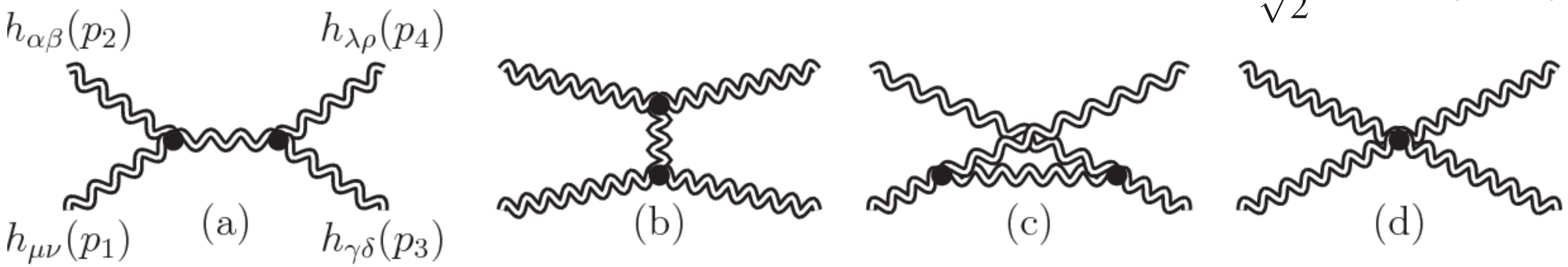

The amplitude grows as s/M_{Pl}^2 Consistent with semi-classical gravity

Formal expansion of Massless Gravity

$$S = \int d^4x \sqrt{g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + \mathcal{L}_{matter} \right\}$$

Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

Diffeomorphism : $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

Equivalent to transformation for Gauge Theories $A_\mu \rightarrow A_\mu + \partial_\mu \xi$

D.O.F counting in d dimensions for the massless graviton

$$d(d+1)/2 - 2d = d(d-3)/2$$

d=3, D.O.F =0
d=4, D.O.F=2
d=5, D.O.F=5

D.O.F counting in d dimensions for the massless gauge boson

d-2

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Let's think of a Massive Photon

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu A^\mu$$

Mass term explicitly breaks the gauge redundancy

Propagator $\frac{-i}{p^2+m^2}\left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$

Stuckelberg Trick $A_\mu \to A_\mu + \partial_\mu \phi$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - mA_\mu \partial^\mu \phi - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + A_\mu J^\mu - \frac{1}{m}\phi\partial_\mu J^\mu$$

$$\delta A_\mu = \partial_\mu \Lambda$$

$$\delta\phi = -m\Lambda$$

Propagators $\frac{-i\eta_{\mu\nu}}{p^2+m^2}, \quad \frac{-i}{p^2+m^2}$

Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

Let's think of a Massive Graviton

$$S_G = \int d^4x \sqrt{g} R + m^2((h_{\mu\nu})^2 - h^2) + \kappa h_{\mu\nu} T^{\mu\nu}$$

Fierz-Pauli Theory: 1940

Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

Let's think of a Massive Graviton $S_G = \int d^4x\sqrt{g}R + m^2((h_{\mu\nu})^2 - h^2) + \kappa h_{\mu\nu}T^{\mu\nu}$ Fierz-Pauli Theory: 1940

Stuckelberg $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$ $A_\mu \to A_\mu + \partial_\mu \phi$

5 propagating degrees of freedom
2 transverse + 3 longitudinal

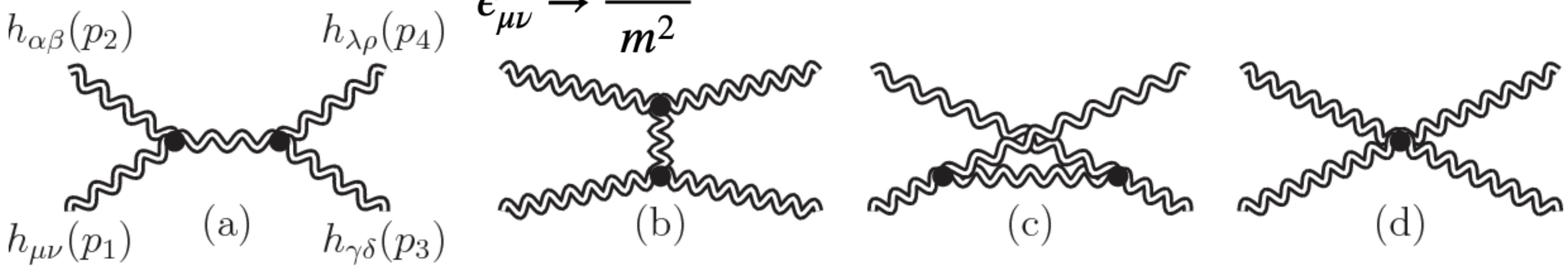
rescale $A_\mu \to \frac{1}{m}A_\mu,\ \phi \to \frac{1}{m^2}\phi$ Assume source is conserved , vanishing $\partial_\nu T^{\mu\nu}$

$$S = \int d^Dx\ \mathcal{L}_{m=0}(h') - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - 2\frac{D-1}{D-2}\partial_\mu\phi\partial^\mu\phi + \kappa h'_{\mu\nu}T^{\mu\nu} + \frac{2}{D-2}\kappa\phi T$$

1. Scalar couples to the trace of the stress energy tensor and does not decouple.
2. Behaves like a Scalar-Tensor/Brans-Dicke Theory. Affects the Newtonian Potential
3. vanDam-Veltman-Zakharov Discontinuity. M-> 0 limit not smooth under Stuckleberg

Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

$$\epsilon^0_{\mu\nu} \to \frac{k_\mu k_\nu}{m^2}$$



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$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

$$\epsilon^{\mu\nu}_{0} = \frac{1}{\sqrt{6}}[\epsilon^{\mu}_{+1}\epsilon^{\nu}_{-1} + \epsilon^{\mu}_{-1}\epsilon^{\nu}_{+1} + 2\epsilon^{\mu}_{0}\epsilon^{\nu}_{0}]$$

$$\epsilon^{\mu}_{\pm 1} = \pm\frac{e^{\pm i\phi}}{\sqrt{2}}(0, -c_\theta c_\phi \pm i s_\phi, -c_\theta s_\phi \mp$$

$$\epsilon^{\mu}_{0} = \frac{E}{m}\left(\sqrt{1-\frac{m^2}{E^2}}, \hat{p}\right),$$

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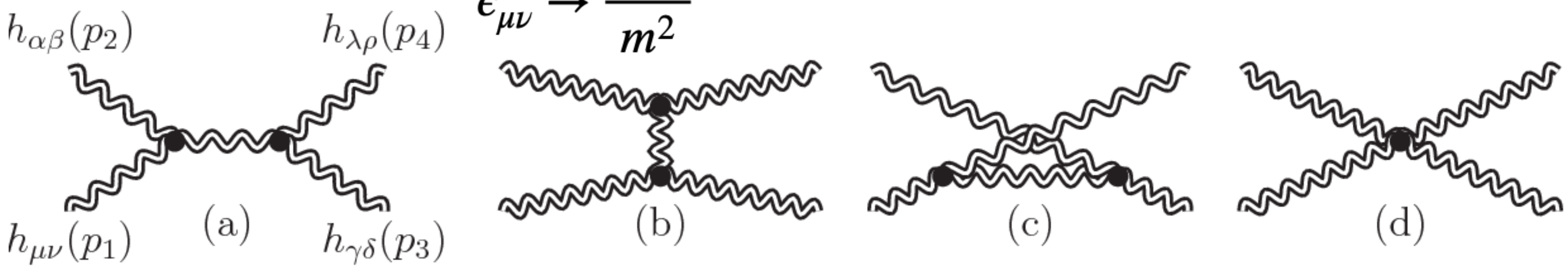

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Power Counting

1. Each external polarization grows as s/m^2
2. Each vertex grows as s
3. The propagator grows as $1/s$

Gravity as an Effective Field Theory : Diffeomorphism and Mass Terms

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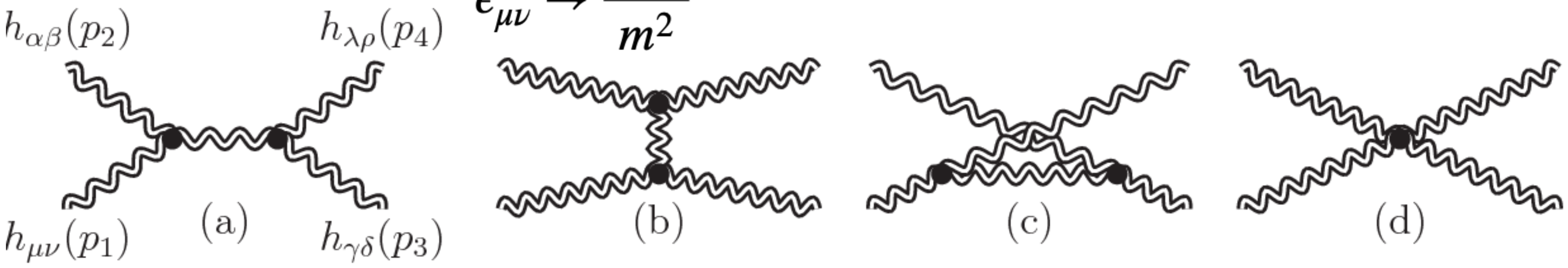

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2. Each vertex grows as s
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Amplitude grows as $\dfrac{s^5}{m^8 M_{pl}^2}$ Discontinuity as m-> 0. Does not reduce to Einstein-Hilbert action

Unitarity is violated at a scale $\Lambda_5 = (M_{pl} m^4)^{1/5} \ll M_{pl}$

De-Rham, Gabadadze, Tolley (2010)
Cheung and Remen (2017)
Bonifacio, Rosen, Hinterbichler (2019)
Georgi, Arkani-Hamed, Schwartz (2001)
Schwartz (2003)

Non-linear massive gravity

Non-linear massive gravity

Most general potential

$$S = \frac{1}{2\kappa^2}\int d^Dx \, \left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h)\right]$$

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V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
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&\quad +f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5,
\end{aligned}$$

Tune coefficients to raise the scale, avoid ghosts

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\end{aligned}$$

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$$c_1 = 2c_3 + \frac{1}{2}, \qquad c_2 = -3c_3 - \frac{1}{2},$$

$$d_1 = -6d_5 + \frac{3}{2}c_3 + \frac{5}{16}, \qquad d_2 = 8d_5 - \frac{3}{2}c_3 - \frac{1}{4},$$

$$d_3 = 3d_5 - \frac{3}{4}c_3 - \frac{1}{16}, \qquad d_4 = -6d_5 + \frac{3}{4}c_3,$$

De-Rahm, Gabadadze, Tolley 2011, …
Cheung and Remen, 2020,..
Bonifiacio, Rosen, Hinterbichler, 2021, …

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Cut-off scale raised to Λ_3

Can show that cut-off can't be raised above Λ_3

Realizations of this set up : dRGT gravity, Bi/Multigravity

Compactified 5D theory

$$\mathcal{S}_{5D}^{EH} \propto \frac{1}{M_5^3} \int d^5x \sqrt{-g} R_{5D}$$

5D diffeomorphism with a 5D Planck mass

Compactification (IR phenomenon) should not change the high energy (UV) behavior,

High energy growth s/M_{Pl}^2

Coupled channel analysis $s^{3/2}/M_5^3$

Examine Strong coupling scale

A. Flat Extra dimension compactified on a torus
B. The Randall Sundrum Model (ADS)

Understanding the problem : Geometrical Deconstruction of Dimensions

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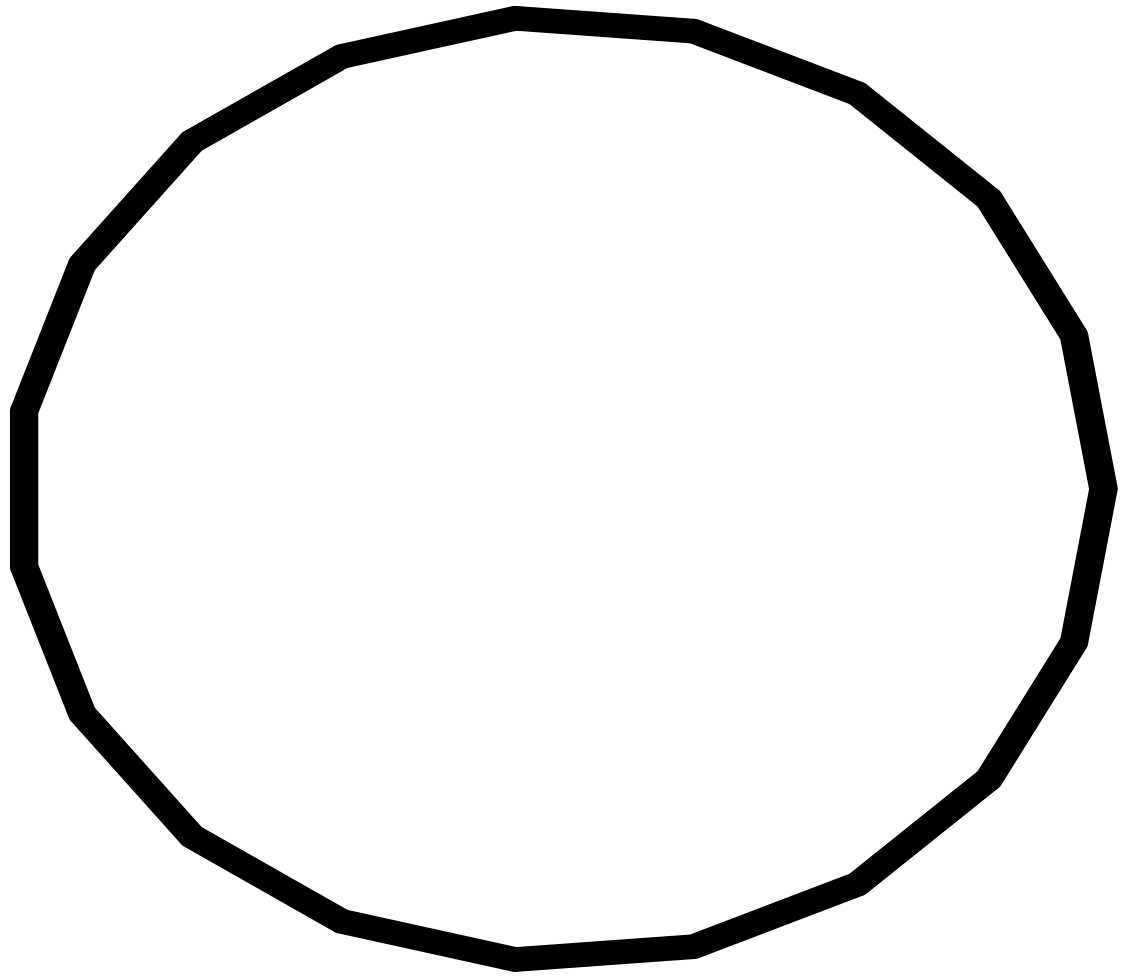


minimal discretization

Discretize a dimension with nearest neighbour interaction

$$\mathcal{L}_{\rm min} = \sum_j M^2 \sqrt{g^j} R[g^j] + M^2 m^2 \sqrt{g^j} (g^j_{\mu\nu} - g^{j+1}_{\mu\nu})(g_j^{\mu\rho} g_j^{\nu\sigma} - g_j^{\mu\nu} g_j^{\rho\sigma})(g^j_{\rho\sigma} - g^{j+1}_{\rho\sigma})$$

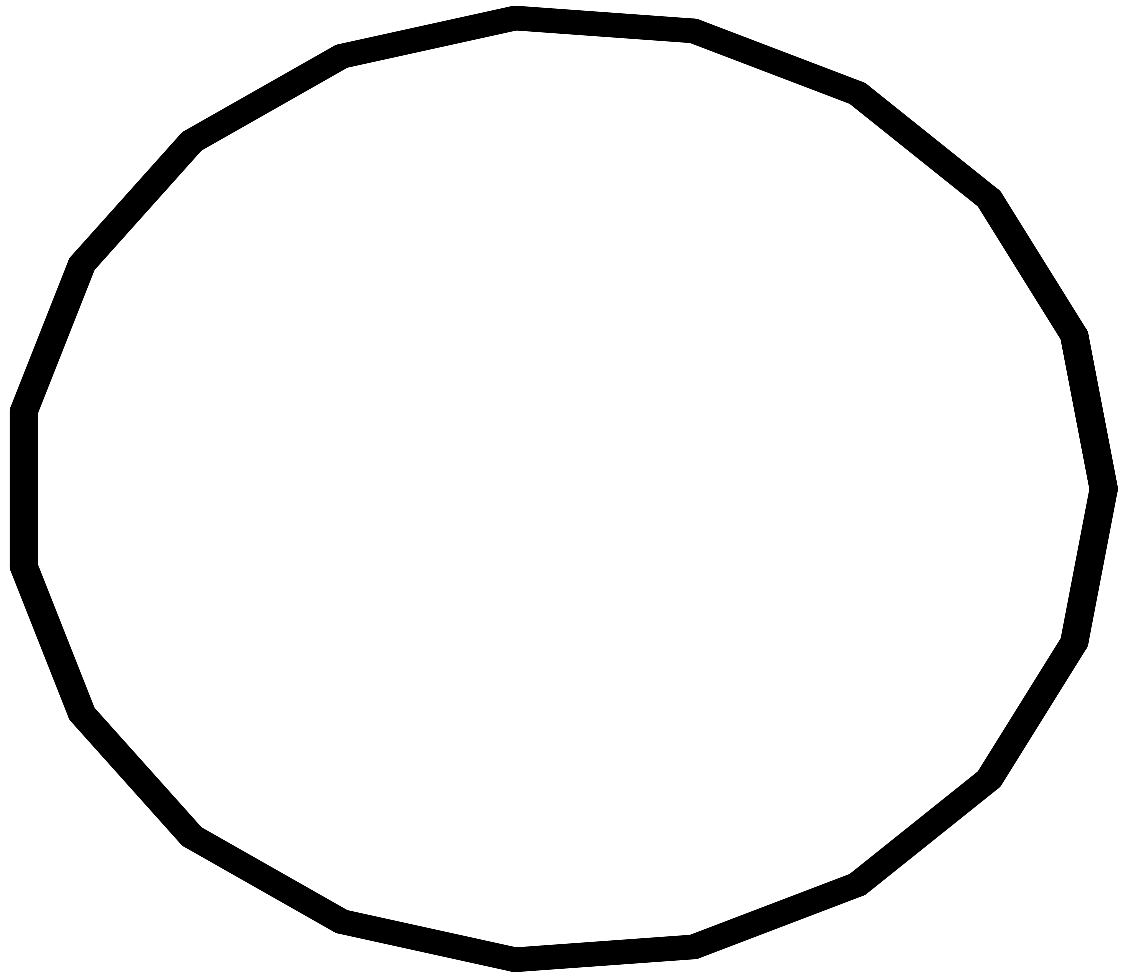
The mass terms correspond to broken diffeormorphisms, which can be replaced by Stuckelberg/Goldstone fields.

$$\Lambda_{\rm min} = (N m_1^4 M_{\rm Pl})^{1/5}$$

$$\Lambda_{\rm min} < \Lambda_{\rm max} = M_{\rm 5D} (R M_{\rm 5D})^{-5/8}$$

Formally never recovers the full 5D

Understanding the problem : Geometrical Deconstruction of Dimensions



minimal discretization

Discretize a dimension with nearest neighbour interaction

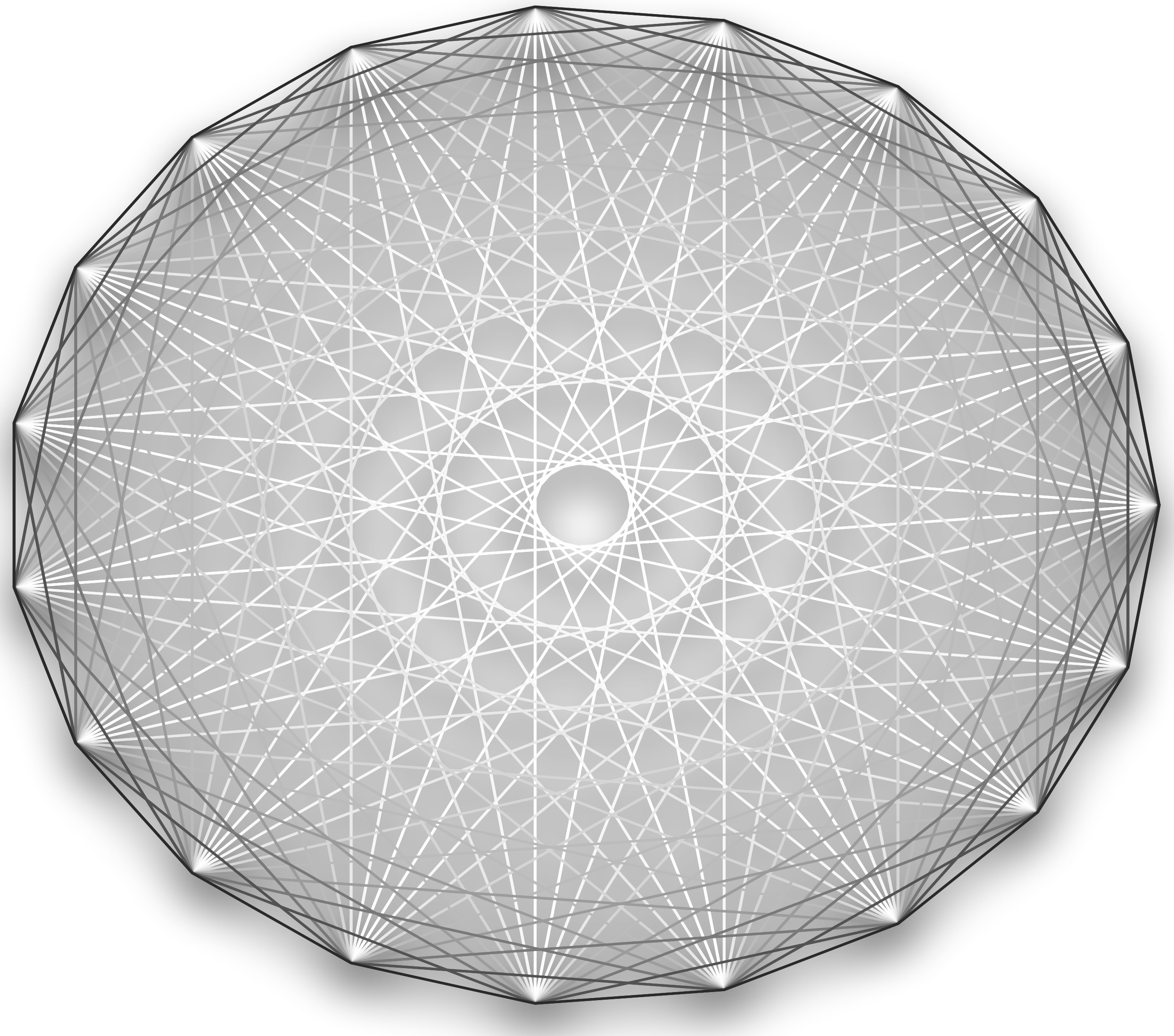
$$\mathcal{L}_{\min} = \sum_j M^2 \sqrt{g^j} R[g^j] + M^2 m^2 \sqrt{g^j} (g^j_{\mu\nu} - g^{j+1}_{\mu\nu})(g_j^{\mu\rho} g_j^{\nu\sigma} - g_j^{\mu\nu} g_j^{\rho\sigma})(g^j_{\rho\sigma} - g^{j+1}_{\rho\sigma})$$

The mass terms correspond to broken diffeormorphisms, which can be replaced by Stuckelberg/Goldstone fields.

$$\Lambda_{\min} = (N m_1^4 M_{\rm Pl})^{1/5}$$

$$\Lambda_{\min} < \Lambda_{\max} = M_{5\rm D} (R M_{5\rm D})^{-5/8}$$

Formally never recovers the full 5D



truncated KK theory

In a full/truncated KK theory,
we need all interactions and not just nearest neighbour ones

Arkani-Hamed, Georgi, Schwartz 2002
Arkani-Hamed, Schwartz 2003
Schwartz 2003

Compact Extra Dimensions : A primer

Anstaz- The Fundamental Theory is 5 or D dimensional

$$ds^2 = (G_{MN}) dx^M dx^N$$

$$S_{4+n} = -M_*^{n+2} \int d^{4+n}x \sqrt{g^{(4+n)}} R^{(4+n)}$$

$$M_{Pl}^2 = M_*^{n+2} V_{(n)} = M_*^{n+2} (2\pi r)^n$$

Fluctuations in 5D $\quad g_{MN}(x, x^5) = \begin{pmatrix} g_{\mu\nu} & g_{\mu 5} \\ g_{5\mu} & g_{55} \end{pmatrix} = \begin{pmatrix} \eta_{\mu\nu} + h_{\mu\nu} & h_{\mu 5} \\ h_{\mu 5} & 1 + h_{55} \end{pmatrix}$

5D gauge invariance $\quad h_{MN} \to h_{MN} + \delta h_{MN} = h_{MN} + \partial_N \epsilon_M + \partial_M \epsilon_N$

- There exists a number of n new spacial compact dimensions. For instance a simple manifold could be just $\mathcal{M}_4 \times T^n$.
- The fundamental Planck scale of the theory is very low $M_D \sim$ TeV.
- The SM degrees of freedom are localized on a 3D-brane stretching along the 3 non-compact space dimensions.

$$\frac{1}{r} = M_* \left(\frac{M_*}{M_{Pl}}\right)^{\frac{2}{n}} = (1\text{TeV}) 10^{-\frac{32}{n}}$$

$r \sim 2 \cdot 10^{-17} 10^{\frac{32}{n}}$ cm $\qquad n = 1 \qquad r = 2 \cdot 10^{15}$ cm

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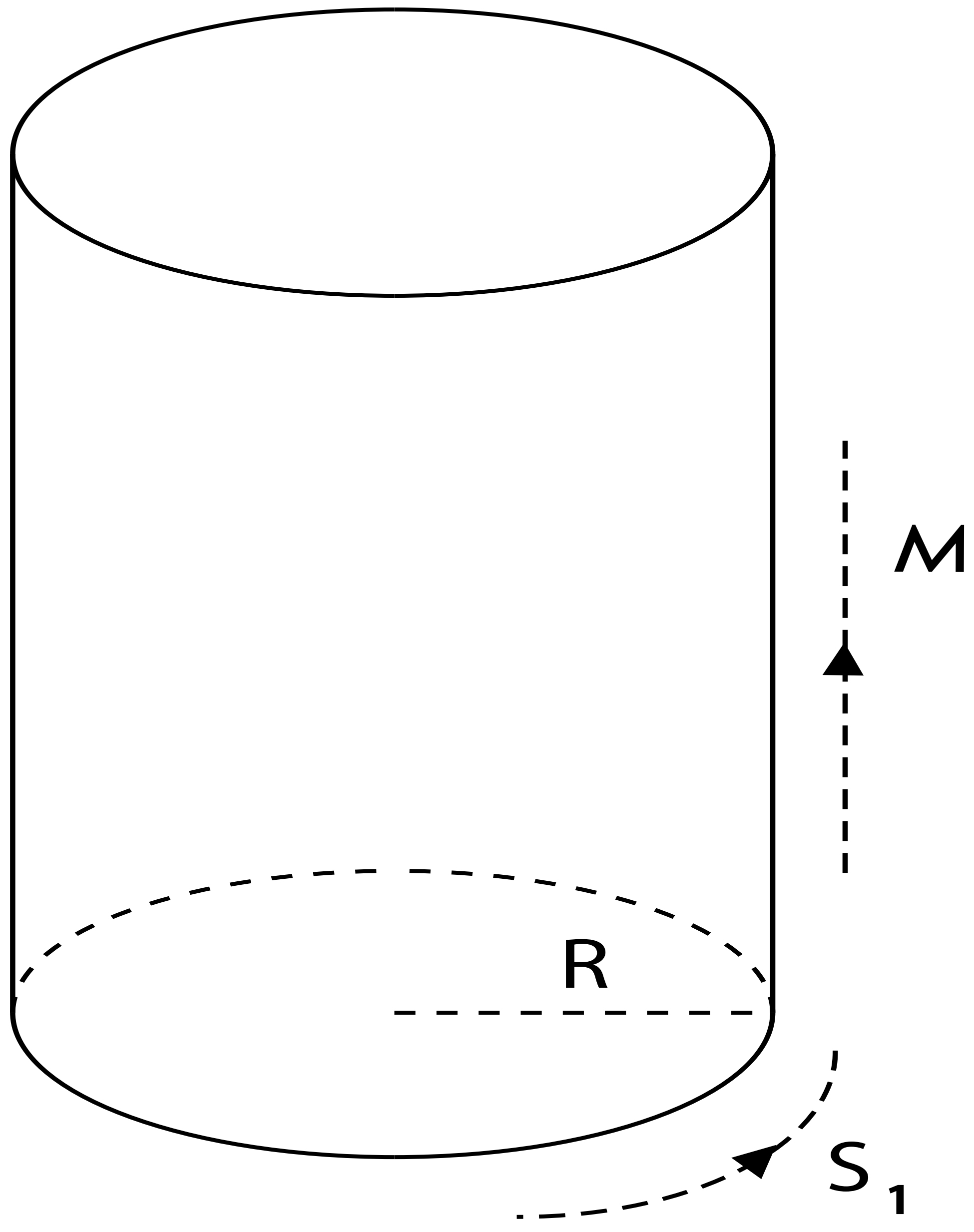

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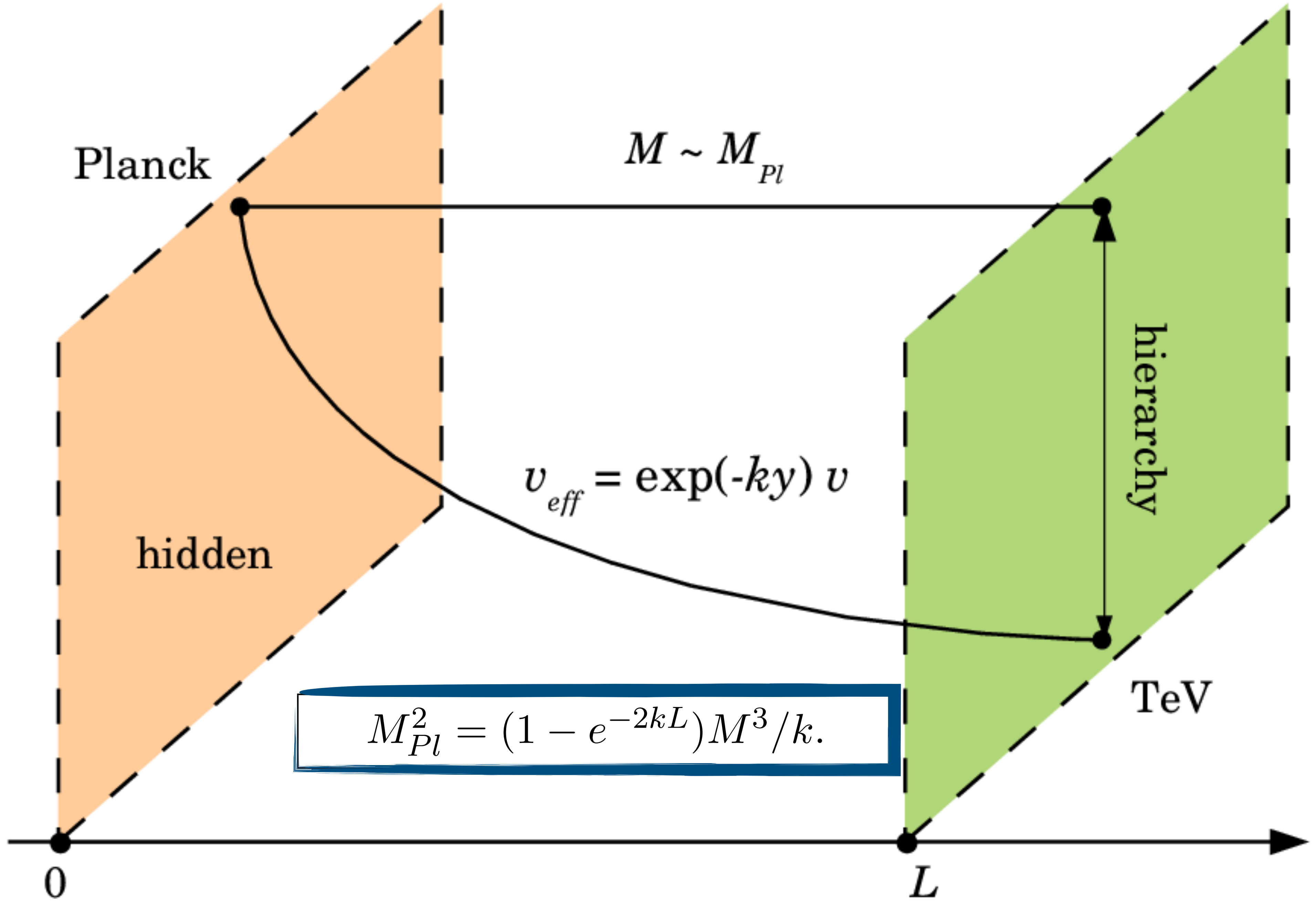
Compact Extra Dimensions : Randall-Sundrum Models



Randall-Sundrum 1999

Compact Extra Dimensions : Randall-Sundrum Models

$$\eta^{(RS)}_{MN} \equiv \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

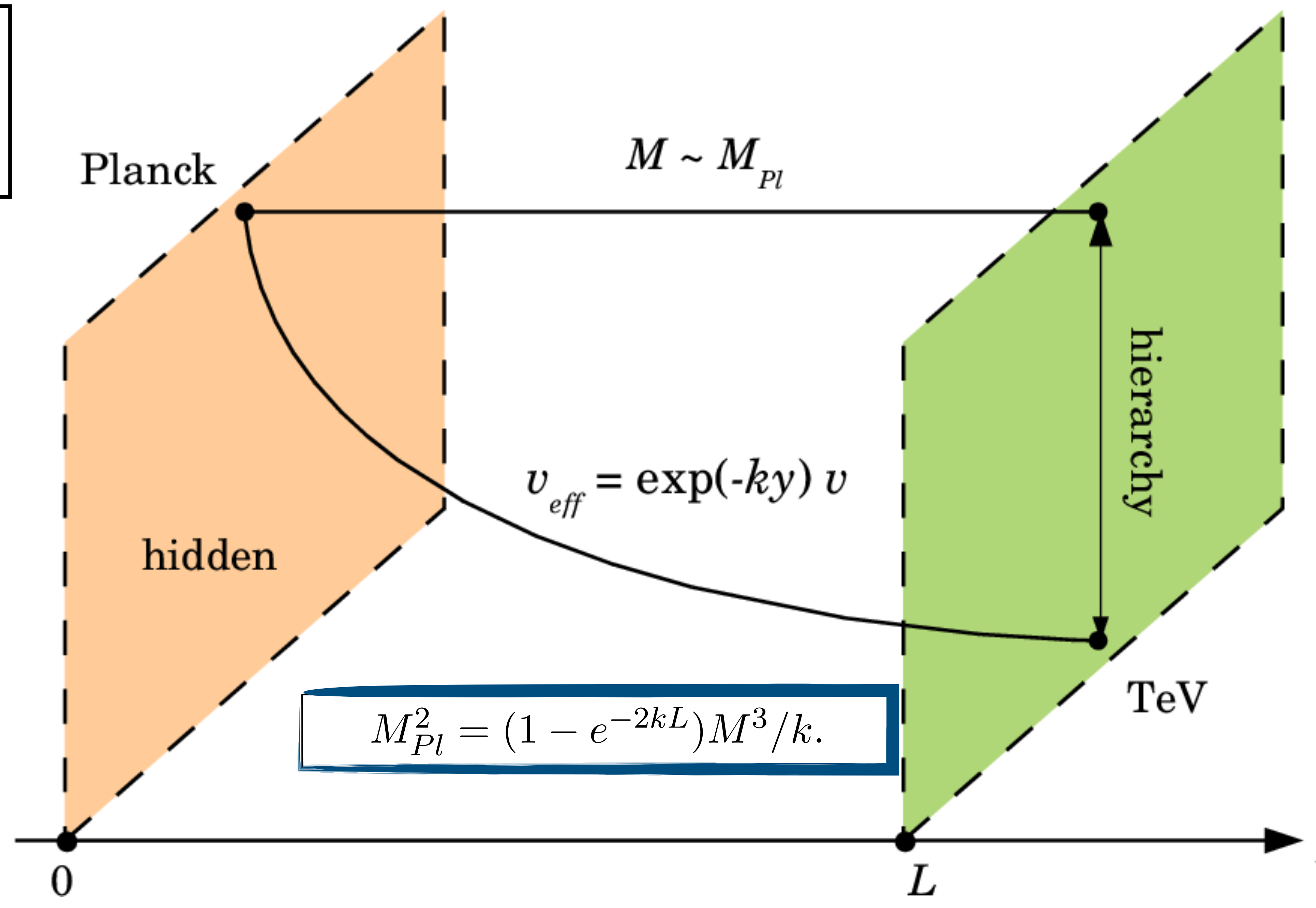
Einstein Frame

$$G^{(RS)}_{MN} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix}$$

$$m_n = k x_n e^{-k r_c \pi}$$

$$\hat{u}(x,y) \equiv \frac{\kappa\, \hat{r}}{2\sqrt{6}}\, e^{+k(2|y|-\pi r_c)}$$

TeV scale masses for kr_c =11-12



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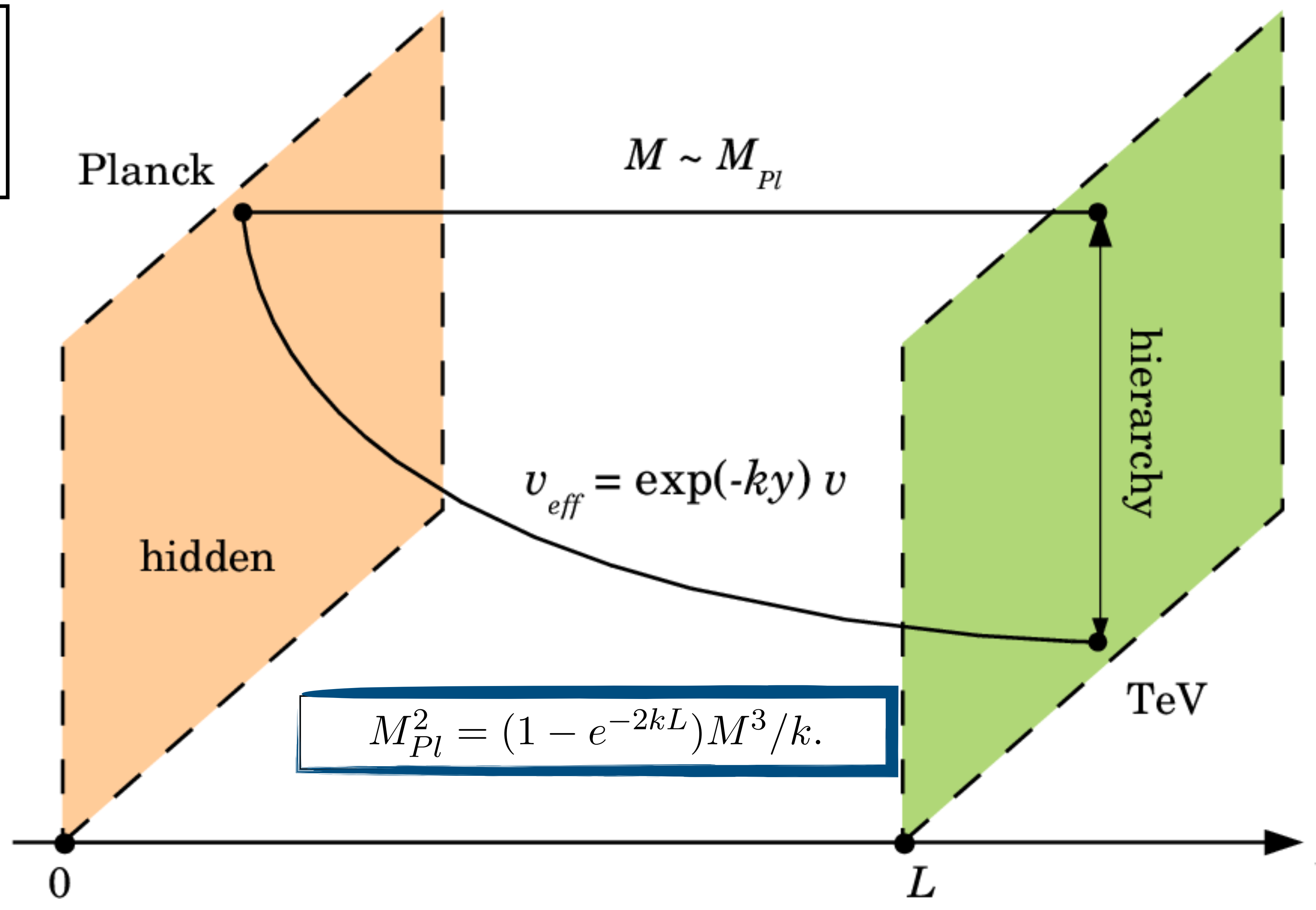
TeV scale masses for kr_c =11-12

$$S = \int d^4x [dy \mathcal{L}_{5\mathrm{D}}] \equiv \int d^4x \mathcal{L}_{4\mathrm{D}}^{(\mathrm{eff})}$$

$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}_{\mu\nu}^{(n)}(x) \psi_n(\varphi)$$

$$\underbrace{\hat{r}(x)}_{\text{5D field}} = \frac{1}{\sqrt{\pi r_c}} \underbrace{\hat{r}^{(0)}(x)}_{\text{4D fields}} \underbrace{\psi_0}_{\text{wfxn}}$$

Radion KK Decomposition



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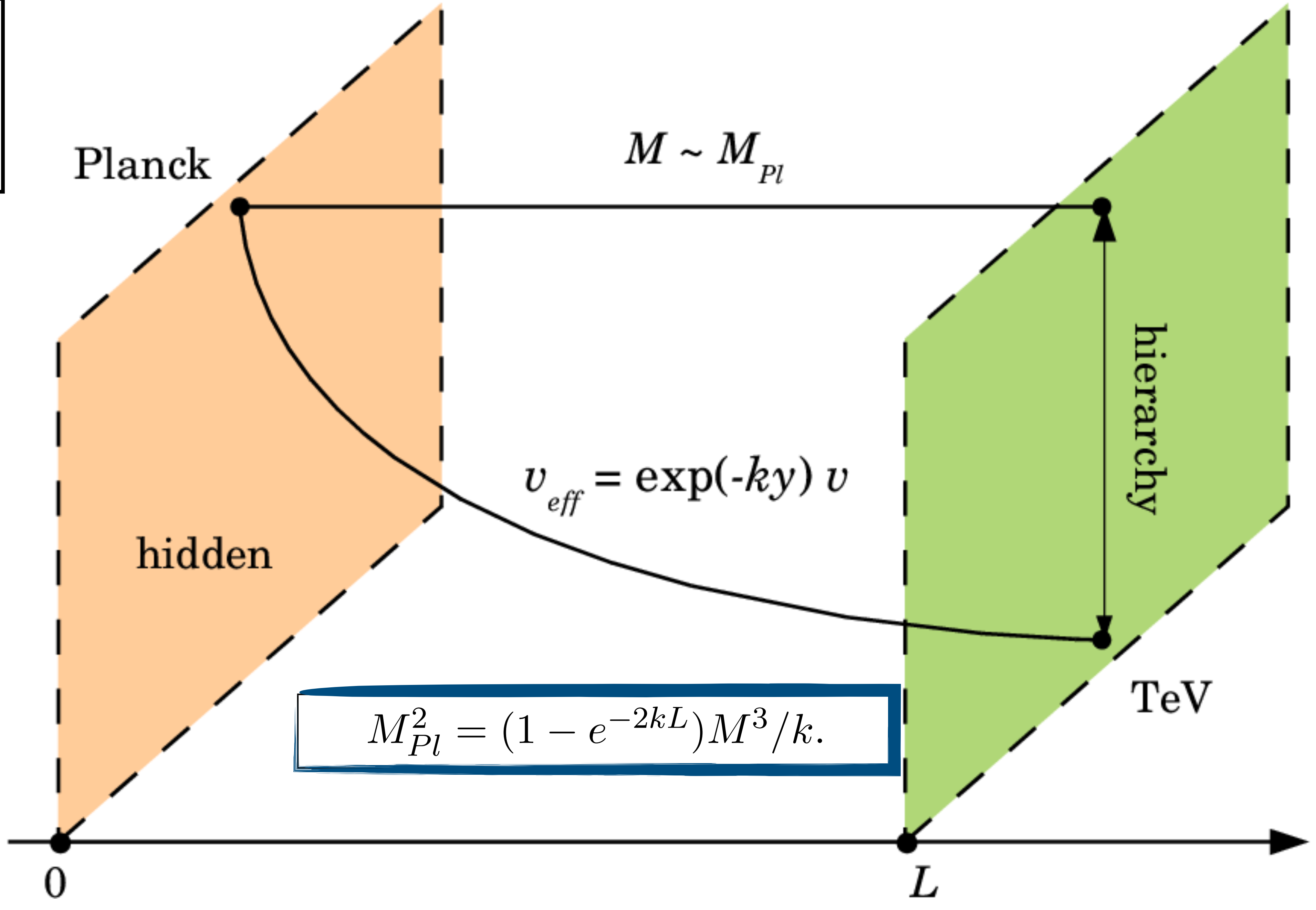

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Radion KK Decomposition

- is **orthonormal+complete** with **discrete spectrum** μ_n

Randall-Sundrum 1999

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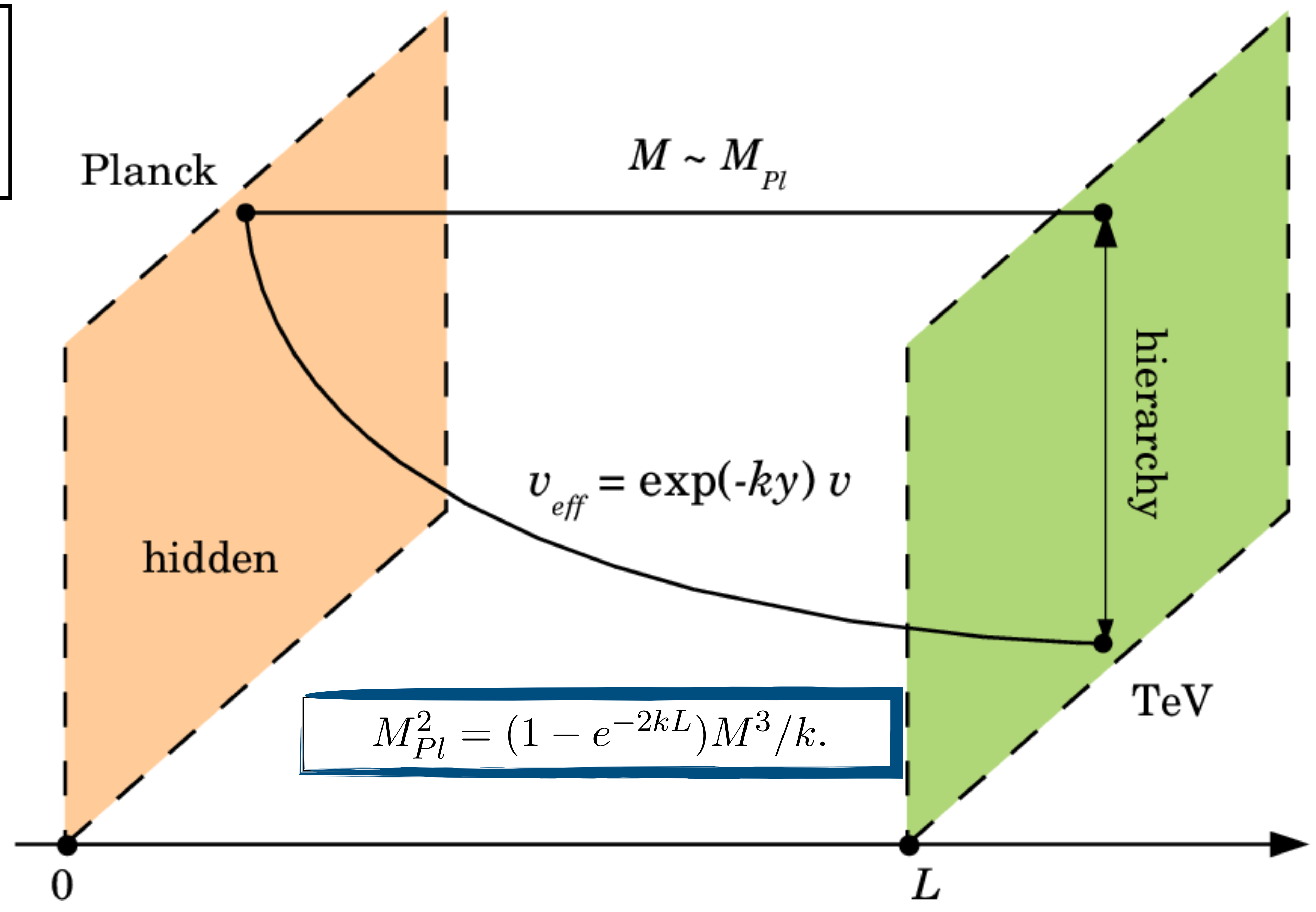

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Radion KK Decomposition

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Schrodinger Equation

$$-\frac{d}{dy}\left[e^{-4k|y|}\frac{d\psi_n}{dy}\right] = m_n^2 e^{-2k|y|}\psi_n$$

Orthonormality

$$\frac{1}{\pi r_c}\int_{-\pi r_e}^{+\pi r_e} dy e^{-2k|y|}\psi_m(y)\psi_n(y) = \delta_{mn}$$

Completeness

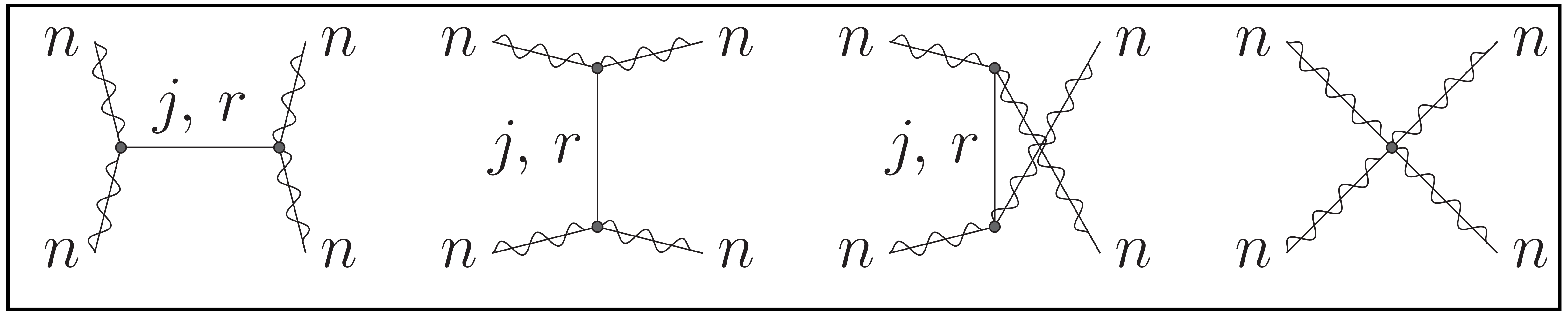
$$\frac{1}{\pi r_c}e^{-2k|y|}\sum_j \psi_j(y)\psi_j(y') = \delta(y-y')$$

Randall-Sundrum 1999

Amplitudes and Coupling Structures on a Torus/ADS

Amplitudes and Coupling Structures on a Torus/ADS

An Elastic scattering process in compactified theories



$$R_{5D} = \tilde{G}^{MN} R_{MN}$$

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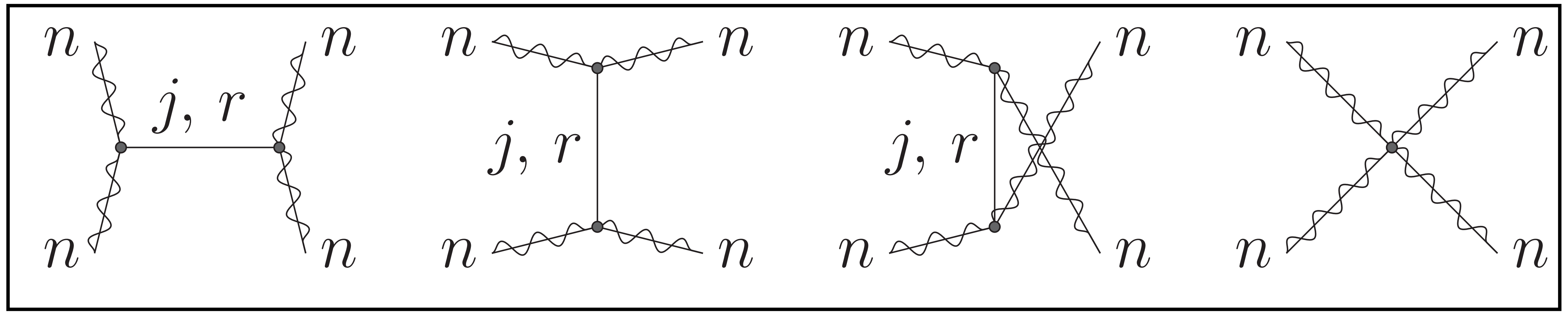
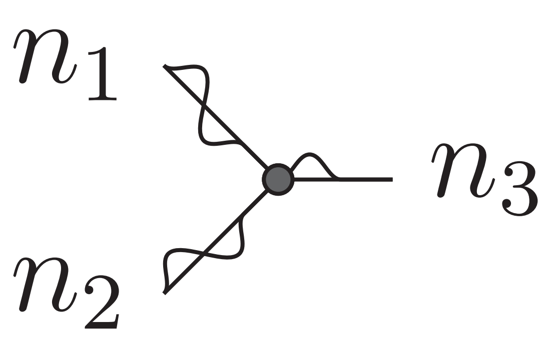
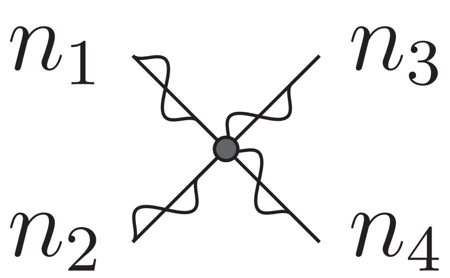


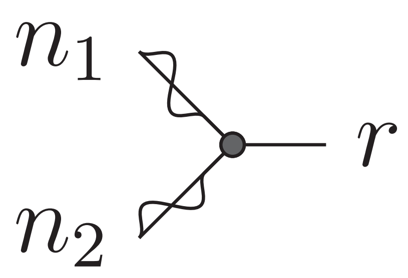
Expansion of Ricci gives two different types of coupling structures



$\supset \quad a_{n_1 n_2 n_3} \quad b_{\mathcal{P}[n_1,n_2,n_3]}$



$\supset \quad a_{n_1 n_2 n_3 n_4} \quad b_{\mathcal{P}[n_1,n_2,n_3,n_4]}$



$\supset \quad b_{n_1 n_2 r}$

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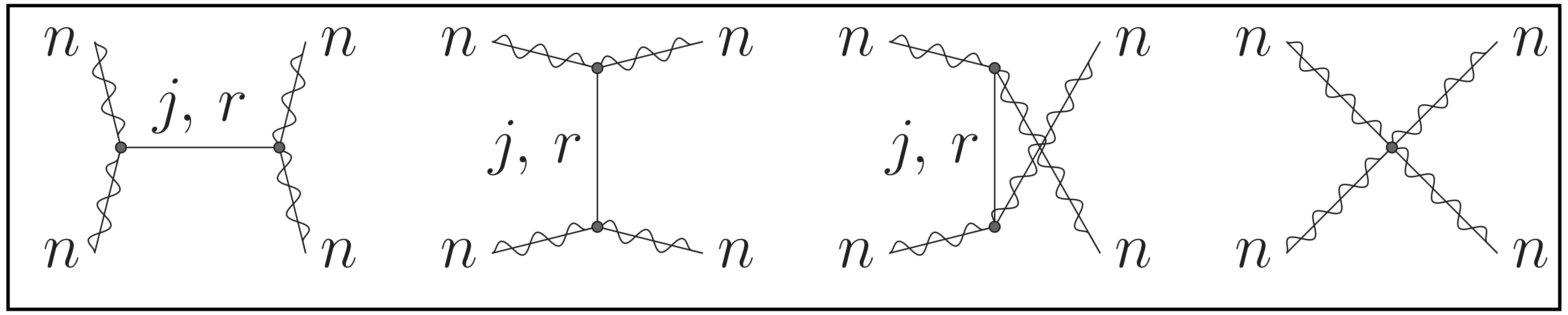

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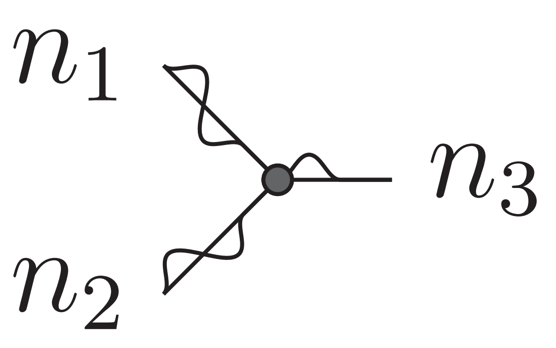

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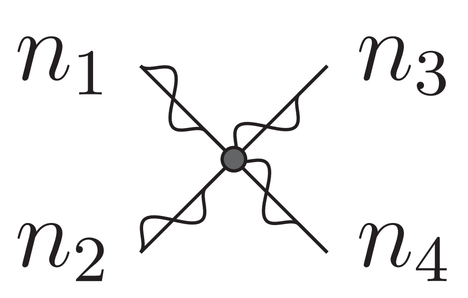

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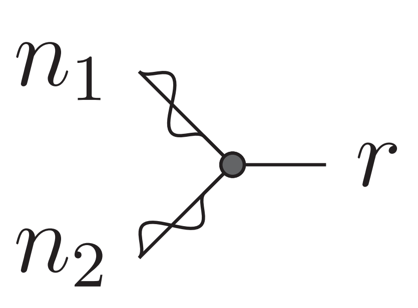

$\supset \quad b_{n_1 n_2 r}$

Define $\mathcal{L}^{(\mathrm{RS})}_{h^H r^R} \equiv$ all terms in $\mathcal{L}^{(\mathrm{RS})}_{5\mathrm{D}}$ with H graviton fields and R radion fields. By construction, each term in this set is either

- **A-Type:** has two spatial derivatives $\partial_\mu \partial_\nu$, or
- **B-Type:** has two extra-dimensional derivatives ∂_y^2

$$\mathcal{L}^{(\mathrm{RS})}_{h^H r^R} = \mathcal{L}^{(\mathrm{RS})}_{A:h^H r^R} + \mathcal{L}^{(\mathrm{RS})}_{B:h^H r^R}$$

$$= \kappa^{(H+R-2)} \left[e^{k[2(R-1)|y| - R\pi r_c]} \, \overline{\mathcal{L}}^{(\mathrm{RS})}_{A:h^H r^R} + e^{k[2(R-2)|y| - R\pi r_c]} \, \overline{\mathcal{L}}^{(\mathrm{RS})}_{B:h^H r^R} \right]$$

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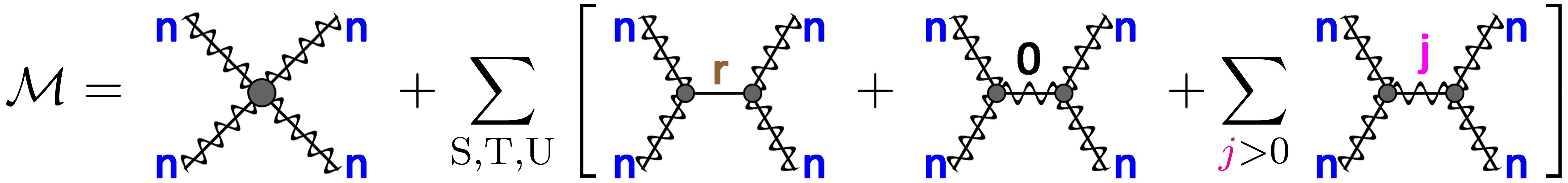
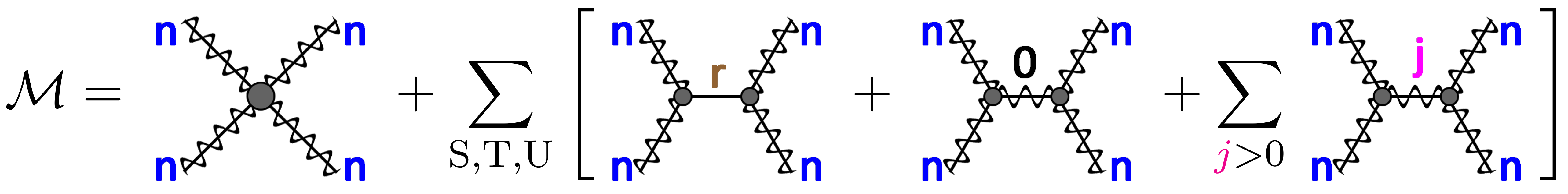


Amplitudes and Coupling Structures on a Torus/ADS



$$\mathcal{M}(s, \cos\theta) \equiv \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)}(\cos\theta) \cdot s^k$$

$$\epsilon^{\mu\nu}_{\pm 2} = \epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{\pm 1},$$

$$\epsilon^{\mu\nu}_{\pm 1} = \frac{1}{\sqrt{2}}[\epsilon^{\mu}_{\pm 1}\epsilon^{\nu}_{0} + \epsilon^{\mu}_{0}\epsilon^{\nu}_{\pm 1}],$$

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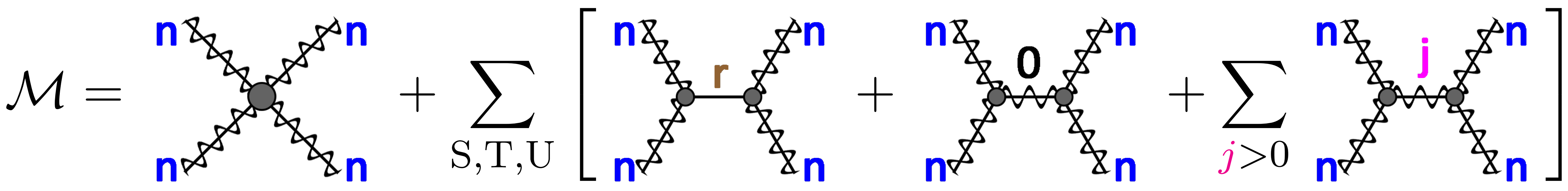
Amplitudes and Coupling Structures on a Torus/ADS



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O(s⁵) & O(s⁴) Sum Rules

$$\mathcal{M}^{(5)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)\sin^2\theta}{2304 m_n^8}\cdot\left(a_{nnnn} - \sum_j a_{nnj}^2\right)$$

$$m_n^2 a_{nnnn} = \frac{3}{4}\sum_j m_j^2 a_{nnj}^2$$

$$\mathcal{M}^{(4)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)^2}{27648 m_n^8}\cdot\left(4 m_n^2 a_{nnnn} - 3\sum_j m_j^2 a_{nnj}^2\right)$$

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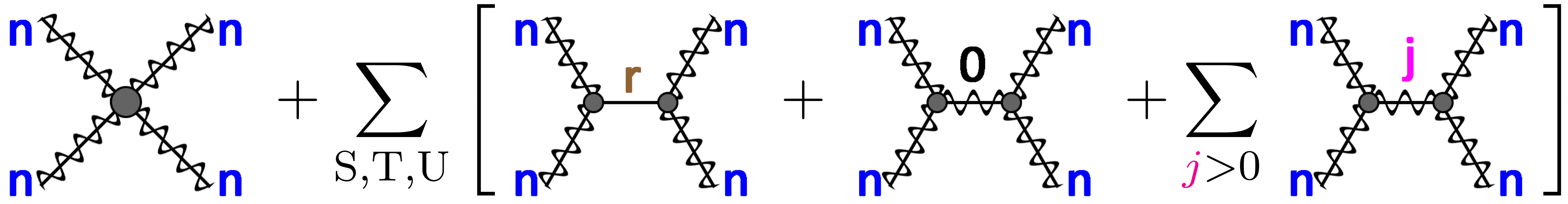
Amplitudes and Coupling Structures on a Torus/ADS

$\mathcal{M} =$ 

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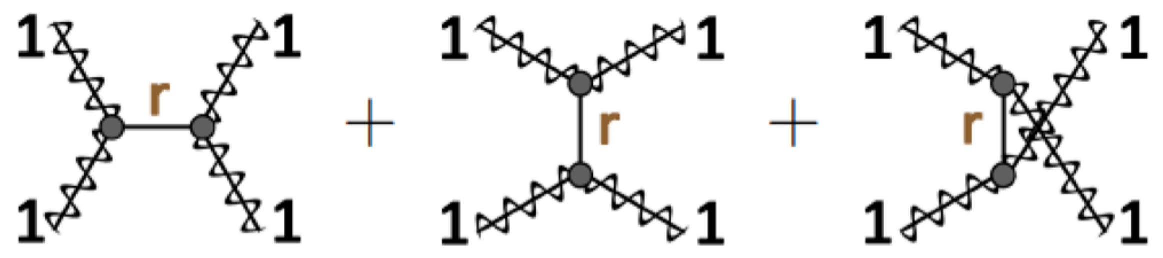
O(s³): Radion Coupling

$\mathcal{M}_r =$ 

$$\mathcal{M}^{(3)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{\sin^2\theta}{3456 m_n^8}\cdot\left(-108\frac{b_{nnr}^2}{r_c^4} + 12 m_n^4 a_{nn0}^2 - 16 m_n^4 a_{nnnn} + 15\sum_j m_j^4 a_{nnj}^2\right)$$

$$\epsilon_{\pm 2}^{\mu\nu} = \epsilon_{\pm 1}^{\mu}\epsilon_{\pm 1}^{\nu},$$

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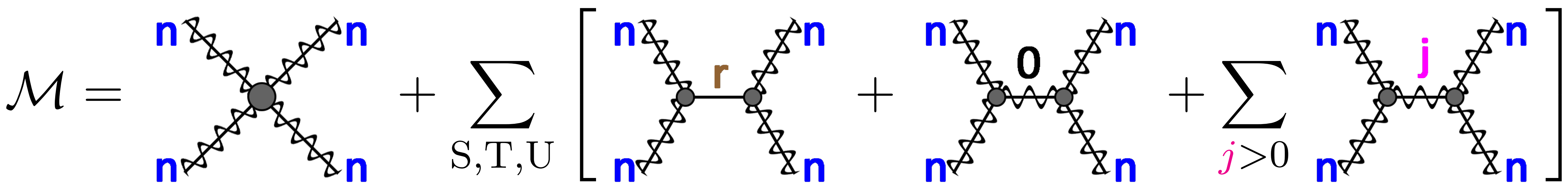
Amplitudes and Coupling Structures on a Torus/ADS



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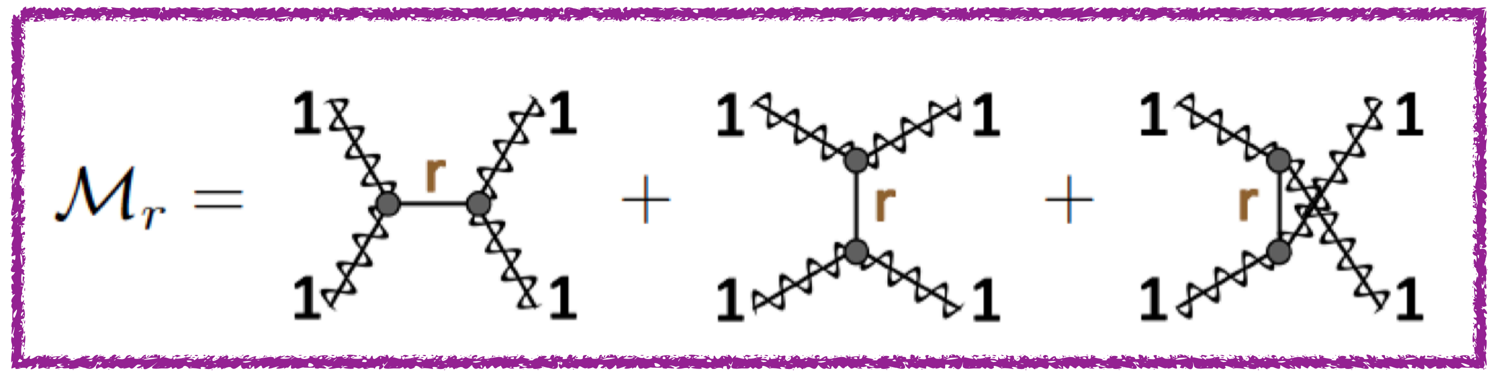


$$\mathcal{M}^{(3)}(\cos\theta) = \frac{\kappa^2}{\pi r_c}\frac{\sin^2\theta}{3456 m_n^8} \cdot \left(-108\frac{b_{nnr}^2}{r_c^4} + 12 m_n^4 a_{nn0}^2 - 16 m_n^4 a_{nnnn} + 15\sum_j m_j^4 a_{nnj}^2\right)$$

O(s²)

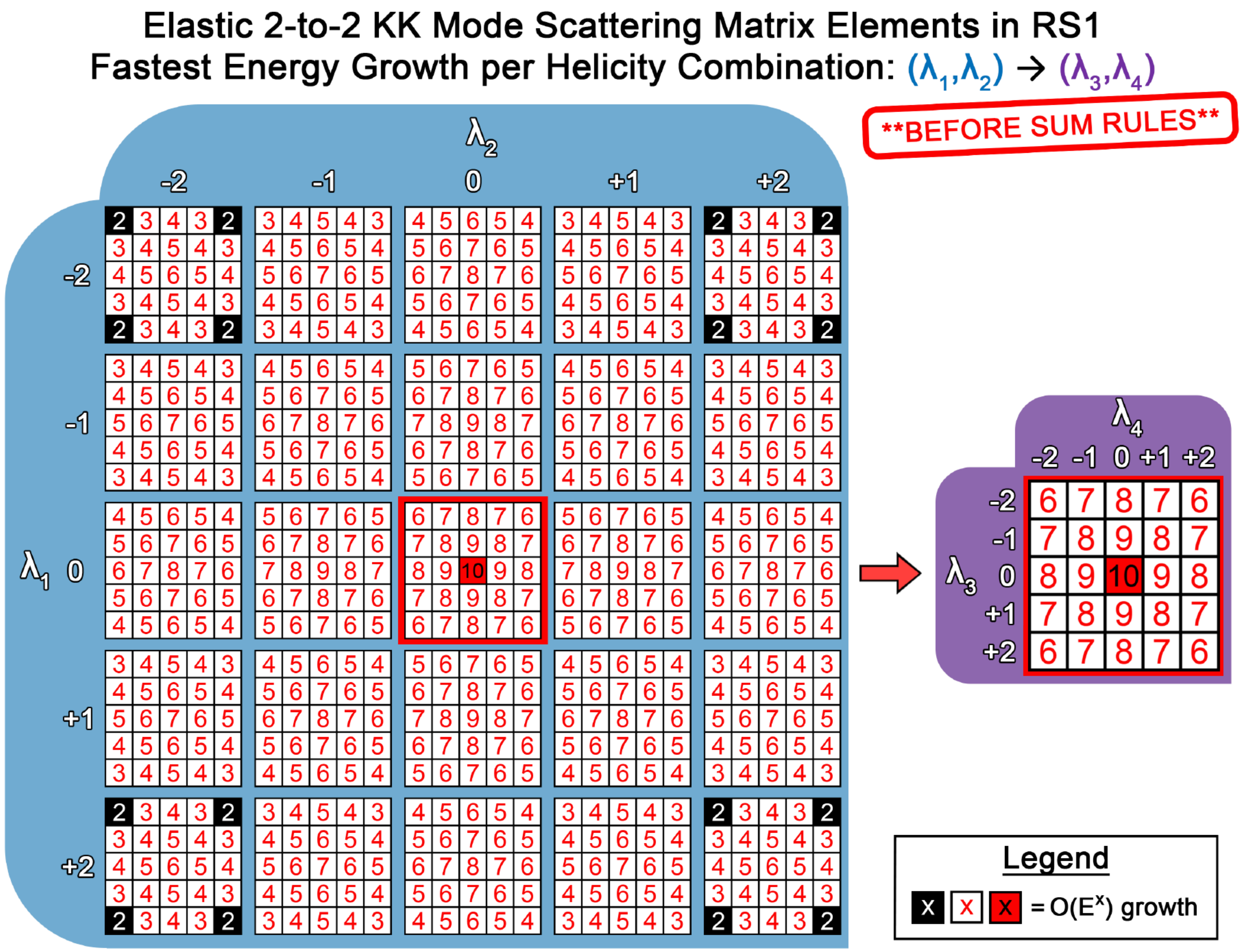
$$\mathcal{M}^{(2)}(\cos\theta) = -\frac{\kappa^2}{\pi r_c}\frac{(7+\cos 2\theta)}{5184 m_n^8} \cdot \left(-108\frac{b_{nnr}^2 m_n^2}{r_c^4} + 12 m_n^6 a_{nn0}^2 + 16 m_n^6 a_{nnnn} - 15 m_n^2 \sum_j m_j^4 a_{nnj}^2 + 6\sum_j m_j^6 a_{nnj}^2\right)$$

$$\sum_j m_j^6 a_{nnj}^2 = 5 m_n^2 \sum_j m_j^4 a_{nnj}^2 - \frac{16}{3} m_n^6 a_{nnnn}$$

Amplitudes and Coupling Structures on a Torus/ADS

Amplitudes grow as s/M_{Pl}^{2} : Consistent with gauge-invariance expectations

Phys.Rev.D 100 (2019) 11, 115033   Phys.Rev.D 101 (2020) 7, 075013   Phys.Rev.D 101 (2020) 5, 055013



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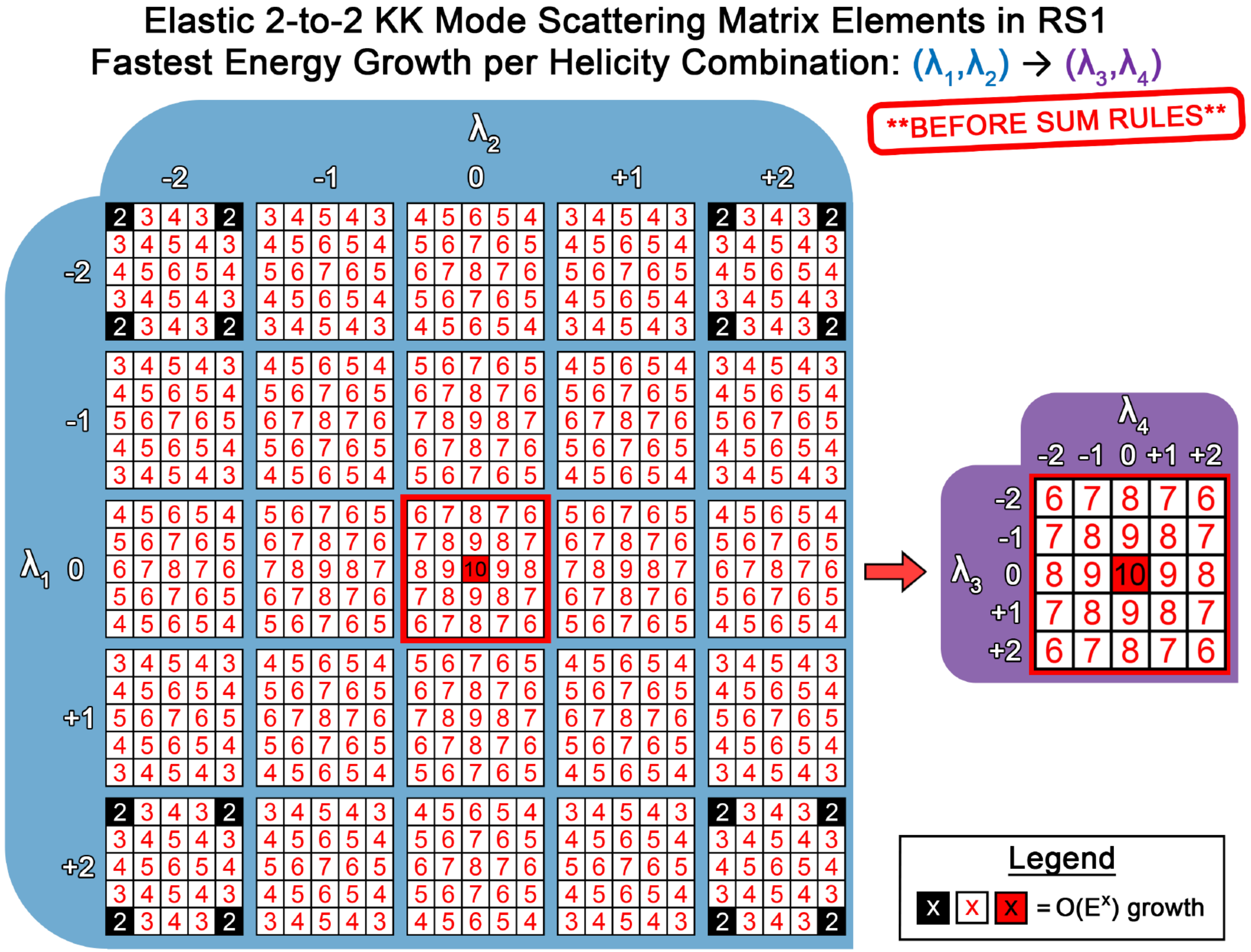

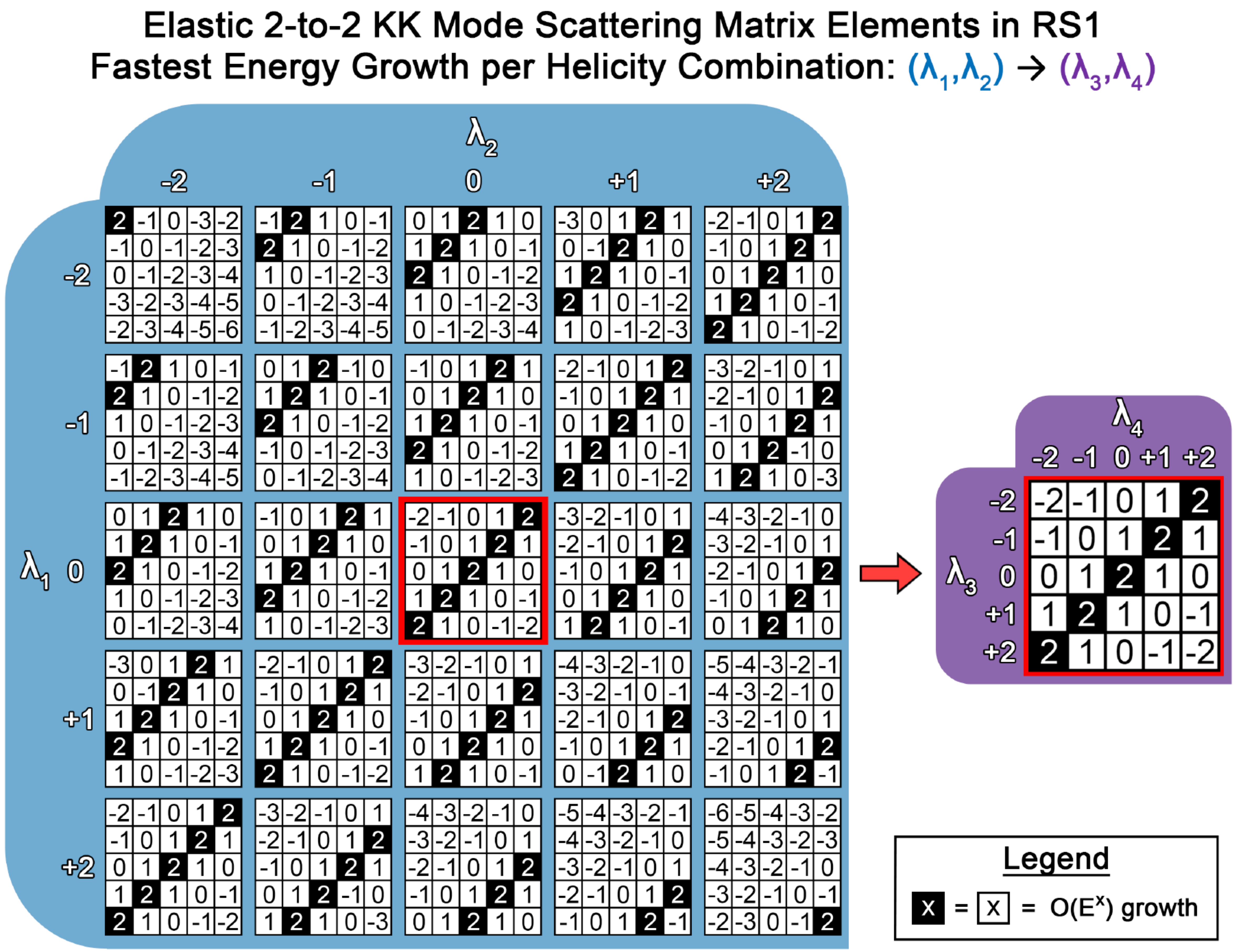

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Goldstone Boson Equivalence theorems for compact extra dimensions

$$G_{MN} = e^{2A(z)} \begin{pmatrix} e^{-\kappa\varphi/\sqrt{6}}(\eta_{\mu\nu} + \kappa h_{\mu\nu}) & \frac{\kappa}{\sqrt{2}} A_\mu \\ \frac{\kappa}{\sqrt{2}} A_\mu & -\left(1 + \frac{\kappa}{\sqrt{6}}\varphi\right)^2 \end{pmatrix}$$

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

$$A(z) = -\ln(kz)$$

$$h_{\mu\nu}(x^\alpha, z) = \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x^\alpha) f^{(n)}(z)$$

$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A^{(n)}_\mu(x^\alpha) g^{(n)}(z)$$

$$\varphi(x^\alpha, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x) k^{(n)}(z)$$

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$$A_\mu(x^\alpha, z) = \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\alpha) g^{(n)}(z)$$ → Un-physical KK graviphotons/goldstones

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$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{\rm CC} + \Delta\mathcal{L} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm m}$$

$$\mathcal{L}_2 = \frac{1}{2} h_{\mu\nu}^n \mathcal{D}_h^{\mu\nu\rho\sigma} h_{\rho\sigma}^n + \frac{1}{2} A_\mu^n \mathcal{D}_A^{\mu\nu} A_\nu^n + \frac{1}{2}\varphi D_\varphi \varphi$$

Propagators

$$\begin{aligned} \mathcal{D}_h^{\mu\nu\rho\sigma} &= \frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\left(-\Box - m_n^2\right) \\ \mathcal{D}_A^{\mu\nu} &= -\eta^{\mu\nu}(-\Box - m_n^2), \\ \mathcal{D}_\varphi &= -\Box - m_n^2. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\rm GF} &= F^\mu F_\mu - F_5 F_5, \\ F_\mu^n &= -\left(\partial^\nu h_{\mu\nu}^n - \frac{1}{2}\partial_\mu h^n + \frac{1}{\sqrt{2}} m_n A_\mu^n\right), \\ F_5^n &= -\left(\frac{1}{2} m_n h^n - \frac{1}{\sqrt{2}}\partial^\mu A_\mu^n + \sqrt{\frac{3}{2}} m_n \varphi^n\right) \end{aligned}$$ → Gauge Fixing Lagrangian

Goldstone Boson Equivalence theorems for compact extra dimensions

Ward Identities $\langle \mathbf{T} F_\mu(x) \Phi \rangle = \langle \mathbf{T} F_5(x) \Phi \rangle = 0$

→ Any External on-shell physical fields after LSZ amputation

$$\langle \mathbf{T} \left(\partial^\nu (h^n_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^n) + \frac{1}{\sqrt{2}} m_n A^n_\mu \right) \Phi \rangle = 0,$$

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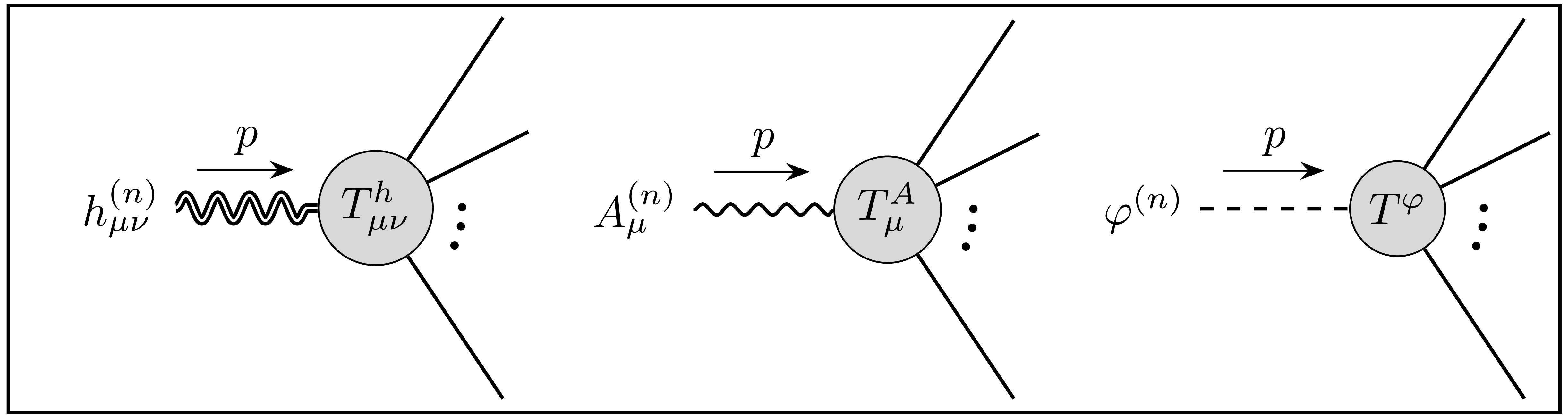
Consider an amplitude of the form



$$\mathcal{M}^h = \epsilon^{\mu\nu}(p) T^h_{\mu\nu}, \qquad \mathcal{M}^A = \epsilon^\mu(p) T^A_\mu, \qquad \mathcal{M}^\varphi = T^\varphi$$

$$\frac{i}{2} p^\nu \left(T^h_{\mu\nu} + T^h_{\nu\mu}\right) - \frac{1}{\sqrt{2}} m_n T^A_\mu = 0$$

$$-\frac{1}{2} m_n T^{h\mu}_\mu + \frac{i}{\sqrt{2}} p^\mu T^A_\mu + \sqrt{\frac{3}{2}} m_n T^\varphi = 0$$

Goldstone Boson Equivalence theorems for compact extra dimensions

Consider the longitudinal polarizations of the KK graviton

$$\epsilon_0^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\epsilon_+^\mu \epsilon_-^\nu + \epsilon_-^\mu \epsilon_+^\nu + 2\epsilon_0^\mu \epsilon_0^\nu\right)$$

The bad high energy growth part comes from the last term $\epsilon_0^{\mu\nu} \sim \mathcal{O}(E^2/m^2)$

Using the polarization sum $\sum_{\lambda=\pm,0} \epsilon_\lambda^\mu \epsilon_\lambda^{\nu *} = -\eta_{\mu\nu} + \frac{p^\mu p^\nu}{m^2}$

Rewrite the longitudinal polarizations as $\epsilon_0^{\mu\nu} = \frac{1}{\sqrt{6}}\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m^2} + 3\epsilon_0^\mu \epsilon_0^\nu\right)$

And separate out the bad high energy behaviour $\epsilon_0^\mu \equiv \frac{p^\mu}{m} + \tilde{\epsilon}_0^\mu$ $\qquad \tilde{\epsilon}_0^\mu \equiv -\frac{m}{E + |\mathbf{p}|}(1, -\mathbf{p}/|\mathbf{p}|) \sim \mathcal{O}(m/E)$

$\frac{p^\mu}{m}$ → Bad High Energy growth

$\tilde{\epsilon}_0^\mu$, $\mathcal{O}(m/E)$ → This is fine

$$\epsilon_0^{\mu\nu} = \tilde{\epsilon}_0^{\mu\nu} + \frac{1}{\sqrt{6}}\left(\eta^{\mu\nu} + 2\,\frac{p^\mu p^\nu}{m^2} + 3\,\frac{p^\mu \tilde{\epsilon}_0^\nu + p^\nu \tilde{\epsilon}_0^\mu}{m}\right)$$

$\tilde{\epsilon}_0^{\mu\nu}$ → $\mathcal{O}\left(\frac{m^2}{E^2}\right)$

Goldstone Boson Equivalence theorems for compact extra dimensions

Use Ward Identities to write the scattering amplitude as

$$T^h_{\mu\nu}\epsilon_0^{\mu\nu} = T^\varphi - i\sqrt{3}\,T^A_\mu \tilde{\epsilon}_0^\mu + T^h_{\mu\nu}\tilde{\epsilon}_0^{\mu\nu}$$

$\tilde{\epsilon}_0^\mu \sim \mathcal{O}(m/E)$ $\quad$ $\tilde{\epsilon}_0^{\mu\nu} \sim \mathcal{O}(m^2/E^2)$

No Bad High Energy growth

$$T^h_{\mu\nu}\epsilon_0^{\mu\nu} = T^\varphi + \mathcal{O}(s^0)$$

KK Goldstone Theorem :

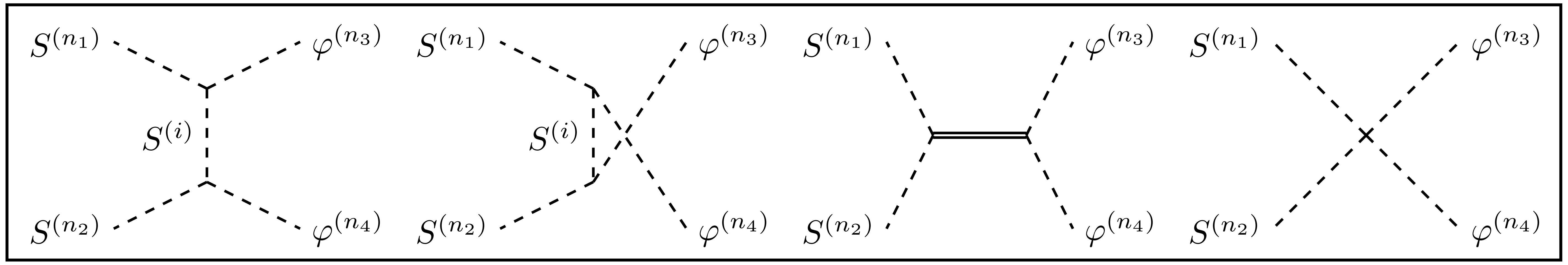
Scattering Amplitude of longitudinally polarized KK graviton=Scalar Goldstones in the High Energy Limit

$$T^h_{\mu\nu}\epsilon_{\pm 1}^{\mu\nu} = -iT^A_\mu \epsilon_\pm^\mu + \mathcal{O}(s^0)$$

→ Similarly for Helicity 1 one states

Goldstone Boson Equivalence theorems for compact extra dimensions

Example 1: Scattering of two bulk scalars with two KK gravitons



$$\mathcal{L}_m = \sqrt{G}\left(\frac{1}{2}G^{MN}\partial_M S \partial_N S - \frac{1}{2}M_S^2 S^2\right)$$

$$S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)} h_L^{(n_4)}$$

Expectation from Goldstone Equivalence Theorem

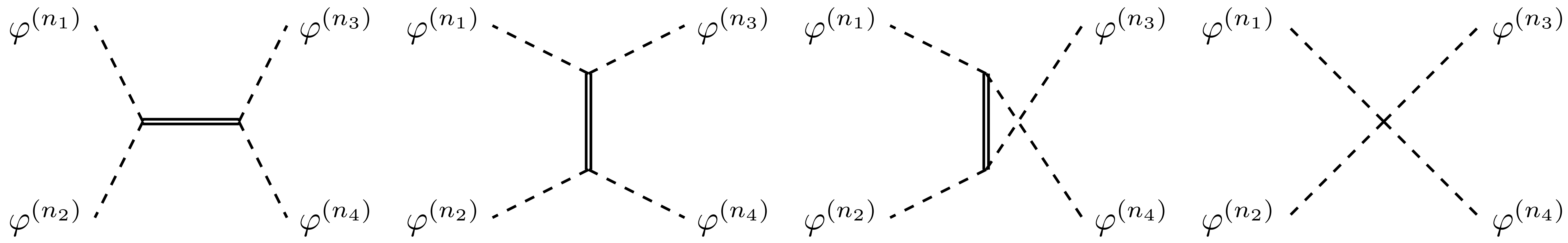
$$\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to h_L^{(n_3)} h_L^{(n_4)}\right] = \mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] + \mathcal{O}(s^0)$$

$$\mathcal{M}\left[S^{(n_1)}S^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] = \frac{\kappa^2 s}{32}(1-\cos 2\theta)\left\langle k^{(n)}k^{(n)}f_S^{(n)}f_S^{(n)}\right\rangle + \mathcal{O}(s^0)$$

Agrees with Unitary gauge calculation

Goldstone Boson Equivalence theorems for compact extra dimensions

Example 2: Scattering of two gravitons with two KK gravitons



Expectation from Goldstone Equivalence Theorem

$$\mathcal{M}\left[h_L^{(n_1)}h_L^{(n_2)} \to h_L^{(n_3)}h_L^{(n_4)}\right] = \mathcal{M}\left[\varphi^{(n_1)}\varphi^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] + \mathcal{O}(s^0)$$

Agrees with Unitary gauge calculation

$$\mathcal{M}\left[\varphi^{(n_1)}\varphi^{(n_2)} \to \varphi^{(n_3)}\varphi^{(n_4)}\right] = \frac{\kappa^2 s}{64}\frac{(\cos 2\theta + 7)^2}{\sin^2\theta}\langle k^{(n)}k^{(n)}k^{(n)}k^{(n)}\rangle + \mathcal{O}(s^0)$$

Chivukula, Gill, Mohan, DS, Simmons Wang et al, **Phys. Rev. D 109 (2024), 1** **Phys. Rev. D 109 (2024), 7**

A Short Summary of Double Copy: BCJ double copy : Bern-Carrasco-Johannson 2010

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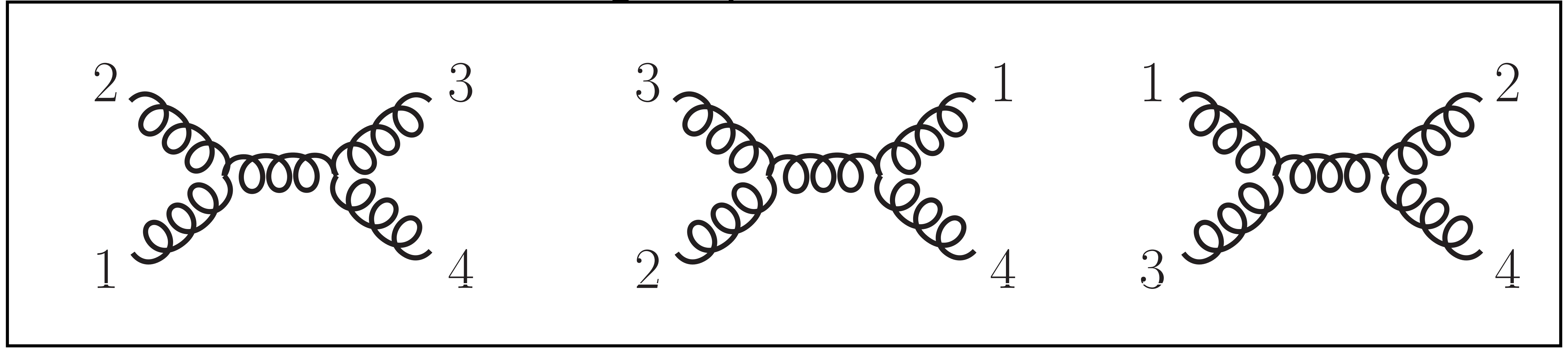
Consider Gluon Scattering Amplitudes



$$iA_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

$$c_s = -2f^{a_1 a_2 b} f^{b a_3 a_4} \qquad c_t n_t = c_s n_s\Big|_{1\to2\to3\to1}$$

$$c_u n_u = c_s n_s\Big|_{1\to3\to2\to1}$$

$$n_s = -\frac{1}{2}\Big\{\Big[(\varepsilon_1\cdot\varepsilon_2)p_1^\mu + 2(\varepsilon_1\cdot p_2)\varepsilon_2^\mu - (1\leftrightarrow 2)\Big]\Big[(\varepsilon_3\cdot\varepsilon_4)p_{3\mu} + 2(\varepsilon_3\cdot p_4)\varepsilon_{4\mu} - (3\leftrightarrow 4)\Big] + s\Big[(\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4) - (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\Big]\Big\}, \quad (1$$

The Contact Interaction is factored into the above definition by suitable kinematic reshuffling

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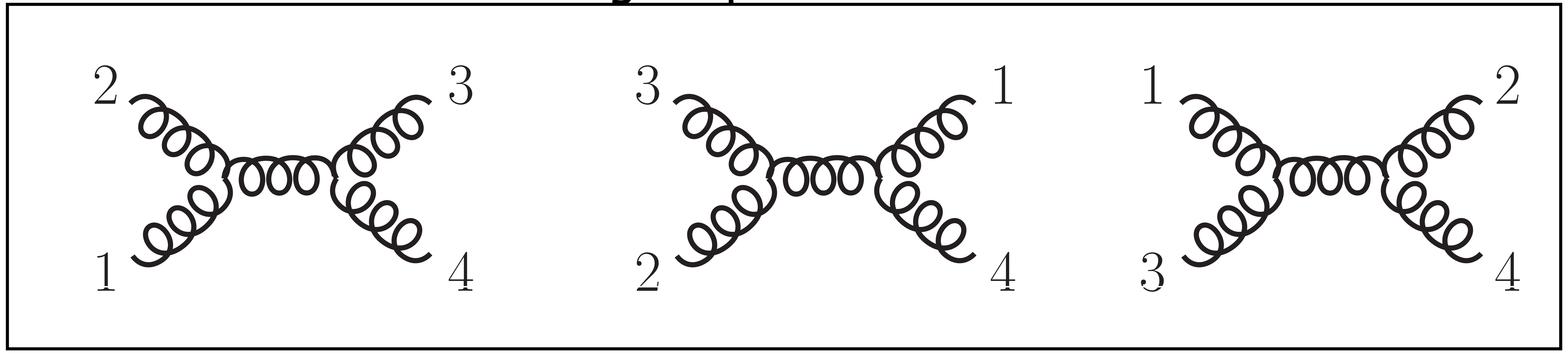

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The Contact Interaction is factored into the above definition by suitable kinematic reshuffling

Gauge Invariance demands that the amplitude must vanish under polarization to momentum replacement

$$n_s\Big|_{\varepsilon_4\to p_4} = -\frac{s}{2}\Big[(\varepsilon_1\cdot\varepsilon_2)\big((\varepsilon_3\cdot p_2) - (\varepsilon_3\cdot p_1)\big) + \text{cyclic}(1,2,3)\Big] \equiv s\,\alpha(\varepsilon, p)$$

$$\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\Big|_{\varepsilon_4\to p_4} = (c_s + c_t + c_u)\,\alpha(\varepsilon, p)$$

Color Jacobi Identity

$$c_s + c_t + c_u = -2\left(f^{a_1 a_2 b} f^{b a_3 a_4} + f^{a_2 a_3 b} f^{b a_1 a_4} + f^{a_3 a_1 b} f^{b a_2 a_4}\right) = 0$$

On-Shell Conditions →

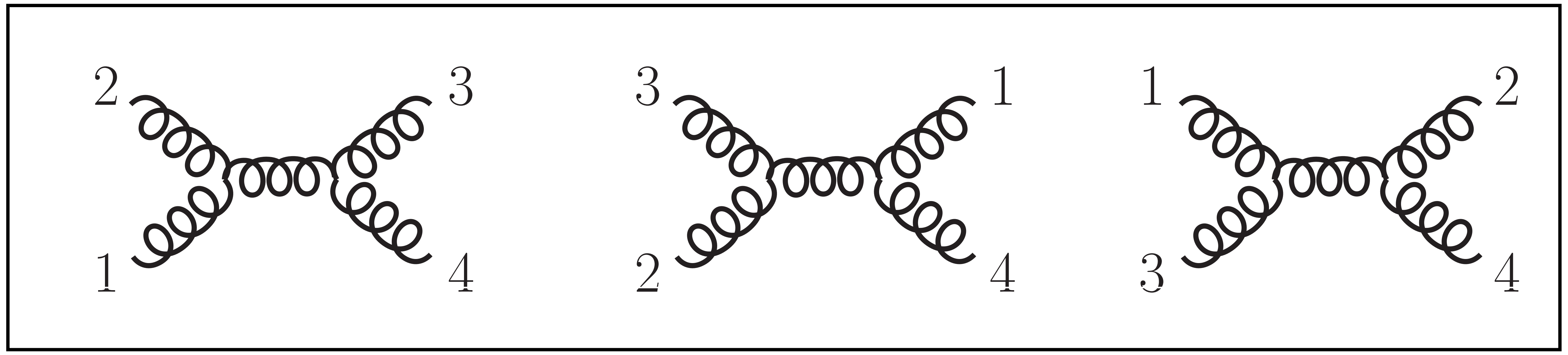
Kinematic Jacobi Identity

$$n_s + n_t + n_u = 0$$

Amplitudes and Double Copy

Amplitudes and Double Copy

Color and Kinematic Factors are mutually Interchangable



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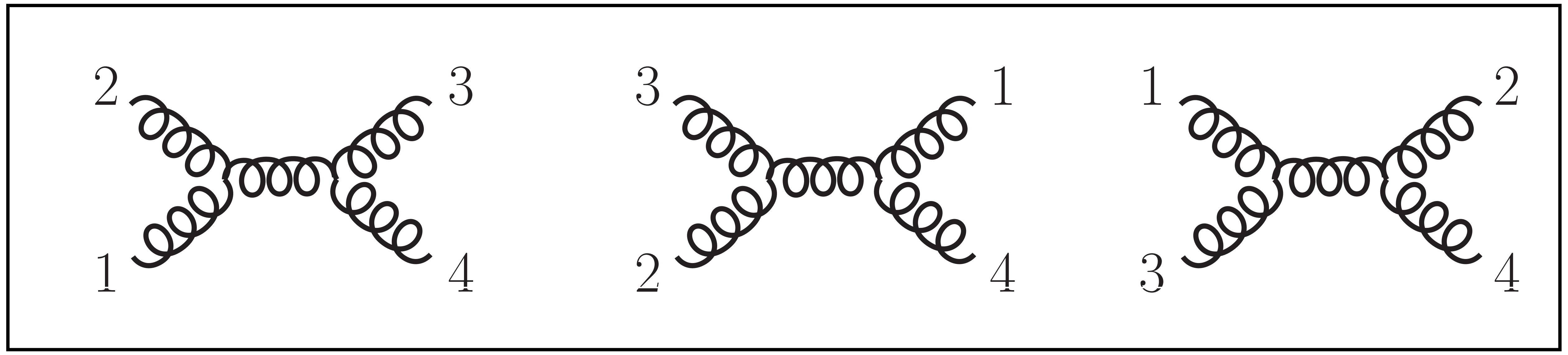


$$i\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

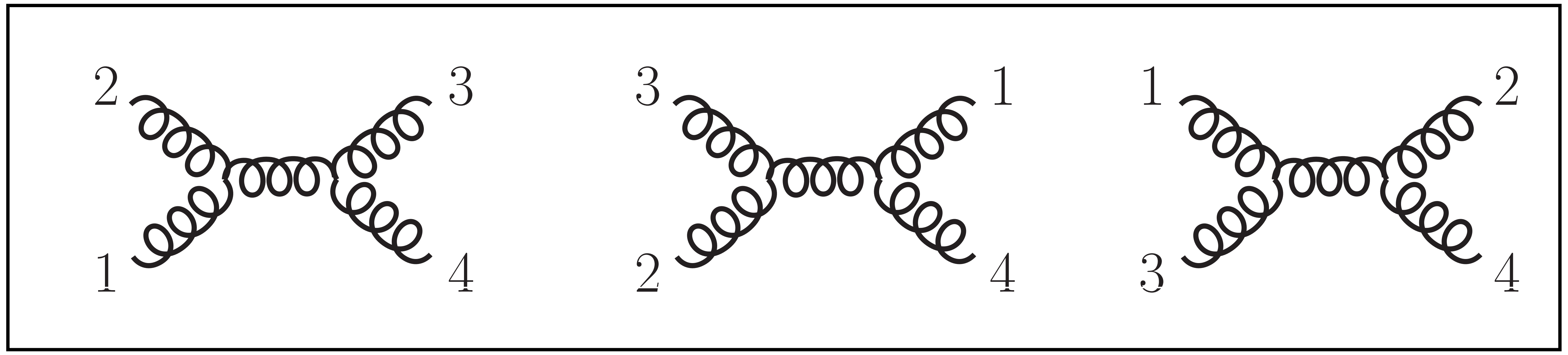
Massless Gravity amplitude

Diffeomorphism Invariance

$$\left.\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right|_{\varepsilon_4^{\mu\nu} \to p_4^\mu \varepsilon_4^\nu + p_4^\nu \varepsilon_4^\mu} = 2(n_s + n_t + n_u)\,\alpha(\varepsilon, p) = 0\,.$$

Amplitudes and Double Copy

Color and Kinematic Factors are mutually Interchangable



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Massless Gravity amplitude

$$i\mathcal{A}_4^{\text{tree}}\Big|_{\substack{c_i\to\tilde{n}_i \\ g\to\kappa/2}} \equiv i\mathcal{M}_4^{\text{tree}} = \left(\frac{\kappa}{2}\right)^2\left(\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}\right)$$

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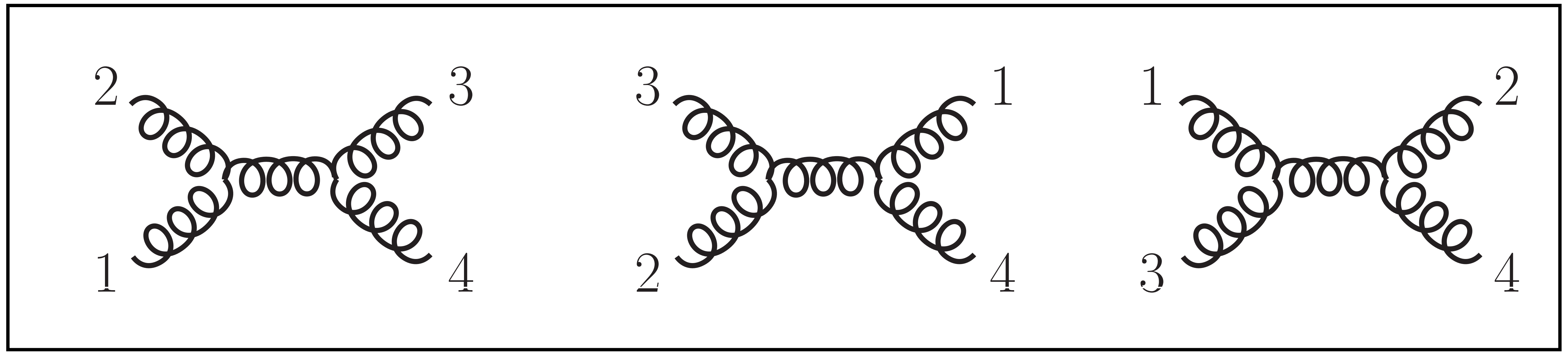

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Amplitudes/double copies and equivalence theorems

$$\mathcal{M}[h^L_{n_1}(k_1), \ldots, h^L_{n_N}(k_N), \Phi] = \mathcal{M}[\phi_{n_1}(k_1), \ldots, \phi_{n_N}(k_N), \Phi]$$

→ Statement of Gravitational Equivalence Theorem

$$\mathcal{T}[A^{aL}_{n_1} A^{bL}_{n_2} \to A^{cL}_{n_3} A^{dL}_{n_4}] = \mathcal{T}[A^{a5}_{n_1} A^{b5}_{n_2} \to A^{c5}_{n_3} A^{d5}_{n_4}]$$

→ Statement of 5D Gauge theory Equivalence Theorem

↓

$$\mathcal{T}'[4A^n_L] = g^2 \left(\frac{\mathcal{C}_s \mathcal{N}'_s}{s_0} + \frac{\mathcal{C}_t \mathcal{N}'_t}{t_0} + \frac{\mathcal{C}_u \mathcal{N}'_u}{u_0} \right)$$

$$\widetilde{\mathcal{T}}'[4A^n_5] = g^2 \left(\frac{\mathcal{C}_s \widetilde{\mathcal{N}}'_s}{s_0} + \frac{\mathcal{C}_t \widetilde{\mathcal{N}}'_t}{t_0} + \frac{\mathcal{C}_u \widetilde{\mathcal{N}}'_u}{u_0} \right)$$

Kinematic Jacobi Identity

$$\mathcal{N}'_s + \mathcal{N}'_t + \mathcal{N}'_u = 0$$

Amplitudes/double copies and equivalence theorems

$$\mathcal{M}[h^L_{n_1}(k_1),\ldots,h^L_{n_N}(k_N),\Phi] = \mathcal{M}[\phi_{n_1}(k_1),\ldots,\phi_{n_N}(k_N),\Phi]$$

→ Statement of Gravitational Equivalence Theorem

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Kinematic Jacobi Identity →

$$\mathcal{N}'_s+\mathcal{N}'_t+\mathcal{N}'_u=0$$

Amplitudes/double copies and equivalence theorems

Color -> Kinematic replacement $(\mathcal{C}_s, \mathcal{C}_t, \mathcal{C}_u) \to (\mathcal{N}_s, \mathcal{N}_t, \mathcal{N}_u)$

$$\mathcal{M}_0[h_L^n h_L^n \to h_L^n h_L^n] = c_0 g^2 \left[\frac{(\mathcal{N}_s^0)^2}{s_0} + \frac{(\mathcal{N}_t^0)^2}{t_0} + \frac{(\mathcal{N}_u^0)^2}{u_0} \right]$$

$$\widetilde{\mathcal{M}}_0[\phi_n \phi_n \to \phi_n \phi_n] = c_0 g^2 \left[\frac{(\widetilde{\mathcal{N}}_s^0)^2}{s_0} + \frac{(\widetilde{\mathcal{N}}_t^0)^2}{t_0} + \frac{(\widetilde{\mathcal{N}}_u^0)^2}{u_0} \right]$$

- Works for Compactified Torus on flat space-times -> Hang-He PRD 2022
- Works for a compactified spacetime with an ADS background -> Chivukula, DS, Gill, Wang et al, PRD 2023

Further works in this direction

- Supersymmetric Structure of Compactified theories -> Chivukula, Simmons, Wang 2021
- Scattering amplitudes for Moduli/radius stabilized geometries -> Chivukula, Foren, Mohan, DS, Simmons 2021,2023
- Goldstone Equivalence Theorem with Matter couplings and generalized Poincare symmtries (Kac-Moody Algebras) -> Chivukula, Gill, Mohan, DS, Simmons, Wang 2023 (Josh Gill's Poster)
- Scattering Amplitudes in models with curvature localized on D-branes/ DGP model correspondence -> Chivukula, Mohan, DS< Simmons, Wang 2024
- Scattering Amplitudes and matter couplings in massive gravity -> Gill, DS, Williams 2022 (Josh Gill's Poster)
- Applications for KK portal dark matter models -> To appear soon

Conclusions

- Compactified theories of extra dimensions -> No low energy cut-off
- Cancellations due to different diagrams reduce $O(s^5)$ growth to $O(s)$.
- No low energy cut-off for consistent models of stabilization
- Uncovered sum rules enforcing this cancellation
- Can show -> Analysis extends to matter on brane or bulk
- Consistent with literature on massive gravity.
- Possible to double-copy a compactified gauge theory to compactified gravity for flat toroidal compactification
- Pheno papers : Doing an unitarity analysis for DM models, ultralight radion as a candidate …
- Theory papers : Spinor Helicity/Goldstone Equivalence calculation ?
- More connections with massive gravity community …

Invitation


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19-24 August 2024 (inclusive)
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DARK MATTER
QCHSC2024
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER
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